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Steven Haider and Jacob Alex Klerman

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DYNAMIC PROPERTIES OF THE WELFARE CASELOAD

STEVEN HAIDER
Department of Economics
Michigan State University
101 Marshall Hall
East Lansing, MI 48824
haider@msu.edu

JACOB ALEX KLERMAN
RAND
1700 Main Street
Santa Monica, CA 90407
klerman@rand.org

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ABSTRACT

The welfare caseload evolves through a process of flows onto and off of welfare that can be described with a Markov Chain model. Using formal results for Markov models, this paper examines the dynamic properties of the welfare caseload. In particular, we examine steady states, the speed of convergence, and the relative importance of entry and exit for changes in the caseload. Implementing these models with administrative data for California, we find that the welfare caseload has considerable momentum and that adjustments are far from instantaneous. In addition, we find that changes in the entry rate are empirically more important than changes in the exit rate for explaining changes in the overall caseload. These findings have considerable implications for the conventional methods that are used to study the changing caseload.

Keywords: Welfare Caseload, Markov Chain, Dynamic Models, and Convergence

Researchers and policy makers have been very interested in why the welfare caseload changes, leading to numerous academic studies.¹ One group of studies has examined changes in the aggregate caseload level to make inferences about the source of these changes, while a second group of studies examined individual flows on and off the caseload to make similar inferences.² A third group of studies has combined the two approaches by examining individual flows on and off the caseload and then inferring the impact on the caseload level with a standard Markov Chain framework.³

This paper extends the third group of studies by using the mathematical properties of Markov Chain models to shed light on the dynamic properties of the welfare caseload. By dynamic properties, we mean the process of how contemporaneous changes impact the future caseload. Several papers suggest that such issues play a central role in interpreting the econometric studies of caseload changes (see Figlio and Ziliak, 1999; CEA, 1999; Bell, 2001; Klerman and Haider, 2001). These mathematical properties can shed light on several substantive questions regarding caseload dynamics. For example, how much momentum exists in the welfare caseload? What is the speed of its adjustment? What is the relative importance of each of the flows?

Using California data on individual flows, we find that the welfare caseload contains considerable momentum. For example, if the conditions in 1993 had persisted indefinitely, the caseload would have continued to increase dramatically. Similarly, if the conditions in 1998 had persisted indefinitely the caseload would have continued to decrease dramatically. Second, a related result implies that the caseload converges to its steady state relatively sluggishly. When the system is out of its steady state, we find that the “half-life” in convergence is approximately two years, i.e., half of the gap between the current state and the steady-state caseload is closed in two years and three-quarters of

¹ By welfare, we are referring to the cash aid assistance to the poor, previously the Aid for Families with Dependent Children (AFDC) program and currently the Temporary Assistance for Needy Families (TANF) program.

² Moffitt (1992) reviews both types of studies. More recent examples of the first type include CEA (1997), Levine and Whitmore (1998), Blank and Wallace (1999), CEA (1999), Figlio and Ziliak (1999), Moffitt (1999), Ziliak et al. (2000), Schoeni and Blank (2001), and Blank (2001). More recent examples of the second type include Bane and Ellwood (1994), Blank and Ruggles (1996), Hoynes (2000), Mueser et al., (2000), Gittleman (2001), and Ribar (2001).

³ For example, see Boskins and Nold (1975), Hutchens (1981), Gittleman (2000), Mueser et al., (2000), and Klerman and Haider (2001).

the gap in four years. Third, our estimates of the dynamic properties are very sensitive to the specification of the state-space. Inferences from the simple two-state models that have been previously used differ dramatically when compared to inferences from richer (and more realistic) models. Finally, much of the caseload change during the 1990s was due to changes in the new entry rate; changes in the continuation rate and the re-entry rate were less important.

The rest of the paper is organized as follows. In the next section, we review the basic properties of Markov Chains models. In the second section, we discuss our data and develop three tractable versions of the welfare caseload. In the third section, we examine the implied steady state of the system to understand the underlying momentum in the welfare caseload. The fourth section examines the speed at which the caseload converges towards its steady state. The fifth section considers the relative role of entry, continuation, and re-entry. We summarize the findings and discuss their implications for welfare policy and welfare policy research in the final section.

1. A DYNAMIC MODEL OF THE WELFARE CASELOAD

In this section, we first develop a Markov Chain representation of the welfare caseload. Then, we discuss the standard mathematical properties of Markov Chain models.

A Markov Chain Representation of the Welfare Caseload. Generically, a Markov Chain model describes how individuals transit between various “states.” Such a model can be represented as

$$(1) \quad \underset{(Q \times 1)}{S(t)} = \underset{(Q \times Q)}{M(t)} \underset{(Q \times 1)}{S(t-1)},$$

where $S(t)$ is a vector that contains the share of individuals in each of Q states and $M(t)$ is the transition matrix between the states, both in period t . Because the transition matrix M is allowed to be time varying, equation 1 is simply an accounting identity describing how the system changes over time.

As a very simple example, suppose individuals were considered to be in one of two “states” in a given period, either receiving welfare or not receiving welfare. Formally, let $s(r, t)$ denote the share of the population that are (r)ecipients and $s(n, t)$ denote the share of the population that are (n)on-recipients. We can then represent the welfare caseload with the 2-state model,

$$(2) \quad \begin{bmatrix} s(r, t) \\ s(n, t) \end{bmatrix} = \begin{bmatrix} c(t) & e(t) \\ 1 - c(t) & 1 - e(t) \end{bmatrix} \begin{bmatrix} s(r, t-1) \\ s(n, t-1) \end{bmatrix},$$

where $e(t)$ is the probability that an individual not on welfare enters aid and $c(t)$ is the probability that a person on welfare continues on aid (i.e., one minus the conventional exit rate), all in time t . Because the sum of the share of recipients and non-recipients must be one ($s(n, t) + s(r, t) = 1$), we can represent equation 2 more as,

$$(3) \quad s(t) = c(t)s(t-1) + e(t)[1 - s(t-1)].$$

where $s(t)$ is the share of the population receiving aid.

