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DOCUMENT

SIZE AND COST OF SHELTER DOORS SUBJECTED TO LONG DURATION NUCLEAR BLAST LOADING

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SUMMARY

This report contains formulas and charts useful in making rough estimates of sizes and costs of shelter doors subjected to the pressure loading of large yield nuclear explosions. Nuclear and thermal radiation effects have been neglected.

The material herein presented is a result of the application of ultimate load theory and simplified dynamic analysis. Dimensionless graphs give the thickness to span ratio required to provide a given static resistance. Other graphs are presented which give cost and weight factors dependent on the resistance provided.

Tables are also provided giving the required thickness, weight and cost of the different types of doors for various spans and degrees of hardness. Six types of door construction and two orientations are considered.

No attempt is made to design and cost the door hardware or opening mechanism.

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SIZE AND COST OF SHELTER DOORS SUBJECTED TO LONG DURATION NUCLEAR BLAST LOADING

I. INTRODUCTION

Doors and closures are critical elements of shelters and their design largely determines the overall resistance of protective structures.

In this report formulas and charts are presented which can be used in estimating the required materials, weights and costs of doors spanning 15' to 30', and capable of survival up to 200 psi peak static overpressure generated by MT yields.

Two door orientations are of interest; the horizontal door, Figure 1, and the vertical door, Figure 2.

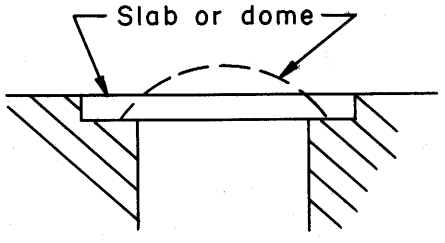


Fig. 1 - Horizontal Door

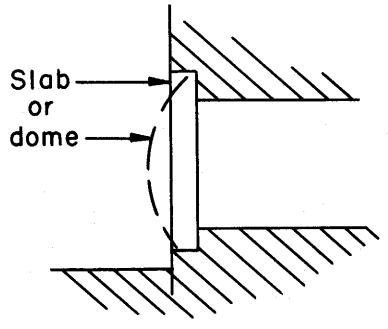


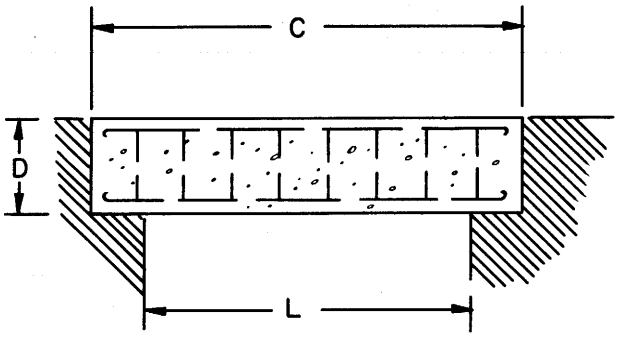
Fig. 2 - Vertical Door

Although a variety of materials and geometry are possible, only 4 configurations of door designs are studied. These are:

1. Reinforced concrete slabs (Fig. 3).
2. Solid Steel slabs (Fig. 4).
3. Steel wide-flange beams (Fig. 5).
4. Steel domes (Fig.6)

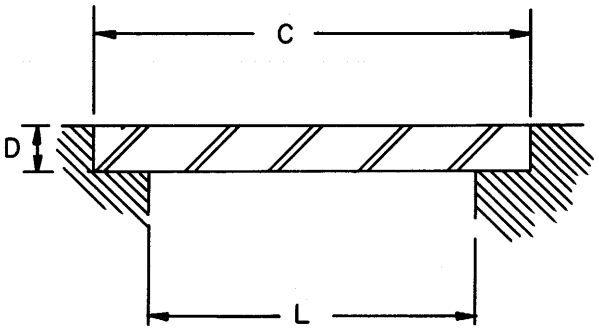
In some doors it may be desirable to limit the dynamic and permanent

deflection to small amounts while in others a larger deformation is permissible. The allowable deflection is a function of response, mechanical operation, fragility of shelter contents, etc. Depending on these variables, slab type doors fall in two design classes, i.e. "hard" doors whose calculated response is limited to the elastic range and "soft" doors whose calculated response is allowed to enter the plastic range. Dome-shaped doors on the other hand are not assumed to exceed this elastic limit.



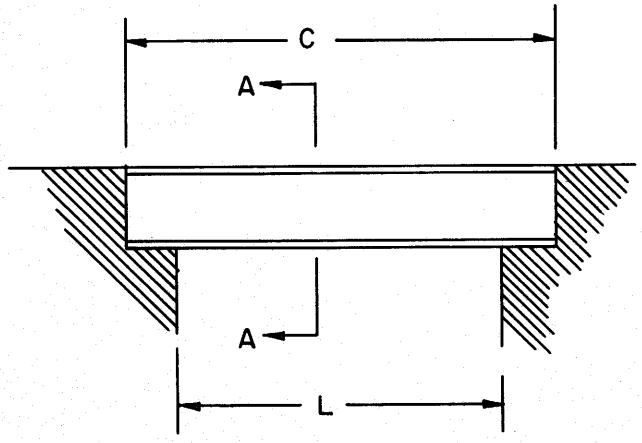
Reinforced concrete

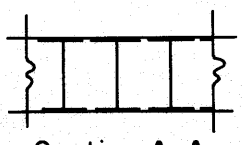
Fig. 3



Solid steel slab

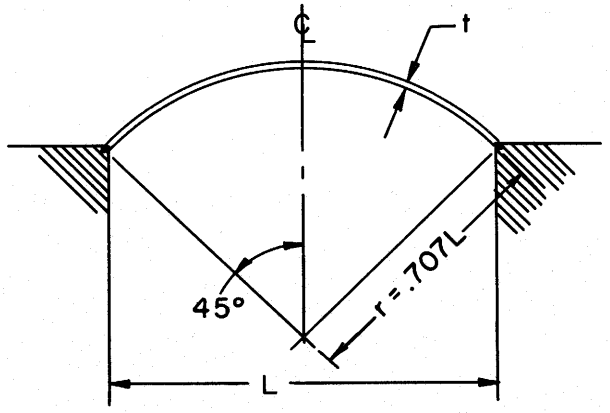
Fig. 4





WF beams

Fig. 5



Steel dome

Fig. 6

Door types

Horizontal orientation

II. OBJECT

The object of this report is to provide ways and means of estimating the structural dimensions and costs of several types of shelter doors for a range of blast loads.

III. ASSUMPTIONS

A. Loading

1. Slabs and Beams

a. Slabs and beams in horizontal position, top surface flush with grade. The load, due to blast, acting on the surface of a horizontal slab door is equal to the static overpressure Δp_1. A typical pressure vs. time diagram is shown in Fig. 7.

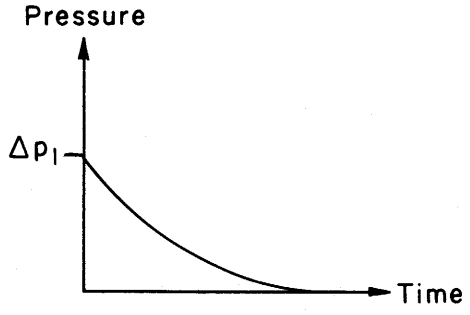


Fig. 7

b. Slabs and beams in vertical position.

The load, due to blast, acting on the surface of beams and slabs in a vertical position facing ground zero is a combination of reflected pressure, static overpressure, Δp_1, and drag pressure, q. The pressure vs. time diagram is shown in Fig. 8.

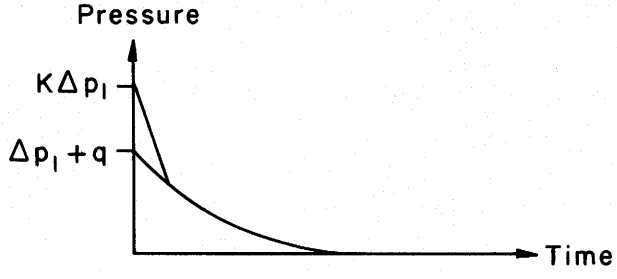


Fig. 8

2. Domes

a. Domes in horizontal position.

Domes in a horizontal position present an obstacle to the pressure wave due to blast and are subjected to reflection and drag effects. The loading is complex and there is little basic data, even in the low pressure regions. In this report it is assumed that the static load shown in Figure 9 will cause maximum stress in the dome shell. The design pressure, P_d, is an average of the reflected pressure and drag pressure acting on the loaded portions of the dome. A method of estimating P_d is given in Appendix II.

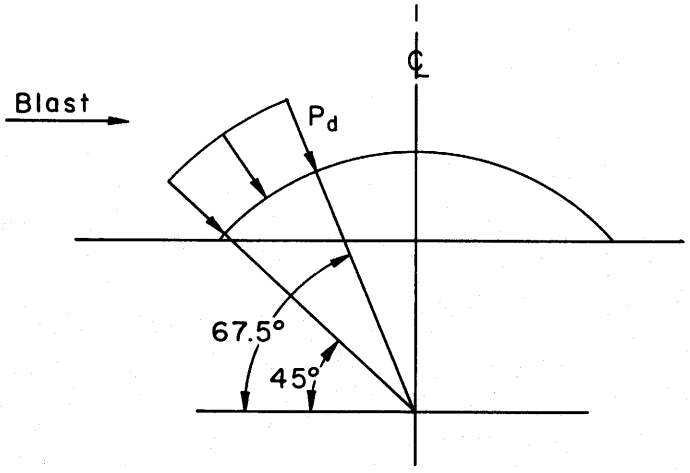


Fig. 9

A tie ring is required around the base of the dome to carry the horizontal component, H, of the reactions, due to the load acting on the dome surface, Figure 10. The vertical components are carried by the bearings. The pressure load, P_n, causing maximum tie ring stress is assumed to be the net load resulting from the uniformly applied

overpressure Δp_1 and the drag load q which has a sinusoidal variation, Figure 11.

