TECHNICAL DESCRIPTION OF THE OPTIMIZATION MODEL

This appendix provides a technical description of the optimization model developed to examine the options presented in the body of the report. The model uses a linear programming construct and is coded in the GAMS software package.

MODEL DIMENSIONS

There are five dimensions, or subscripts, used in the model:

- Subscript i represents the two Army components, AC and RC.
- Subscript j represents the 51 potential home locations (50 states plus the District of Columbia) of the students.
- Subscript k represents the 111 maintenance-related courses included in the analysis. A list of the courses is provided in Appendix C.
- Subscript l represents the 27 different AC (10 schools and NCO Academies) and RC (17 RTS-Ms) schools that conduct maintenance-related courses. A list of the schools is provided in Appendix B.
- Subscript g represents the 11 different course groupings. The relationship of courses to groups is based on department structure at Aberdeen or on the courses offered at the other AC schools. The MOSs or functional courses in each group are shown in Table A.1.

Table A.1

Relationship Between Groups and MOSs

Group Name	MOSs/Functional Courses
ANCOC	All AC 63 level
Construction equipment	62B
Field artillery	45D, 63D
Metalworking	44B, 44E
Quartermaster	92A, TAMMS, ULLS
Tactical support equipment	52C, 52D, 52F, 63J
Tanks and Bradleys	45E, 45T, 63E, 63N, 63T
TOW/Dragon repair	27E
Weapons	45B, 45G, 45K
Wheel and track vehicle (DS/GS)	63G, 63H, 63W, 63Y
Wheel vehicle (organizational)	63B, 63S

DECISION VARIABLES

There are four decision variables in the model:

- $X_{i,j,k,l}$ is the number of students in component *i* from home location *j* taking course *k* at school *l*.
- $W_{k,l}$ is a binary (0,1) variable indicating that course *k* is offered at school *l*. This variable can be preset in the model to force certain courses to be offered at certain schools (option 1), or the model can determine the optimal assignment of courses to schools (options 2 and 3).
- Y_l is a binary (0,1) variable indicating school *l* is open. This variable can be preset in the model to force a school to be open (options 1 and 2), or the model can determine the optimal set of schools to use for the course offerings (option 3).
- $T_{g,l}$ is a binary (0,1) variable indicating that course group *g* is offered at school *l*. This variable can be preset to indicate that a set of courses are offered at a school (option 1 and 2 multi-functional), or the model can determine the optimal assignment of course groupings to schools (option 2 specialized and option 3).

As indicated above, some of the decision variables can be preset, or not included in the model, depending on the specific option being examined.

PARAMETERS

There are a number of parameters included in the model:

- $D_{j,l}$ is the round-trip distance in miles from a student's home location j to school l. Data from the ATRRS files provide the home state for RC students and the state where AC students' units are located. The model uses the latitude and longitude for the centroid of each state and the latitude and longitude for each school to calculate the straight-line distance.
- E_k is the length of course k in days. These data are from ATRRS.
- $\$D$ is the cost per mile. A factor of .30 is used.
- $\$F$ is the fixed annual cost for offering a course. This cost includes initial supplies and courseware and the required training for the course instructor. We use a value of $50,000 based on earlier research.[1]
- $\$_l$ is the fixed annual cost for having school l open. It includes administrative staff personnel, utilities, and annual facilities maintenance costs. We use a factor of $370,000 for all schools and RTS-Ms.
- $R_{i,j,k}$ is the number of students from component i at home location j requiring course k. These data are from the fiscal year 1996 ATRRS files and represent the actual number of students who were trained.
- M_k is the minimum number of students required to offer course k. We use a value of 5 students for all courses except those courses where the annual demand was less than 5 students. For those low-demand courses, we use the annual demand as the minimum class size.

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[1]See Shanley et al. (1997) for a discussion of how this $50,000 cost was derived.

- M_g is the minimum number of students in group g in order for a school to offer courses in that group. We use a factor of 50 for all groups.
- $Q_{k,l}$ is the maximum number of students in course k at school l. These data are from the ATRRS schedule file.
- Q_l is the maximum number of student days at RTS-M l. We use a value of 15,600 for all RTS-Ms. This value is based on an average of data from the RTS-Ms at Forts Stewart, Bragg, and Dix. It considers the number of classrooms and instructors available at an RTS-M. It relates to 1,200 students a year taking an average 13-day course.
- $S_{k,g}$ is a binary variable set to one if course k is in group g, and zero otherwise.

