Chapter Four

KNOWLEDGE-ENHANCED LANCHESTER

The Lanchester attrition processes are perhaps the best-known models of combat. They were developed by F. W. Lanchester just prior to U.S. involvement in World War I and were first published in his now famous book, *Aircraft in Warfare: The Dawn of the Fourth Arm*.[1] Lanchester distinguishes two forms of warfare: ancient and modern. The former is characterized by his linear law and the latter by his square law. In this chapter, we discuss both and present a third, information-enhanced variant which we refer to as the Lanchester *mixed law*. This third law is an attempt to assess the implications of information superiority for ground combat by referring to an established body of work other than game theory. Unit effectiveness, force survivability, and force size as well as force structure may change as a result of better information. The Lanchester laws provide a useful set of models to examine these changes.

Lanchester hypothesized basic "laws" that describe combat in "ancient times" and under "modern conditions." Taylor summarized Lanchester's laws as follows:

> In "ancient times," warfare was essentially a sequence of one-on-one duels so that the casualty-exchange ratio during any period of battle did not depend on the combatants' force levels. But under "modern conditions," however, the firepower of weapons widely separated in firing location can be concentrated on surviving targets so that each side's casualty rate is proportional to the number of

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[1] Lanchester [1916] (1956).

> enemy firers and the casualty-exchange ratio consequently depends inversely on the force ratio.[2]

The fundamental difference between ancient and modern warfare, then, is that in modern warfare there is a decided advantage to be gained from concentrating forces, whereas in ancient warfare there is no such advantage.[3] In ancient warfare, for example, if 1,000 combatants were arrayed against 500 enemy combatants, the number of possible engagements would be proportional to the product of the two force sizes, and each engagement would be identical. Thus there is no particular advantage to the larger committed force. In modern warfare, however, concentrating the 1,000 against the enemy's 500 provides a decided advantage in that each combatant is capable of being involved in an engagement, providing essentially a two-to-one advantage to the larger committed force. Ancient warfare conforms to what is referred to as the Lanchester linear law, and modern warfare conforms to what is referred to as the Lanchester square law.[4]

Applications of Lanchester processes include both rigorous mathematical development, which assumes conflict is continuous, and simulation, which treats conflict as a series of discrete events. The mathematical approach has emphasized the use of continuous functions, particularly differential equations, though some work has been done with difference equations. The simulation approach is usually tied to discrete-time processes. The major theater-level warfare

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[2] Taylor (1983), vol. 1.

[3] In reviewing this document, RAND colleague Paul Davis observed: "Most of the usual discussion of Lanchester equations is simply wrong. Except perhaps with circular logic or subtle footnotes, distinctions between the equations do not correspond simply to ancient versus modern warfare, to aimed versus unaimed fire, or even to the nominal ability of the sides to concentrate force—much less to a cartoon of how the combatants are lined up at an instant. The form of the aggregate attrition equations depends on a complex averaging over minibattles separated by minimaneuvers according to some set of tactics. Even qualitative features of the resulting average depend on details. For example, we might expect many so-called modern warfare battles to look in the aggregate more like linear-law battles because attrition rates will depend on either shooter-level or small-maneuver-unit search processes dependent on the density of targets. We should also expect profound asymmetries, as discussed later in the [report], and not merely because of modern information systems."

[4] There has been considerable debate on just how well these laws represent ancient and modern combat. Bracken has shown, for example, that the linear law more accurately models the Ardennes battle, a battle considered to be "modern."

simulations all have fixed time steps. Even if they draw upon data from more detailed models, say at the division level or brigade level, the underlying models typically involve fixed time steps.

In this work we take both approaches. Our initial insights were formed on the basis of adding information to time-stepped simulations. We constructed a Lanchester square simulation and a Lanchester linear simulation. In the former case, each shooter has an opportunity to detect and engage targets during each period, and this opportunity stays the same over time. In the latter case, each shooter has an opportunity to detect and engage targets during each period, but this opportunity is dependent on the number of targets.

We observed that adding possible encounters to the Lanchester square process, by hypothesizing more information, had a striking effect, as did reducing possible encounters by hypothesizing less information. We were able to demonstrate that if one side had considerable information and the other side very little, then the former followed the linear law while the latter followed the square law. We then turned to a mathematical investigation in which we developed a theory for the complete range of cases.

This chapter presents the mathematical theory and conceptual discussion first. It results in a table of all of the Lanchester laws (Table 4.1), including a number of different mixed cases. Next, the chapter presents simulation results based on varying the main parameters. These results show transitions from linear to mixed and from square to mixed. We then investigate the mixed cases in more detail.

The overall goal of this chapter is to demonstrate how adding centrally supplied information changes the dynamics of a battle and results in different tradeoffs between quality and quantity of forces.