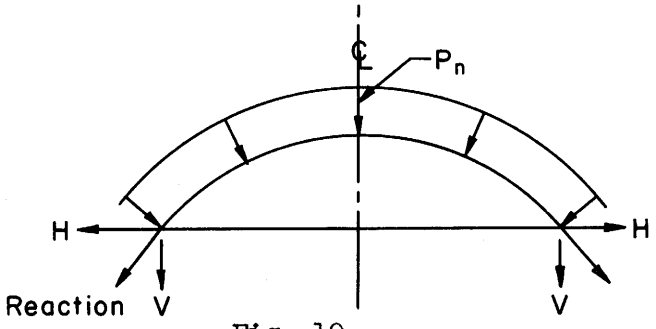


Fig. 10

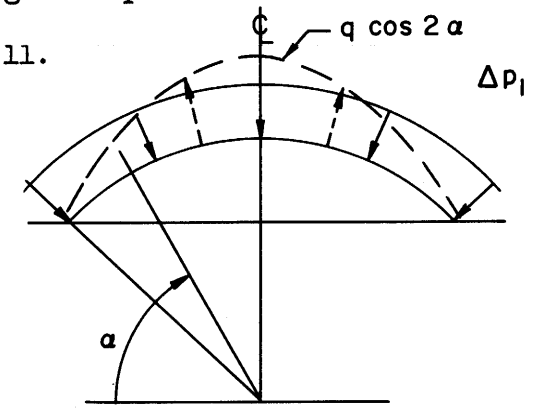


Fig. 11

The method used in estimating Pn is given in Appendix III-D.

b. Domes in a vertical position.

The dome shaped doors in a vertical position facing ground zero are assumed to be subjected to the same loading as the vertical slabs, viz., a combination of reflected pressure, static overpressure and drag pressure.

B. Structural Design

1. Slabs and Beams

All slabs, whether one or two-way, are designed in accordance with limit-load method. The dead load is neglected.

In the course of the design it was found that the required thickness of reinforced concrete slabs is governed by diagonal tension above a certain limiting value of static resistance. This limit is 350 psi for one-way slabs and 456 psi for two-way slabs. Web reinforcing is provided to carry all computed shear stress. Reinforced concrete slabs in which flexure governs are designed with 1.5% tensile reinforcement. Reinforced concrete slabs in which shear governs are provided with a reduced percentage so that in spite of greater depth,

yielding of the tensile reinforcement will occur at a load equal to the computed shear resistance.

The bearing surfaces on which the doors are supported must be large enough to prevent undue settlement. It is assumed that bearings are of reinforced concrete and that the reactions are uniformly distributed. Then the total length of the door, c, Figure 2, which includes the bearing surface is obtained by multiplying the door span, L, by the factor $\frac{f_b}{fb - R}$, where f_b is the assumed maximum allowable bearing stress for concrete and R is the static resistance of the door.

Formulas, based on the above assumptions, for door thickness, weight and cost are given in Appendix II.

2. Domes

Steel domes are designed to meet either of two criteria which may govern:

(1) The shell thickness is determined by the maximum allowable unit stress which is assumed to be 50,000 psi, or,

(2) The shell thickness to radius (or span) ratio is determined by the critical buckling stress and the safety factor (assumed as 1.5).

Criterion (1) governs the design of domes for the higher pressure region. Formulas for the required thickness, based on the above criteria, and the location of the transition point for buckling are given in Appendix III.

The tie ring around the base of the dome is designed to carry the tension caused by the horizontal component, H, Figure 11, of the reactions at a unit stress of 50,000 psi.

Formulas for the weight and cost of dome shell and tie ring are also included in Appendix III.

IV. RESULTS

Figure 12 is a plot of the thickness to span ratio vs. static resistance. The static resistance is not the overpressure that the doors are designed to resist but is determined by the allowable deflection which is computed according to methods explained in Section V. However, once the resistance is determined, the thickness, D, of a given door may be computed by multiplying the corresponding ordinate of Figure 12 by the span, L. The units of D are of course the same as of L.

Figure 13 is a plot of door weight vs. static resistance. This chart is applicable to square openings for slab doors and circular openings for dome shaped doors. Weight is an important factor in the door costs since for equal operational requirement the cost of motive power is a function of weight. The weight, in pounds, of a door capable of furnishing a given static resistance is obtained by multiplying the corresponding ordinate of Figure 13 by the cube of the span, L^3, in ft.^{3}.

Figure 14 is a plot of the cost/(span)^3 vs. static resistance. This cost covers the door structure only. No hardware or mechanism is included.

Figures 15 and 16 give weight and cost for doors made up of steel wide-flange beams. It was necessary to plot these curves separately because of the limitations due to the fixed depths and widths of standard sections. The curves were plotted for the heaviest rolled sections of 18", 24", 30" and 36" nominal depth. Figure 16 was drawn for a unit cost of $.18 per lb. of steel. For other unit costs the results may be readily adjusted.

Figure 17 is a plot of thickness vs. overpressure for steel domes in a horizontal position.

Figure 18 is a plot of weight and cost of steel dome and tie ring vs.

overpressure for horizontal domes.

Figure 19 shows cost vs. overpressure for "hard" horizontal doors. Solid steel doors are the most expensive and it is doubtful that such large steel slabs would be readily available. The thickest slab available in the Los Angeles area is 14 inches with a maximum width of 146 inches. Slabs of fifteen feet would have to be specially rolled and the cost might be higher than shown. However, steel slabs work out quite well for very small spans and low pressures as they require very little fabrication.

Reinforced concrete slabs are less expensive than wide-flange beams and they provide protection against nuclear and thermal radiation whereas the steel beams and domes require additional insulation. The great weight and thickness creates problems in operation and layout. These slabs are designed under rather conservative assumptions as to their strength in diagonal tension and future tests may show them to be more economical than appears in this report.

Reinforced concrete beams and domes appear to be almost competitive in cost but the assumptions as to loading of domes are probably over conservative so this may represent an upper limit for this type of door.

The cost data shown on Figures 14, 16 and 18 are tabulated on Tables I, II and III.

