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Full Title: The Cross-sectional Average Length of Life (CAL): A Cross-sectional Mortality Measure That Reflects the Experience of Cohorts

Short Title: The Cross-sectional Average Length of Life (CAL)

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ABSTRACT

This paper presents the implications of a summary mortality index, the Cross-sectional Average Length of Life (CAL). By combining the mortality experience of various cohorts in a cross-sectional fashion, CAL complements traditional one-period or one-cohort indexes and enriches our understanding of population processes. First, CAL provides an alternative insight into the analysis of mortality. By taking into account the real mortality conditions to which individuals in a population have been subject to, it tends to yield less favorable mortality levels than e_0 and produces different rankings of mortality levels across countries. Second, CAL is a relevant index for the study of population dynamics. In particular, change in CAL over time shows the direct impact of mortality change on population growth, and the e_0 /CAL ratio for a given year shows the mortality-induced growth that can be expected given current mortality levels. It illustrates that mortality can play a non-negligible role in future population growth, even in the absence of future mortality improvements.

INTRODUCTION

A classic issue in demography relates to the relative advantages of the period and cohort approaches. Both in mortality and fertility studies, authors have been discussing which dimension is more relevant for the analysis of population processes (Ryder 1983; Caselli 1990; Ní Brochláin 1992). Cohort measures reflect the real experience of individuals, since human life develops within the cohort framework whereby age and time vary simultaneously. Although cohort measures correspond better to reality, they are not as often calculated as period measures for reasons that are well known to demographers. First, in order to compute summary cohort measures, one needs to wait for cohorts to have completed fertility (for fertility measures) or to be extinct (for mortality measures). Thus cohort summary measures require data spanning for many years. Such data requirements make cohort measures more difficult to compute. A related but more substantive drawback of cohort summary measures is that they refer to conditions stretching over several decades. For policy purposes, it is sometimes necessary to obtain information in a more timely manner and in a way that reflects current conditions.

It is both because of data availability and for analytical purposes that demographers have designed period measures, which rely on the concept of synthetic cohorts. A synthetic cohort is a hypothetical cohort of people who would be subject at each age to the age-specific rates of one specific period. In reality, unless vital rates are constant over time, no individual will face rates observed during one period, and thus period measures are not realistic measures of people's experience. However, period

measures provide a convenient way of summarizing current conditions. They also allow one to identify “period effects”, which occur when various cohorts experience simultaneous, sudden and temporary changes in their demographic behavior as a result of particular period conditions (war, epidemic, economic crisis, etc).

In brief, cohort and period measures answer different types of questions in demography, and thus they are both relevant (Wilmoth 1990). In fact, there is an important literature comparing period and cohort measures. For fertility, in particular, there are a number of studies dealing with the relationship between cohort and period indicators (Ryder 1964; Bongaarts and Feeney 1998). The main purpose of such research is to understand the biases between period measures and the actual cohort experience of individuals.

One reason for preferring cohort summary measures over period measures is that they may be more relevant for the understanding of population dynamics. This statement is particularly true for mortality, which is the focus of this paper. Indeed, if one is interested in the effect of mortality change on population growth, cohort mortality is more relevant because it tells about how cohorts of individuals actually survive over time. In order to understand how age-groups vary in size over time as a result of mortality change, one needs to use cohort mortality indexes (Horiuchi and Preston 1988; Arthur and Vaupel 1984).

Yet cohort mortality measures do not perfectly translate into population growth either. The population growth rate is affected by the experience of a multitude of cohorts

altogether, and therefore a comparison of the experience of single cohorts will tell little about the dynamics of the entire population.

In this paper, I use an alternative mortality index, the Cross-sectional Average Length of Life (CAL). By combining the mortality experience of various cohorts in a cross-sectional fashion, CAL complements traditional one-period or one-cohort mortality indexes. The CAL index was first proposed by Brouard (1986) to examine the gap between cohort and period mortality and to show how a population's age structure is affected by past mortality conditions. The term Cross-sectional Average Length of Life is adapted from Brouard's French term "*durée de vie moyenne actuelle*" (Brouard 1986, p.165). CAL was then used in several instances to identify the potential biases involved in current-status measures of disability (Brouard and Robine 1992, p.89), to examine prospects for future survival (Caselli 1992:19-20), and to analyze the impact of mortality on changes in population size and changes in the population sex ratio (Guillot 1999, 2000). Recently, it has been used to illustrate tempo effects in mortality (Bongaarts and Feeney 2002). However, the various properties and implications of this index for mortality analysis and population dynamics have not been fully explored. I will show here that the advantage of CAL is twofold. First, CAL provides an alternative insight into the analysis of mortality levels and trends, and second, CAL is useful for studying the effect of mortality change on population growth.

DEFINITION OF CAL

The Cross-sectional Average Length of Life, $CAL(t)$, refers to a particular period t , but takes into account the actual mortality conditions experienced by the various cohorts surviving to that particular period t . $CAL(t)$ is defined in the following equation:

$$CAL(t) = \int_0^{\omega} p_c(x, t-x) dx = \int_0^{\omega} e^{-\int_0^x \mu(a, t-x+a) da} dx \quad (1)$$

where $p_c(x, t-x)$ = probability of surviving from age 0 to x according to the mortality conditions prevailing in the cohort born at time $t-x$ (i.e., aged x at time t),

$\mu(a, t)$ = force of mortality at age a and at time t ,

T = maximum age at death (i.e., age at which $p_c(x, t-x) = 0$)

We can see from equation (1) that $CAL(t)$ is the sum of proportions of survivors at time t among the various cohorts present in the population at time t . Instead of summing survival probabilities (or proportions of survivors) for one period as in the case of the classical period life expectancy at birth (e_0), CAL involves probabilities of surviving for a multitude of cohorts.

Another way to interpret CAL is to view it as the average number of person-years that would be lived between time t and $t+dt$ in a closed population where a constant number of births (by time unit, or, to simplify, by year) would have been exposed to actual cohort mortality conditions. (More precisely, $CAL(t)$ is a density function; the

number of hypothetical person-years lived during the interval t to $t+dt$ is $CAL(t)dt$.)

Therefore, it is an index which accounts for the real mortality conditions which have been experienced by the various birth cohorts whose survivors are present in a population at a particular time.

$CAL(t)$ can also be interpreted as the life expectancy at birth of a cohort aged a at time t , (i.e., born at time $t-a$), whose survival advantage relative to the population's younger or older cohorts is equal to its survival disadvantage. The mortality experience of this cohort, summarized in $CAL(t)$, can thus be considered as an "average" of the population's mortality experience. This third interpretation of $CAL(t)$ is developed further in the paper.

On the Lexis diagram shown in Figure 1, we see that $CAL(T)$ involves death rates located along diagonals crossing the age axis at time T . This measure differs from the traditional life expectancy at birth, $e_0(T)$, which uses death rates located on one single age axis at time T , and from the cohort life expectancy at birth, $e_0^c(T)$, which is computed using death rates located on one single diagonal corresponding to one particular cohort born at time T . Equation (1) can be compared to corresponding equations for e_0 and e_0^c in expressions (2) and (3):

$$e_0(t) = \int_0^{\omega} p(x, t) dx = \int_0^{\omega} e^{-\int_0^x \mu(a, t) da} dx \quad (2)$$

where $p(x, t)$ = probability of surviving from age 0 to x according to the mortality conditions prevailing at time t .

$$e_0^c(t) = \int_0^{\omega} p_c(x, t) dx = \int_0^{\omega} e^{-\int_0^x \mu(a, t+a) da} dx \quad (3)$$

where $p_c(x, t)$ = probability of surviving from age 0 to x according to the mortality conditions prevailing in the cohort born at time t .

