Chapter Three

CRUISE MISSILE AND BALLISTIC MISSILE DEFENSE

This chapter examines the information aspects of ship defense against ASCMs while those ships conduct TBMD. Overall, the defense problem is analyzed as a double-queuing problem. First, the launched ASCMs and ballistic missiles in a given time period, T, enter an initial engagement queue based on an assessment of the likelihood that cruise missiles will be a threat to the defenders (two Aegis cruisers) or that the ballistic missiles will be a threat to critical infrastructure targets. Second, if no interceptor missile defeats the incoming attack missiles, one of two things will occur: ASCM leakers will join a second queue to be "serviced" by the Close-In Weapon System (CIWS) on board the cruisers, or ballistic missile damage to land targets will be assessed.[1]

We begin by discussing the development of the "initial engagement queue" and the rate at which that queue is populated or the arrival rate of missiles for each time period. Next, we develop the appropriate "service" rates depending on the shooting policy established. Finally, we address the leaker or terminal queue.

[1]http://www.chinfo.navy.mil/navpalib/factfile/weapons/wep-phal.html. Phalanx provides ships of the U.S. Navy with a "last-chance" defense against antiship missiles and littoral warfare threats that have penetrated other fleet defenses. Phalanx automatically detects, tracks, and engages antiair warfare threats, such as antiship missiles and aircraft, while the Block 1B's man-in-the-loop system counters the emerging littoral warfare threat.

INITIATING EVENTS

Two Aegis cruisers are assigned to cover the area of operations as depicted in Figure 3.1 in order to defend against a likely enemy cruise and ballistic missile attack. Given their role in defending friendly territory, the cruisers are also likely to be targets and therefore they are prepared to defend against such an attack.

Although it is likely that other ships would be in the area of operations, for purposes of this analysis, we assume that only the two Aegis cruisers are involved in the attack and in the defensive operations.

Measures of Performance and Force Effectiveness

The Aegis cruisers have two (competing) missions: defend against cruise missile attacks against themselves and prevent enemy ballistic missiles from destroying key allied infrastructure targets. For both missions, the obvious measure of success is survivability; that is, *the fraction of the critical infrastructure targets that survive the attack and the "fraction" of the cruisers that survive the attack.* In the case of the cruisers, the "fraction surviving" may not be meaningful in that a

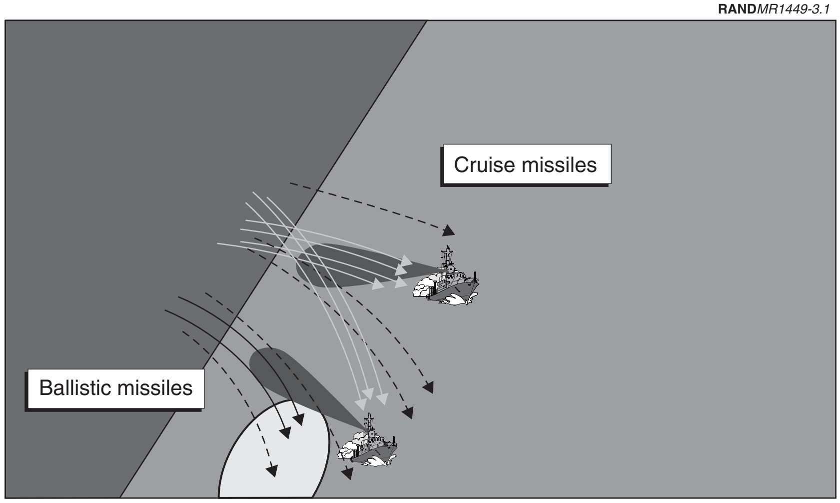

Figure 3.1—Coordinated Enemy Attack

single hit by a cruise missile is likely to result in damage sufficient to render the ship useless.

Given the two missions, priority is clearly given to defending the two Aegis cruisers. If they fail to defend themselves, they cannot conduct TBMD. Depending on the nature of the attack, this can pose serious problems for the defense of allied infrastructure targets.

Few uncertainties are considered in this analysis. We assume that the AN/SPY-1 radars on board the cruisers will detect and track all missiles the enemy is able to launch with certainty.[2] Only those missiles considered likely to hit infrastructure targets and the cruisers are considered a threat.