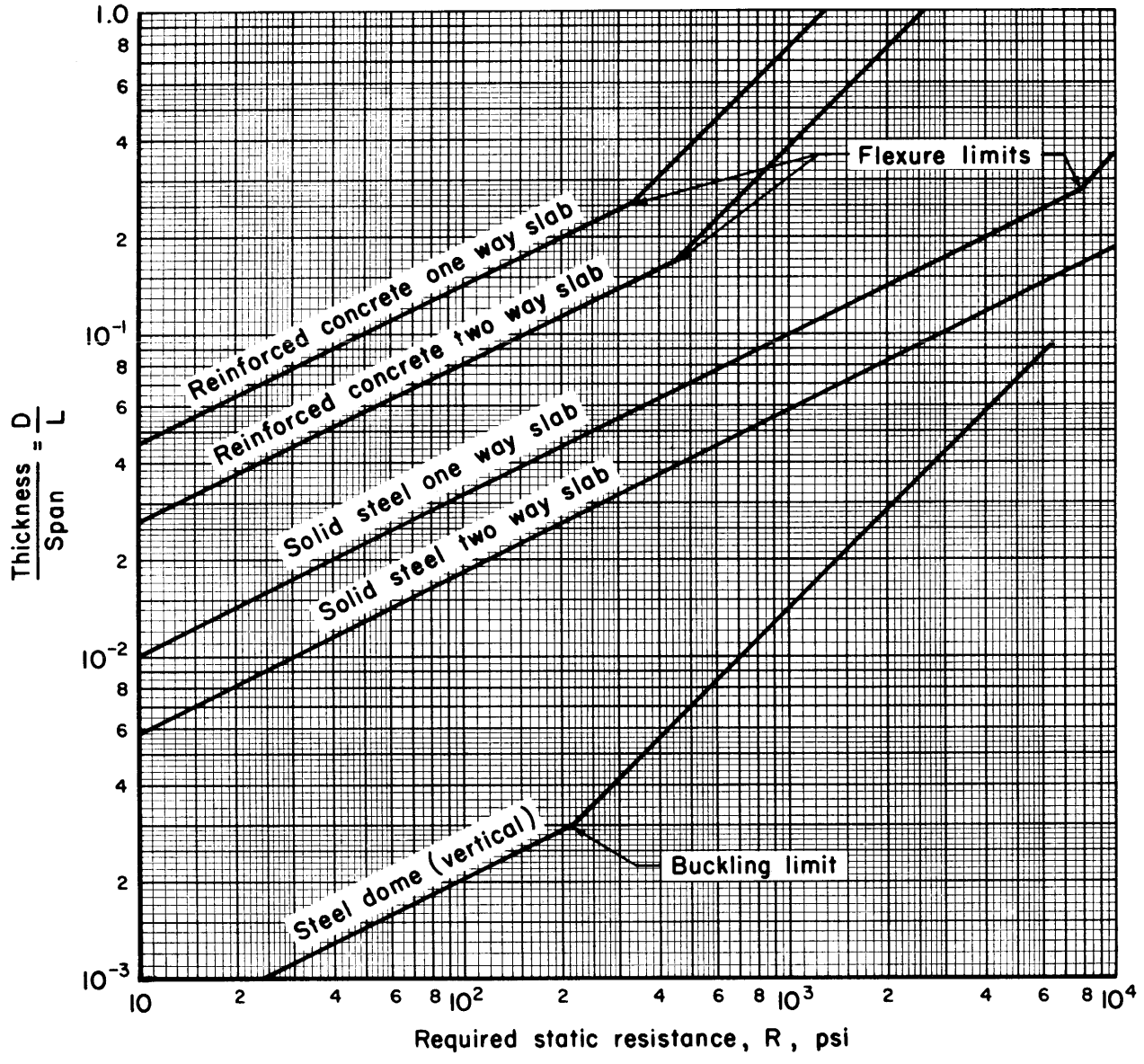


Fig. 12—Thickness of doors

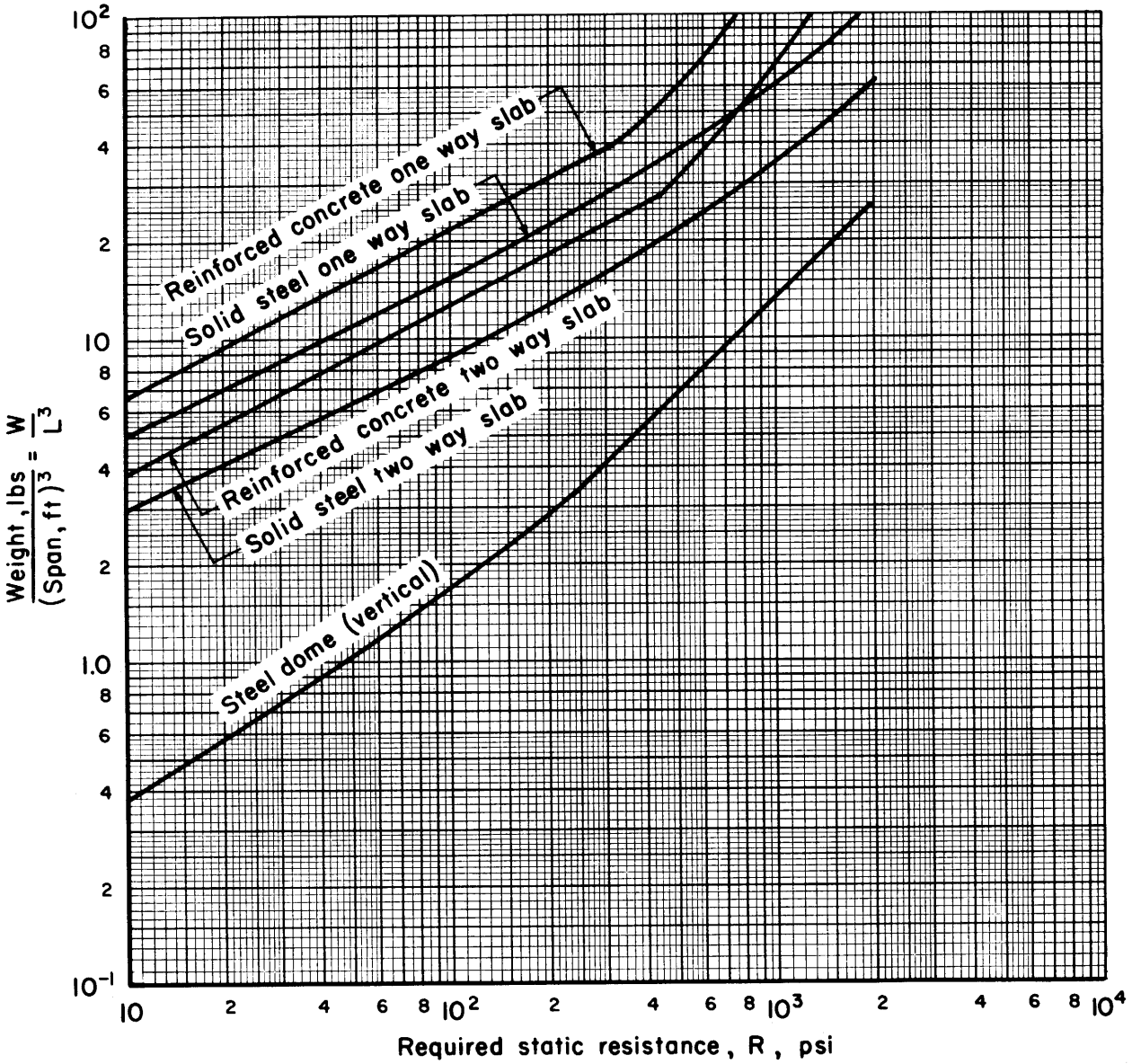


Fig. 13—Weight of doors

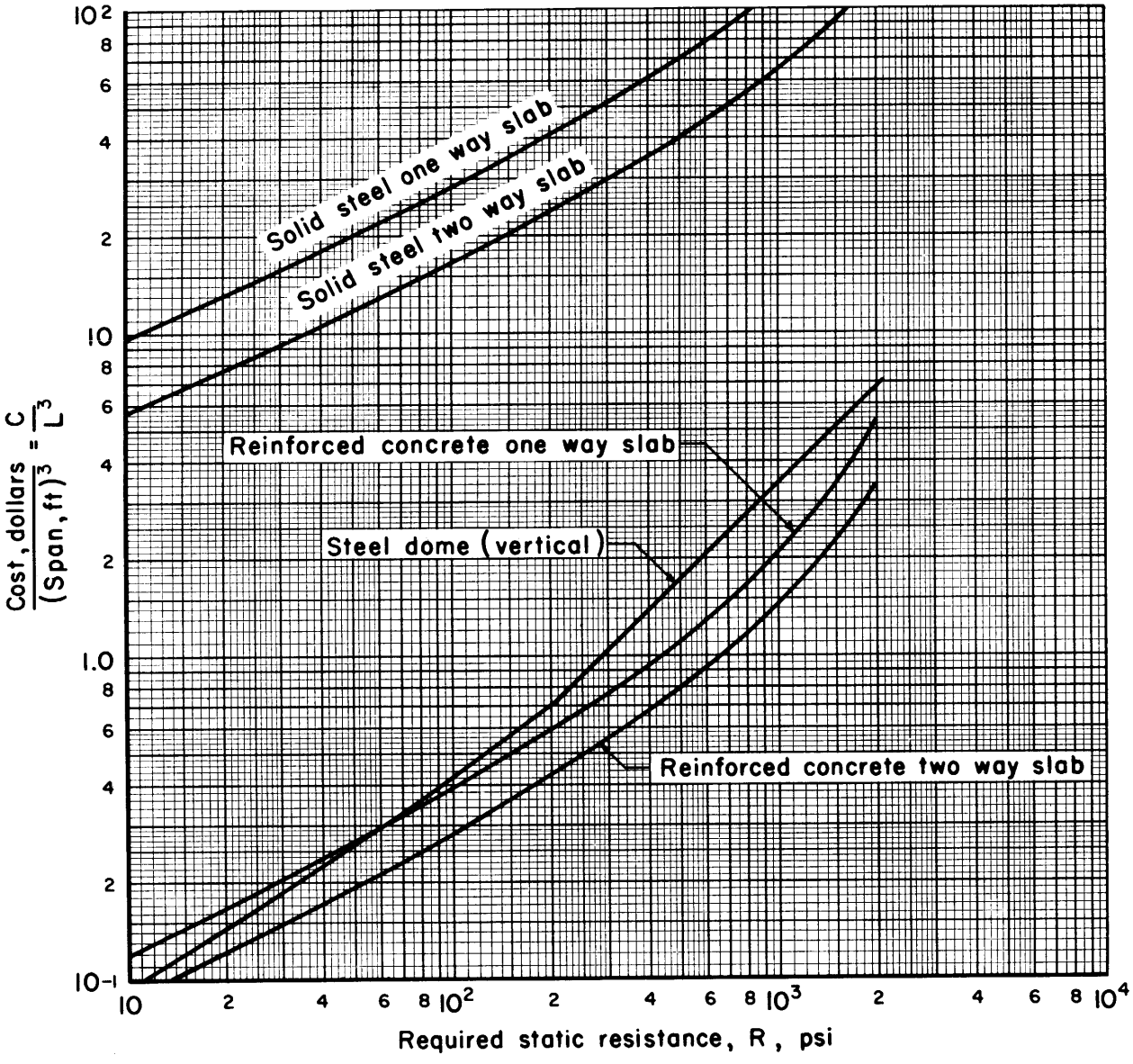


Fig. 14—Cost of doors

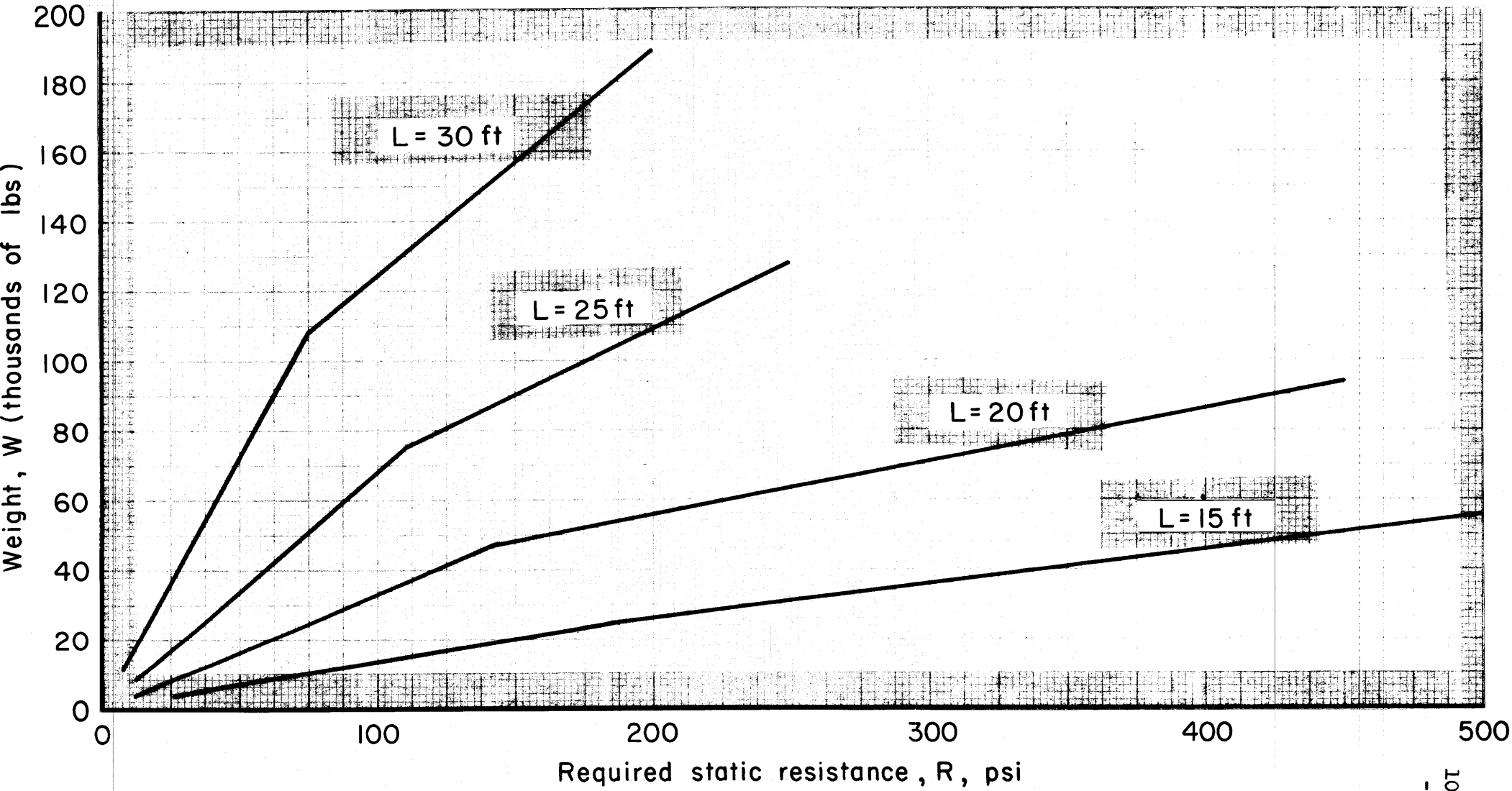


Fig. 15—Weight of wide-flange beams for square openings

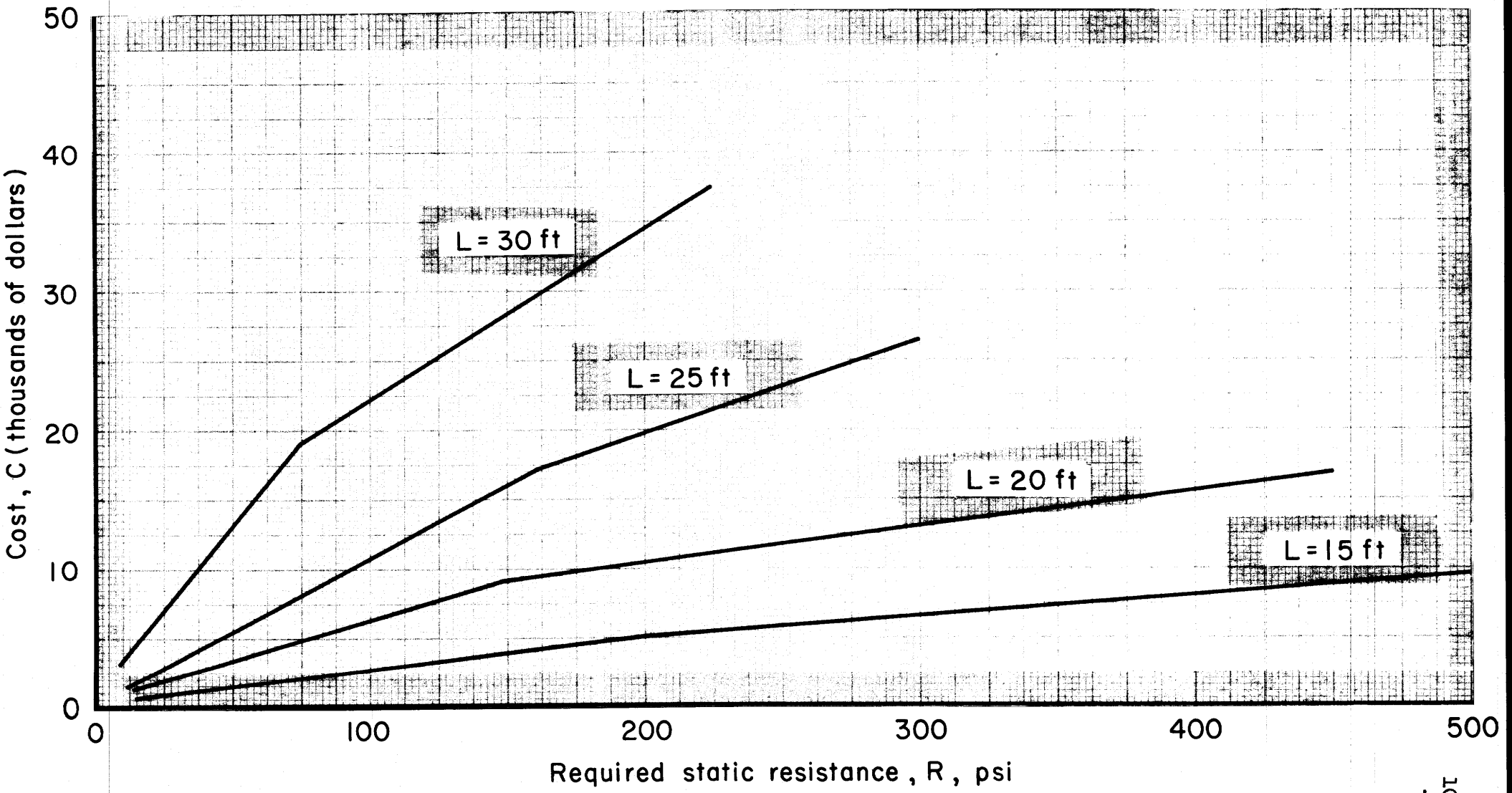


Fig. 16—Cost of wide-flange beams for square openings

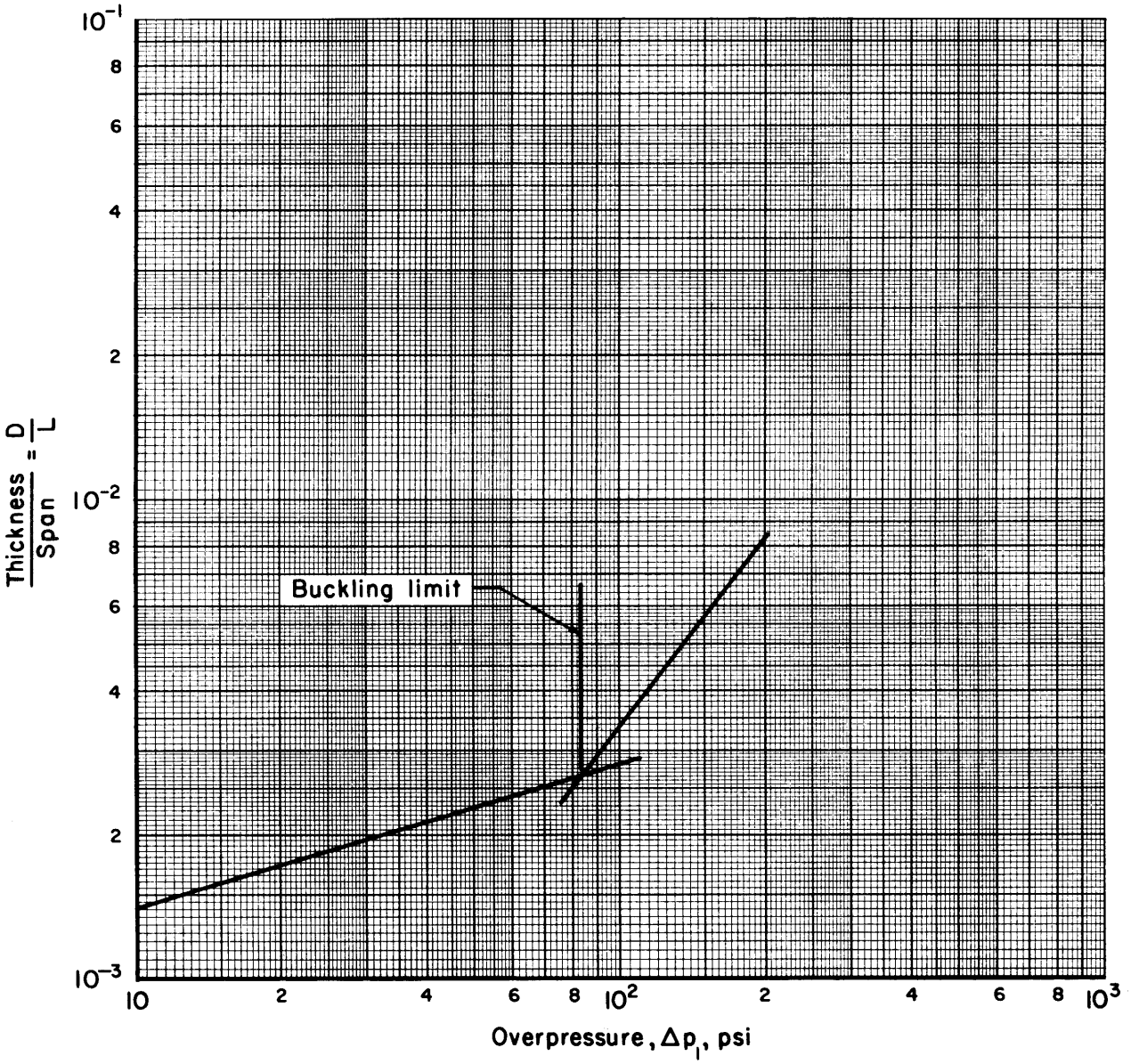


Fig. 17—Thickness of steel domes for horizontal orientation

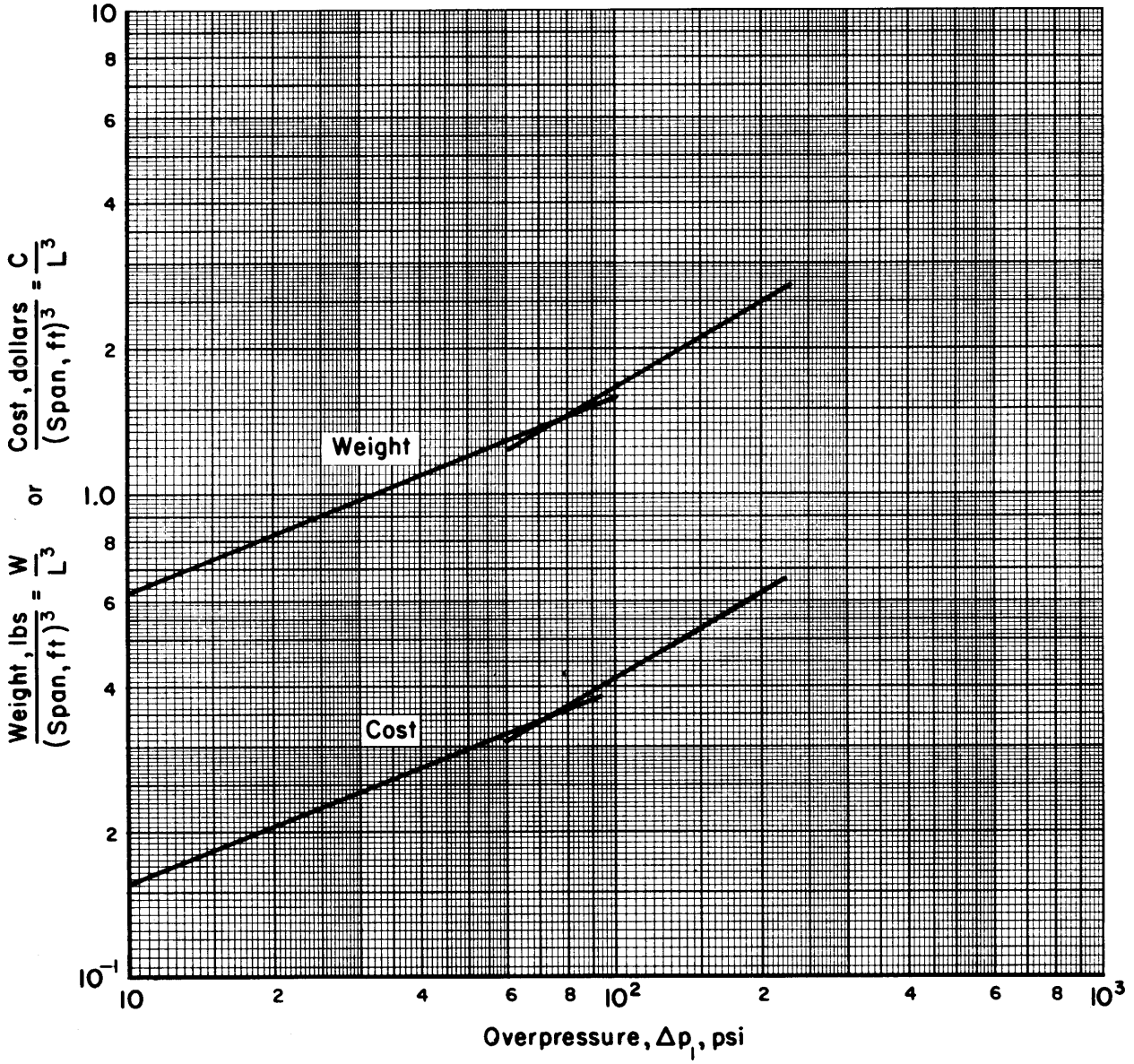


Fig. 18—Weight and cost of steel dome and tie ring for horizontal orientation

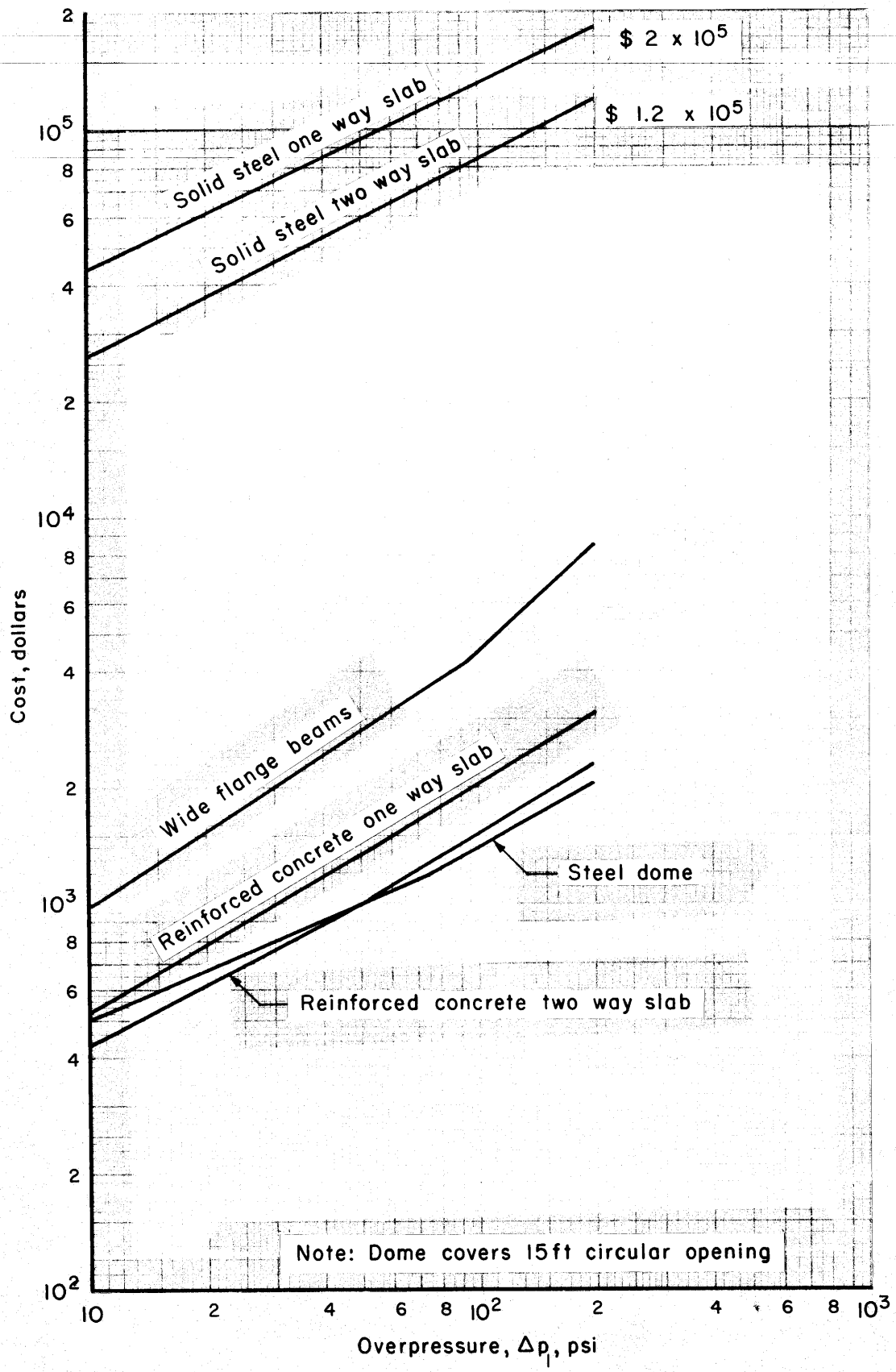


Fig. 19 — Cost of horizontal doors vs overpressure for 15ft square openings (hard or elastic response)

Table I

COST (IN $M) OF HORIZONTAL SLAB DOORS FOR SQUARE OPENINGS

Reinforced Concrete One Way Slab				
ΔP_1 Overpressure, psi \ Span, L, Ft.	Hard		Soft	
	15	30	15	30
10	.54	4.32	.37	3.00
50	1.32	10.50	.88	7.00
100	2.03	16.20	.132	10.50
200	3.21	25.70	.203	16.20