[Figure 1 about here]

CAL is a mortality index that refers to one period because it refers to person-years lived during one period, t to $t+dt$, in our hypothetical constant birth population. However, CAL takes into account the past mortality of cohorts, because the time location of the death rates in equation (1), $t-x+a$, refers also to periods before t as we integrate with respect to a and x . Therefore, CAL will tend be lower than e_0 if mortality has been declining. It will also be durably affected by particular mortality conditions during specific years (due to a war or an epidemic, for example), even after normal mortality conditions are reestablished. CAL has a slow inertia which, as we will see below, parallels that of population growth.

The main feature of CAL is that unlike e_0 , it takes into account the true mortality experience of birth cohorts whose survivors are still present in a particular population. In a period life table, survival probabilities are obtained by linking mortality rates at

different ages in a synthetic cohort, even though no cohort may ever experience such survival probabilities. In the case of CAL, the survival probabilities used have been experienced by actual cohorts. However, the corresponding function $p_c(x, t-x)$ does not aim to describe or simulate the life course of individuals, unlike the traditional $p_c(x,t)$ or $p(x, t)$ functions of cohort and period life tables, respectively. As we will see later, the $p_c(x, t-x)$ function is not necessarily monotonic, whereas probabilities of surviving from birth within one cohort, synthetic or actual, can not increase with age.

It is important to note here that CAL is not equal to mean age at death in a constant-birth population exposed to actual cohort mortality conditions. Unlike stationary populations where the area under the $p(x, t)$ curve is also equal to the mean age at death, it is not the case for the curve $p_c(x,t-x)$. Recently, it has been argued that the area under the $p_c(x,t-x)$ curve is equal to the average age at death in a population with constant births and changing mortality (Bongaarts and Feeney 2002). This conclusion is reached by stating that the density of deaths at time t in such a population is produced by a force of mortality, $\mu^*(a,t)$, defined as $-\{M p_c(a,t-a)/M a\}/p_c(a,t-a)$. It should be noted that (1) deaths at time t in any population is produced by $\mu(a,t)$ (the actual force of mortality at time t) applied to the survivors, not by $\mu^*(a,t)$; and (2) $\mu^*(a,t)$ is not strictly speaking a force of mortality because, as we will see later in the paper, it can take on negative values. The difference between CAL(t) and the mean age at death at time t in a population with constant births and changing mortality can be quite large. For French males, for example, the difference was 2.51 years in 2001, and it was even larger during earlier decades (9.24 years in 1954).

In order to estimate CAL for a particular year, one needs mortality rates (or survival probabilities) for all cohorts whose members are still present in a population. For practical purposes, one can use data for cohorts born up to about 100 years before the date for which CAL is estimated, because earlier cohorts will contribute very little to its final value.

In this paper, I have calculated CAL for males and females in Sweden, France and the U.S. where long series of mortality rates are available. I used ${}_1q_x$ series available in the Berkeley Mortality Data Base (<http://demog.berkeley.edu/wilmoth/mortality>), and computed cohort survival probabilities (or cohort survivors) at each age and time. CAL is then estimated by summing all cohort survivors at a particular time t , using the following equation, which is the equivalent of equation (1) in discrete terms:

$$CAL(t) = .5 + \frac{\sum_{x=1}^{\omega} l_x^c(t-x)}{l_0^c}$$

where $l_x^c(t-x)$ = number of survivors at exact age x in the life table for the annual birth cohort born on average at exact time $t-x$ (i.e., between exact time $t-x-.5$ and $t-x+.5$).

l_0^c = life table radix

Alternatively, CAL(t) can be calculated using cohort person-years in the following manner:

$$CAL(t) = \frac{\sum_{x=0, n}^{\omega} {}_nL_x^c [t-x-n, t-x]}{l_0^c}$$

where ${}_nL_x^c$ = number of person-years lived between age x and $x+n$ in the life table for the cohort born between exact time $(t-x-n)$ and $(t-x)$.

The combination of cohort survivors for the computation of CAL is illustrated in Figure 2, where cohort survivors for French males are plotted at the time when age x is reached. CAL corresponds to the sum of these cohort survivors (divided by 100,000) for one specific year t .

[Figure 2 about here]

For Sweden, it was possible to compute CAL from 1861 to 1999 because ${}_1q_x$'s are available for every year since 1751. For France, ${}_1q_x$'s are available starting in 1806, which enabled me to compute CAL from 1905 to 2001. For the United States, where ${}_1q_x$'s are available since 1900, I was able to compute CAL for a few years only, from 1997 to 2001. For comparison purposes, I also computed the two traditional mortality summary indexes, e_0 and e_0^c , in these three countries. Brouard has often claimed that CAL is the mortality index of the 21st century, because it requires at least 100 years of mortality data for its computation. It should be noted, however, that it is not necessary to

wait for cohorts to be extinct in order to compute CAL. This explains why CAL can be computed for recent years. By comparison, the most recent cohort for which we can confidently estimate e_0^c in France is the 1902 cohort.

[Figure 3 about here]

Figure 3 presents results for French males. As illustrated in this Figure, a feature of CAL is that it is almost always lower than e_0 . As was stated earlier, this reflects mortality decline: the past mortality of cohorts surviving to a particular period is generally higher than that of a fictive cohort subject to the mortality conditions of that period. Another situation where CAL would be lower than e_0 is a situation where mortality would have been constant, except for one period where mortality was exceptionally high.

A second feature of Figure 3 is that the curve for CAL is much smoother than the curve for e_0 or e_0^c . This slow inertia is due to the fact that the number of age-specific death rates included in the computation of CAL is much larger than for e_0 or e_0^c (about 5,000 single-age mortality rates for one CAL value vs. about 100 rates for one e_0 or e_0^c value). Therefore, the CAL value during a high-mortality year will not be affected as much as the value for e_0 or e_0^c . However, this does not imply that excess period mortality does not have a large impact on CAL. The effect of a mortality crisis on e_0 is large in magnitude but it is also short in duration. Typically, the overall trend in e_0 is reestablished shortly after the crisis. For CAL, however, the effect of excess period

mortality is not as large as for e_0 but the impact is much longer in duration. The trend for CAL is durably affected by a mortality crisis, whereas e_0 forgets its past. Among French males, there were large drops in life expectancy at birth during WWI and WWII, but the overall trend was reestablished shortly after war periods. For CAL, the short-term decreases or slowdowns were not as large, but the secular trend in CAL, up to present, has been durably affected by the two wars.

Due to their different perspectives on mortality trends, CAL and e_0 provide different conclusions regarding national mortality levels. Table 1 shows levels of CAL and e_0 for 1999 in Sweden, France and the U.S. We can see that these three countries rank differently depending on the index used for the comparison. France, in particular, always ranks lower in CAL than in e_0 , due to the durable mark that the two wars have left on its CAL levels. For males in France and the US, CAL implies that birth cohorts whose survivors are present in 1999 were subject on average to higher mortality in France than in the U.S., whereas e_0 implies that mortality conditions for the survivors in 1999 are less favorable in the US than in France.