LANCHESTER SQUARE LAW

The effect of concentrating the force is reflected by the fact that the casualty rate is assumed to depend only on the size of the shooting force. This is due to the firepower delivery available with modern weapons. If we let R and B represent the initial size of the Red and Blue forces (number of units) respectively, and N and M $(0 \leq N, M \leq 1)$ be the effectiveness of each Red and Blue unit respectively, the rate at which each of the two forces is depleted is given by the relations

$$\frac{dr(t)}{dt} = -Mb(t)$$
$$\frac{db(t)}{dt} = -Nr(t),$$

where *r(t)* and *b(t)* represent the Red and Blue force sizes at time *t* and $r(0) = R$ and $b(0) = B$. The attrition to each side depends on the effectiveness of the shooting side's units and the remaining size of the shooting force. Dividing the two equations, we get

$$\frac{\frac{dr(t)}{dt}}{\frac{db(t)}{dt}} = \frac{dr(t)}{db(t)} = \frac{Mb(t)}{Nr(t)}.$$

Rearranging, we get

$$b(t)db(t) = \frac{N}{M} r(t)dr(t).$$

Integrating from time 0 to time *t*, we get

$$b(t)^2 - B^2 = \frac{N}{M}(r(t)^2 - R^2).$$

This formulation allows us to examine the requirements for Blue (or Red) to win. For Blue to win, we must have that at time *T*, $r(t) = 0$ and $b(t) > 0$. Rewriting the above equation with $t = T$ and solving for $b(T)^2$, we get

$$b(T)^2 = B^2 - \frac{N}{M} R^2 > 0.$$

Solving the inequality, we get

$$\frac{M}{N} > \left(\frac{R}{B}\right)^2 .$$

For Blue to win, the relative effectiveness of the two forces must exceed the square of the initial force ratio.

One type of battle described by a Lanchester square law occurs when both sides can employ constant fractions of their forces and have target-rich environments. The size of the force the friendly commander commits to the battle determines the amount of enemy attrition attained rather than the size of the enemy force committed.

FRACTIONAL LOSS EXCHANGE RATE

We make use of the force loss exchange ratio (FLER) later in Chapter Six. It is useful to introduce it here in that it can be defined in Lanchester equation terms. The FLER is simply the ratio of Red fractional losses to Blue fractional losses, or

$$\text{FLER } = \frac{\dfrac{dr(t)}{r(t)dt}}{\dfrac{db(t)}{b(t)dt}} = \frac{dr(t)}{db(t)}\frac{b(t)}{r(t)} .$$

We can use the FLER, then, to determine who is winning. If the FLER = 1, then $dr(t)B = db(t)R$, and the sides can cause attrition to each other but are not able to improve their force ratio: it is a stalemate. If FLER > 1, Blue wins, and if FLER < 1, Red wins.

Note that for the square law,

$$\text{FLER } = \frac{dr(t)}{db(t)}\frac{b(t)}{r(t)} = \frac{M}{N}\left(\frac{b(t)}{r(t)}\right)^2 .$$

At time $t = 0$, we have

$$\text{FLER} = \frac{M}{N}\left(\frac{B}{R}\right)^2 .$$

Therefore, stalemate occurs when

$$\left(\frac{R}{B}\right)^2 = \frac{M}{N} .$$

To compensate for an adverse force ratio, *R*/*B*, Blue must achieve a unit effectiveness advantage equal to the square of the force ratio (M/N).

LANCHESTER LINEAR LAW

The linear law reflects the inability, or more accurately the futility, of either side to mass its forces effectively. Lanchester referred to this as a characteristic of ancient warfare:

> In olden times, when weapon directly answered weapon, the act of defence was positive and direct, the blow of sword or battleaxe was parried by sword and shield. . . . Under [these] conditions, it was not possible by any strategic plan or tactical manoeuver to bring other than equal numbers of men into the actual fighting line; one man would ordinarily find himself opposed to one man.[5]

Under these conditions, attrition depends solely upon the effectiveness of the individual combatant.

Another, more modern interpretation of the linear law is that it represents area fires. That is, we assume that the attacker knows the enemy is located within an area, but that he is unable to target each combatant individually. The best he can do is launch indirect fires into the area. In this case, the effectiveness of the attacker depends not only on the effectiveness of the weapon, but also on the number of attackers (number of weapons), the effectiveness of each attacker,

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[5] Lanchester [1916] (1956).

and the number of targets in the area fired upon. Both of these cases result in a linear law.

As above, we let M and N be the effectiveness of each combatant, with $r(0) = R$ and $b(0) = B$, the original size of the Red and Blue forces. The number of firing opportunities for Blue is proportional to $b(t)r(t)$, and the number of Red firing opportunities is proportional to $r(t)b(t)$:[6]

$$\frac{dr(t)}{dt} = -M[b(t)r(t)]$$
$$\frac{db(t)}{dt} = -N[r(t)b(t)]\,.$$

The effectiveness scores refer to the effectiveness of the individual combatant. Dividing the two equations as above, we get

$$\frac{\frac{dr(t)}{dt}}{\frac{db(t)}{dt}} = \frac{dr(t)}{db(t)} = \frac{M}{N}.$$

Rearranging, we get

$$db(t) = \frac{N}{M}dr(t).$$

Integrating from time 0 to time t, we get

$$b(t) - B = \frac{N}{M}(r(t) - R).$$

For Blue to win, we again must have that at time T, $r(T) = 0$ and $b(T) > 0$. Rewriting the above equation with $t = T$ and solving for $b(T)$, we get

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[6] There are two ways this can come about: (1) If Blue units are searching for Red units and, when they find them, they can shoot them under target-rich conditions, then the encounter rate is proportional to the density of Red units and the kills per unit time is proportional to $r(t)b(t)$. (2) If Blue is merely firing blind, the fraction of the time it hits something is proportional to the density of Red units.

$$b(T) = B - \frac{N}{M} R > 0 .$$

Solving the inequality, we get

$$\frac{M}{N} > \left(\frac{R}{B} \right).$$

In this case, to win, the effectiveness ratio need only exceed the initial force ratio. In the linear case, the impact of the force size on combat outcome is significantly less than in the square case.