The simple separation of entry and continuation is sufficient to make the welfare caseload a dynamic process. Denoting the change in an arbitrary variable v as $\Delta v(t) = v(t) - v(t-1)$, the change in the caseload can be rewritten as,⁴

$$(4) \quad \Delta s(t) = \Delta c(t)s(t-1) + \Delta e(t)[1 - s(t-1)] + [c(t-1) - e(t-1)]\Delta s(t-1).$$

This expression decomposes the change in the welfare caseload into three components: (1) contemporaneous changes in the continuation rate, (2) contemporaneous changes in the entry rate, and (3) past changes in the caseload level. Thus, a one-time policy change that affects the entry rate today would cause a change in this period’s caseload (because of the second term) and in next period’s caseload (because of the last term). Moreover, the lagged effects would disappear if the entry and continuation rates were equal because

⁴ Equation 4 follows from equation 3 by adding and subtracting the two terms $c(t-1)s(t-1)$ and $e(t-1)s(t-1)$, and then simplifying appropriately.

the first two terms would collapse to be the common flow rate and the last term would be zero.

It is straightforward to extend the 2-state model to allow for more realistic processes. One useful extension is to allow the continuation rate to vary with time on aid. This pattern in the data has been noted by many previous authors and is often referred to as “duration dependence.” This empirical regularity can be incorporated by expanding the state-space to distinguish individuals by length of time on aid. In other words, individuals who have been on aid for two periods are considered to be in a different state than those who have been on aid for three periods, and thus their continuation probabilities could differ. The model can similarly be extended to allow entry onto welfare to differ for those who have recently left welfare, as well as differ with the length of time since an individual leaves welfare. We will consider such models below.

Properties of Markov Chain Models. A Markov Chain model can be used to examine how the caseload changes over time by simulating a “counterfactual” caseload path. For example, we could simulate what would happen if the current entry and continuation rates were to persist. Similarly, we could simulate what would happen if the entry rate were cut in half, perhaps due to the passage of more stringent welfare laws. Such simulations follow directly from the recursive structure of equation 1.

The dynamic properties of simulations with Markov Chain models have been studied extensively, and thus we can rely on a general theorem to describe how the caseload would evolve. The following theorem states the general results for an arbitrary Q -state Markov matrix in a form that is directly applicable to the models we consider here:⁵

Theorem (Cinlar, 1975, p. 378). Let P be an irreducible, aperiodic, and non-negative Markov matrix. Then,

(a) There exists a steady state π for the system where, for any initial vector x_0 ,

$$\pi = \lim_{t \rightarrow \infty} P^t x_0, \text{ and all elements of } \pi \text{ are strictly positive.}$$

⁵ Simon and Blume (1994) provides a useful, elementary development of these concepts. Cinlar (1975) provides more complete treatments.

- (b) The vector π is the unique solution of $P\pi = \pi$ and $1\pi = \pi$; in other words, the vector π is the eigenvector of the matrix P associated with the eigenvalue 1, normalized so that the elements sum to one.
- (c) The convergence in (a) is geometric: there exists constants $\alpha > 0$ and $1 > \beta \geq 0$ such that $|P^k(i, j) - \pi(j)| \leq \alpha\beta^k$, $k = 1, 2, \dots$, for all i, j . Here β can be taken to be the largest number amongst the absolute values of the eigenvalues of P excluding the eigenvalue 1.

The theorem is directly applicable to the transition matrix M in equation 1 because M satisfies the restrictions on P .⁶

To make clear the content of this theorem, we develop these ideas explicitly for the two-state model. The steady state can be computed directly by solving for the fixed point s^* — the value such that $s(t) = s(t-1)$ — for equation 3,

$$(5) \quad s^* = \frac{e}{1 - c + e}.$$

This expression demonstrates part (a) of the theorem: the steady state is a function only of the transition rates and not of the initial distribution.

The general method for analyzing Markov Chain models follows from decomposing the Markov matrix into its eigenvalues and eigenvectors. For the two-state case, equation 2 can be re-written as

$$(6) \quad \begin{bmatrix} s(r, t) \\ s(n, t) \end{bmatrix} = \lambda_1' \begin{bmatrix} w_{11} \\ w_{12} \end{bmatrix} i_1 + \lambda_2' \begin{bmatrix} w_{21} \\ w_{22} \end{bmatrix} i_2,$$

where λ_1 and λ_2 are the eigenvalues of M , $\begin{bmatrix} w_{11} \\ w_{12} \end{bmatrix}$ and $\begin{bmatrix} w_{21} \\ w_{22} \end{bmatrix}$ are the eigenvectors of M ,

and i_1 and i_2 are constants which are determined by the initial state of the system ($t = 0$).

Under the assumptions of the theorem, the largest eigenvalue of the system (call it λ_1) is

⁶ The matrix M is non-negative and Markov because all of its elements are transition probabilities and the columns sum to one. It is aperiodic and irreducible because individuals from any starting state have a positive probability of transiting through every other state and that the path to visit other states is not fixed.

identically equal to one and the other eigenvalue (call it λ_2) is strictly less than one in absolute value.

The rest of the theorem follows from the representation in equation 6. First, as t becomes large, the second term on the right hand side of equation 6 vanishes because λ_2 is strictly less than one in absolute value, while the first term remains constant because λ_1 is equal to one. Thus, the first eigenvector gives the steady state of the system as t becomes arbitrarily large. Second, the system converges to this steady state at a speed governed by the second eigenvalue raised to the t th power, and thus the rate of convergence to the steady state is geometric.