[Table 1 about here]

Which index is a more appropriate measure of the mortality conditions of a population? The answer depends on how we define mortality conditions. In Table 1, e_0 summarizes mortality rates for 1999 only, whereas CAL summarizes the mortality rates to which individuals living in a population in 1999 have been subject to. We often adhere to

period-based measures to compare mortality levels across countries, without questioning this approach. If by mortality levels in 1999, we mean age-specific mortality rates prevailing in 1999, then e_0 is more appropriate. If, however, we mean actual cohort survivorship prevailing among individuals in the population in 1999, CAL is more appropriate. CAL is not a better or unbiased summary measure reflecting current mortality conditions (Bongaarts and Feeney 2002). It is, however, a better estimate of the real mortality conditions to which individuals in a population have been subject, experience which spans over several past decades.

A CLOSER LOOK AT CAL AND e_0

As specified in equation (1) and (2), $CAL(t)$ corresponds to the area under the curve $p_c(x, t-x)$ (= proportion of survivors in each cohort at time t), and $e_0(t)$ corresponds to the area under the curve $p(x, t)$ (= proportion of survivors at each age in the period life table at time t). The contrasting features of CAL and e_0 can thus be best analyzed by comparing $p_c(x, t-x)$ and $p(x, t)$ at a given time t . These two functions are shown in Figure 4 for French males in 1999.

[Figure 4 about here]

The survival probability from birth to age x in the cohort aged x at time t (or born at time $t-x$) is specified in the following expression:

$$p_c(x, t-x) = e^{-\int_0^x \mu(a, t-x+a) da} \quad (4)$$

This function can be expressed in terms of period survival probabilities with the following equation:

$$p_c(x, t-x) = p(x, t) \cdot e^{\int_0^x [\mu(a, t) - \mu(a, t-x+a)] da} \quad (5)$$

Equation (5) is a simple expression that shows the relationship between the survival probability of a cohort aged x at time t and the survival probability to age x in the period life table pertaining to time t . We can see that the factor relating cohort and period survival probabilities from birth to age x is a function of differences in age-specific death rates at time t and corresponding rates in the cohort aged x at time t .

To examine how this function varies with age, the following ratio can be derived, as x tends towards 0:

$$\frac{p_c(x+\Delta x, t-x-\Delta x)}{p_c(x, t-x)} = e^{-\int_x^{x+\Delta x} \mu(a, t) da} \cdot e^{\int_0^x [\mu(a, t-x+a) - \mu(a, t-x-\Delta x+a)] da} \quad (6)$$

We compare this ratio to the similar ratio for the survival curve in a period life table:

$$\frac{p(x+\Delta x, t)}{p(x, t)} = e^{-\int_x^{x+\Delta x} \mu(a, t) da} \quad (7)$$

Equations (6) and (7) show that, at a given time t , whereas the factor by which

$p(x, t)$ decreases with age is a function of (a, t) for one period t only, $p_c(x, t-x)$ varies with age because of both (a, t) for one period t and the difference in the age-specific death rates between the cohort born at time $t-x$ and the cohort born at time $t-x-x$. Simply put, one can say that proportions of survivors in a population are typically smaller among older cohorts because of two factors: (1) older cohorts have been exposed to mortality risks for longer time periods than younger cohorts because of their higher age, and (2) older cohorts have been exposed to higher mortality risks than younger cohorts because they were born earlier. This contrasts with period survivors which are affected only by the first factor. As a result, the function $p_c(x, t-x)$ will decrease faster with age than $p(x, t)$ if mortality has been declining.

Equations (6) and (7) also imply that, whereas $p(x, t)$ is a monotonically decreasing function of age, the curve $p_c(x, t-x)$ can increase or decrease with age. An increase between age x and $x+\Delta x$ will occur if death rates below age x have been increasing between the two cohorts, to an extent that it overcompensates for the force of mortality at age x , i.e., if the following inequality holds:

$$\int_0^x [\mu(a, t-x+a) - \mu(a, t-x-\Delta x+a)] da > \int_x^{x+\Delta x} \mu(a, t) da \quad .$$

Figure 4 illustrates the fact that $p_c(x, t-x)$ is typically lower than $p(x, t)$ and that $p_c(x, t-x)$ does not necessarily decrease monotonically with age. For example, among French males, the cohort born in 1945 have experienced lower survival probabilities than the 1944 cohort, mostly because of an exceptionally high infant mortality rate in 1945.

Although in 1999, the 1945 cohort had experienced one fewer year of exposure to mortality relative to the 1944 cohort, this shorter exposure was not enough to compensate for their higher mortality. Thus, in 1999, there are more survivors on average among the 1944 cohort than among the 1945 cohort, and the curve $p_c(x, t-x)$ goes up between age 54 and 55.

Because the curve $p_c(x, t-x)$ is not always monotonic, it should not be interpreted as an actual survivorship function, representing the unbiased life course of individuals (Bongaarts and Feeney 2002). An increase in $p_c(x, t-x)$ between two ages produces a function $\mu(a, t)$, as defined earlier, with negative values. This violates the definition of the force of mortality, and results in survival probabilities that can be greater than one, which is not humanly possible. Bongaarts and Feeney (2002) estimated $\mu(a, t)$ by applying a steady mortality decline from generation to generation, in which case the $p_c(x, t-x)$ declines monotonically. Empirical data from France show that it is not necessarily the case, and demonstrate the inadequacy of $\mu(a, t)$ as a force of mortality function.

The examination of the function $p_c(x, t-x)$ also allows to develop an interpretation of $CAL(t)$ that is meaningful in terms of the actual life expectancy at birth of a cohort. Figure 5 shows the $p_c(x, t-x)$ curve in 1948 for Swedish females. In 1948, $CAL(t)$ was about 60.35 years, which is equal to the life expectancy at birth for the 1898 cohort, aged 50 in 1948. The survival curve for this cohort ($p_c(x, t-a)$) is also shown in Figure 5. Since in this case $CAL(1948) = e_0^c(1898)$, the area under both curves is equal.

The comparison of these curves in Figure 5 illustrates the additional interpretation

of $CAL(t)$ mentioned earlier. For the cohort aged 50 in 1948, the survival disadvantage relative to younger cohorts (i.e., aged less than 50 years in 1948) is equal to the survival advantage that this cohort will experience relative to older cohorts (aged more than 50 years in 1948). As illustrated in Figure 5, the negative and positive differences between the two curves are equal. The 1898 cohort can thus be considered as an “average” cohort in terms of the population’s mortality experience, and $CAL(1948)$ can be interpreted as the life expectancy at birth of this “average” cohort. Naturally, it is not possible to identify this cohort for recent years, because the future mortality experience of cohorts present in the population is not known. Nonetheless, the identification of this “average” cohort for past years provides an additional interpretation of CAL that is also valid for recent years and perhaps easier to conceptualize than other interpretations.

[Figure 5 about here]

Theoretically, there is a possibility that no cohort present in a population will ever experience a life expectancy at birth that is equal to current CAL levels. This could happen if there were a very large, sudden and continued increase or decrease in mortality that would make the life expectancy at birth of each cohort higher or lower than what would be expected given the survival that they have experienced up to that year. While this scenario is theoretically possible, cohort mortality varies more progressively in actual populations, even in situations with large fluctuations in period mortality like in France. Thus, in our empirical examples, the “average” cohort was identifiable whenever data

availability permitted to do so. On the other hand, it is possible that several cohorts will experience a life expectancy at birth equal to current CAL levels. This can happen in situations where cohort mortality has been fluctuating, like in France for example, or if, on the contrary, mortality has not been changing. In fact, if mortality has been absolutely constant, all cohorts will have life expectancy levels equal to CAL. If mortality has been declining steadily, however, e_0^c will be consistently higher among younger cohorts, and only one cohort will experience life expectancy levels equal to current CAL levels. In Sweden, this “average” cohort, identifiable between 1862 and 1948, was born between 30 and 50 years before the date at which CAL was calculated.