Reinforced Concrete Two Way Slab				
ΔP_1 Overpressure, psi \ Span, L, Ft.	Hard		Soft	
	15	30	15	30
10	.44	3.51	.30	2.43
50	.88	8.10	.68	5.40
100	1.52	12.20	1.00	8.10
200	2.37	18.90	1.52	12.20

Solid Steel One Way Slab				
ΔP_1 Overpressure, psi \ Span, L, Ft.	Hard		Soft	
	15	30	15	30
10	44.0	351.0	32.4	260.0
50	95.5	763.0	67.0	534.0
100	133.0	1070.0	95.5	763.0
200	199.0	1580.0	133.0	1070

Solid Steel Two Way Slab				
ΔP_1 Overpressure, psi \ Span, L, Ft.	Hard		Soft	
	15	30	15	30
10	26.0	208.0	19.0	153.0
50	55.4	443.0	40.0	321.0
100	80.2	641.0	55.4	443.0
200	119.0	946.0	80.2	641.0

Table I (cont'd.)

Wide Flange Beams

Span, L, Ft. / ΔP_1 Overpressure, psi	Hard		Soft	
	15	30	15	30
10	1.7	6.2	1.0	2.8
50	2.8	22.0	1.7	13.0
100	5.1	33.9	2.8	22.0
200	8.5	--	5.1	33.9

Table II

COST (in $M) OF STEEL DOMES IN HORIZONTAL ORIENTATION

Span, L, Ft. / ΔP_1 Overpressure, psi	15	30
10	.53	4.3
50	1.0	8.0
100	1.4	11.3
200	3.2	25.6

Table III

COST (in $M) OF STEEL DOMES IN VERTICAL ORIENTATION

Span, L, Ft. / ΔP_1 Overpressure, psi	15	30
10	.4	2.7