The area-fires interpretation results in the following attrition rates:

$$\frac{dr(t)}{dt} = -[b(t)M]r(t)$$
$$\frac{db(t)}{dt} = -[r(t)N]b(t) ,$$

reflecting the effects of force size, weapon effectiveness, and targets available. Here $[b(t)M]$ can be interpreted as the firing effectiveness of Blue and $[r(t)N]$ can be interpreted as the firing effectiveness of Red. Dividing the two equations as above, we get exactly the same results as above.

THE LANCHESTER MIXED LAW

We now consider adapting the Lanchester laws to account for knowledge. One approach is to consider knowledge to be a subcomponent of the unit's effectiveness score, M or N, so that $M = P(d)P(k|d)$, where $P(d)$ is the probability that a target will be detected (knowledge) and k is the effectiveness of the weapon system selected to engage the target. In this construct, we take $P(d)$ to be a measure of *local knowledge*, that is, knowledge of the enemy obtained from sources organic to the unit.

The problem is in selecting the appropriate Lanchester law. If we select the square law, we can examine the effect of an increase in Blue knowledge on Blue's ability to win. Rearranging the winning condition equation, for Blue to win, we must have that

$$\frac{B}{R} > \left(\frac{N}{M}\right)^{0.5} .$$

Let us assume that M is doubled due to an increase in Blue's knowledge. If all other variables remain the same, this has the effect of increasing the force ratio by a factor of $\sqrt{2}$ and thus enhancing the Blue win. Performing the same calculation in the linear law increases the force ratio by a factor of 2, a considerable difference. However, it is not clear which of these more closely models the effects of knowledge on combat outcomes.

As an alternative, suppose we link the *maximum possible number of encounters a unit may have in a combat cycle* to the information available to the unit from external as well as organic sources. If we let $c_r \leq b(t)$ and $c_b \leq r(t)$ represent the total number of encounters each Red unit can have with Blue units and Blue with Red respectively, then clearly $c_r = f[K_R, b(t)]$ and $c_b = g[K_B, r(t)]$. The quantities K_B and K_R represent the knowledge available to each side from external sources such as imagery from national assets, information from higher or adjacent commands, etc. These quantities are developed in Chapter Two. The cases $c_b = 1$ and $c_r = 1$ imply no external knowledge, and the sides rely on their organic sensors and sources to engage the enemy. The result is a single engagement per combat cycle. The number of encounters depends upon the information available to the unit (organic and external) and the size of the opposing force. This means that the enemy attrition rate is now dependent upon the number of units attacking, the effectiveness of the attacking unit, and the maximum number of encounters (number of targets possibly presented). This leads to the attrition that looks very much like the linear area-fires case:

$$\frac{dr(t)}{dt} = -[b(t)M]c_b$$

$$\frac{db(t)}{dt} = -[r(t)N]c_r .$$

Dividing the two equations as before, we get

$$\frac{\frac{dr(t)}{dt}}{\frac{db(t)}{dt}} = \frac{dr(t)}{db(t)} = \frac{Mb(t)c_b}{Nr(t)c_r}.$$

Rearranging, we get

$$b(t)db(t) = \frac{Nc_r}{Mc_b} r(t)dr(t).$$

Integrating from time 0 to time t, we get

$$b(t)^2 - B^2 = \frac{Nc_r}{Mc_b}[r(t)^2 - R^2].$$

For Blue to win, we again must have that at time T, $r(T) = 0$ and $b(T) > 0$. Rewriting the above equation with $t = T$ and solving for $b(T)$, we get

$$b(T)^2 = B^2 - \frac{Nc_r}{Mc_b} R^2 > 0.$$

Solving the inequality, we get

$$\frac{Mc_b}{Nc_r} > \left(\frac{R}{B}\right)^2.$$

Although this is clearly a square law representation in this form, we can make some interesting observations by examining the nature of the Red and Blue encounters. First we observe that information has a greater effect than the effectiveness scores, in that the encounter values are not fractions, but rather numbers of units.

More interesting, however, are the results obtained by examining some extreme values for c_r and c_b. Table 4.1 summarizes the results obtained through this process. We also include an illustrative interpretation for each of the cases.