CAL AND POPULATION DYNAMICS

In this section, I suggest an alternative interpretation of CAL. Let us take again the simple fictive case of a closed population where constant annual births would have been subject to actual, changing cohort mortality. As shown in equation (8), CAL is the factor by which one can multiply these annual births to obtain the total size of this fictive population at a particular time:

$$N(t) = \int_0^{\omega} B \cdot p_c(x, t-x) dx = B \cdot CAL(t) \quad (8)$$

where $N(t)$ = total population at time t ,

B = constant annual number of births.

Equation (8) resembles that of the stationary population ($N = B e_0$) except that it is applicable to populations with changing mortality as well, unlike the stationary model. If we examine Table 1 again and compare CAL for males in France and the U.S., the results can be interpreted in a different fashion. If both France and the U.S. had an identical and constant number of births each year, more people would be present in 1999 in the U.S. than in France, despite almost two additional years of e_0 in France. Clearly, survivorship up to year 1993 has been better in the U.S. than in France overall, as reflected by this greater number of survivors.

Therefore, in addition to providing an alternative approach for the analysis of mortality change, CAL has the advantage of showing the direct impact of mortality on population size and growth. In particular, the factor by which CAL varies between two dates can be interpreted as the factor by which the population has changed as a direct effect of mortality change. By keeping births constant, the CAL approach isolates direct mortality growth effects, independently of other factors.

Table 2 shows that, with CAL changing from 41.20 to 68.49 between 1905 and 1999, the French male population has been multiplied by 1.66 between 1905 and 1999 because of overall mortality decline. Figure 3 illustrates, however, that this growth has not been monotonic, due to mortality fluctuations during the two war periods. With the CAL analysis, the direct effect of this excess mortality on population growth can be directly estimated on a yearly basis. During the same period, the Swedish male population has increased by the factor 1.54 as a result of mortality change. Table 2 shows the growth factor for the French and Swedish female populations as well.

[Table 2 about here]

The conclusion that CAL change corresponds to direct mortality growth effects can be illustrated by performing a decomposition of population change. The total population size of a country closed to migration is the product of past births and cohort survival probabilities, as shown in equation (9):

$$N(t) = \int_0^{\omega} B(t-x) \cdot p_c(x, t-x) dx \quad (9)$$

Thus, the absolute change in $N(t)$ between two dates can be decomposed into (1) change attributable to changes in the birth series, $B(t-x)$; and (2) change attributable to changes in cohort survival probabilities, $p_c(x, t-x)$. Using actual annual births and cohort survival probabilities for Swedish males and standard decomposition techniques (Das Gupta 1993), I decomposed the trend in $N(t)$ between 1858 and 1999. Results are shown in Figure 6. (The estimated $N(t)$ series is not exactly equal to the actual population size in Sweden because of migration, but the differences are minor).

[Figure 6 about here]

Taking the 1858 population as a reference, the population growth factor in Figure 6 shows the factor by which the Swedish male population has increased since 1858. The curve for birth effects shows the factor by which the population has increased

since 1858 as a result of changes in the birth series, and the curve for mortality effects shows growth effects due to changes in cohort survival probabilities. At any two dates, the product of birth-effect and mortality-effect factors equals the population growth factor. For comparison, I included the factor by which CAL has grown since 1858. It is clear that CAL change is very similar to the mortality growth effects in the above decomposition. The two curves do not coincide exactly, because mortality growth effects in the decomposition are weighted by the $B(x,t-x)$ function, whereas CAL does not apply any weights to $p_c(x, t-x)$. Indeed, in the decomposition approach, interactions between the two functions do have an impact on the results, whereas CAL shows the “un-interacted” effect of mortality change on population growth. Figure 6 confirms the relative non-importance of interactions in this particular case. Therefore the CAL approach allows to focus on direct mortality growth effect in a fashion that is comparable to the decomposition of changes in population using equation (9). (In reality, the annual number of births in equation (9) is also in part the result of past mortality levels, because in addition to fertility levels, it is affected by the number of males and females surviving to reproductive ages. Mortality has therefore both direct and indirect effects on population growth. Indirect mortality effects operate in combination with fertility through the annual number of births, whereas direct mortality effects on population growth operate independently through the survival of these births. CAL corresponds to this latter direct effect.)

The decomposition of population growth into a birth component and a mortality component using some version of equation (9) is not new. Similar decomposition has

been used in several instances to study the growth of specific age groups (Horiuchi and Preston 1988; Preston, Himes and Eggers 1989; Caselli and Vallin 1990; Vaupel and Jeune 1995) and for the entire population (Horiuchi 1995). The contribution of the CAL approach is (1) the analysis of mortality growth effects for all ages combined, and (2) the fact that CAL can be estimated independently of other growth factors and also has a meaning in terms of the survival experience of the population.

This simple strategy for analyzing mortality growth effects also contrasts with other methods that have been used to estimate the impact of mortality change on the growth of the entire population. One approach is the “comparative statics” approach, which compares two steady states, i.e., before and after mortality change (Schoen 1986; Vaupel 1997). Through this approach, it is expected that a 10% increase in e_0 will lead to a 10% increase in the population due to direct mortality growth effects. In reality, however, mortality is constantly changing, which violates the steady state assumption, and the pattern of these changes has effects on population growth that is not captured by e_0 change. CAL change, however, permits to analyze mortality growth effects between any two dates, whether or not the population is experiencing the steady state at these two dates. Unlike the e_0 , CAL also permits to estimate the growth trajectory between these two dates of interest.

Another approach used to link mortality change to population change is based on population projections. It is a common demographic exercise to compare the actual population size of a country to the same population projected with constant mortality in order to examine how mortality change affects population growth and structure

(Hermalin 1966; Vaupel and Gowan 1986; Kannisto 1996; White and Preston 1996; Heuveline 1999). Such an approach does not completely isolate mortality growth effects, because the conclusions are affected in part by the shape of the age structure at the beginning of the time period (which is itself a function of past demographic trends) and by fertility and migration levels used for the projection period. Using counterfactual projections, Heuveline (1999) estimated that mortality decline between 1955 and 2000 is responsible for an additional 1.5 billion people in the world in 2000 under observed fertility rates for the period 1955-2000. By keeping fertility constant at the 1950-55 levels, however, it is estimated that mortality decline is responsible for an additional 2.2 billion people (Heuveline 1999, p.687). This illustrates the fact that with the projection approach, results depend in part on levels of fertility and migration. The analysis of CAL change, however, is totally independent of fertility and migration trends. The mortality growth effects analyzed with the CAL approach are completely endogenous to mortality. Thus the CAL methodology would provide the exact same result regarding the direct impact of mortality change on population growth for two countries with same mortality history, regardless of their respective fertility and migration levels. When estimating mortality growth effects, it seems desirable to distinguish the effect of mortality from the effect of other variables. The use of CAL allows the estimation of direct mortality growth effects which are not confounded by variables other than mortality.