We further assume that through the Intelligence Preparation of the Battlespace (IPB) process, the size of the enemy missile attack inventory is known. It is also possible that the fixed launch sites would be known as a result of the same process. The locations of sea- or land-based mobile launch sites are not likely to be known. However, we assume air and sea supremacy, and therefore these are no longer a serious threat. The minimum time required to launch an attack can also be estimated. What is not known, however, is the attack distribution for ballistic missiles and cruise missiles—i.e., how the enemy will schedule the attack to ensure that the friendly infrastructure targets are destroyed while minimizing interference from the defending cruisers.

Knowing the attack distribution contributes directly to the allocation of missile interceptors and therefore to the effective defense of both the cruisers and the friendly infrastructure targets. Defenders may determine that they have ample defensive weapons, for example, and therefore respond more aggressively. A measure of performance therefore is *the degree to which the friendly commander "knows" the enemy's attack distribution.* Before the attack begins little is known, and at the end of the attack the distribution is known with certainty.

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[2]The heart of the Aegis system is an advanced, automatic detect and track, multifunction phased-array radar, the AN/SPY-1. This high-powered (four-megawatt) radar is able to perform search, track, and missile guidance functions simultaneously with a track capacity of more than 100 targets. The computer-based command and decision element is the core of the Aegis combat system.

As the attack unfolds and the enemy completes its attack, more information is obtained allowing the friendly commander to increase his knowledge concerning the distribution of the remaining attack.

ALTERNATIVES

Network-centric operations include connectivity, equipment, and operating procedures (information and sensor grids). The emphasis here is on the operating procedures—i.e., the degree of cooperation between the two cruisers in servicing the targets. This in turn depends on the nature of the connectivity between the two and the command and control arrangements in place. In this simple arrangement, operations consist primarily of how the two cruisers function to accomplish the desired mission. For this study, we examine three alternatives: operations with divided duties, independent operations using a shared COP, and coordinated operations using CEC.

Cruiser Operations

The Aegis cruisers operate in pairs with one cruiser, $A^{(b)}$, directing its SPY-1 radar to detect and track ballistic missiles, while the other cruiser, $A^{(c)}$, directs its SPY-1 radar to detect and track ASCMs as depicted in Figure 3.2. This is necessary to ensure that both threats are covered given that both types of coverage cannot be provided simultaneously by a single radar.

Platform-Centric Operations—Divided Duties

In this case, the two cruisers operate almost autonomously (see Figure 3.3). That is, no mechanism on board either ship automatically shares information on the arriving threat and/or firing solutions and no central authority directs the defensive response. Control is decentralized and managed in the Command Information Center (CIC) and therefore each ship acts alone: one ($A^{(c)}$) against all incoming cruise missiles and the other ($A^{(b)}$) against all incoming ballistic

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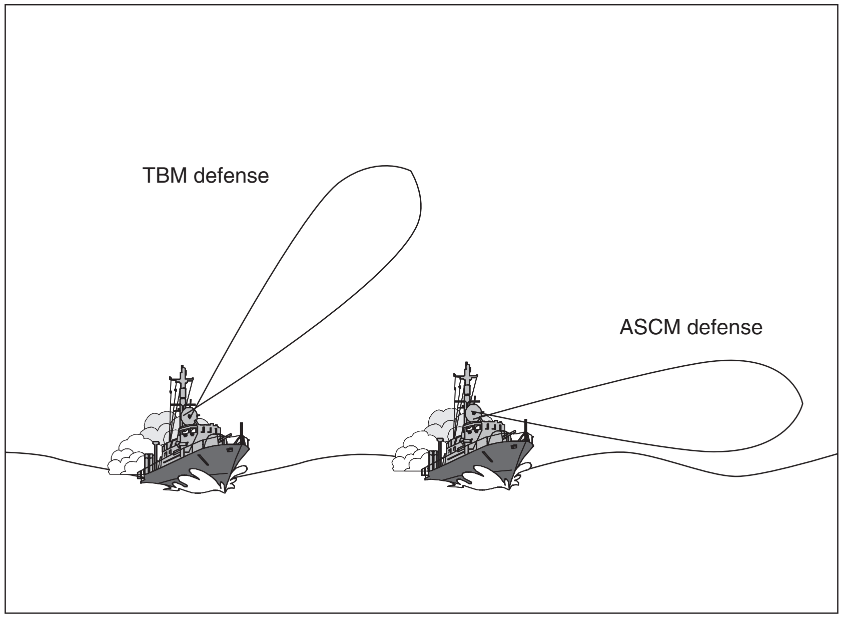