Table 4.1

Lanchester Information Laws

c_r	c_b	Condition for a Blue win	Law	Illustrative Interpretation
1	1	$\frac{M}{N} > \left(\frac{R}{B}\right)^2$	Square	Both sides rely solely on organic collection assets. No information is available from higher headquarters.
1	g	$\frac{Mg}{N} > \left(\frac{R}{B}\right)^2$	Square	In these two cases, one side has only organic collection assets and the other receives some information from higher headquarters ($0 < g < R$, and $0 < h < B$).
h	1	$\frac{M}{Nh} > \left(\frac{R}{B}\right)^2$	Square	
B	R	$\frac{M}{N} > \frac{R}{B}$	Linear	Both sides have complete information from higher headquarters as well as information from their organic collection assets. This is the best either can do with respect to information.
B	g	$\frac{Mg}{N} > \frac{R^2}{B}$ [a]	Mixed	In these two cases, one side has knowledge of its opponent's entire force whereas the other side has only some knowledge of its opponent's force ($0 < g < R$, and $0 < h < B$). Both sides have information available from their own organic collection assets.
h	R	$\frac{M}{Nh} > \frac{R}{B^2}$	Mixed	
B	1	$\frac{M}{N} > \frac{R^2}{B}$	Mixed	These last two cases illustrate extreme mismatches. One side receives information from higher headquarters concerning the entire enemy force whereas the other only has information from organic collection assets.
1	R	$\frac{M}{N} > \frac{R}{B^2}$	Mixed	

[a] This result was also obtained by Smith (1997) by assuming that Red attrition is proportional to the size of the Blue force, b, whereas Blue attrition is proportional to the size of both the Blue force and the Red force, mn. Thus, we get the following Lanchester differential equations: $\frac{db(t)}{dt} = -Mr(t)b(t)$ and $\frac{dr(t)}{dt} = -Nb(t)$. Following the usual derivation, we have that for a Blue win, we must have that $\frac{2M}{N} > \frac{R^2}{M}$. Reversing the argument produces the next case, namely $\frac{M}{2N} > \frac{R}{M^2}$.

Note that if we let $c_r = f[K_R, B] = K_R B$ and $c_b = g[K_B, R] = K_B R$, i.e., the number of encounters is directly proportional to the side's external knowledge, we get

$$\frac{\frac{dr(t)}{dt}}{\frac{db(t)}{dt}} = \frac{dr(t)}{db(t)} = \frac{Mb(t)K_B R}{Nr(t)K_R B} = \frac{Mb(t)RK_B}{Nr(t)BK_R}.$$

This is a linear model similar to the fourth case in Table 4.1. The requirement for a Blue victory then is

$$\frac{MK_B}{NK_R} > \frac{R}{B}.$$

The effect of knowledge plays a much greater role in that the force size has only linear effects. In this case, therefore, we have answered the question concerning the applicable Lanchester law.

We can also examine the effects of information dominance in this construct. Recall from Chapter Two that if, for example, Blue dominates Red, then $\delta_B \le K_B \le 1$ and $K_B > K_R$, where $0 \le \delta_B \le 1$ is the minimum knowledge required for Blue information dominance. If, at one extreme, we have that $K_B = 1.0$ and $K_R = \gamma$, where $\gamma < 1$, the requirement for a Blue victory becomes

$$\frac{M}{N\gamma} > \frac{R}{B}.$$

Even for large values of γ, it is clear that information dominance allows for a Blue victory with a less favorable (to Blue) force ratio. If both K_B and K_R are too close to γ, the effects of information dominance on winning are negligible and we have case 4 in Table 4.1.

SIMULATING THE MIXED LAW

To illustrate the effects of information on combat outcomes using the Lanchester models, we resort to a simple simulation of a stochastic process. Table 4.1 is the focus of the simulations in that we attempt to simulate conditions similar to those presented in the table. A brief description of the process is presented below.

Resources and Effectiveness Parameters

We first define a stochastic process, $\{F(t),\ t \in T\}$, where $F(t)$ is the force ratio of Blue to Red forces at the end of time t and T is the maximum number of time steps. Therefore, we have that the starting force ratio is $F(0) = B/R$ and, in general, $F(t) = b(t)/r(t)$, where both $b(t)$ and $r(t)$ are random variables.

Each Red and Blue unit is characterized by three effectiveness parameters: the number of targets (opposing units) each is able to encounter in a given time period, the probability that an encountered unit is detected, and the probability that a detected unit is destroyed. These last two parameters are included in the effectiveness scores, M and N above. In the stochastic process notation, we set the following:

- **Encounters.** $c_b \le r(t)$ and $c_r \le b(t)$ represent the maximum number of units a Blue and Red force can encounter during any time period respectively. As discussed above, this represents the level of knowledge available to the fighting units from sources external to the unit (usually higher headquarters) and the size of the opposing force. Although these parameters are fixed for any given simulation, a dependence on t exists because of the upper bound conditions.
- **Detections.** For these examples, we assume that local knowledge is the probability that a unit can detect a target, or $P_b(d) = d_b$ and $P_r(d) = d_r$. We further assume that these quantities are time invariant, i.e., they are independent of the relative force sizes. This represents the knowledge available to each unit based on its organic ability to detect enemy units.
- **Attrition.** $P_b(k|d) = e_b$ and $P_r(k|d) = e_r$ are the time-invariant probabilities that Blue and Red forces are able to destroy an opponent given that a target is detected.

Therefore, force effectiveness is simply the product of these last two quantities so that $N = d_r e_r$ and $M = d_b e_b$. Recall also that $c_b = f(K_B, r(t))$ and that $c_r = g(K_R, b(t))$. That is, the number of encounters allowed depends upon the encountering side's knowledge from external sources and the size of the opposing force.