CAL, e_0 AND FUTURE POPULATION GROWTH

The fact that in Figure 3, periods of excess mortality correspond to periods of slowdown in CAL is intuitive with the population dynamics interpretation of CAL. If period mortality suddenly increases relative to the previous year, we expect the population not to grow as fast as before. To understand this correspondence more formally, I analyze the derivative of $CAL(t)$. It can be demonstrated that:

$$\frac{\partial CAL(t)}{\partial t} = \bar{\mu}(t) \cdot [e_0(t) - CAL(t)] \quad (10)$$

where $\bar{\mu}(t)$ is a mean value of $\mu(a, t)$, whose sign is always positive.

Equation (10), whose full derivation is described in Appendix 1, shows that the sign of the derivative of $CAL(t)$ is a function of the difference between $e_0(t)$ and $CAL(t)$. Most commonly in our empirical examples, $CAL(t)$ tends to increase with time because $e_0(t)$ tends to be greater than $CAL(t)$ as a result of mortality decline. If $CAL(t)$ and $e_0(t)$ are equal (which can happen, for example, if mortality has been constant) $CAL(t)$ stays constant. On the contrary, if $CAL(t)$ is greater than $e_0(t)$, which can happen during periods of mortality crisis (for example, for French males in 1915), then CAL decreases. The CAL analysis shows, however, that an increase in death rates during a specific period does not necessarily have a negative impact on population growth. As long as $e_0(t)$ remains greater than or equal to $CAL(t)$, CAL will continue increasing or will remain

constant. For example, France experienced substantial increases in death rates in 1926 and 1929, resulting in a 1.5-year decrease in e_0 in each case. This did not result in CAL declines because $e_0(t)$ did not drop below corresponding CAL levels.

As a general rule, the direct impact of mortality on population change at time t is simply due to the disparity between the functions $p_c(x, t-x)$ and $p(x, t)$, summarized in $CAL(t)$ and $e_0(t)$. It therefore depends both on mortality levels at time t and on mortality levels before time t . Figure 3 illustrates that CAL change is a function of the difference between $e_0(t)$ and $CAL(t)$. Among French males, $CAL(t)$ actually increased or decreased during a given year depending on whether $e_0(t)$ was greater or smaller than $CAL(t)$.

This principle has important implications for future population growth induced by mortality. In particular, $e_0(t)$ indicates the level towards which $CAL(t)$ is tending. In the most extreme scenario, if period mortality were to stop changing in the future and stay constant at its current levels (i.e., those prevailing at time t), $CAL(t)$ would continue changing until it equates the level of $e_0(t)$, and it would stay constant at this value thereafter. The ratio of e_0 to CAL for a given year can therefore be considered as the mortality-induced growth that one can expect in a population, given its current mortality levels. If period mortality continues decreasing in the future, which is a likely scenario in many populations, actual mortality growth effects will be even greater. In such populations, the e_0/CAL ratio can be considered as the minimum mortality-induced growth that one can expect in the future.

Table 3 shows the e_0/CAL ratio for Sweden, France and the U.S. in 1999. If mortality were to freeze at its current levels, the French male population would still grow

by about 10% just because of the disparity between past cohort mortality and 1999 period mortality. The US male population would augment by 6%. This “momentum of mortality decline”, or population momentum attributable to direct mortality growth effects, illustrates that mortality can play a non-negligible role in future population growth, even in the absence of future mortality improvements.

[Table 3 about here]

The momentum of mortality decline can be expressed in terms of the difference between cohort and period mortality rates. Using equation (5), we can express the life expectancy in terms of cohort survival probabilities:

$$e_0(t) = \int_0^{\omega} p(x, t) dx = \int_0^{\omega} p_c(x, t-x) \cdot e^{\int_0^x [\mu(a, t-x+a) - \mu(a, t)] da} dx$$

Therefore, the ratio of e_0 to CAL can be expressed in the following manner:

$$\begin{aligned} \frac{e_0(t)}{CAL(t)} &= \frac{\int_0^{\omega} p_c(x, t-x) \cdot e^{\int_0^x [\mu(a, t-x+a) - \mu(a, t)] da} dx}{\int_0^{\omega} p_c(x, t-x) dx} \\ &= \int_0^{\omega} c_{CAL}(x, t) \cdot e^{\int_0^x [\mu(a, t-x+a) - \mu(a, t)] da} dx \end{aligned} \quad (11)$$

where $c_{\text{CAL}}(x, t)$ = proportion of survivors aged x at time t in the CAL age structure

$$(\text{= } p_c(x, t-x)/\text{CAL}(t))$$

Equation (11) shows that the e_0/CAL ratio is a direct function of changes in age-specific mortality rates between cohorts surviving to time t and period mortality at time t , in the same manner as $p_c(x, t-x)$ and $p(x, t)$ were related in equation (5). This e_0/CAL ratio expresses the gap that needs to be filled for cohort mortality to equate period mortality. It is function of the difference between mortality rate at age a for a cohort age x at time t , : $(x, t-x+a)$, and the mortality rate at age a at time t , : (a, t) . The larger is this difference, the larger is the momentum of mortality decline. Indeed, larger past declines in age-specific mortality rates imply more “room” for cohort mortality to improve, and thus for population to grow as a result of this mortality improvement.

CONCLUSION

This paper has presented the implications of a summary mortality index, CAL, which I believe complements traditional one-period or one-cohort indexes and enriches our understanding of population processes. First, CAL provides an alternative insight into the analysis of mortality. By taking into account the real mortality conditions to which individuals in a population have been subject to, it tends to yield less favorable mortality levels than e_0 and produces different rankings of mortality levels across countries.

Second, it is a relevant index for the study of population dynamics. In particular, change in CAL over time shows the direct impact of mortality change on population growth, and the e_0/CAL ratio shows the mortality-induced growth that can be expected given current mortality levels. It illustrates that mortality can play a non-negligible role in future population growth, even in the absence of future mortality improvements.

One drawback of the CAL approach is that, unless some assumptions are made about the pattern of mortality decline (Bongaarts and Feeney 2002), it requires detailed historical mortality information for its computation. This contrasts with e_0 which can be easily computed with mortality information for one period only. Unlike the cohort life expectancy at birth (e_0^c), however, CAL does not require cohorts to be extinct for its computation.

The purpose of this paper is not to argue that one index is better than the other. CAL and e_0 are two different and complementary approaches that are both instructive as long as they are correctly interpreted.

APPENDIX 1

Derivation of $MCAL(t)/Mt$ (using the Landau notation, $o(h)$)

$$\frac{\partial CAL(t)}{\partial t} = \lim_{h \rightarrow 0} \frac{CAL(t+h) - CAL(t)}{h}$$

$$\begin{aligned} CAL(t+h) &= \int_0^{\omega} p_c(y, t+h-y) dy \\ &= \int_0^h p_c(y, t+h-y) dy + \int_h^{\omega} p_c(y, t+h-y) dy \\ &= h \cdot p_c(0, t+h-0) + \int_h^{\omega} p_c(y, t+h-y) dy + o(h) \\ &= h + \int_h^{\omega} p_c(y, t+h-y) dy + o(h) \end{aligned}$$

Define $x = y-h$. Then:

$$\begin{aligned} CAL(t+h) &= h + \int_0^{\omega} p_c(x+h, t+h-x-h) dx + o(h) \\ &= h + \int_0^{\omega} p_c(x+h, t-x) dx + o(h) \\ &= h + \int_0^{\omega} p_c(x, t-x) \cdot e^{-\int_x^{x+h} \mu(a, t) da} dx + o(h) \end{aligned}$$