2. IMPLEMENTING THE MODELS WITH CALIFORNIA DATA

In this section, we first briefly discuss our data; see Haider, et al (1999) and Klerman and Haider (2001) for more details. We then describe the three specific nested versions of a Markov Chain model of the welfare caseload that we analyze in the balance of the paper.

MEDS Data. Because our model specification is influenced by the available data, we begin by briefly describing our data. Our empirical work uses individual-level administrative data for California from the Medi-Cal Eligibility Determination System (MEDS). The MEDS provides a monthly roster of all welfare participants in California from 1987 to 1999. A different extract from the same underlying data were used in Hoynes (2000).

We construct an analysis file by drawing a random sample of approximately 3 percent of the MEDS, stratified so that we obtain approximately an equal number of observations

in each of California's counties. This scheme results in an analysis file that contains 282,381 people and 10,966,420 person-months during the interval 1989 to 1998.⁷

We cannot construct entry rates onto welfare directly from the MEDS because the MEDS is a random sample of individuals receiving welfare. To estimate entry, we additionally rely on the Intercensal Population estimates generated by the United States Census Bureau. These data provide estimates of the annual population by age group and by county; we use simple linear interpolation to estimate monthly population. Combining these data then allow us to estimate the population at risk and the subsequent entry rate. The population at risk is defined as all individuals under the age of 50.

Figure 1 plots the aggregate monthly caseload estimated from the MEDS analysis file, as well as the official California caseload counts. The MEDS data track the official California caseload counts very well. Moreover, the state's caseload broadly follows the national pattern, peaking in the mid 1990s and then declining rapidly.

Specifying the Markov Models. We consider three nested versions of the Markov chain model. The first specification is the two-state model described above. Similar models were used in previous papers that examined the welfare caseload (e.g., Boskin and Nold, 1975; Hutchens 1982, Mueser, et al 2000, and Gittleman 2000). The second specification allows the continuation rate to vary with time on aid. As others have found, the continuation rate in our data varies with time on aid, initially declining and then increasing steadily approximately for 20 months (see Figure 2). The third model further extends the second model to allow the probability of initially entering welfare to differ from re-entering welfare and to vary with time since last receiving aid. Again, this extension is consistent with previous research and our data. In the MEDS, recent welfare leavers have a probability of returning to welfare well above that of the general population and the probability declines with time off of welfare (see Figure 3).

⁷ Previous research indicates that there is considerable "churning" on and off welfare in the MEDS data. This churning is likely due to administrative record keeping rather than "real" entrances and exits (see Hoynes, 2000). To mitigate such concerns, we recode one-month spells on and off of aid as not having occurred, following Hoynes (2000).

For the latter two models, the structure of our data induces an initial conditions problem. The MEDS includes information only on recipients' current welfare status, so we do not know how long welfare recipients have been on aid or if non-recipients were previously on aid in the initial period of our data. Therefore, we cannot always assign recipients to their true state (e.g., on aid for two periods). We address this problem by assuming that the continuation rate is constant after a given number of months and that the process of re-entry and new entry is identical after a given number of months. With these assumptions and dropping sufficient initial months of data, we can place individuals into their proper state.

As an example of this structure, consider the case where the continuation rate is constant after three periods and re-entrants are identical to entrants after three periods. Then, the third model from above (i.e., allowing for duration dependence in entry and re-entry) could be described with the Markov chain model,

$$(7) \quad \begin{bmatrix} s(r,1,t) \\ s(r,2,t) \\ s(r,3,t) \\ s(o,1,t) \\ s(o,2,t) \\ s(o,3,t) \\ s(n,t) \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0 & b(1) & b(2) & b(3) & e \\ c(1) & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & c(2) & c(3) & 0 & 0 & 0 & 0 \\ 1-c(1) & 1-c(2) & 1-c(3) & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1-b(1) & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1-b(2) & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1-b(3) & 1-e \end{bmatrix} \begin{bmatrix} s(r,1,t-1) \\ s(r,2,t-1) \\ s(r,3,t-1) \\ s(o,1,t-1) \\ s(o,2,t-1) \\ s(o,3,t-1) \\ s(n,t-1) \end{bmatrix},$$

where $c(k)$ is the continuation rate for individuals who have been on aid for k periods, $b(l)$ is the re-entry rate for individuals who have been off aid for l periods, $s(r,k,t)$ is the share of the population who have received aid for k consecutive periods, $s(o,l,t)$ is the share of the population who have who have been off aid for l consecutive periods, and $s(n,t)$ is the share of the population that has not been on aid for at least three periods.⁸ The structure of the transition matrix places individuals who have been on welfare for three periods in the same state as those individuals who are beginning their third period on aid ($s(r,3,t)$). Similarly, individuals who have completed their third period on aid are

⁸ We suppress the dependence of the flow rates on the time period for compactness of notation.

placed in the same bin as those individuals who have not ever been on aid ($s(n,t)$). For this specification, we can place everyone in the appropriate state after dropping the first three periods of data.

To empirically implement the second model, we use monthly data and assume that the continuation rate is constant after 24 months. Thus, the second model is comprised of 25 states: being on welfare for 1 month, on for 2 months, ..., on for 23 months, on for 24 or more months, and off welfare. The third model additionally allows for the re-entry rate to vary for 24 months and thus contains 49 states: being off welfare for 1 month, off for 2 months, ..., off for 24 months, and the same 25 states in the second model. In the discussion that follows, we refer to these models as the 2-state, 25-state, and 49-state models, respectively.