50	1.4	10.8
100	2.7	21.6
200	6.1	48.6

V. USE OF CHARTS

A. Slabs and Beams

1. Determination of Required Static Resistance

The design of structural members subjected to blast loading is usually accomplished by trial and error methods. This is because of the dependence of response on duration, period of vibration and magnitude of load. However, the problem is simplified in the case of extremely long duration (i.e. long, with respect to the period of vibration of the structure or structural element). Then, the loading may be considered as a rectangular, suddenly applied pulse of infinite duration, Figure 20.

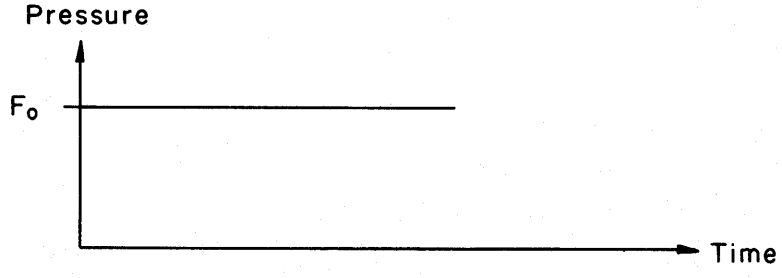


Figure 20

In reference 1, Newmark presents an expression for determining the required static resistance, R, of simple, one degree of freedom structures subjected to a long duration loading whose peak is F_o. This formula is given in the following form:

$$\frac{F_o}{R} = 1 - \frac{.5}{u}$$

where u = ratio of maximum deflection (x_m) to yield point deflection (x_e) (Figure 21).