Therefore:

$$\begin{aligned} \lim_{h \rightarrow 0} CAL(t+h) &= h + \int_0^{\omega} p_c(x, t-x) \cdot (1 - \mu(x, t) \cdot h) dx + o(h) \\ &= h + \int_0^{\omega} p_c(x, t-x) dx - \int_0^{\omega} p_c(x, t-x) \cdot \mu(x, t) \cdot h \cdot dx + o(h) \end{aligned}$$

and

$$\lim_{h \rightarrow 0} [CAL(t+h) - CAL(t)] = h - \int_0^{\omega} p_c(x, t-x) \cdot \mu(x, t) \cdot h \cdot dx + o(h)$$

Since $\lim_{h \rightarrow 0} \frac{o(h)}{h} = 0$:

$$\frac{\partial CAL(t)}{\partial t} = 1 - \int_0^{\omega} p_c(x, t-x) \mu(x, t) dx$$

Since $\int_0^{\omega} p(x, t) \mu(x, t) = 1$, one can write:

$$\frac{\partial CAL(t)}{\partial t} = \int_0^{\omega} \mu(x, t) \cdot [p(x, t) - p_c(x, t-x)] dx$$

Using the mean value theorem:

$$\frac{\partial CAL(t)}{\partial t} = \bar{\mu}(t) \cdot [e_0(t) - CAL(t)]$$

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Table 1: e_0 and CAL in France, Sweden and the U.S., 1999

1999	Males				Females			
Rank	e_0		CAL		e_0		CAL	
1	Sweden	76.90	Sweden	72.56	France	82.71	Sweden	78.07
2	France	75.15	USA	69.17	Sweden	81.74	France	76.01
3	USA	73.30	France	68.49	USA	79.53	USA	75.66

Table 2: Factor by which the population has increased due to the direct effect of mortality change, 1905-1999

Population		CAL		Growth Factor
		1905	1999	
Males	France	41.20	68.49	1.66
	Sweden	47.16	72.56	1.54
Females	France	43.92	76.01	1.73
	Sweden	50.25	78.07	1.55

Table 3: e_0 /CAL ratio in France, Sweden and the US, 1999

Country	e_0	CAL	e_0 /CAL
<i>Males</i>			
France	75.15	68.49	1.097
Sweden	76.90	72.56	1.060
USA	73.30	69.17	1.060
<i>Females</i>			
France	82.71	76.01	1.088
Sweden	81.74	78.07	1.047
USA	79.53	75.66	1.051

Figure 1: Lexis diagram showing the location of death rates used in the computation of cross-sectional average length of life, $CAL(T)$, period life expectancy at birth, $e_0(T)$, and cohort life expectancy at birth, $e_0^c(T)$.

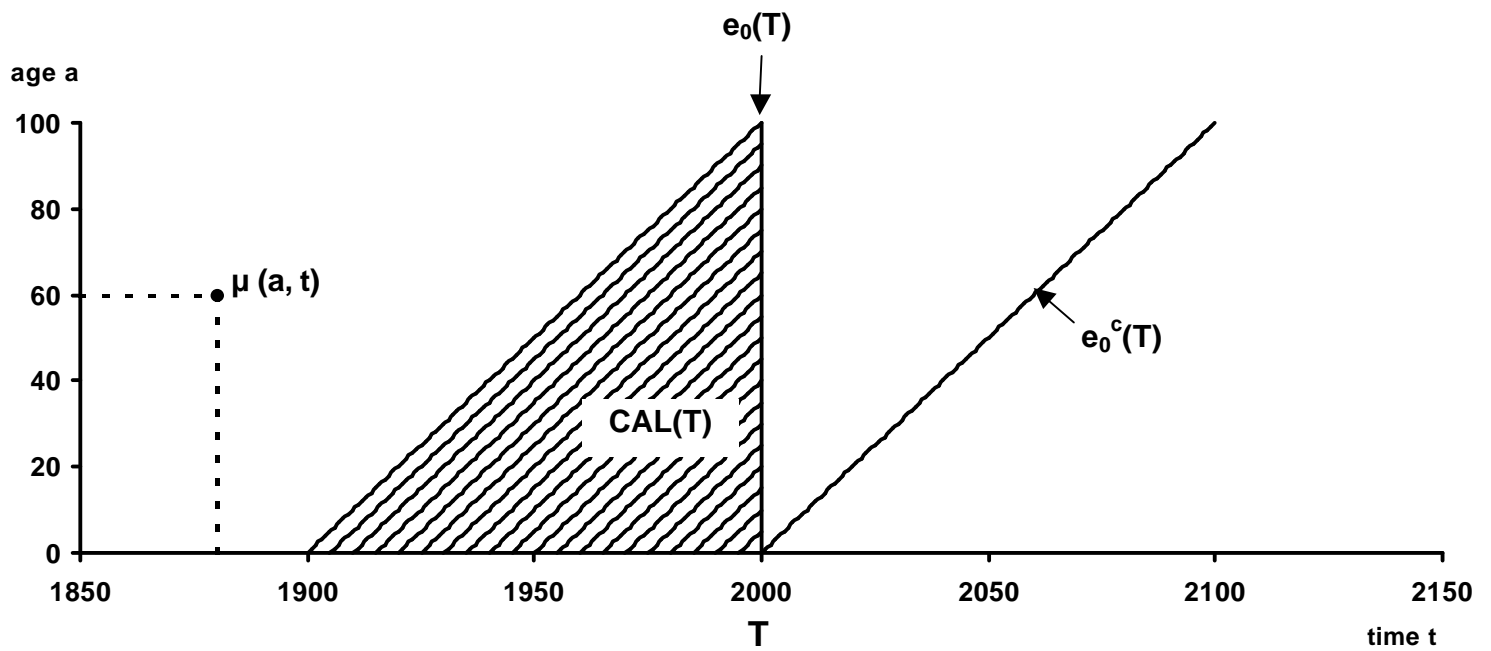


Figure 2: Cohort survivors at age x (per 100,000 individuals at birth), $l_x^c(x, t-x)$, plotted at the time when age x is reached. France, Males, 1900-2000.