3. THE STEADY-STATE OF THE CASELOAD

We first exploit the fact that Markov Chain models have well defined steady states to describe the amount of momentum in the welfare caseload. For example, we can compute what the caseload would be if the flow rates of a particular period were to continue indefinitely. If the steady state were much different than the actual caseload in a particular period, we would consider there to be much momentum remaining in the system.⁹

Two-State Model Results. For each year from 1989 to 1999, Table 1 presents the underlying flow rates, the actual share on aid, and the steady-state caseload implied by the 2-state model (based on equation 5). For 1999, the average monthly entry rate was 0.0019, and the average monthly continuation rate was 0.9609.¹⁰ These flow rates imply a steady state in which 5.3 percent of the population is on aid. Comparing this steady state to the actual share on aid of 6.9 percent, the two-state model implies that the

⁹ These calculations are not intended to be forecasts of what the caseload would actually be in future years. Such forecasts would more usefully be done by forecasting flow rates into the future and then simulating period by period changes.

caseload would decrease to a level 34 percent below its 1999 level if the 1999 entry and continuation rates were to persist indefinitely. Similar calculations for the peak of the steady-state caseload in 1993 suggest that the caseload had the potential of increasing another 18 percent (from 0.1035 to 0.1224). Such calculations suggest that much momentum can remain in the caseload at any particular time.

Although these calculations follow immediately from standard properties of dynamic processes, it is useful to provide a heuristic description of how these processes work. Consider a change that lowers the entry rate, perhaps due to a change in policy or the economy. The full impact of this change in the entry rate on the caseload will not be observed instantaneously. Instead, the full impact on the caseload will only occur over time as the smaller cohorts entering under the new regime replace the larger cohorts that entered aid under the old regime. Such a process will cause the caseload to continue to fall in the future, without further changes to the entry rate. However, these caseload declines will not continue indefinitely. The larger cohorts that entered under the old regime will eventually be replaced completely by the new, smaller cohorts. When this occurs, the caseload will achieve its new long-run level. This new long-run level is the steady state we calculate in Table 1.

Another implication of Table 1 is that there need not be a simple relationship between changes in the flows and changes in the stocks. For example, if the flow rates of 1999 were to continue unchanged into 2000, the caseload would fall in 2000. A more dramatic example can be observed between 1994 and 1995. Between these two years, both of the flow rates declined but the caseload increased. The Markov Chain model provides a clear interpretation for this perhaps puzzling divergence. The comparison of the actual caseload to the steady state caseload for 1994 (0.1089 vs. 0.1179) suggests that substantial upward momentum was left in the system. Thus, even though the declining flow rates would suggest that fewer people were entering and fewer people were

¹⁰ There is a notable seasonable component to California's welfare caseload, which is not surprising given the state's large agricultural sector. As a rough correction for this seasonality, we compute average monthly flow rates and per capita caseload over a calendar year.

continuing on the caseload, it appears that momentum (i.e., the last term of equation 4) was sufficient to cause the caseload to increase.

This divergence between the direction of change in the aggregate caseload and the direction of change in the underlying flows has disturbing implications for conventional static empirical analyses of the caseload. If one were to monitor the actual caseload to try to determine why it declined, one would be led to examine what was different around 1995 in California. However, if one were to monitor the flows, one would look around 1993, two years earlier. Because most researchers who have examined recent declines in the welfare caseload have been monitoring stocks, it is possible that they are looking too late for the true causes.

Comparing the Results from the Three Models. Table 2 presents steady-state calculations for all three empirical models: the 2-state model just analyzed, the 25-state model that allows for duration dependence in welfare use, and the 49-state model that allows for duration dependence in welfare use and returning to welfare. We calculate the steady states by computing the eigenvector (appropriately normalized) associated with the eigenvalue equal to one.¹¹ Rather than presenting the entire distribution across all of the states (on aid for one period, on aid for two periods, etc), we simply report the steady state share of individuals on aid in each year for the three models.

The variation in the steady-state caseloads across the three models is striking. The magnitude of the swings in the steady state share on aid grows as we enrich the specification of the state space. At the steady state peak in 1993, the implied share of the population on welfare grows from 12.2 percent in the 2-state model, to 13.2 percent in the 25-state model, and to 14.8 percent in the 49-state model. Similarly, for 1999, the steady-state share on aid falls from 4.5 percent in the 2-state model, to 4.0 percent in the 25-state model, and to 3.2 percent in the 49-state model.

¹¹ Alternatively, the steady state could be computed by simulating the caseload forward a sufficient number of periods. As a computational matter, such a solution is far easier to implement.

Because the larger state models nest the smaller state models, the results based on the larger state models are the preferred estimates. For example, the 2-state model is the 25-state model with the restriction that the continuation rates are constant across the 24 durations. Thus, the 25-state model can potentially capture all of the variation in the 2-state model, but not vice versa. The fact that the steady states diverge between the two models is another indication that the restriction is not true, as was observed in Figure 2.

It is instructive to consider mechanically why the steady states diverge differentially over time when the restrictions in the 2-state model are not satisfied. Suppose the 25-state model were the true model, but a 2-state model were used to examine the data. It is straightforward to see that the 2-state continuation rate would be a weighted sum of the 24 continuation rates in the 25-state model,

$$(8) \quad c^{2s}(\cdot, t) = \sum_{k=1}^{24} w(k) c^{25s}(k, t) = \sum_{k=1}^{24} \left\{ \frac{s^{25s}(r, k, t)}{\sum_{k=1}^{24} s^{25s}(r, k, t)} \right\} c^{25s}(k, t),$$

where the superscript Qs denotes quantities for the Q -state model. The weights are the distribution of individuals across the on-aid states. This equation makes clear that, even if the 25-state continuation rates were constant, the steady-state caseload computed from the two-state model would change whenever the weights would change. In other words, the two-state model would not return a steady state until the true 25-state process was already in a steady state. The weights on the 24 continuation rates would correspond to the steady state distribution of those on aid.