Process

The process modeled is essentially an attrition process that is modified by knowledge manifested by a restricted target set and probabilities of detection. The key to the process is determining the outcomes of the several engagements at each time period. The likelihood that a Blue/Red unit engages an opponent is based on the relative residual sizes of the forces and the values of c_b and c_r.

At each time step, the ratio $Q(t) = b(t)/[b(t) + r(t)]$ is calculated and compared to a random number, ρ, drawn from a uniform distribution defined on the interval [0, 1]. If $\rho \leq Q(t)$, a Blue unit has the opportunity to engage a Red unit, i.e., the Blue unit has encountered the Red target. It now remains to apply the detection probability to determine if the encountered target will be engaged. If $\rho > Q(t)$, then Red has the opportunity to engage Blue in the same way.

This continues at each time step until either the maximum allowable number of encounters on both sides have been examined or until one side or the other has no surviving units.

RESULTS FOR THE MIXED LAW

In the results that follow, we have selected force sizes and detection probabilities such that the starting force ratios are always 1.0 for varying values of c_b and c_r. In the first case, we start from the pure linear law conditions ($c_b = R$ and $c_r = B$) and then we proceed to degrade c_r, thus illustrating rows 4, 5, and 6 in Table 4.1. In the second case, we do the same for the square law case ($c_b = c_r = 1$) and cause c_r to increase, thus illustrating rows 1, 2, and 3 in Table 4.1. Finally, we treat the last two (rows 7 and 8) separately. In all cases, we treat K_B and K_R implicitly in that the functional relationships $c_b = f(K_B, r(t))$ and $c_r = f(K_R, b(t))$ are unknown.

Linear to Mixed Cases

For all the cases in this set, we assume that $e_b = e_r = 0.5$, $d_b = 0.02$, and $d_r = 0.01$. Thus, although the probability of kill given a detection is the same for both sides, Blue is twice as likely to detect a target as Red, or relative knowledge is $\Gamma = .02/.01 = 2.0$.

We begin with the pure linear case (the fourth row in Table 4.1). We first assume that both sides have access to information from sensors with a global view of the battlespace. This might be from unmanned aerial vehicles (UAVs), JSTARS satellite imagery (and the Red equivalent), or perhaps a combination of all. The implications are that this type of coverage provides external information about the location of the entire enemy force, or $c_b = R = 200$ and $c_r = B = 100$. We can easily verify that under these conditions, the outcome should result in a draw, i.e., we should have that $Nr(t) = Mb(t)$. Indeed, at time $t = 0$ we get

$$Nr(0) = d_r e_r c_b = (.01)(0.5)(200) = Mb(0) = d_b e_b c_r = (.02)(0.5)(100) = 1.0\,.$$

In addition, the beginning force ratio is $F(0) = 100/200 = 0.5$.

A total of 5 cycles ($T = 5$) were evaluated 100 times. The resulting average force sizes at the end of cycle 5 are $r(5) = 27.13$, and $b(5) = 14.17$. Thus we get $Mb(5) = 0.142$, and $Nr(5) = 0.136$. So we conclude that the equality condition holds approximately for this case. The final average force ratio is $F(5) = 14.17/27.13 = 0.522$.

We now examine the effect on the ending ($T = 5$) force ratio when the information available to Red from external sources deteriorates, that is, we assume that its nonorganic sensor coverage deteriorates. We also assume that both sides' organic sensors are the same as before. That is, their ability to detect and kill a target remains constant. In addition, we assume that the external information available to Red remains fixed for all 5 cycles, i.e., c_r remains fixed. Whenever $c_r \geq b(t)$, we allow the number of encounters per cycle to increase to $c_r / b(t)$ Table 4.2 lists the results of 5 cases in which we allow the number of Red encounters to decrease from $c_r = B = 100$ to $c_r = 20$. The number of Blue encounters remains fixed at $c_b = R = 200$, i.e., Blue continues to enjoy global coverage of the AO.

The data illustrate how decreasing external information affects the ending force ratio. When the Red commander receives information on the location of between 40 and 50 Blue units, the advantage swings dramatically in favor of Blue. Clearly, Blue achieves information superiority between these points, and at $c_r = 20$ it might be argued that Blue achieved information dominance.

Table 4.2

Linear to Mixed Cases

c_r	$b(5)$	$r(5)$	$F(5) = b(5)/r(5)$
100	14.17	27.13	0.522
80	14.40	27.88	0.517
60	17.76	22.22	0.799
40	31.18	12.87	2.423
20	68.21	3.92	17.401

Figure 4.1 compares the ending force sizes as fractions of the initial force sizes. The c_r values are plotted along the horizontal axis, and the fraction of the force remaining at the end of the 5 combat cycles is plotted along the vertical axis. The figure illustrates the deteriorating effect of reduced external knowledge on Red's survivability. Beyond 80 encounters per cycle, the gap between Blue and Red survivability widens rapidly, as illustrated by the sharply rising Blue curve and the rapidly declining Red curve.