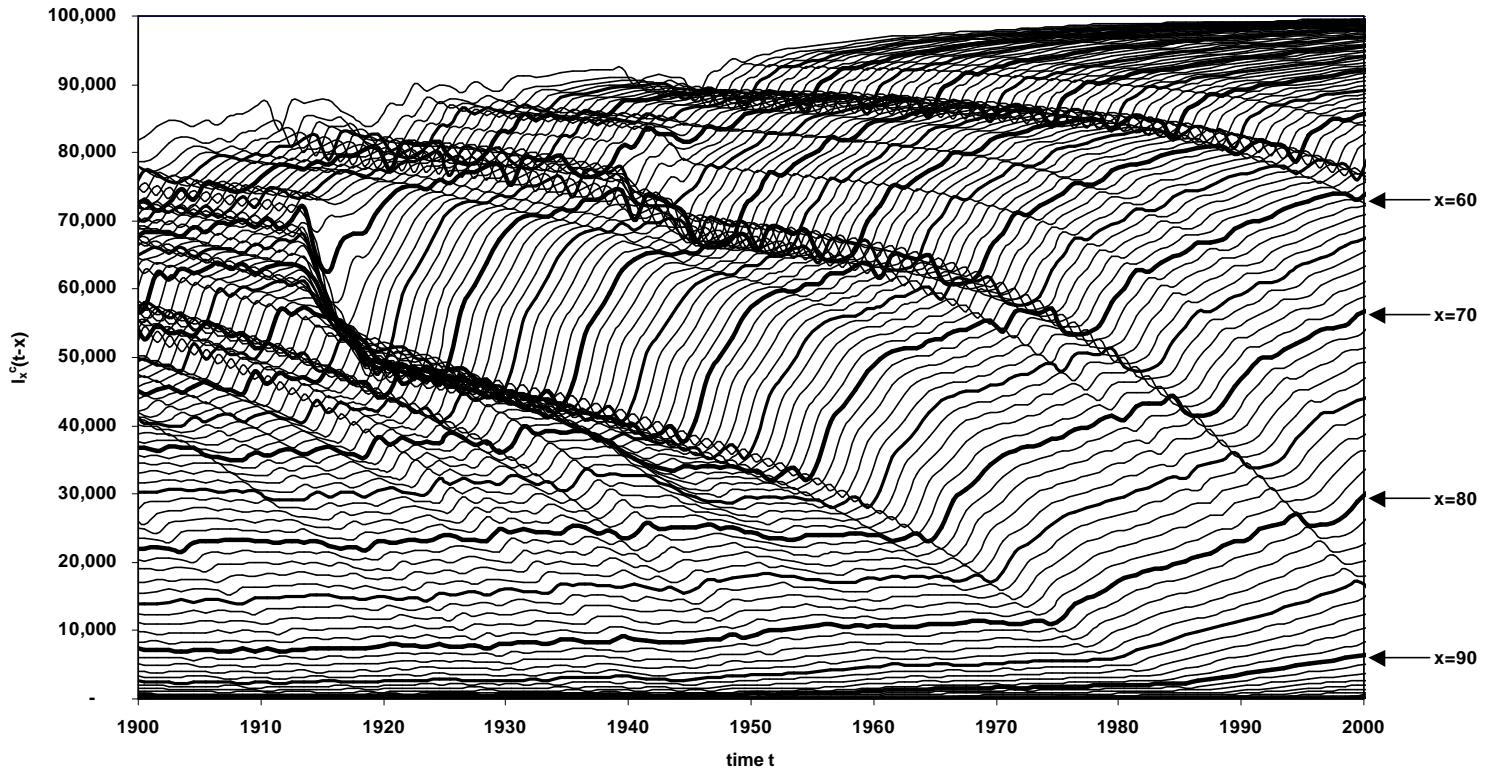


Figure 3: Cross-sectional average length of life, $CAL(t)$, period life expectancy at birth, $e_0(t)$, and cohort life expectancy at birth, $e_0^c(t)$. France, Males, 1806-2000.

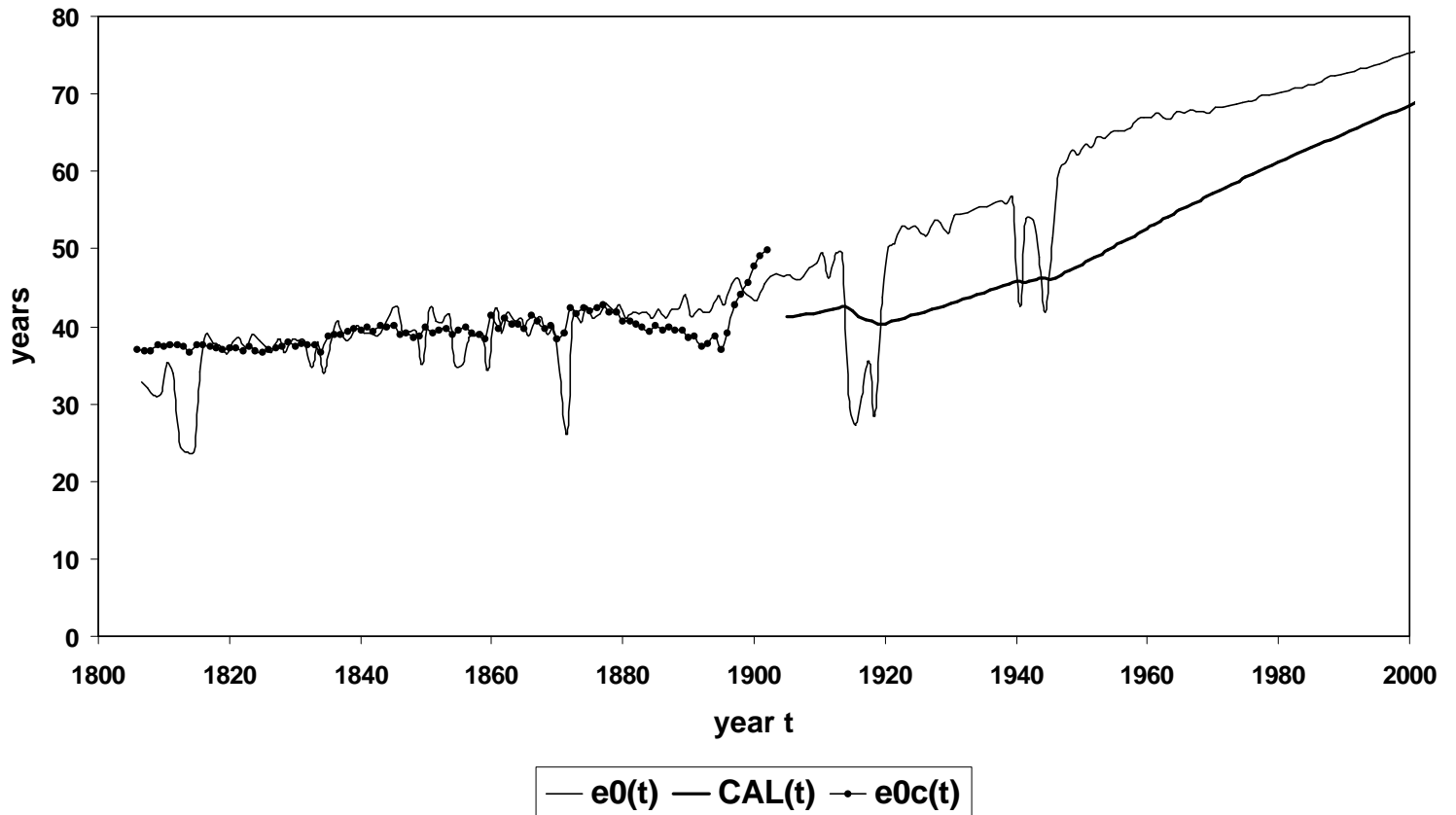


Figure 4: $p(x, t)$ and $p_c(x, t-x)$ functions. France, Males, 1999.