The larger swing in the steady states computed with the 25-state and 49-state models is caused by the systematic change in the weights. Figure 4 plots the difference between the steady state and actual distribution of the caseload by length of time on aid for 1993 (the year with the largest implied steady-state caseload) and 1999 (the year with the smallest implied steady-state caseload). In 1993, the actual distribution was weighted more towards short-term users, who tend to have lower continuation rates (see Figure 2). Thus, the 2-state continuation rate would tend to be lower than the 2-state continuation rate in steady state. This lower 2-state continuation rate implies that the computed 2-state

steady state would be lower than the 25-state steady state, exactly consistent with the findings in Table 2. The story is exactly the opposite in 1999: the observed distribution has more long-term recipients than the implied steady-state distribution, so the composite continuation rate is higher than the composite continuation rate implied by the 25-state steady-state distribution. Thus, the estimated 2-state steady state should be higher than the 25-state steady state, again consistent with the findings in Table 2. A similar analysis can explain the divergence (and higher amplitude) in the fraction of the caseload on welfare implied by the 25-state model and the 49-state model, where the observed distribution between entry and re-entry differs from the steady-state distribution.

This analysis has important implications for understanding the changing caseload. Our results suggest that there is a substantial amount of momentum in the welfare caseload. For example, if the 1999 entry, continuation, and re-entry rates were to persist, the welfare caseload would decline by 50 percent from its 1993 caseload level. A corollary to this point is that the caseload might be changing in one period because of events that happened in previous periods. For example, the continued caseload decline that has been observed during the late 1990s could be due to changes in the mid-1990s.

4. RATES OF CONVERGENCE: CALCULATING HALF-LIVES

Although steady states provide a useful description of the long-term implications of current transition rates, these long-term implications might be viewed as irrelevant if the convergence towards the steady state were sufficiently slow. Moreover, the rate of convergence will also be directly informative regarding the length of time one must wait to see the impact of a policy reflected in the total caseload.¹²

As noted in Section 1, rates of convergence of a Markov process towards its steady-state can be characterized by the eigenvalues of the Markov matrix. However, this characterization is difficult to interpret. Fortunately, the speed of convergence of a

¹² As should be clear, such an analysis only applies to policies that restrict the underlying flows. Certain policies would have an immediate impact. For example, if the welfare program were cancelled, the impact of the policy change would be immediately observed as a substantial and immediate decline in the welfare rolls.

Markov Chain model can be described using a more easily interpreted metric.¹³ A standard result regarding Markov processes is that they converge toward their steady state at a geometric rate (see part (c) of Cinlar's theorem). In addition, it is straightforward to show that geometric convergence has a half-life that can be calculated as,

$$(9) \quad h = \frac{\ln 0.5}{\ln |\beta|},$$

where β is the rate of geometric decay (see the appendix). In other words, from any initial distribution, a half-life of h months implies that the system moves half the distance towards its new steady state in h months, half of the remaining distance in $2h$ months (i.e., 75 percent), and then half of the remaining distance in $3h$ months (i.e., 87.5 percent), etc. A relatively long half-life would imply that it would take a long-time for a change in transition rates to be reflected in the aggregate caseload.

The results of Section 1 imply that the rate of geometric decay for a two-state model is given by the second eigenvalue. Standard results for Markov processes suggest that the second eigenvalue for the two-state model is given by $\lambda_2 = c - e$.¹⁴ Combining this expression with equation 9 implies that the half-life for the two-state model can be computed as follows,

$$(10) \quad h = \frac{\ln 0.5}{\ln |c - e|}.$$

Thus, the half-life is increasing in the continuation rate and decreasing in the entry rate (assuming that the continuation rate is greater than the entry rate).

¹³ We are aware that half-lives are commonly used to describe exponential decay and that exponential and geometric decay are closely related. However, we are unaware of previous studies that have calculated half-lives in the context of Markov Chain models to describe the rate of convergence.

¹⁴ We obtain this expression by relying on two results: that the sum of the eigenvalues equals the trace of the Markov matrix (see equation 2) and that the largest eigenvalue is equal to one for a Markov matrix. Furthermore, we note that if the continuation rate and the entry rate are equal, then equation 10 is undefined and the methods we describe are not applicable.

The intuitive explanation for equation 10 is straightforward. Consider two Markov processes that describe the welfare caseload, where the first process has a larger continuation rate than the second. Furthermore, suppose that both processes were out of steady-state because more people were on aid than was implied by the steady-state; for example, suppose that a policy was recently instituted that resulted in a lower entry rate. The first process will have a relatively longer half-life because the cohorts that entered aid under the earlier regime (with a higher entry rate) will remain on aid longer because of the larger continuation rate. Thus, the cohorts that entered under the old regime will not be replaced as quickly. A similar argument can be made for differences in the entry rate.

We present the half-lives for the two-state model in Table 3. Over the period 1989 to 1999, the half-life ranges between 16 and 22 months for each observed combination of entry and continuation rates. Thus, it would take about 18 months for half the impact of a change in the underlying flows (to the levels in the particular year) to be reflected in the aggregate caseload, and 36 months for 75 percent of the eventual impact to be observed in the flows.

Using simulation (rather than analytic expressions), Table 4 reports the half-lives for the three different models in each year. Once again, expanding the state-space affects the results, usually increasing the implied half-life. Once again, the disparity in results can be understood by the implicit distributions that are used to calculate composite continuation and entry rates. For example, the composite continuation rate in the latter years tends to be too high because the caseload is weighted towards many long-term recipients than would be implied by the steady state weights, and higher continuation rates are associated with longer half-lives (see equation 10). Due to the same nesting argument made in the previous section lead us to prefer the, our preferred results are those based on the 49-state model.

These half-life results have important implications for evaluating the impact of anything that changes the caseload through the underlying flows. Our results imply that any change in the flows will only slowly affect the aggregate caseload. Conventional

models of the aggregate caseload that use a difference-in-difference identification strategy implicitly assume that the entire effect of a policy is observed immediately (e.g., CEA, 1997, 1999). Thus, such models will likely underestimate the true impact of the policy or any other predictor of the caseload change.

