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Optimal Commercial Satellite Leasing Strategies

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Project AIR FORCE

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Preface

The Department of Defense needs far more satellite communications capacity than it owns and thus must lease satellite communications services. This report examines a "rule of thumb" that communications planners can use to make efficient satellite leasing decisions while facing uncertain demand for satellite services.

The research is a continuation of work originally undertaken at the request of Headquarters, United States Air Force (SC), the Assistant Secretary of the Air Force for Acquisition (SAF/AQS), and the Air Force Space Command (XP and SC). It is part of the Employing Commercial Communications task within Project AIR FORCE's Aerospace Force Development Program.

This research should interest defense analysts concerned with obtaining satellite communications within the Air Force, the other military services, and the defense agencies. It should also be of general interest to those making procurement decisions while facing uncertain demand.

Project AIR FORCE

Project AIR FORCE, a division of RAND, is the Air Force federally funded research and development center (FFRDC) for studies and analyses. It provides the Air Force with independent analyses of policy alternatives affecting the development, employment, combat readiness, and support of current and future aerospace forces. Research is performed in four programs: Aerospace Force Development; Manpower, Personnel, and Training; Resource Management; and Strategy and Doctrine.

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Summary

There is a gap that will extend into the foreseeable future between the military requirement for long-haul wideband communications and communications satellite capacity the military owns.[1] The United States government will need to bridge the gap by leasing commercial communications satellite services. Military communications planners are faced with the difficult task of choosing the appropriate amount of communications capacity to lease, and the appropriate length of the lease, given uncertainty over future communications demand.

This report presents a simple, graphical technique to help communications planners determine the appropriate amount of communications capacity to lease when facing uncertain demand. A simple mathematical model shows why the graphical technique works. Extensions to the basic model show how price uncertainty and the ability to salvage unused capacity change the appropriate amount of capacity to lease. Finally, a multiple-period version of the basic model shows how communications planners can consider the trade-offs between long- and short-term leases when demand grows over time.

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[1]Headquarters, United States Space Command, *Advanced Military Communications Capstone Requirements Document*, Peterson Air Force Base, Colorado, April 24, 1998.

Acknowledgments

I would like to thank RAND Graduate School Fellow David Persselin for providing me with his inventory theory bibliography and for his tenacity in tracking down the literature on "optimal rules of thumb." I also thank the director of Project AIR FORCE's Aerospace Force Development Program, Tim Bonds, who offered useful feedback and encouragement in a critical phase of this work, and RAND colleague Edward Chan for his thoughtful review of an earlier draft of this report.

1. Introduction

United States Space Command projects that long-haul wideband communications required for daily military operations will grow from 1 gigabit per second (Gbps) today to roughly 9 Gbps in 2008.[1] Crisis response demand is projected to grow from less than 1 Gbps to 4 Gbps. The current capacity of military communications satellites for wideband communications is about 1 Gbps; the capacity will grow to less than 4 Gbps in 2004 when the Gapfiller satellite system comes into operation. Thus, the projected military-owned supply will fall well short of the projected demand into the foreseeable future. The United States government will need to bridge the gap by leasing commercial communications satellite services.

Commercial systems currently supply 200 to 250 Gbps of long-haul, wideband capacity to both commercial and government users, at a variety of lease length and price combinations. Prices, as one might suppose, decrease with increasing lease length. Military communications planners must choose the appropriate amount of satellite communications capacity to lease, and the appropriate length of the lease, in the face of an uncertain future demand. Because longer leases purchased in advance tend to be less expensive, the decisionmaker must find the appropriate amount of capacity to lease in advance of the actual realization of demand, knowing that he will have to meet any demand overages with short-term leases on the spot market.

This is a difficult problem. In this report, I identify a simple graphical technique that communications planners could use to gain insight into the amount of communications capacity to lease. The graphical technique is based on a static model that employs a well-known result in the optimal inventory literature, and I show how, with some restrictions, a dynamic model can be solved by the same graphical technique as a series of static problems.