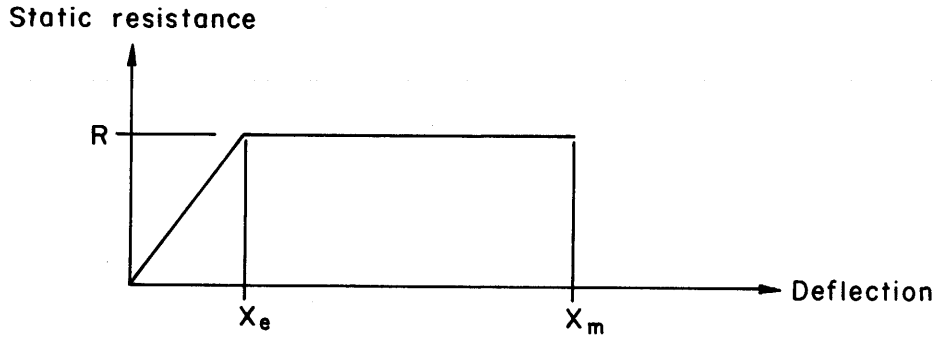


Figure 21

The required resistance is dependent on the amount of allowable deflection. The allowable deflection must be fairly small if a structure is to remain relatively undamaged, that is if the structure is classified as "hard" or "elastic." In this case a value of u = 1 is not excessive and will be used in this report. Then, under the assumed loading, the door will deflect just to the yield point, x_e. If we substitute u = 1 into Newmark's formula, we have:

$$\frac{F_o}{R} = 1 - .5 = .5$$

and,

$$R = 2F_o$$

This means that the structure must be designed to resist a statically applied load equal to twice the peak dynamic load.

If more deflection is permissible, that is, if a structure is classified as "soft" or "plastic" the value of u may be increased. If we assume u = 10 and again apply Newmark's formula we have:

$$\frac{F_o}{R} = 1 - .05 = .95$$

and,

$$R = 1.05\ F_o$$

The required resistance in this case must be equal to 1.05 times the peak load. For purpose of this report it is accurate enough to take the ratio as equal to one. Then, we have $R = F_o$ for soft doors.

2. Peak Dynamic Load

Now that we have the relationships between static resistance and peak dynamic load we must define the peak dynamic loads for the two orientations.

a. Horizontal slabs and beams - The peak dynamic load is equal to the overpressure, Δp_1. Then for soft doors,

$$F_o = R = \Delta p_1$$

This means that Figures 12 through 16 may be read directly in obtaining thickness, weights and costs of soft horizontal slab doors.

For hard doors, the required resistance, R, is twice the peak dynamic load, F_o, and

$$R = 2 F_o = 2 \Delta p_1$$

that is, values of thickness, weight and cost are taken from Figures 12 through 16 by using a static resistance equal to twice the overpressure.

b. Vertical slabs and beams

The peak dynamic load for vertical doors is the reflected pressure, $K \Delta p_1$.* This pressure drops off rapidly to a pressure equal to the static overpressure plus the drag pressure. The

*Values of K are available in Reference 2.

time required for this drop to occur is the clearing time. Its value is three times the least dimension of the face of the structure divided by the velocity of the shock front. If the clearing time is long, that is, if the door is situated in the face of a very high and wide cliff, the reflected pressure may be considered of infinite duration. In this case the peak load, F_o, would be the reflected pressure and elastic doors would have to be designed to resist twice this pressure.

If the door is situated in an isolated building the clearing time is very short and the effect of the reflected pressure may be considered to be an impulse which imparts an initial velocity to the door, followed by a rectangular forcing function whose peak value is the overpressure plus the drag pressure. The design of doors under these conditions is fairly complex and it is felt that for rough estimating purposes, the sizes and costs may be assumed to be those required to resist a peak load equal to 3.0 $(\Delta p_1 + q)$ in the case of elastic doors and 1.5 $(\Delta p_1 + q)$ in the case of plastic doors.

Values of K and q corresponding to various overpressures are tabulated here for convenience.

Overpressure, Δp_1 Psi	Reflection Factor, K	Drag Pressure, q Psi
10	2.5	2.2
50	3.9	44
100	4.8	125
200	5.7	330

B. Domes

1. Horizontal Domes

Figures 17 and 18 which give thickness, weight and cost for domes are presented as functions of overpressure and it is not necessary to compute a required static resistance. These figures were plotted for hard domes. There is little information about the response of domes to nuclear blast loading and the simplified differences between hard and soft structures as explained in section V.A.1 may be inapplicable and the use of the charts for domes, in either orientation is recommended for the hard case only.

2. Vertical Domes

The required static resistance in this case is determined in the same manner as for hard vertical slabs.

C. Example

As an example, let us determine the thickness, weight and cost of a reinforced concrete two-way slab for the following conditions:

span $L = 20 \text{ ft} \times 20 \text{ ft}$

overpressure, $\Delta p_1 = 100 \text{ psi}$

$u = 1$

horizontal orientation.

Solution:

The door is a "hard" structure,

$\therefore R = 2F_o = 2 \Delta \bar{p}_1 = 200 \text{ psi}$

To obtain thickness

Enter Figure 12 with R = 200 psi and find

$D/L = .11$

$\therefore D = .11 (20) = 2.2 \text{ ft.}$

To obtain weight, enter Figure 13 with R = 200 psi and find $W/L^3 = 18$

$$W = 18\ (20)^3 = 144000 \text{ lbs.}$$

To obtain cost, enter Figure 14 with R = 200 psi and find $C/L^3 = .45$

$$C = .45\ (20)^3 = \$3{,}600$$

The same method is used to obtain sizes and cost of steel slabs and domes.

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Appendix I

NOMENCLATURE

L = size of door opening, ft.

R = static resistance of door, psi.

Δp_1 = static overpressure, psi

K = reflection factor

q = drag pressure, psi

Q = total drag force on dome, lbs.

F_o = peak load, psi

D = effective depth of concrete section or total depth of steel section, ft.

p = percentage of steel tensile reinforcing in concrete doors

f_s = tensile unit yield stress in steel, 50,000 psi

f_v = tensile unit yield stress in shear reinforcing steel, 50,000 psi

f_w = shear yield stress in structural steel, 21,000 psi

f_c' = ultimate compressive unit stress in concrete, 5000 psi

f_b = ultimate bearing unit stress in concrete, 5000 psi

$\surd$ = ultimate shearing unit stress in concrete, 750 psi

b = width of wide flange beams, inches

t = web thickness of wide flange beams, inches

W = weight, lbs.

w = weight of wide flange beams, lbs. per ft.

C = cost, in dollars

c = total length of door, including bearing area, ft.

v = Poisson's ratio for steel = .3

u = ratio of maximum deflection to yield point deflection

S = section modulus for wide flange beams, inches^3.

r = radius of curvature of dome

W_D = weight of dome, lbs.

W_R = weight of tie ring, lbs.

P_d = design pressure for domes, psi

P_n = design pressure for dome tie rings, psi

Appendix II

THICKNESS, WEIGHT AND COST OF SLABS

1. Unit costs and unit weights used in deriving the weight and cost formulas are as follows:

   a. Unit Weights

Concrete	150 lbs per cu. ft.
Steel	490 lbs per cu. ft.

   b. Unit Costs

Concrete	$30 per cu. yd.
Reinforcing steel	$.16 per lb.
Structural steel	$.18 per lb.

2. Reinforced concrete one-way slab.

   Thickness,

$$D = 1.38 \times 10^{-2} \sqrt{R}\, L \qquad R \leq 350 \text{ psi} \qquad (1)$$

$$D = 7.4 \times 10^{-4} R\, L \qquad R \geq 350 \text{ psi} \qquad (2)$$

   Weight per foot of width

$$W = 2.06 \times \left(\frac{5000}{5000-R}\right) \sqrt{R}\, L^2 \qquad R \leq 350 \text{ psi} \qquad (3)$$

$$W = 1.14 \times 10^{-1} \left(\frac{5000}{5000-R}\right) R\, L^2 \qquad R \geq 350 \text{ psi} \qquad (4)$$

   Cost per foot of width

$$C = (34.2 + .446 \sqrt{R}) \left(\frac{5000}{5000-R}\right) 10^{-6} \sqrt{R}\, L^2 \qquad R \leq 350 \text{ psi} \qquad (5)$$

$$C = (1.44 + \frac{286}{R}) \left(\frac{5000}{5000-R}\right) 10^{-6} R\, L^2 \qquad R \geq 350 \text{ psi} \qquad (6)$$

3. Reinforced concrete two-way square slab

   Thickness,

$$D = 7.9 \times 10^{-3} \sqrt{R}\, L \qquad R \leq 456 \text{ psi} \qquad (7)$$

$$D = 3.7 \times 10^{-4} R\, L \qquad R \geq 456 \text{ psi} \qquad (8)$$

Weight, entire slab

$$W = 1.19 \times \left(\frac{5000}{5000-R}\right) \sqrt{R}\ L^3 \qquad R \leq 456 \text{ psi} \qquad (9)$$

$$W = 5.7 \times 10^{-2} \quad \frac{5000-}{5000-R} \quad RL^3 \qquad R \geq 456 \text{ psi} \qquad (10)$$

Cost, entire slab

$$C = (27.5 + .298\sqrt{R}) \left(\frac{5000}{5000-R}\right) 10^{-6} \sqrt{R}\ \ L^3 \qquad R \leq 456 \text{ psi} \qquad (11)$$

$$C = \left(.72 + \frac{382}{R}\right) \left(\frac{5000}{5000-R}\right) 10^{-6}\ R\ L^3 \qquad R \geq 456 \text{ psi} \qquad (12)$$

4. Solid Steel One-Way Slab

Thickness,

$$D = 3.16 \times 10^{-3} \sqrt{R}\ \ L \qquad R < 7{,}900 \text{ psi} \qquad (13)$$

Weight, per foot of width

$$W = 1.55 \times \left(\frac{5000}{5000-R}\right) \sqrt{R}\ L^2 \qquad R < 7{,}900 \text{ psi} \qquad (14)$$

