Exploring Imprecise Probabilities

Stephen Broomell and David V. Budescu
Department of Psychology, University of Illinois, Champaign-Urbana, IL 61820

Introduction

Many real world decision problems (such as abrupt climate change, stock market predictions, and insurance) involve imprecise probabilities, and it is important to study human behavior in the presence of such vague risks. Through the use of an interactive decision aid that uses prior distributions provided by the DM to calculate expected payoffs, we explore the nature and use of these distributions.

Research Questions

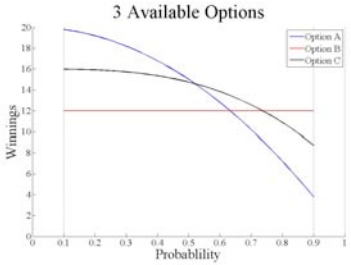
(A) How do DMs specify probability distributions over a certain range of values?
Ignorance Prior Hypothesis – The tendency of people to place equal weight on each partition given in the decision task. (Fox & Rottenstreich 2003)

(B) How do these prior distributions affect choice?
Vagueness Aversion Hypothesis – People prefer making decisions with precise probabilities, even at a premium. (Ellsberg 1961; Camerer & Weber 1992)

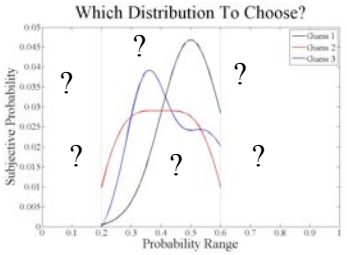
Method

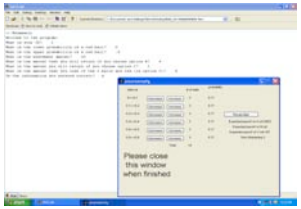
Participants in a study dealing with decision making in the presence of vague probability were given access to a decision aid that allowed them to interactively explore possible outcomes (Budescu et al., 2006). The subjects knew the range of relevant probabilities, $[P_L, P_U]$, and several payoff parameters defining 3 options, a risky option, a safe (riskless) option, and a reduced risk option. They are described in Figure 1 for the case $P_L = 0.1$, $P_U = 0.9$.

Option A – Risky
Option B – Safe
Option C – Reduc



Subjects input **up to three** "hypothetical distributions" within the allowed range to receive information about the expected payoff of three options.



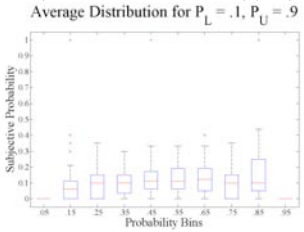


Results

We analyzed 1,723 distributions generated by 71 subjects. Subjects generated an average of 2.18 (out of 3) distributions. Fifty four percent of them assign non-zero probability to each category, and 11% placed all the probability mass on a single category.

How do DMs specify hypothetical distributions?

- The mean of the distributions follow the imprecise probability midrange (Mean difference = .022 ; $t(295) = .75$; $p > 0.05$).
- Standard deviations of the distributions are proportional to the range (Mean log(ratio) = −1.55), but biased downward ($t(295) = -6.06$; $p < 0.05$).



- Most distributions are not symmetric.

	Negative Skew	Single Category	Symmetric	Positive Skew
Percent	41	11	12	36

- Cases where subjects generated 2, or more, distributions per decision commonly comprised of both positively and negatively skewed distributions. (N = 573)

	Both	All Positive	All Negative	Other
Percent	42	13	17	28

- At the aggregate level we can not reject the assumption of uniformity by Kolmogorov-Smirnov tests of goodness of fit.

Results

How do these distributions affect choice?

In 89% of the cases the feedback from the decision aid identified one option as best (having the highest EV) in a majority of the cases. Do the subjects' choices follow the advice in these cases?

	Advice			
Choice	a	b	c	Total
a	**181**	9	24	214
b	**47**	**32**	**56**	135
c	76	12	**74**	162
Total	304	53	154	511

Subjects tend to follow the prescription of the decision aid (56% agreement, $K = .30$). However, deviations from the prescription tend toward the less risky option (p-value < .001 Stuart-Maxwell statistic).

Discussion

Support for the Ignorance Prior Hypothesis

The distributions generated are close to uniformity in terms of their parameters, but have smaller variance ("edge aversion")

Support for the Vagueness Aversion Hypothesis

Subjects' choices are consistent with the EV advice in most cases, but subjects chose the "safe" option more often than prescribed by expected value

References

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