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[1]Headquarters, United States Space Command, *Advanced Military Communications Capstone Requirements Document*, Peterson Air Force Base, Colorado, April 24, 1998.

The Graphical "Rule of Thumb"

A communications planner faces the following basic problem:

Demand for future satellite communications capacity is uncertain. You can buy some fixed capacity now before you find out the actual demand. You can buy any additional capacity you need on the spot market after you learn the actual demand. How much fixed capacity should you buy now to minimize expected cost?

To solve this problem, first draw a curve that indicates the probability that demand will be greater than or equal to a particular value (see Figure 1). The horizontal axis indicates the level of demand, and the vertical axis indicates the probability.

Then, to find the expected cost-minimizing amount of capacity to purchase in advance, find the ratio of the price of advance capacity to spot market capacity, and locate the corresponding point on the vertical axis. Draw a horizontal line from the point on the vertical axis to the curve, and then drop a vertical line to the horizontal axis. The line will intersect the horizontal axis at the cost-minimizing amount of capacity to be purchased in advance.

An explanation of why this technique works is detailed in the second chapter, along with some extensions examining the effects of being able to sell off excess capacity and price uncertainty. The third chapter shows where this approach can be used to evaluate leases of varying lengths in a dynamic environment. The final chapter offers conclusions and recommendations for further work.

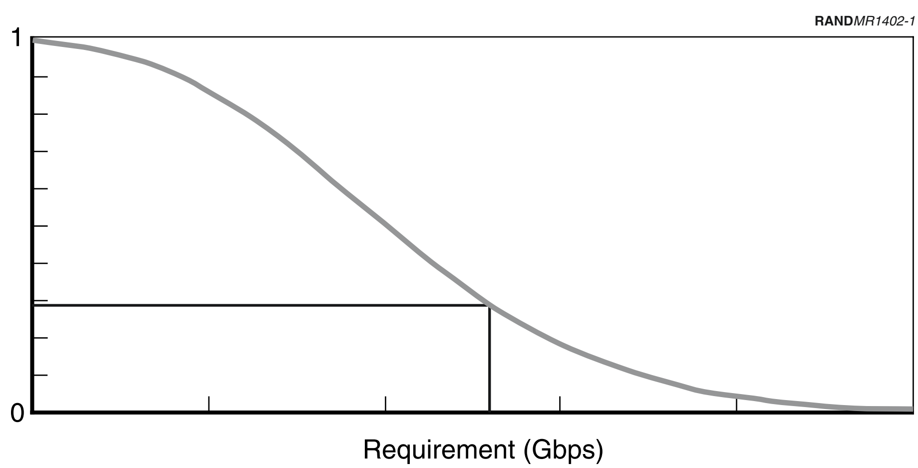

Figure 1—Graphical Solution

Related Literature

This report touches on two strands in the literature. The first strand concerns optimal capacity or inventory decisions under uncertainty, and the second strand deals with determining optimal rules of thumb for economic actors.

Optimal Capacity

The optimal capacity decision problem is closely related to the optimal inventory problem, and indeed, in a one-period problem, can be identical with the inventory problem. The communications capacity problem described above is essentially identical to the classic newsvendor problem, where there are costs of purchasing or holding inventory, of being short of inventory, and possibly a salvage value on excess inventory. Arrow, Karlin, and Scarf (1958) provide an exhaustive treatment of the basic inventory problem. Other important references on the problem are Arrow, Harris, and Marschak (1951), Whitin (1952), and Dvoretzky, Kiefer, and Wolfowitz (1952a, b). Scarf (1959) provides an exposition of a Bayesian approach to solving the inventory problem when the underlying probability distribution of demand is unknown. Veinott (1966) provides a good tutorial on and a literature review of standard mathematical inventory theory. Ridder, Laan, and Salomon (1998) provide a modern treatment of the newsvendor problem.

The dynamic optimal capacity problem under uncertain demand is treated in Manne (1961). In Manne's paper, demand evolves according to a geometric Brownian motion process, and capacity, once purchased, endures forever. These assumptions allow Manne to solve the problem by considering finite subintervals where the starting or regeneration points are determined by when demand has grown by a particular amount.