5. THE RELATIVE IMPORTANCE OF ENTRY, CONTINUATION, AND RE-ENTRY

Understanding the relative role of the various transition rates is important for two reasons. First, the relative role can provide evidence regarding the potential cause of underlying changes. For example, it is likely that strong welfare-to-work programs will impact the caseload primarily through changing the continuation rate, whereas any effect of more stringent eligibility conditions would operate primarily through the entry rate. Second, the relative contribution of changes in the various rates will have important implications for how caseload changes should be studied. Standard randomized trials of welfare programs have estimated the effect of programs on the future welfare participation of those randomized into the program. Because many experiments of welfare policies only randomize those already on welfare into treatment, these experiments estimate only the impact of treatment on continuing on welfare (and perhaps re-entering welfare). To the extent that changes in entry are relatively important, then such randomized trials will be less useful to understanding general caseload changes (Moffitt, 1991; Grogger, Karoly, and Klerman, 2002).

Analyzing the relative contributions of the underlying flows is not straightforward, however, because of the sluggish adjustment of the caseload. For example, consider again the actual caseload change from 1993 to 1994 in Table 1. Despite the fact that the entry and continuation rates are declining, the actual caseload is increasing during this interval. If we were to focus on the actual caseload changes for such a sub-period, we would conclude that the contribution of entry and continuation to the caseload increase is negative, an unappealing conclusion.

To avoid such problems, we examine the importance of entry and continuation for changes in the steady state caseload. Specifically, we compute the entry rate, continuation rate, and steady state for two years of interest. Then, we simulate the steady state caseload under the counterfactual assumption that only one of the flows changed. For example, we define the change in the fraction of the caseload on aid due to the entry rate $\Delta s^*(\Delta e)$ as,

$$(11) \quad \Delta s^*(\Delta e) = \frac{s^*(e_2, c_1) - s^*(e_1, c_1)}{s^*(e_2, c_2) - s^*(e_1, c_1)}$$

where the denominator is the total change in steady state between the first set of rates (e_1, c_1) and the second set of rates (e_2, c_2) and the numerator is the implied change in the steady-state allowing only the entry rate to change. For this calculation, we consider year 1 to be the base year. We compute the impact of changing the continuation rate analogously.

It should be noted that these decompositions are highly non-linear, and thus the percentage of the caseload change attributed to entry and continuation need not add to one hundred percent. Moreover, when assessing the change between two years, it will generally matter which year was assigned the role of the base year. Because there is not a correct method to assigning the base year, we calculate results using both years as the base year.¹⁵

Table 4 reports the results of these calculations for two time periods: the period of increasing flow rates (1989 to 1993) and the period of decreasing flow rates (1993 to 1998). We present results using all three models and using each of the years as the base year. For the 25-state and 49-state models, we change the continuation or re-entry rates at each duration simultaneously.

¹⁵ The decomposition would add to 100 if we instead changed the various quantities sequentially. However, the order in changing the rates would matter.

From Table 4, it is clear that the individual contributions of the rates do not add to one hundred percent and that the value of the contributions vary with the chosen base year. However, the estimates tell a consistent story: changes in the entry rate are more important than changes in the continuation rates. For the period of the caseload increase, the estimated contribution of changes in the entry rate is consistently fifty percent larger than the estimated contribution of changes in the continuation rate. Changes in the re-entry rate have little impact. The relative role of changes in the entry rate is not as great when examining the caseload decline, but it is always greater than the role of changes in the continuation rate.

Again, these results have important implications for aggregate caseload modeling. They suggest that those looking for the causes of the caseload decrease (and the increase which preceded it) should consider factors that change the entry rate at least as much as factors that change the continuation rate. Moreover, over the last ten years, considerable resources have been invested in random assignment evaluations of various caseload policies. The analysis presented here suggests that such randomized trials are missing a large part of the potential cause of the change in the caseload. Similarly, econometric studies that only examine continuation and re-entry miss much of the story (e.g., Hoynes, 2000).

6. CONCLUSIONS AND DISCUSSION

Using standard results on Markov Chain models, this paper examines the dynamic properties of the welfare caseload. This exercise yields several empirical findings with important implications for the study of welfare policy. First, there is considerable momentum in the welfare caseload. The caseload would have increased much more (40 percent) if the flows of 1993 had persisted and the caseload will decline much more (by another 50 percent) if the flows of 1999 persist. Such momentum can cause the caseload to change substantially for several periods following a one time change in the rates.

Second, the magnitude of the mis-specification from considering the stock rather than the flows (i.e., the transition rates) will depend on the speed of convergence to the steady-

state implied by the new transition rates. If the transition to the new steady-state was rapid, the distinction would be unimportant. However, we find that the adjustment of the caseload to changes in the transition rates is far from instantaneous. Given the flow rates that were generally observed during the 1990s in California, the implicit half-life of the welfare caseload is always more than two years (our preferred results from the 49-state model). These long half-lives imply that an evaluation of any policy that operates through the flows must wait at least two years to see even half of the policy's impact whenever the aggregate caseload is being monitored. Thus, if we are interested in measuring the full impact of policy changes such as welfare reform, then a significant amount of time must be allowed to pass before such impacts will be observed in the aggregate caseload level. Conventional models of the aggregate caseload that use a difference-in-difference identification strategy such as CEA (1999) implicitly assume that all of the effect of a policy is observed immediately (i.e., in the two periods for which the policy changes).

Third, our estimates of the dynamic properties are very sensitive to the specification of the state-space. Inferences from the simple two-state models that have been previously used differ dramatically when compared to inferences from richer (and more realistic) models. Since the two-state models are nested in the larger state models, we conclude that those using Markov models need to consider models with duration dependence.