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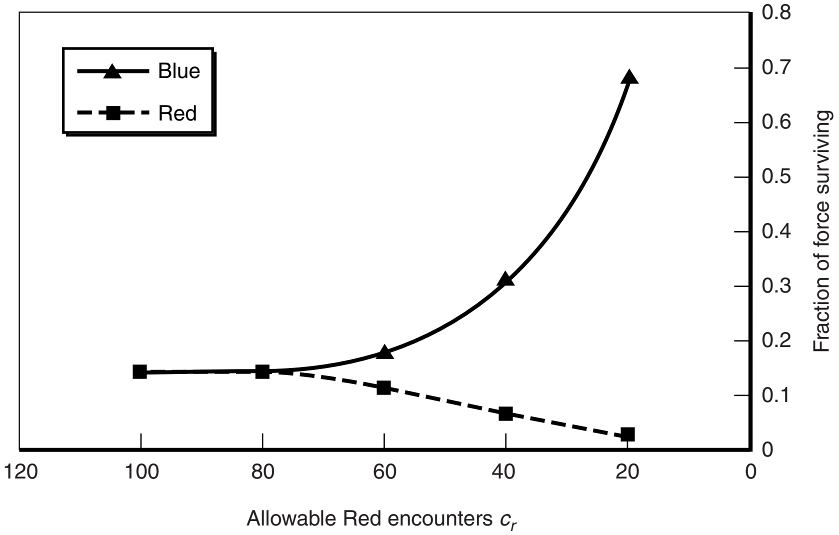

Figure 4.1—Ending Force Levels (Linear to Mixed)

Square to Mixed Cases

As with the linear to mixed cases, we assume that $e_b = e_r = 0.5$. However, we change the relative knowledge by increasing the Blue detection probability to $d_b = 0.08$ and the Red detection probability to $d_r = 0.02$. Now, relative knowledge is $\Gamma = .08/.02 = 4.0$. As in the previous case, Blue has local information superiority. However, this time his advantage has doubled. We begin with the pure square case (row 1 in Table 4.1). As in the previous case, if we assume that both Red and Blue have information on the location of all enemy units in the AO so that $c_b = R = 200$ and $c_r = B = 100$, we can verify that this describes a case in which the two sides fight to a draw, or $N[r(t)]^2 = M[b(t)]^2$:

$$N[r(0)]^2 = (.02)(0.5)(200)^2 = M[b(0)]^2 = (.08)(0.5)(100)^2 = 400.0\,.$$

The beginning force ratio is still $F(0) = 100/200 = 0.5$.

We again evaluate a total of 5 cycles ($T = 5$) 100 times for each variant. This time however, we incrementally increase Blue's information from external sources from $c_b = 1$ to $c_b = 10$, at the same time holding Red's external information to $c_r = 1$. For the first case, in which each side receives essentially no information from external sources, the average force sizes at the end of cycle 5 are $r(5) = 180.62$ and $b(5) = 90.27$. Thus we get $M[b(5)]^2 = 324$ and $N[r(5)]^2 = 324$. Therefore, this also leads to a draw. The final average force ratio is $F(5) = 90.27/180.62 = 0.500$.

We now examine the effect on the ending ($T = 5$) force ratio when the information available to Blue from external sources improves. As above, we assume that both sides' ability to detect and kill a target remains constant. We further assume that although Blue's information from external sources increases, Red's remains at the same low level. Table 4.3 lists the results.

As the number of allowable Blue encounters increases from 1 to 10, there is a linear decrease in the Red ending force levels, while the ending Blue force levels remain approximately the same. What is interesting about these cases is the fact that the dramatic decrease in the ending Red force size was caused by only modest increases in the number of allowable Blue encounters. The major effect appears to

Table 4.3

Square to Mixed Cases

c_b	$b(5)$	$r(5)$	$F(5) = b(5)/r(5)$
1	90.27	180.6	0.500
2	90.58	161.5	0.561
3	90.94	141.2	0.644
4	91.18	122.4	0.745
5	91.97	102.1	0.901
6	92.23	82.7	1.116
7	92.74	61.8	1.500
8	92.95	44.7	2.079
9	93.05	26.4	3.529
10	93.44	11.2	8.365

be attributable to Red's inability to "know" the location of more than one enemy target at each combat cycle.

In Figure 4.2 we plot the varying levels of Blue external information along the horizontal axis. The vertical axis is the fraction of Blue and

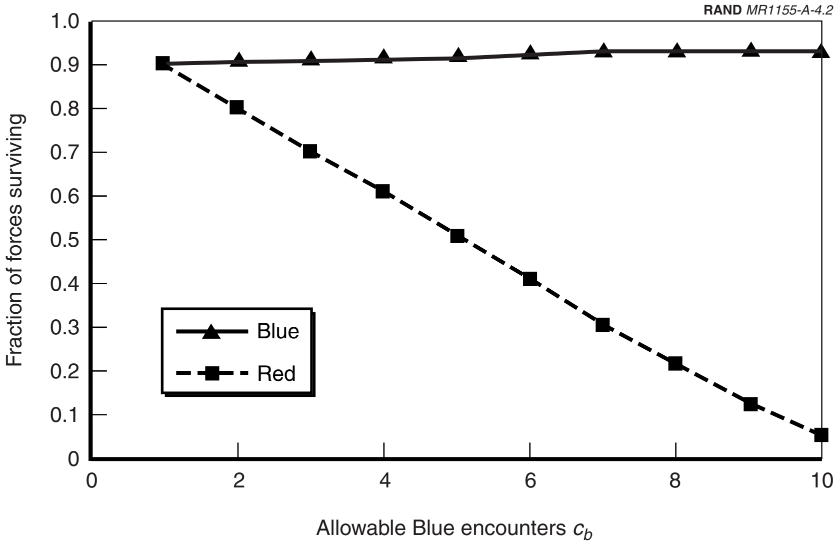

Figure 4.2—Ending Force Levels (Square to Mixed)

Red forces remaining after 5 cycles of combat. When both sides have minimal information from outside sources, they fight to a draw with 90 percent of their forces surviving. But as Blue sensor coverage increases even slightly, the fraction of Red's force surviving drops off precipitously at the rate of 10 percent for each unit of increase in c_b. The fraction of Blue's force surviving, however, remains rather constant at near 90 percent.