Luss (1982) provides a detailed review of the capacity expansion operations research literature.

A more recent vein of the literature considers capacity investment as a special case of irreversible investment under uncertainty, and uses an options theory or real options approach. These papers generally model the demand curve as shifting over time stochastically, with the demand shift parameter following a geometric Brownian motion process. Pindyck (1988, 1991) and Abel and Eberly (1994, 1996) typify this approach.

Rules of Thumb

Baumol and Quandt (1964), in their seminal paper "Rules of Thumb and Optimally Imperfect Decisions," list several desirable attributes of rules of thumb.[2] They advocate a simulation approach for evaluating the effectiveness of a particular rule of thumb with the optimal solution. Smith (1991) demonstrates a simulation method for determining the optimal parameter values for a specific rule of thumb for solving a stochastic dynamic programming problem. Smith (1992) explores the implications of agents using near-optimal rules of thumb for the ability of economists to empirically distinguish between the "standard" real business cycle model and alternatives. Krusell and Smith (1996) examine the robustness of the general equilibrium stochastic growth model to agents using near-optimal rules of thumb. Lettau and Uhlig (1999) model how agents would choose among competing rules of thumb, and explore how the performance of the equilibrium rules of thumb differs from dynamic programming. Quite frequently in this literature, researchers identify relatively simple decision rules that are either optimal or are only pennies away from optimality. I examine one such rule.

[2] ". . . we define a rule of thumb to be a set of rules describing a decision procedure with the following characteristics:

(a) The variables which are employed in the decision criteria are objectively measurable.

(b) The decision criteria are objectively communicable, and decisions do not depend on the judgment of individual decision-makers.

(c) As a corollary to (b), every logically possible configuration of variables corresponds to a (usually unique) determinate decision.

(d) The calculation of the appropriate decision is simple, inexpensive, and well suited for frequent repetition and for spot-checking by management in higher echelons."

2. The One-Period Model

After I explain why the graphical technique outlined in the introduction gives the cost-minimizing solution to a class of planning problems, I show two extensions to the basic model: one where the government can obtain a partial refund on capacity contracted for but not used, and one where there is some uncertainty over the spot-market price.

To reiterate, the basic problem a communications planner faces is:

Demand for satellite communications capacity is uncertain. You can buy some fixed capacity before you find out the actual demand. You can buy any additional capacity you need on the spot market after you find out the actual demand. How much fixed capacity should you buy to minimize expected cost?

A slightly more formal statement of the problem is:

Suppose that demand for satellite communications x is distributed as $f(x)$. You can buy fixed capacity a for total cost $p_a a$ before you find out the actual demand. You can buy any residual capacity needed on the spot market for $p_s(x-a)$, if x is greater than a. What is the value of a that minimizes expected cost?

We can write the following expression for the expected cost C:

$$C = p_a a + p_s \int_a^{\infty} (x-a) f(x) dx \qquad (1)$$

where p_a is the price of a,

a is the amount of fixed capacity,

p_s is the spot market price,

x is communications demand, and

$f(x)$ is the distribution of demand x.

The first term in the expression, $p_a a$, gives the total cost of fixed capacity a. The second term in the expression,

$$p_s \int_a^{\infty} (x-a) f(x) dx,$$

gives the expected expenditure on spot-market communications capacity, given that fixed capacity is a.

We can find the value of a that minimizes the expected cost C by using the first-order condition to solve for the optimal value a^*. The first-order condition is simply that the first derivative of C with respect to a is equal to zero. Equation (2) gives the first-order condition.

$$\frac{\partial C}{\partial a} = p_a - p_s \int_a^\infty f(x)dx = 0\,. \tag{2}$$

We can rewrite Eq. (2) in terms of $F(a)$, 1 – the cumulative density function,

$$\frac{\partial C}{\partial a} = p_a - p_s F(a) = 0 \tag{3}$$

where

$$F(a) = \int_a^\infty f(x)dx\,.$$

Solving Eq. (3) for a gives

$$a^* = F^{-1}\left(\frac{p_a}{p_s}\right) \tag{4}$$

where $F^{-1}(\cdot)$ denotes the inverse of $F(\cdot)$ (i.e., $F^{-1}[F(x)] = x$). a^* is the amount of fixed capacity that minimizes the expected cost.