Fourth, a large portion of the caseload level change is due to changes in the entry rate.¹⁶ Thus, random assignment evaluations and some previous dynamic studies that focus on changes in the continuation and re-entry rates miss much of the story. Furthermore, this finding has implications for understanding the causes of the observed caseload decline. Many changes in welfare programs would be expected primarily to affect continuation rates. For example, more intensive welfare-to-work programs, more aggressive sanction policies, and time limits would all be expected mainly to lower

¹⁶ The national importance of this finding is in need of more study. California did not implement significant and broad changes to its welfare-to-work programs or diversion until late 1998, and then California's changes were less severe than in other states (e.g., Klerman, et al 2001). It is possible that the role of entry was great in states that adopted policies earlier and that adopted more severe policies.

continuation rates (i.e., increase exit rates). Our finding that changes in entry are more important suggest that other factors are likely to be important in explaining the caseload decline—perhaps the economy or policies that discourage entry (e.g., formal or informal diversion programs and the deterrent effect of participation requirements on entry).

APPENDIX: Geometric Decay and Half-lives

Suppose that a particular quantity $A(0)$ is decaying geometrically towards zero with each time period t . Then, the distance from zero of the quantity in time period t is,

$$(A1) \quad |A(t)| = |\beta|^t |A(0)| = |\beta|^t |A(0)|,$$

where the absolute value of β is between zero and one. If this process has a “proper half-life”, then there exists a constant time period h that it takes to traverse the remaining distance towards zero. Letting N be the number of half-lives, then the following equation should hold,

$$(A2) \quad 0.5^N |A(0)| = |\beta|^{hN} |A(0)|,$$

with the half-life h being independent of N and $A(0)$. It is straightforward to show that this is the case with (A1) implying that

$$(A3) \quad h = \frac{\ln 0.5}{\ln |\beta|}.$$

REFERENCES

- Bane, Mary Jo and David Ellwood (1994). *Welfare Realities: From Rhetoric to Reform: Preface*. Cambridge, MA: Harvard University Press.
- Bell, Stephen (2001). "Why Are the Caseloads Falling?" *Urban Institute Discussion Paper, 01-02*.
- Blank, Rebecca (2001). "What Causes Public Assistance Caseloads to Grow?" *Journal of Human Resources*, 36:1, 85-118.
- Blank, Rebecca and Patricia Ruggles (1996). "When Do Women Use AFDC and Food Stamps? The Dynamics of Eligibility vs. Participation." *Journal of Human Resources*, 31:1, 57-89.
- Blank, Rebecca and Geoffrey Wallace (1999). "What Goes Up Must Come Down?" In *Economic Conditions and Welfare Reform* (ed. Sheldon Danziger), Kalamazoo, MI: W.E. Upjohn Institute for Employment Research.
- Boskin, Michael and Frederick Nold (1975). "A Markov Model of Turnover in Aid to Families with Dependent Children." *The Journal of Human Resources*, 10:4, 467-481.
- Cinlar, Ehnan (1975). *Introduction to Stochastic Processes*. Englewood Cliffs, NJ: Prentice-Hall, Inc.
- Council of Economic Advisers (1997). "Technical Report: Explaining the Decline in Welfare Receipt, 1993-96." A Report by the Council of Economic Advisers, May.
- Council of Economic Advisers (1999). "Technical Report: The Effects of Welfare Policy and the Economic Expansion on Welfare Caseloads: An Update." A Report by the Council of Economic Advisors, August.
- Figlio, David and James Ziliak (1999). "Welfare Reform, the Business Cycle, and the Decline in AFDC Caseloads." In *Economic Conditions and Welfare Reform* (ed. Sheldon Danziger), Kalamazoo, MI: W.E. Upjohn Institute for Employment Research.
- Figlio, D. N., Gunderson, C., & Ziliak, J. P. (2000). "The effects of the macroeconomy and welfare reform on food stamp caseloads." January 2000.
- Fitzgerald, John (1995). "Local Labor Markets and Local Area Effects on Welfare Duration," *Journal of Policy Analysis and Management*, 14:1, 43-67.
- Gittleman, Maury (2001). "Declining Caseloads: What Do the Dynamics of Welfare Participation Reveal?" *Industrial Relations*, 40:4, 537-70.

- Grogger, J. K., L.A. Karoly, and J.A. Klerman. 2002. "The Effects of Welfare Reform: A Research Synthesis," Final Report to DHHS-ACF, Santa Monica: RAND.
- Haider, Steven, Jacob Alex Klerman, Jan Hanley, Laurie McDonald, Elizabeth Roth, Liisa Hiatt, and Marika Suttorp (1999). "Welfare Reform in California: Design of the Impact Analysis, Preliminary Investigations of Caseload Data." RAND DRR(L)-2077/1-CDSS, May.
- Hoynes, Hilary Williamson (2000). "Local Labor Markets and Welfare Spells: Do Demand Conditions Matter?" *Review of Economics and Statistics*, 82(3), 351-368.
- Hutchens, Robert (1981). "Entry and Exit Transitions in a Government Transfer Program: The Case of Aid to Families with Dependent Children." *Journal of Human Resources*, 16(2).
- Klerman, Jacob Alex and Steven J. Haider (2002). "A Stock-Flow Analysis of the Welfare Caseload." July 2002 manuscript.
- Levine, Phillip and Diane Whitmore (1998). "The Impact of Welfare Reform on the AFDC Caseload" *National Tax Association Proceedings*. Washington, DC: Nation Tax Association, 24-33.
- Moffitt, Robert (1991). "Evaluation Methods for Program Entry Effects." In *Evaluating Welfare and Training Programs* (eds., C. Manski and I. Garfinkel), Cambridge: Harvard University Press.
- Moffitt, Robert (1992). "Incentive Effects of the U.S. Welfare System: A Review." *Journal of Economic Literature*, 30, 1-61.
- Moffitt, Robert (1999). "The Effect of Pre-PRWORA Waivers on AFDC Caseloads and Female Earnings, Income, and Labor Force Behavior." In *Economic Conditions and Welfare Reform* (ed. Sheldon Danziger), Kalamazoo, MI: W.E. Upjohn Institute for Employment Research.
- Mueser, Peter R., Julie L. Hotchkiss, Christopher T. King, Phillip S. Rokicki, and David W. Stevens (2000). "The Welfare Caseload, Economic Growth and Welfare-to-Work Policies: An Analysis of Five Urban Areas." July manuscript.
- Ribar, David (2000). "Transitions from Welfare and the Employment Prospects of Low-skill Workers." March manuscript.
- Simon, Carl and Lawrence Blume (1994). *Mathematics for Economists*. New York: W. W. Norton and Company.
- Ziliak, James, David Figlio, Elizabeth Davis, Laura Connolly (2000). "Accounting for the Decline in AFDC Caseloads: Welfare Reform or Economic Growth?" *Journal of Human Resources*, 35(3), 570 - 586.