MODEL OBJECTIVE FUNCTION

The objective of the model is to minimize costs. For the general model, the mathematical representation is

$$\text{Minimize } \sum_i \sum_j \sum_k \sum_l X_{i,j,k,l} \, (D_{j,i} \$D) \; + \; \sum_k \sum_l \$F \, W_{k,l} + \sum_l \$_l \, Y_l \, .$$

The first term represents the travel cost and the variable course costs (currently set to zero) associated with sending X students in component i from location j to school l to take course k. The second term captures the fixed cost of offering a course at a school. The last term represents the fixed cost in having a school open.

In option 1 and the multifunctional case of option 2, we use only the first cost element, the travel cost (i.e., we set $\$F$ and $\$_l$ equal to zero). For the specialized case of option 2, we use the first two elements of cost, the travel and fixed course costs (i.e., we set $\$_l$ equal to zero). For option 3, we use all three elements of cost.

MODEL CONSTRAINTS

There are several constraints in the model that define the potential solution space for the objective function. We list these constraints below and describe how they are, or are not, used for the specific options we analyzed.

Supply Equals Demand

$$\sum_l X_{i,j,k,l} = R_{i,j,k} \quad \text{for all } i, j, k\ .$$

In each of the options, we ensure that the AC and RC students who were trained in fiscal year 1996 in the maintenance-related courses we are considering are assigned to an AC school or an RTS-M.

Assign Students to Schools Where the Course Is Taught

$$\sum_i \sum_j X_{i,j,k,l} \leq \sum_i \sum_j R_{i,j,k}\ W_{k,l} \quad \text{for all } k, l\ .$$

In each of the options, students can only take courses at the AC schools or the RTS-Ms where the courses are offered. In option 1 and the multifunctional case of option 2, we use the fiscal year 1996 assignment of courses to schools as reflected in ATRRS to predefine the $W_{k,l}$ binary variable. For the specialized case in option 2 and for option 3, we allow the model to determine the least-cost assignment of courses to schools based on the local demand for a course.

Ensure Minimum Course Size Requirements Are Met

$$\sum_i \sum_j X_{i,j,k,l} \geq M_k\ W_{k,l} \quad \text{for all } k, l\ .$$

This constraint is used for all options. It prevents the model from assigning less than a minimum number of students to a school for a specific course.

Ensure Maximum Course Size Is Not Exceeded

$$\sum_i \sum_j X_{i,j,k,l} \leq Q_{k,l} \quad \text{for all } k, l\ .$$

This constraint is used for all options and prevents the model from assigning more students to a school for a specific course than the maximum number that a school can accommodate.

Ensure Maximum Capacity of a RTS-M Is Not Exceeded

$$\sum_i \sum_j \sum_k X_{i,j,k,l} E_k \leq Q_l \quad \text{for all RTS-Ms.}$$

This constraint is used for all options. It ensures that the capacity of an RTS-M, expressed in annual student days, is not exceeded. That is, the total number of student days associated with a number of students taking various courses must be within the capacity of the RTS-M.

Assign Courses to Schools Where Their Groups Are Taught

$$\sum_i \sum_j \sum_k X_{i,j,k,l} S_{k,g} \leq 10{,}000\ T_{g,l} \quad \text{for all RTS-Ms } l \text{ and groups } g.$$

This constraint is used for the specialized case of option 2 and for option 3. It makes sure that courses can be taught at schools that have the responsibility for the group to which the course belongs. The 10,000 factor is an arbitrarily large number that ensures if $T_{g,l}$ is one, then courses in that group can be taught at the school.

Comply with the Minimum Number of Student Days Needed for a Course-Group to Be Taught at an RTS-M

$$\sum_i \sum_j \sum_k X_{i,j,k,l} S_{k,g} \geq 50\ T_{g,l} \quad \text{for all RTS-Ms } l \text{ and groups } g.$$

This constraint is used for the specialized case of option 2 and for option 3. It ensures that groups are assigned to schools only when there are at least 50 students taking courses within that group.

Teach Courses Only at Schools That Are Open

$$W_{k,l} \leq Y_l \quad \text{for all } k \text{ and } l.$$

This constraint is used in option 3. It ensures that courses are offered only at schools that remain open (i.e., that are assigned maintenance-related courses).