We can verify that we are at a local minimum by examining the second-order condition:

$$\frac{\partial^2 C}{\partial a^2} = -p_s F'(a) \tag{5}$$

which is positive, because $-p_s$ is negative and $F'(a)$ is negative.

Graphical Interpretation of the Solution

Figure 2 gives a graphical interpretation of the solution. It shows an illustrative cumulative distribution function, where the vertical axis gives the probability that demand will be greater than or equal to x. The price ratio is also read off the vertical axis. The graphical process of drawing a line from the price ratio on the vertical axis to the cumulative distribution function line and then dropping a line

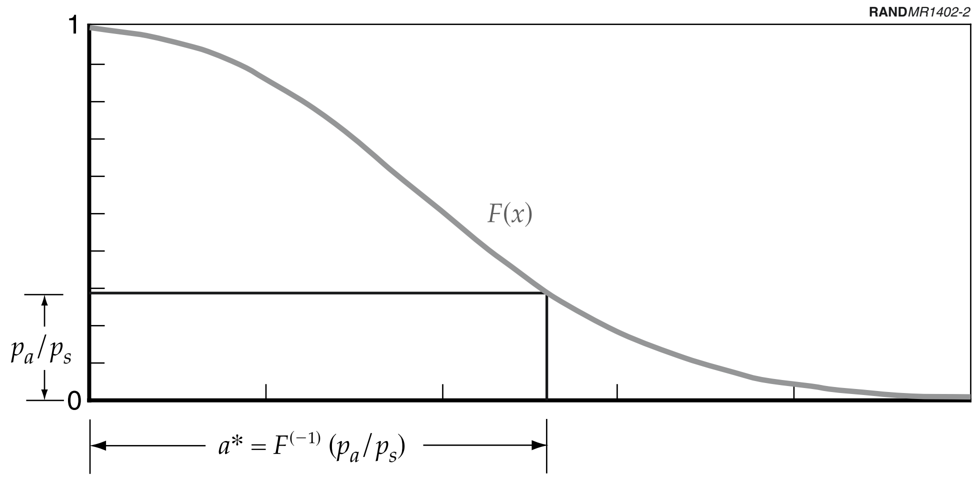

Figure 2—Graphical Interpretation of the Optimal Solution

to the horizontal axis to find the optimal capacity is equivalent to solving the equation $a^* = F^{-1}(p_a / p_s)$ to find the optimal capacity.[1]

Figure 3 gives an alternative graphical interpretation of the solution. In Figure 3, the hatched area to the right of the optimal capacity is the probability that demand will exceed the capacity. This probability is exactly equal to the price ratio.[2] We are led to another point: if an expected cost-minimizing