Cost, per foot of width

$$C = 2.80 \times 10^{-4} \left(\frac{5000}{5000-R}\right) \sqrt{R}\ L^2 \qquad R \leq 7{,}900 \text{ psi} \qquad (15)$$

5. Solid Steel Two-Way Square Slab

Thickness,

$$D = 1.83 \times 10^{-3} \sqrt{R}\ L \qquad (16)$$

Weight, entire slab

$$W = 8.96 \times \left(\frac{5000}{5000-R}\right) \sqrt{R}\ L^3 \qquad (17)$$

Cost, entire slab

$$C = 1.63 \times 10^{-4} \left(\frac{5000}{5000-R}\right) \sqrt{R}\ L^3 \qquad (18)$$

6. Wide Flange Beams

The resistance of a given wide flange beam is obtained from one of the following formulas:

$$R = \frac{2\ f_w\ t_w\ d}{b\ell} \quad \text{, in case shear governs} \qquad (19)$$

or,

$$R = \frac{8 f_s Z}{bL^2} \quad \text{, in case flexure governs} \tag{20}$$

The weight per foot of width is,

$$W = \frac{12000 \, w \, c}{b} \tag{21}$$

The cost per foot of width is,

$$C = \frac{2.17 \, w \, c}{b} \tag{22}$$

Appendix III

LOADING AND DESIGN OF DOMES

A. Loading for Horizontal Domes

The load acting on domed shape is more complicated than that acting on flat surfaces. A simplified load pattern has been assumed for use in this report.

1. Reflection of Overpressure

The presence of the dome in the path of the blast wave creates an obstacle to the flow and the pressure acting on the windward edge will be increased to a value larger than the free air overpressure. This increased pressure is known as the reflected pressure. The severity of the increase is dependent on the shock strength and the angle of incidence of the shock wave. The reflected pressure decays almost linearly to a value equal to the static overpressure plus drag pressure in a very short time, t_c. "t_c" is the clearing time and may be estimated by Equation (23).

$$t_c = \frac{3h}{U} = \frac{.62\ L}{U} \qquad (23)$$

where h = height of the dome, ft. = .207L
U = velocity of sound in reflected region, ft. per sec.

By trail and error calculation it appears that a maximum stress condition will exist in the shell of the dome when the blast wave has progressed about one-quarter of the arc length along the prime meridian, Figure 22.

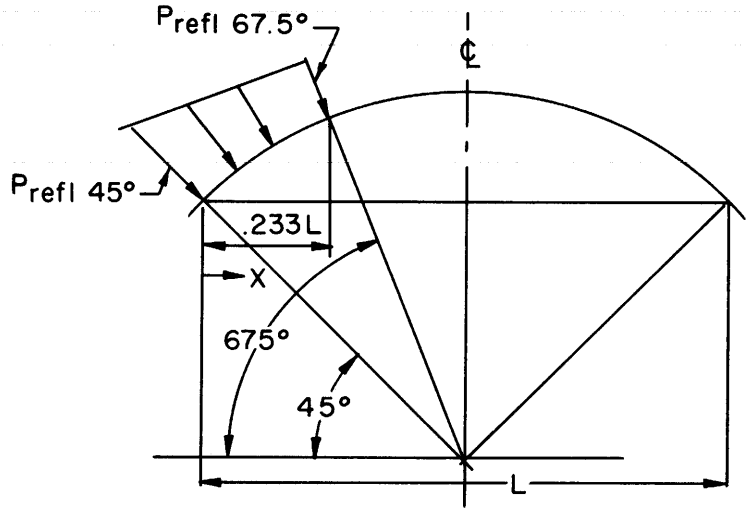


Fig. 22

The value of $P_{refl\ 67.5^o}$ is the static overpressure times the reflection factor for 67.5^o incidence. The value of $P_{refl\ 45^o}$ is somewhat less than the static overpressure times the 45^o reflection factor because it has decayed as the front of the shock wave thas traveled from the edge of the dome, x = 0 to x = .233L, Figure 23:

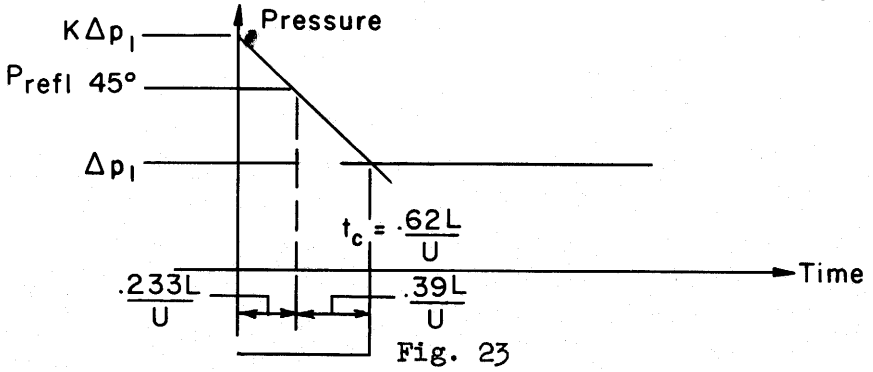


Fig. 23

For design purposes, the average value of $P_{refl\ 67.5^o}$ and $P_{refl\ 45^o}$ is used. The reflection factors are taken from Reference 3. The reflection factors for Δp_1 = 100 psi and 200 psi were obtained by assuming that they are reduced with the angle of incidence in the same proportion as the reflection factors for Δp_1 = 70 psi as shown in Fig. 11.12 of Reference 3. The factors for normal incidence for 100 and 200 psi were taken from Reference 1. The average pressure, using the notation of Figure 23 is given by Equation (24).

$$P_{ave} = \frac{P_{refl\ 45^o} + P_{refl\ 67.5^o}}{2} = \frac{\frac{.39}{.62}\ \Delta P_1 (K_{45^o} - 1) + \Delta P_1 + K_{67.5^o}\ \Delta P_1}{2}$$

$$= \frac{\Delta P_1}{2}\ (.63 K_{45^o} - .37 + K_{67.5^o}) \qquad (24)$$

where, ΔP_1 = static overpressure

K_{45^o}, $K_{67.5^o}$ = reflection factor for 45^o and 67.5^o incidence

P_{ave} will be combined with the average drag pressure to obtain the required design pressure.

2. Drag Pressure

According to Reference 3, the local drag pressure coefficient varies along the meridian as $\cos \frac{15}{8} \alpha$, Figure 24. In this report the variation will be

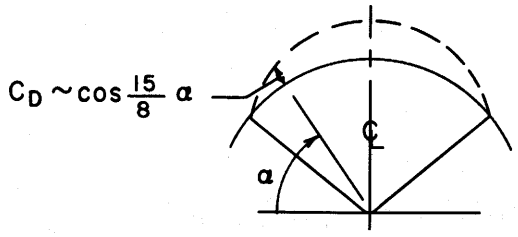


Fig. 24

approximated as $\cos 2 \phi$. The drag pressure acting on the loaded portion of the dome is shown in Fig. 25. The average drag pressure under these assumptions is given by Equation (25),

$$q_{ave} = \frac{qr \int_{\frac{\pi}{4}}^{\frac{3\pi}{8}} \cos 2\alpha\, d\alpha}{r \int_{\frac{\pi}{4}}^{\frac{3\pi}{8}} d\alpha} = -.373\, q \qquad (25)$$

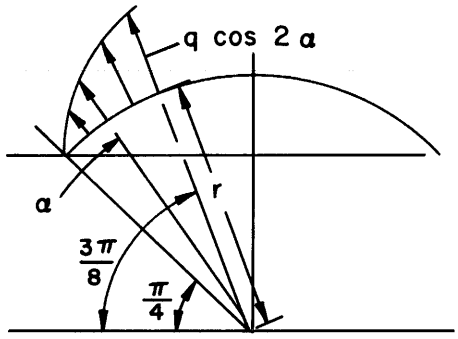


Fig. 25

The negative sign indicates that the load acts in an upward direction.

3. Design Pressure

The design pressure, P_d, is the algebraic sum of P_{ave} and Q_{ave}:

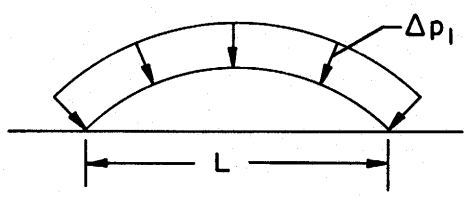
$$P_{ave} + Q_{ave} = P_d = \frac{\Delta P_1}{2} (.63K_{45^o} - .37 + K_{67.5}) - .373q \qquad (26)$$

This pressure is used to determine the required thickness of the dome shell.

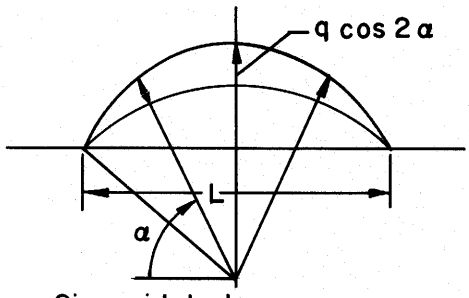
4. Maximum Loading Condition for Tie Ring

Maximum stress condition in the tie ring will probably occur when the dome is fully loaded.

The net vertical force acting on the dome will be the net effect of the static overpressure and drag pressure. This effect can be approximated by assuming a uniform distribution of static overpressure and a sinusoidal distribution of drag pressures, Figures 26 and 27.



Uniform Static Overpressure
Fig. 26



Sinusoidal drag pressure
Fig. 27

The total downward load due to overpressure is:

$$\frac{\pi}{4} \Delta\bar{p}_1 L^2 \tag{27}$$

The total uplift due to drag is

$$2q\pi r^2 \int_{\frac{\pi}{4}}^{\frac{\pi}{2}} (\sin\alpha \cos\alpha - 2 \sin\alpha \cos\alpha) d\alpha = -\frac{1}{4} q\eta r^2 = -\frac{\pi}{8} qL^2 \tag{28}$$

The net load is the sum of these two loads:

$$\frac{\pi}{4} L^2 (\Delta\bar{p}_1 - \frac{q}{2}) \tag{29}$$

For the range of pressures being considered in this report the net load, as given by the preceding equation, is always downward, Table 1.