Pure Mixed Cases

As the base case set of assumptions for examining the pure mixed cases, we assume a greatly outnumbered Blue force with a starting size of 20 against a starting Red force of 400. We further assume that both sides have equal organic knowledge with detection probabilities of $d_b = d_r = .04$, and that the probabilities of kill given detection are $e_b = e_r = .25$. This implies that relative knowledge is $\Gamma = 1.0$, with a beginning force ratio of $F(0) = b(0)/r(0) = 0.5$. Next, we assume that Blue has extensive sensor coverage so that it knows the location of all 400 enemy units and that Red has little or no knowledge about the location of Blue forces, so that $c_b = 400$ and $c_r = 1$. This reflects the conditions described in row 8 of Table 4.1. These assumptions are similar to those proposed by Deitchman (1962) in describing guerrilla warfare such as the United States encountered in Vietnam. Only here, we credit Blue with being the small, well-hidden force with superior information from local networks.

We can verify as before that these assumptions result in a draw, $Nr(t) = M[b(t)]^2$:

$$Nr(0) = d_r e_r r(0) = (.04)(.25)(400) = 4,$$

and

$$M[b(0)]^2 = d_b e_b [b(0)]^2 = (.04)(.25)(20)^2 = 4.$$

Unlike the previous two cases, we examined two runs of 100 observations each for 5 cycles (time periods) of warfare. Table 4.4 records the surviving force sizes for both runs and the resulting average force ratios. The fact that the final force ratios for the two runs are approximately the same as the initial force ratio of 0.05, at the .05 confidence level, confirms the pure mixed law for this case.

Table 4.4

Pure Mixed Case (Base)

Run	$b(5)$	$r(5)$	$F(5) = b(5)/r(5)$
1	6.41	138.7	0.046
2	6.96	126.9	0.055

Next we examine the robustness of these findings across several cases in which the initial conditions are varied.

MIXED LAW SENSITIVITY

We now investigate the sensitivity of variations in local and external knowledge on ending force ratios for the mixed law.

Sensitivity to Local Knowledge

We first examine the sensitivity of these results to variations in knowledge as expressed by the detection probabilities. We continue to assume that both sides have equal knowledge, but at different levels. First we let the probability of detection be set to $d_b = d_r = .01$, below the 0.04 level in the previous case. Next we set it to $d_b = d_r = .10$, above the previous setting. Results for two runs of 100 observations each for the two variations are summarized in Table 4.5.

Although casualties on both sides increase considerably when their local knowledge increases, we note that for the higher detection

Table 4.5

Pure Mixed Case (Knowledge Variant)

Variant	Run	$b(5)$	$r(5)$	$F(5) = b(5)/r(5)$
$d_b = d_r = .01$	1	15.67	309.7	0.051
	2	15.54	310.7	0.050
$d_b = d_r = .10$	1	2.73	64.87	0.042
	2	3.19	57.25	0.056

probability (.10), the ratios are a bit more volatile. In all cases, however, the ending force ratio can be considered equivalent to the beginning force ratios at the .05 level of confidence.

Sensitivity to Blue External Knowledge

We now examine the effect on the surviving forces and the force ratios at $t = 0$ when the information available to Blue from higher headquarters deteriorates, i.e., as c_b decreases from 400 (the initial size of the Red force) to 300. We assume that the probabilities of both sides to detect and to kill given detection remain as in the base case above. In addition, we let the Red external information remain negligible at one encounter per cycle or $c_r = 1$. Table 4.6 and Figure 4.3 summarize the results of 100 repetitions at each encounter level.

The data illustrate the effect on the ending force ratio of decreasing the information available to Blue from external sources. There is a steady decline in the force ratio at the end of 5 periods and a steady increase in the number of surviving Red forces. In Figure 4.3, the deteriorating effects of reduced information on the location of enemy units is reflected in the number of Blue encounters per cycle. The vertical axis again records the fraction of each force surviving at the end of 5 cycles of combat. The graph depicts the rapid divergence of end strengths as Blue loses visibility over the battlespace.