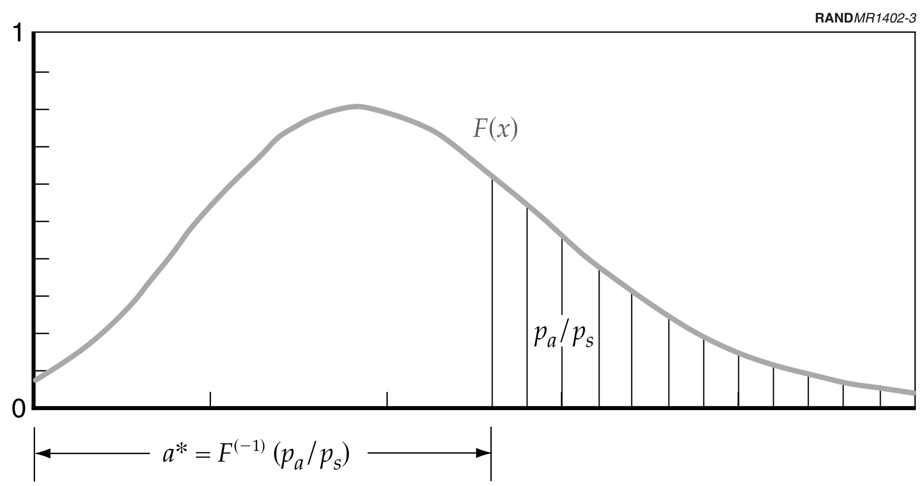

Figure 3—Alternative Graphical Interpretation of the Optimal Solution

[1]Whitin (1952) uses a similar diagram in his analysis of the optimal inventory of seasonal goods.

[2]Arrow, Harris, and Marschak (1951) provide a similar diagram to illustrate the optimal solution to a one-period inventory problem.

decisionmaker chooses a certain capacity, then the decisionmaker is implicitly asserting that the probability of demand exceeding that capacity is no more than a price ratio. If decisionmakers differ in their assessment of the optimal capacity, it is because their probability estimates differ.

Key Assumptions

This model, while giving a very general solution, is based on several key assumptions:

- The probability distribution of demand is known.
- Minimization of present value of expected cost is the appropriate objective.
- Demand is exogenous.
- Relative prices are exogenous.
- Prices are linear in quantity.

Let's briefly discuss each of these assumptions in turn.

The probability distribution of demand is known.[3] This assumption is perhaps the most debatable. Quantifying uncertainty is difficult. Military planners generally have little information on the historical distribution of demand. In addition, the distribution of demand is not stationary over time, adding to the difficulty of estimating the current distribution based on the little historical data available. Also, demand is subject to spikes associated with unique historical events, only some of which can be anticipated by the planner. All these effects combine to make any effort to determine the distribution of demand seemingly futile.

However, even if we cannot get a precise statement of the distribution faced by the decisionmaker, all is not lost. Part of the value of the graphical solution of this model is that it allows decisionmakers to make explicit the relationship between buying decisions and their judgment as to the likelihood of certain

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[3]Dvoretzky, Kiefer, and Wolfowitz (1952a) offer some stern words on this subject: "It may be objected that our method requires one to specify the function *W* [the probability distribution of demand] and that this function may be unknown or difficult to give. We wish to emphasize that the need for a function *W* is inevitable in the sense that any method which does not explicitly use a function *W* simply uses one implicitly. Thus one who selects a method of solving the inventory problem which ostensibly has the advantage of not requiring the specification of *W* is simply relinquishing control of *W*, and may be implicitly using a *W* of which he would disapprove (if he knew it). It is difficult to see what advantage accrues to an ordering agency from deliberately burying its intellectual head in the sand. Even if the function *W* is very difficult to obtain it seems preferable to make some attempt at an intelligent decision about it. A rough approximation or greatly simplified version of the underlying *W* may be preferable to completely ignoring this fundamental datum of the problem."

outcomes. Judgments on the likely distribution of demand can be compared and debated, and the participants in the conversation can clearly see the effect of alternative assumptions on the optimal capacity. The model also makes some relationships more explicit—for example, a decisionmaker's choice to set capacity at a particular level implies that he thinks that the chance of demand exceeding that level is no more than the price ratio.

Minimization of expected cost is the appropriate objective. This assumption means that the objective function considers only expected cost and not the possible variance in cost. If the decisionmaker is risk averse, this would not be an appropriate assumption. Since the decisionmaker in this case is acting in the interests of the U.S. government, and the government is highly diversified, it is reasonable to assume that the government is risk neutral, i.e., that the government cares only about expected cost and not about possible variance in cost.

Demand is exogenous. This assumption means that the demand given by the distribution function is not a function of the price of communications capacity. Demand in this case is set externally and is not a decision variable; the only decision variable available to the decisionmaker is the amount of fixed capacity to buy. The decisionmaker has no effect on the amount demanded.