Figure 1: Welfare Recipients and Unemployment Rate in California

Source: Authors' tabulations from the MEDS and CA237.

Notes: The CA237 is the official California welfare caseload. The MEDS represents an estimate of the caseload based on the 3% sample we analyze in this paper.

Figure 2: Continuation Rate by Time on Aid

Source: Authors' tabulations from the MEDS.

Notes: This figure plots the continuation rate by time on aid. The last category (to the far right) is the continuation rate for those individuals who have been on aid for at least 24 months.

Figure 3: Entry Rate by Period of Time Since off Aid

Source: Authors' tabulations from the MEDS.

Notes: This figure plots the entry rate by time off of aid. The last category (to the far right, labeled "Not") is the entry rate for those who have been off of aid for at least 24 months and who have not been on aid previously. continuation rate for those individuals who have been on aid for at least 24 months.

Figure 4: Deviation of Actual Distribution on Aid from Steady State Distribution, 25 State Model

Source: Authors' tabulations from the MEDS.

Notes: This figure plots the continuation rate by time on aid. The last category (to the far right) is the continuation rate for those individuals who have been on aid for at least 24 months.

Table 1: Average Monthly Caseload Characteristics for the 2-State Model

Year	Share on Aid	Entry Rate	Continuation Rate	Steady State Share on Aid
1989	0.0802	0.0032	0.9674	0.0882
1990	0.0835	0.0036	0.9657	0.0944
1991	0.0905	0.0041	0.9689	0.1156
1992	0.0969	0.0037	0.9705	0.1127
1993	0.1035	0.0040	0.9714	0.1224
1994	0.1089	0.0037	0.9721	0.1179
1995	0.1095	0.0035	0.9702	0.1038
1996	0.1053	0.0031	0.9681	0.0875
1997	0.0939	0.0024	0.9635	0.0625
1998	0.0803	0.0022	0.9616	0.0542
1999	0.0689	0.0019	0.9592	0.0454

Source: Authors' tabulations from the MEDS.

Notes: The first three columns represent tabulations from the MEDS. The steady state is based on equation 5.

Table 2: Steady State Share of the Population Receiving Aid for All Models

Year	Share on Aid	Steady State Share on Aid		
		2 State	25 State	49 State
1989	0.0802	0.0882	0.0917	0.0969
1990	0.0835	0.0944	0.0971	0.1087
1991	0.0905	0.1156	0.1298	0.1514
1992	0.0969	0.1127	0.1251	0.1387
1993	0.1035	0.1224	0.1318	0.1482
1994	0.1089	0.1179	0.1241	0.1337
1995	0.1095	0.1038	0.1038	0.1046
1996	0.1053	0.0875	0.0838	0.0788
1997	0.0939	0.0625	0.0559	0.0460
1998	0.0803	0.0542	0.0476	0.0383
1999	0.0689	0.0454	0.0396	0.0322

Source: Authors' tabulations from the MEDS.

Notes: The first columns represent tabulations from the MEDS. The steady state is based on summing the appropriate elements of the eigenvector associated with the eigenvalue equal to one.

Table 3: Half-life Calculations for All Models.

Year	Entry rate	Continuation rate	Half-lives in months		
			2 State	25 State	49 State
1989	0.0032	0.9674	19	22	43
1990	0.0036	0.9657	18	21	39
1991	0.0041	0.9689	19	25	50
1992	0.0037	0.9705	20	26	50
1993	0.0040	0.9714	21	25	48
1994	0.0037	0.9721	22	25	47
1995	0.0035	0.9702	20	22	41
1996	0.0031	0.9681	20	20	35
1997	0.0024	0.9635	17	17	27
1998	0.0022	0.9616	17	16	25
1999	0.0019	0.9592	16	15	23

Source: Authors' tabulations from the MEDS.

Notes: The 2-state half-lives are calculated with equation 9 (and could be calculated by simulation) and the 25- and 49-state half-lives are calculated by simulation. To be consistent across the quantities, we round the 2-state half-lives to the next highest integer.

Table 4: Decomposition of Caseload Changes Due to Changes in Entry, Continuation and Re-Entry

Model	Base Year	Percent Contribution of		
		Entry	Continuation	Re-Entry
1989-1993 Caseload Increase				
2-State Model	1989	49.0	44.6	--
2-State Model	1993	55.4	51.0	--
25-State Model	1989	53.2	39.0	--
25-State Model	1993	61.0	46.8	--
49-State Model	1989	52.4	32.0	5.8
49-State Model	1993	61.3	40.5	8.1
1993-1999 Caseload Decrease				
2-State Model	1993	72.9	47.1	--
2-State Model	1999	52.9	27.1	--
25-State Model	1993	68.5	57.9	--
25-State Model	1999	42.1	31.5	--
49-State Model	1993	67.8	51.1	22.9
49-State Model	1999	34.4	20.5	6.9

Source: Authors' tabulations from the MEDS.

Notes: The decomposition is based on equation 10. For the 25-state and 49-state models, all of the continuation and re-entry rates are changed as appropriate.