Table 1

P,psi	$\bar{q}$,psi	$\frac{\pi}{4}(\Delta\bar{p}_1 - q/2)$,psi
10	2	7.0
50	44	22.0
100	125	29.0
200	330	27.5

The tie ring must be designed to withstand the stresses due to this net load.

B. Design of Horizontal Dome Shell

The stresses in dome shells due to unbalanced long duration dynamic loads will be greater than the stresses due to static uniform loads for two reasons:

1. The unbalanced nature of the loading causes greater stress at the loaded edge.
2. The dynamic effect, i.e., the suddenness of the load application brings about an increase in stress. In this report the stress amplification factor will be taken as 2.

An approximate method for determining the maximum stress in partially loaded domes is given in Reference 4. This method can be briefly described in the following manner:

The meridional stresses, N_α, and the circumferential stresses, N_β, are related to the external pressure, P_d, by Equation (30),

$$N_\alpha + N_\beta = P_o r \tag{30}$$

where r is the radius of the dome. For the loading condition assumed herein, N_α, is less than N_β and the required thickness, t, may therefore be approximated by Equation (31),

$$t \cong \frac{P_d r - N_\alpha}{\sigma} \tag{31}$$

where σ is the allowable unit stress. Now, referring to Figure 28, we introduce a unit line load, F, whose magnitude is given by Equation (32),

$$F = \frac{r \int_{\alpha_o}^{\alpha_1} P_d \sin 2\alpha \, d\alpha}{2 \sin^2 \alpha} \tag{32}$$

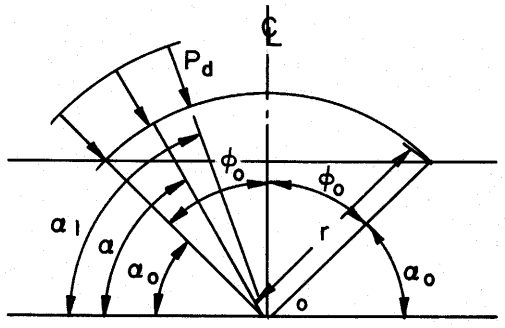


Fig. 28

The normal forces, N_α^1, due to the load are given by Equations (33) and (34)

$$N_\alpha^1 = \frac{r \int_{\alpha_o}^{\alpha} P_d \sin 2\alpha \, d\alpha}{2 \sin^2 \alpha_1}, \quad \alpha < \alpha_1 \tag{33}$$

$$N_\alpha^1 = 0 \qquad \blacksquare\ \alpha > \alpha_1 \tag{34}$$

To these normal forces must be added the normal forces N_α^{11} due to the line load, F. Then,

$$N_\alpha = N_\alpha^1 + N_\alpha^{11} \tag{35}$$

Normal forces N_α^{11} can be approximated by assuming that F divides into two components each inversely proportional to the length of the axis of the prime meridian ahead and behind the point of application, Figure 28. In Figure 28 it is shown that, $\pi - 2\alpha_o = 2\phi_o$, then,

$$N_\alpha^{11} = -\frac{-F(2\phi_o + \alpha_o - \alpha_1)}{2\phi_o}, \quad \alpha < \alpha_1 \tag{36}$$

and

$$N_\alpha^{11} = -\frac{F(\alpha_1 - \alpha_o)}{2\phi_o}, \quad \alpha > \alpha_1 \tag{37}$$

However, we are only interested in $\alpha < \alpha_1$ and then in accordance with Equation (35), N_α is given by Equation (38),

$$N_\alpha = \frac{r}{2\sin^2\alpha_1}\left[\int_{\alpha_o}^{\alpha} P_d \sin 2\alpha d\alpha - \frac{(2\phi_o+\alpha_o-\alpha_1)}{2\phi_o}\int_{\alpha_o}^{\alpha_1} P_d \sin 2\alpha d\alpha\right], \alpha < \alpha_1 \tag{38}$$

Then, the required thickness given by substituting Equation (38) into Equation (31) is,

$$t = \frac{P_d r}{\sigma} - \frac{r}{2\sigma\sin^2\alpha_1}\int_{\alpha_o}^{\alpha} P_d \sin 2\alpha\, d\alpha + \frac{r}{2\sigma\sin^2\alpha_1}\frac{(2\phi_o + \alpha_o - \alpha_1)}{2\phi_o}\int_{\alpha_o}^{\alpha_1} P_d \sin 2\alpha d\alpha \tag{39}$$

In this report we have,

$$\alpha_o = \frac{\pi}{4} = 45^\circ$$

$$\alpha_1 = \frac{3\pi}{8} = 67.5^\circ$$

$$2\phi_o = \frac{\pi}{2} = 90^\circ$$

The maximum value of t occurs for $\alpha = \pi/4$, therefore substituting these values into Equation (39) the required thickness is,

$$t = \frac{1.16\ PR}{\sigma} \tag{40}$$

Equation 40 must be multiplied by the dynamic load factor (=2), since $r = .707L$ and $\sigma = 50{,}000$ psi, the final value of t is,

$$t = 3.28 \times 10^{-5} P_d L \tag{41}$$

Buckling

The possibility of buckling is not considered in the above equations. In this section an approximation will be made to provide for this eventuality.

The critical stress for buckling given in Reference 4 is,

$$\sigma_{cr} = \frac{E}{\sqrt{3(1 - \nu^2)}}\ \frac{t}{r} \tag{42}$$

if we substitute,

$$E = 30 \times 10^6 \text{ psi}$$

$$r = .707\ L$$

$$\nu = .3$$

and multiply the left hand side of Equation (42) by a safety factor of 1.5, which appears to be adequate, we obtain Equation (43) for critical stress,

$$\sigma_{cr} = 17.1 \times 10^6\ \frac{t}{L} \tag{43}$$

Substituting Equation (41) into Equation (43) we obtain Equation (44),

$$t = 31 \times 10^{-5} \sqrt{P_d}\, L \tag{44}$$

Comparing (41) and (44) we find that for $P_d < 89.5$ psi Equation (44) governs (i.e. gives a larger value of t) and that Equation (41) governs for $P_d > 89.5$ psi.

The weight of the dome shell is given by Equations(45) and (46)

$$W_D = 140 \times 10^{-3} \sqrt{P_d}\, L^3 \quad , \quad P_d < 89.5 \text{ psi} \tag{45}$$

and

$$W_D = 13.9 \times 10^{3} \; P_d \, L^3 \quad , \quad P_d > 89.5 \text{ psi} \tag{46}$$

The cost at $.25 per lb. is given by Equations (47) and (48)

$$C_D = 35 \times 10^{-3} \sqrt{P_d}\, L^3 \quad , \quad P_d < 89.5 \text{ psi} \tag{47}$$

and

$$C_D = 3.48 \times 10^{-3} \; P_d \, L^3 \quad , \quad P_d > 89.5 \text{ psi} \tag{48}$$

C. Design of Vertical Dome Shell

The required thickness of a dome shell subjected to a uniform radial load is given by Equation (49),

$$t = \frac{P_d r}{2\sigma} \tag{49}$$

For a suddenly applied load Equation (49) must be multiplied by the dynamic load factor, which in this case is assumed as 2. Then if we substitute $r = .707\ L$, $\sigma = 50{,}000$ and multiply by 2 the required thickness is given by Equation (50),

$$t = 1.41 \times 10^{-5} \; P_d \, L \tag{50}$$

Equation (50) does not take into account the possibility of buckling.

If we assume the critical stress to be given by the same expression as for horizontal domes, $\sigma_{cr} = \frac{E}{\sqrt{3(1-\nu^2)}} \frac{t}{r}$, and substitute this value into Equation (49) we have Equation (51),

$$t = \frac{P_d \, r^2 \sqrt{3(1-\nu^2)}}{2 \, t \, E} \tag{51}$$

Equation (51) must be multiplied by the dynamic load factor (= 2) and by a safety factor against buckling. The safety factor is again taken as 1.5. Then we have,

$$t = 20.4 \times 10^{-5} \sqrt{P_d} \, L \tag{52}$$

By equating (50) and (52) we find that Equation (52) governs for $P_d < 207$ psi and (50) governs for $P_d > 207$ psi.

The weight of the shell is then given by Equations (53) and (54),

$$W_D = 92 \times 10^{-3} \sqrt{P_d} \, L^3, \quad P_d < 207 \tag{53}$$

and

$$W_D = 6.36 \times 10^{-3} \, P_d \, L^3, \quad P_d > 207 \tag{54}$$

The cost, assuming dished steel plate at $.25 per lb. is given by Equations (55) and (56),

$$C_D = 23 \times 10^{-3} \sqrt{P_d} \, L^3, \quad P_d < 207 \tag{55}$$

and,

$$C_D = 1.59 \times 10^{-3} \, P_d \, L^3, \quad P_d > 207 \tag{56}$$

D. Design of Tie Ring

The tie ring must carry the tension, T, due to the horizontal component of the thrust at the edge of the dome. The thrust due to a uniform radial load is $\frac{P_n r}{2}$. The horizontal component, H, of this thrust is $\frac{P_n r}{2} \cos 45^\circ$.

Then the tension $T = \int_0^{\pi/2} H\, r \sin 45^\circ \, d\theta \sin\theta \, d\theta$

$$= \frac{P_n r^2}{4} \qquad (57)$$

This value of T must be multiplied by the dynamic load factor, 2.

The required area is,

$$A = \frac{2\,T}{f_s} \qquad (58)$$

The weight of the tie ring is found by multiplying the area, A, by the length, $2\pi r \sin 45^\circ$, and the unit weight, 490 lbs. per cu. ft. The weight is then given by Equation (59),

$$W_R = .785 \times 10^{-2} P_n L^3 \qquad (59)$$

The cost at $.25/lb is,

$$C_R = .196 \times 10^{-2} P_n L^3 \qquad (60)$$

For tie rings in horizontal domes, $P_n = \frac{\pi}{4}(\Delta p_1 - q/2)$ as shown in Appendix III.A.4. For tie rings in vertical domes P_n is determined by the method given in section V.A.2.b. for hard structures.