Relative prices are exogenous. This assumption means that the decisionmaker is a "price-taker" and his decisions do not influence the market price.

Prices are linear in quantity. This assumption means that the total cost of a particular quantity of communications capacity can be expressed as the product of the quantity and a price. That is, there are no "quantity breaks."

Unfortunately, it seems unlikely that analytic solutions, much less graphical solutions, exist for models that violate these assumptions.

Extending the Basic Model

Redeeming Unused Capacity

A new proposed contractual mechanism would allow the Air Force to redeem unused capacity for some amount less than the original purchase price.[4]

[4]The Air Force is not allowed by law to sell unused capacity on the open market, hence this alternative arrangement has been proposed. For details, see Office of the Under Secretary of Defense (Acquisition & Technology), *Report to Congress on Impediments to the Innovative Acquisition of Commercial Satellite Communications*, Washington, D.C., June 1998.

Suppose that unused capacity can be redeemed at price ρp_s, where $0 < \rho < p_a / p_s$ (that is, the redemption price is always less than the advance purchase price). Then the planner's problem becomes

$$C = p_a a + p_s \int_a^{\infty} (x - a) f(x) dx - \rho p_s \int_{-\infty}^{a} (a - x) f(x) dx . \tag{6}$$

The first-order condition then becomes

$$\frac{\partial C}{\partial a} = p_a - p_s \int_a^{\infty} f(x) dx - \rho p_s \int_{-\infty}^{a} f(x) dx = 0 . \tag{7}$$

Rewriting Eq. (7) in terms of

$$F(a) = \int_a^{\infty} f(x) dx$$

yields

$$\frac{\partial C}{\partial a} = p_a - p_s F(a) - \rho p_s [1 - F(a)] = 0 .$$

Solving for *a**,

$$a^* = F^{-1}\left(\frac{p_a - \rho p_s}{p_s - \rho p_s} \right). \tag{8}$$

Note that $(p_a - \rho p_s) / (p_s - \rho p_s)$ decreases as ρ increases, which implies that *a** increases as ρ increases. This result is intuitively appealing, for it simply means that the higher the redemption value of unused capacity, the higher the amount of capacity you would be willing to buy in advance.

Stochastic Spot-Market Price

In addition to demand uncertainty, planners may also face price uncertainty. Suppose that price p_s is distributed as $h(p_s)$. Then we can write the expected cost as

$$C = p_a a + \int_{-\infty}^{\infty} \left[p_s \int_a^{\infty} (x - a) f(x) dx \right] h(p_s) dp_s . \tag{9}$$

The first-order condition then becomes

$$\frac{\partial C}{\partial a} = p_a \int_{-\infty}^{\infty} p_s h(p_s) dp_s \int_a^{\infty} f(x) dx = 0 . \tag{10}$$

Rewriting Eq. (10) in terms of

$$F(a) = \int_a^{\infty} f(x) dx$$

yields

$$\frac{\partial C}{\partial a} = p_a - \int_{-\infty}^{\infty} p_s h(p_s) dp_s F(a) = 0 . \tag{11}$$

Solving for *a**,

$$a^* = F^{-1}\left(\frac{p_a}{\int_{-\infty}^{\infty} p_s h(p_s) dp_s} \right) \tag{12}$$

which shows that in this static, one-period model, only the expected value of the spot price enters into the expression for the optimal capacity, and the variance of the spot price has no effect on the optimal capacity. This may seem counterintuitive, but it results from the assumption that expected cost is the appropriate measure of merit and that the decisionmaker, when acting for the government, is risk neutral.

3. A Multi-Period Model

The static one-period model is attractive because it is mathematically tractable and has a simple graphical solution that can be easily implemented. However, communications planners face a dynamic, multiple-period world where capacity decisions made in one period can have influence for many later periods. It seems that the optimal capacity will depend not only on the demand distribution of the immediate period but also on the distributions of demand far out into the future. So the question naturally arises, can the graphical solution method still provide useful insight in a multiple-period world?