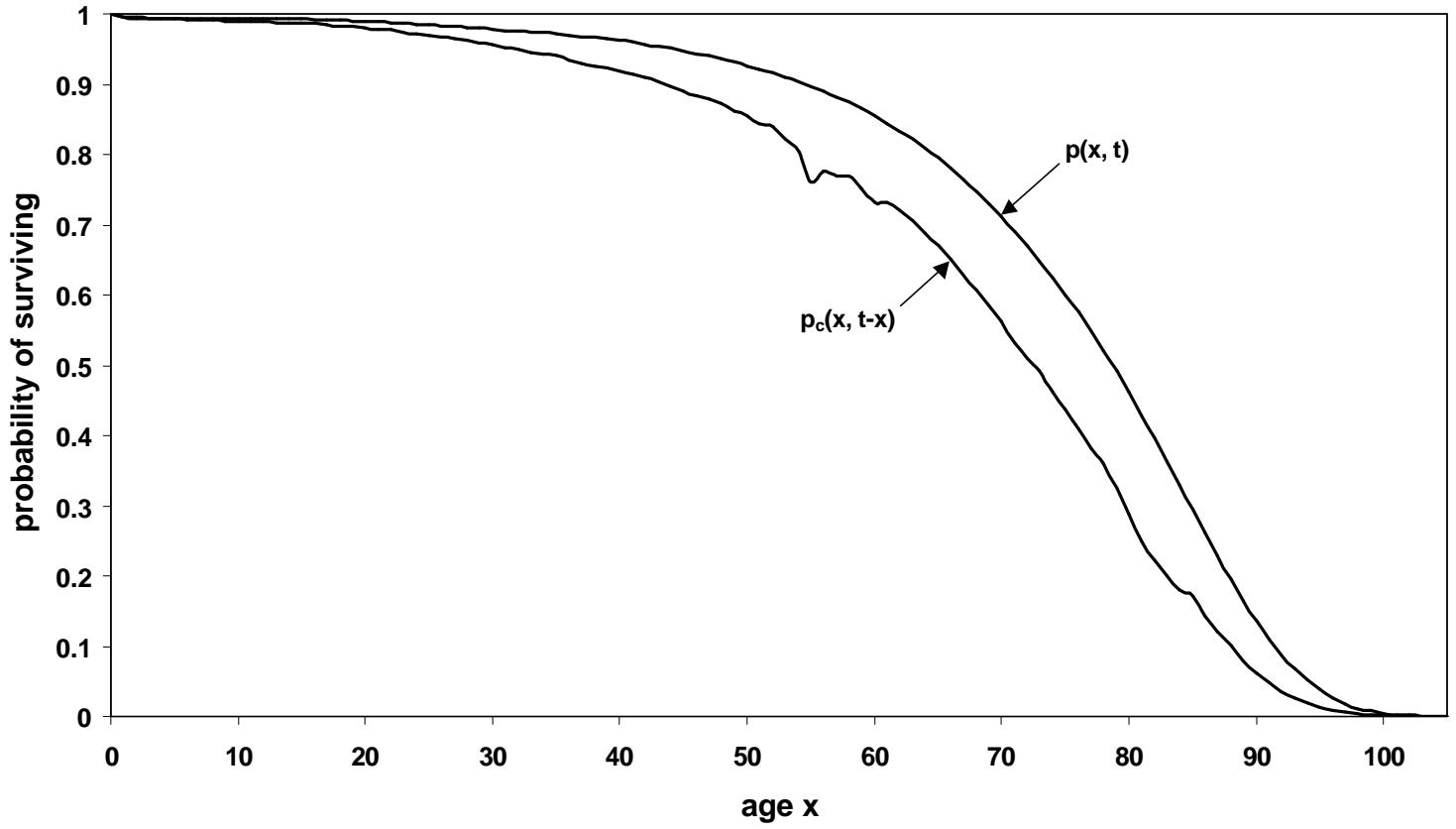
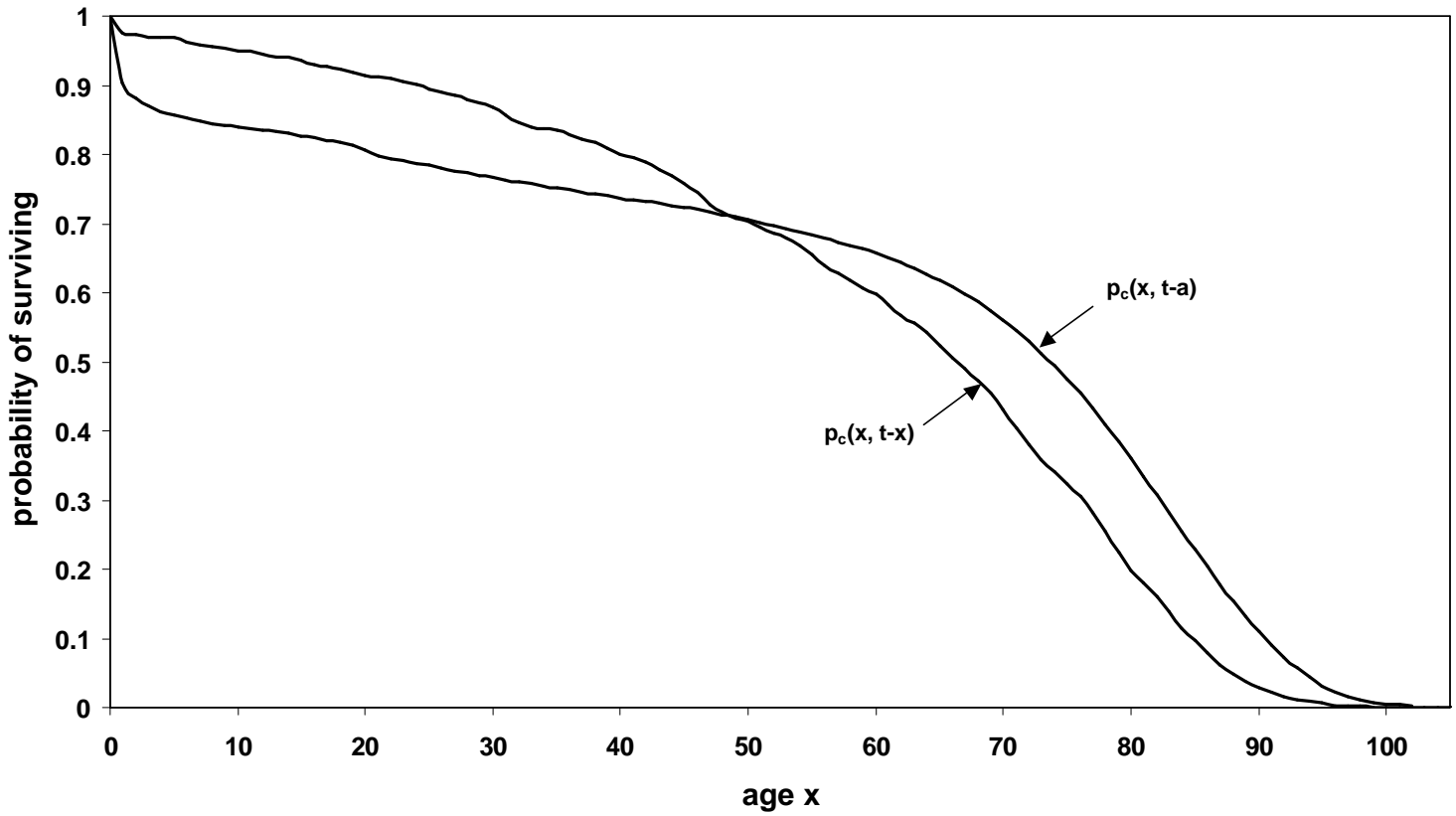


Figure 5: $p_c(x, t-x)$ and $p_c(x, t-a)$. Sweden, Females, $t=1948$, $a=50$.



Note: The curve $p_c(x, t-a)$ in this Figure corresponds to the probability of surviving from birth to age x for the cohort aged 50 in 1948, i.e., born in 1898.

Figure 6: Decomposition of population growth and CAL growth. Sweden, Males, 1858-1999. 1858=1.