Table 4.6

Pure Mixed Case (Variations in Blue External Knowledge)

c_b	$b(5)$	$r(5)$	$F(5) = b(5)/r(5)$
400	6.41	138.8	0.046
380	6.62	143.1	0.046
360	6.00	157.4	0.038
340	5.78	175.9	0.033
320	5.61	187.3	0.030
300	4.72	207.6	0.023

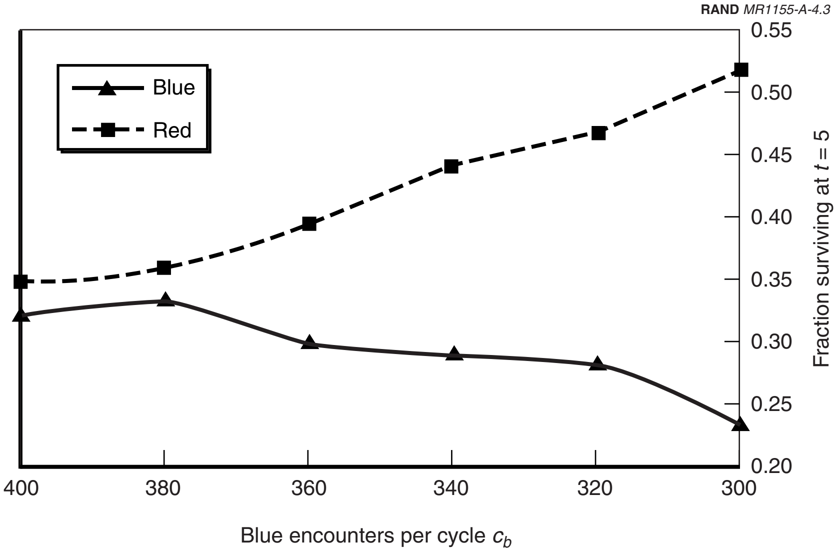

Figure 4.3—Ending Force Levels (Pure Mixed)

Sensitivity to Red External Knowledge

The mixed law as reflected in row 8 of Table 4.1 is dependent on single Red encounters. This models the case in which Red receives no information from higher headquarters and must rely solely on local knowledge to locate targets. In the previous case, we demonstrated a gradual deterioration in force ratio caused by a gradual reduction in the number of Blue encounters. However, this gradual response is not the case when the number of Red encounters is allowed to increase—even by one. That is, we observe dramatic results when the Red commander is able to receive external information that allows for even two encounters per cycle. The results listed in Table 4.7 suggest that it is critical for the Blue commander to prevent Red from receiving external information of any kind. In both cases, Blue encounters are set to 400 ($c_b = 400$).

Variance

It is of interest to observe how the variation in ending force sizes changes as parameters change. From the experimentation conducted, it appears that the outcome variance is quite sensitive to the probability of detection. For the base case with $d_b = d_r = .04$, and the variant $d_b = d_r = .10$, we conducted two sets of runs and calculated the standard deviation of the ending force sizes. The results are in Table 4.8.

Note that the dispersion in the samples is much greater for the larger detection probabilities. For example, for the .04 detection probability case, the standard deviation is approximately 60 percent of the mean Blue force size and 40 percent of the mean Red force size. In contrast, for the .10 detection probability case, the percentages vary from 105 to 136 percent of the mean ending force sizes. As local knowledge increases, it appears that the increase in attrition levels is accompanied by an increased uncertainty in the actual results.

Table 4.7

Pure Mixed Case (Variations in Red External Knowledge)

c_r	$b(5)$	$r(5)$	$F(5) = b(5)/r(5)$
1	6.41	138.7	0.046
2	0.18	239.0	0.001

Table 4.8

Pure Mixed Case (Base Case: Variance Comparisons)

Variant	Run	$b(5)$	$\sigma_{b(5)}$	$r(5)$	$\sigma_{r(5)}$
$d_b = d_r = .04$	1	6.41	3.60	138.7	53.8
	2	6.96	3.82	126.9	55.3
$d_b = d_r = .01$	1	3.79	3.98	52.47	71.45
	2	3.37	4.11	50.10	61.06

Implications

The pure mixed case illustrates how important it is to consider asymmetries in combat. Both Deitchman (1962) and Smith (1997) considered something like the mixed laws we present here, but for different reasons. In general, it challenges the basic assumptions about symmetry that generally accompany discussions about representing combat using the Lanchester models. In this discussion, we have shown how the Lanchester equations can be used to illustrate how the asymmetries associated with access to information can translate to combat advantage for the side possessing information superiority.

Before we leave this subject, it is important to make clear that we are not advocating the use of Lanchester equations to model Information-Age combat. We take the same view as our RAND colleague, Paul Davis, that the proper tools for combat analysis are simulations. Lanchester equations aggregate several combat effects into a single, constant coefficient: effects such as changes in combat posture, decisions to avoid combat, maneuvering, etc. More importantly, they do not account for the fact that these effects may vary over time.[7] In a sense, Lanchester models can be thought of as textbook representations of combat. They are useful for explaining basic principles such as we have done here, but for serious analysis of complex issues, combat simulations should be used. Davis, Blumenthal, and Gaver (1997) make just this point:

> Despite the hundreds of papers written about them, Lanchester equations (as most people understand this term) are largely irrelevant to today's combat modeling by DOD, which uses computer simulation, not simplistic constant-coefficient differential equations such as the Lanchester square law. *Lanchester equations will probably remain quite useful for making particular points in the classroom . . . or theoretical papers*, *but to argue about their more general validity is to chase red herrings. It is the simulations, not the Lanchester equations that should be examined* (p. 226, emphasis added.)

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[7] In some simulations, a Lanchester-like expression is used locally, but the coefficients are adjusted in each time step. In other cases, the local algorithm is not Lanchesterian at all.