The answer is a qualified yes. Under certain restrictions, outlined below, the graphical technique can still provide the optimal answer, even though the decisionmaker takes into account only the demand distribution of the immediate period.

Suppose that demand for satellite communications x at time t is distributed as $f_t(x)$. You can lease one period of capacity a_t for cost $p_a a_t$ or lease two periods of capacity b_t for price $p_b b_t$. You can buy any residual capacity needed on the spot market for

$$p_s\left(x - a_t - \sum_{j=0}^{1} b_{t-j}\right).$$

What values of a_t and b_t, $t = 0 \ldots \infty$ minimize expected cost?

We can write a recursive expression for expected cost from time t forward as

$$V_t(c) - p_a a_t + p_b b_t + p_s \int_{a_t+b_t+c}^{\infty} (x - a_t - b_t - c) f_t(x)dx + \beta V_{t+1}(b_t)$$

where a_t is the amount of one-period contracts purchased at time t,

b_t is the amount of two-period contracts purchased at time t,

p_a is the price of a_t,

p_b is the price of b_t,

c is the stock of two-period contracts purchased in the previous period, i.e., b_{t-1}, for $t \geq 1$,

β is the discounting term, and

$f_t(x)$ is the distribution of demand x at time t.

To solve this problem, we consider two subcases—one where the price of the two-period lease is greater than or equal to the discounted present value of the one-period lease over two periods, i.e., $p_b \geq p_a(1+\beta)$, and one where the price of the two-period lease is less than the discounted present value of the one-period lease over two periods, or $p_b < p_a(1+\beta)$.

Clearly, if $p_b \geq p_a(1+\beta)$, there is no advantage to buying the longer lease, since the discount for buying "in bulk" is outweighed by the time value of money as reflected in the discount rate, and the optimal solution will set $b_t \equiv 0$ for all t and

$$a_t^* = F_t^{-1}\left(\frac{p_a}{p_s}\right)$$

as in the one-period problem. So the multi-period problem becomes simply a sequence of one-period decisions.

Now, let's consider the case where $p_b < p_a(1+\beta)$. If $f_t(x)$ is stationary, i.e., $f_{t+1}(x) = f_t(x)$, or if $f_t(x)$ grows over time, i.e.,

$$\int_s^\infty f_{t+1}(x)dx \geq \int_s^\infty f_t(x)dx$$

for all s and t, then

$$F_t^{-1}\left(\frac{p_b}{p_s(1+\beta)}\right) \geq F_{t-1}^{-1}\left(\frac{p_b}{p_s(1+\beta)}\right).$$

That is, the optimal capacity in period t will always be greater than or equal to the optimal capacity in period $t-1$, so if we buy all our capacity in $t-1$ in two-year contracts, none of it will go to waste in period t.

Thus, in this case, the optimal solution will be to set $a_t \equiv 0$ for all t and

$$b_t^* = F_t^{-1}\left(\frac{p_b}{p_s(1+\beta)}\right) - b_{t-1}$$

for all t.

Hence, for the case where $f_t(x)$ is stationary or grows over time, myopically choosing b_t^* based solely on this period's distribution and the capacity on hand results in the cost-minimizing choice of capacity over time. Figure 4 illustrates this solution.

Unfortunately, if $f_t(x)$ is declining or fluctuating over time, there is generally no analytic closed-form solution, and thus the problem will not be subject to solution by the type of technique examined above (Veinott (1966)). However, problems of this type can be solved by numerical techniques under specific assumptions as to the form and parameters of the demand distributions.

The one-period graphical technique is still useful if demand is stationary or growing in the sense described above, or if the amortized cost of a multi-period lease is greater than that of a single-period lease.[1] Fortunately this seems to be the case for many military communications planning applications. But what about the case where the amortized cost of a multi-period lease is less than that of a single-period lease and demand is declining or may fluctuate over time?

In this case, the optimal solution will involve either all multi-period leases or a mix of multi-period and single-period leases. Unfortunately, not much more can

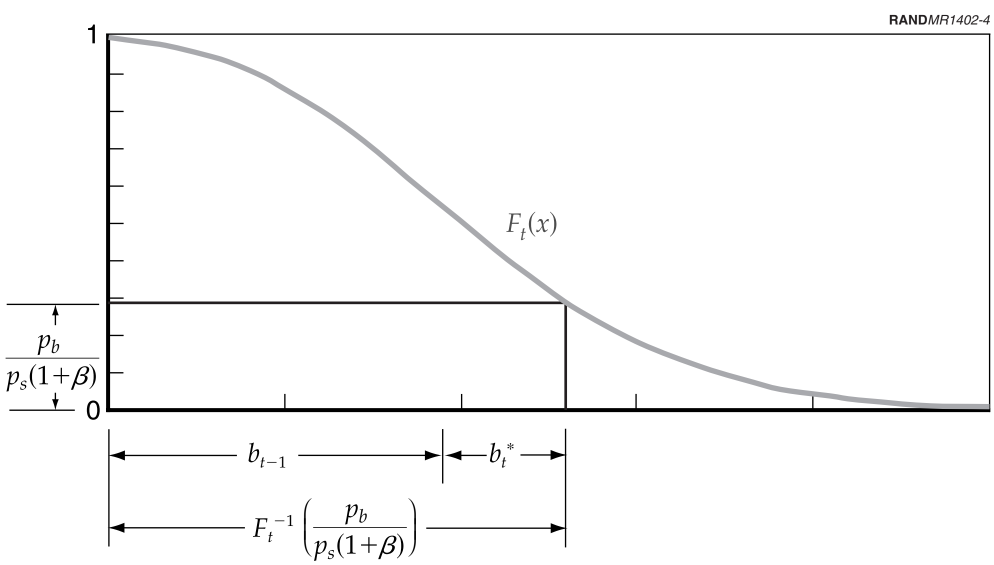

Figure 4—Graphical Interpretation of the Multi-Period Model

[1]This latter case may hold more frequently than one would guess. The Department of Defense has recently negotiated several one-year lease contracts for satellite transponders at rates within a few percentage points of longer-term contracts. Given a sufficiently high discount rate, the Department of

be said without assuming a particular functional form for the evolution of the demand distribution over time, and even then the resulting solution may be difficult to characterize as a rule of thumb. However, if one is willing to postulate specific demand distributions over time, there is at least one technique for finding an "optimal" approximation to the optimal decision rule. Smith (1991), in a numerical experiment, compares the performance of an optimal decision rule for a stochastic dynamic programming problem to the performance of a simple partial-adjustment rule of thumb, where the parameter governing the percentage of the full adjustment to make in each period was chosen to maximize the objective. (In a partial adjustment rule, the planner observes current capacity and current demand, and buys long-term leases of *m* percent of the difference between current demand and current capacity, and buys (100 – *m*) percent as short-term leases.) Smith found the performance of the optimal partial-adjustment rule of thumb to be within a few percentage points of that of the optimal decision rule for the problem he examined. Perhaps this technique could be fruitfully applied to satellite communications planning for those cases not covered by the graphical rule of thumb.

Defense may be facing a regime in which the price of multiple-year contracts is strictly greater than the discounted present-value stream of payments for one-year contracts.

4. Conclusions and Recommendations for Further Investigation

We have demonstrated a graphical technique for solving a particular type of communications capacity problem, illustrating its applicability and limitations in a multiple-period context. Military satellite communications planners should consider using this approach, or perhaps a more elaborate one, for gaining insight in the initial evaluation of leasing decisions.

Further work along the lines of Smith (1991) is needed to address those cases where the distributions of demand are not static or increasing in the sense described in Chapter 3. This will require assumption of specific functional forms and parameters for the evolution of demand over time in order to yield a dynamic program that can be solved by numerical methods. However, it may be possible to find a rule of thumb that would perform well over the scenarios typically faced by communications planners, and come closer to the attributes proposed by Baumol and Quandt for decision criteria to be directly communicable and not dependent upon the judgment of individual decisionmakers.

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