Figure 3.2—Cruiser Operations

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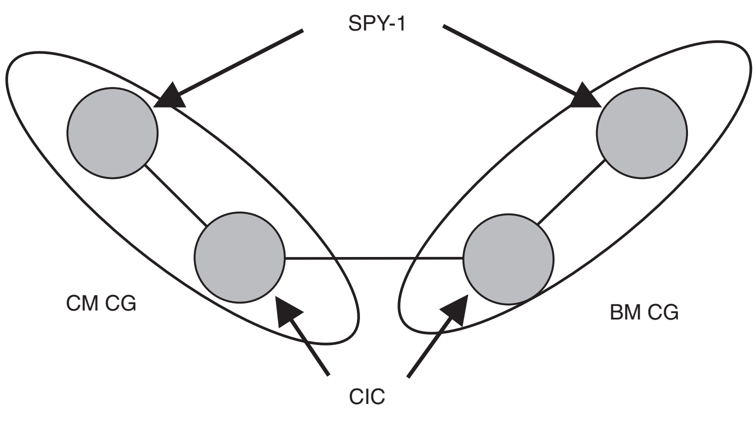

Figure 3.3—Divided Duties Connectivity

missiles.[3] Both Aegis ships employ a first-in-first-out (FIFO) queue-discipline policy for engaging incoming missiles. For cruise missiles, this means that self-preservation is collective. That is, the ship designated to intercept cruise missiles does not give itself priority against attack.

Both ships must take several decisions based on the situation they confront and the information available to them from organic and external sources. Not all of these decisions are modeled in this study. The description of the modeled decisions appears later.

- **The ballistic missile defense ship,** $A^{(b)}$, must prioritize ballistic missile threats. If it concludes that an enemy missile is not going to hit a high-value target, it may not defend against it. If it concludes that an enemy missile will impact a high-value target, it either chooses to engage it or may conclude that it will be defended by another system, such as Patriot, and therefore choose not to defend against it. If it concludes that an incoming enemy missile will impact a target no longer worth defending (because it was destroyed earlier), it will not defend against it. It must decide how many Standard missiles it is willing to assign against a given incoming enemy ballistic missile. If it appears that it could run out of defensive weapons before the ballistic missile attack ends, it must decide if that is a real problem and work to switch roles with the other cruiser (covered separately below). Although it has no inherent self-defense capability, it *can* decide how to operate so that the other ship can defend it.
- **The cruise missile defense ship,** $A^{(c)}$, must prioritize its response to cruise missile threats. It must decide between defending itself and the ballistic missile defense ship based on remaining inventories of anti–cruise missile (ACM) and anti–ballistic missile (ABM) weapons on board both ships. It must decide how many Standard missiles it is willing to assign to a given cruise missile as with the $A^{(b)}$ ship. If it appears that it could run out of defensive

[3]One or two Aegis ships could perform simultaneous cruise missile and ballistic missile defense. Use of two Aegis ships was selected for this analysis because of the more challenging command and control problem associated with using two ships and because two-ship defense was judged more appropriate to the projected threat. This arrangement should be viewed as a "first step" toward developing discrete, collaborative command and control metrics in a network-centric context.

weapons before the cruise missile attack ends, it must decide if it has a real problem and work to switch roles (covered separately below). It can also attempt to maneuver to make it easier to defend itself and the ballistic missile defense ship.

- **Role-switching decision:** The operational sequence for switching roles is illustrated in Figure 3.4. Once it has been determined that a reversal in roles is necessary (based on inventory levels of ACMs and ABMs on both ships), the ballistic missile defense ship "lowers" the regard of its SPY-1 radar to search for cruise missiles (Figure 3.3a). During this period, there is no ballistic missile

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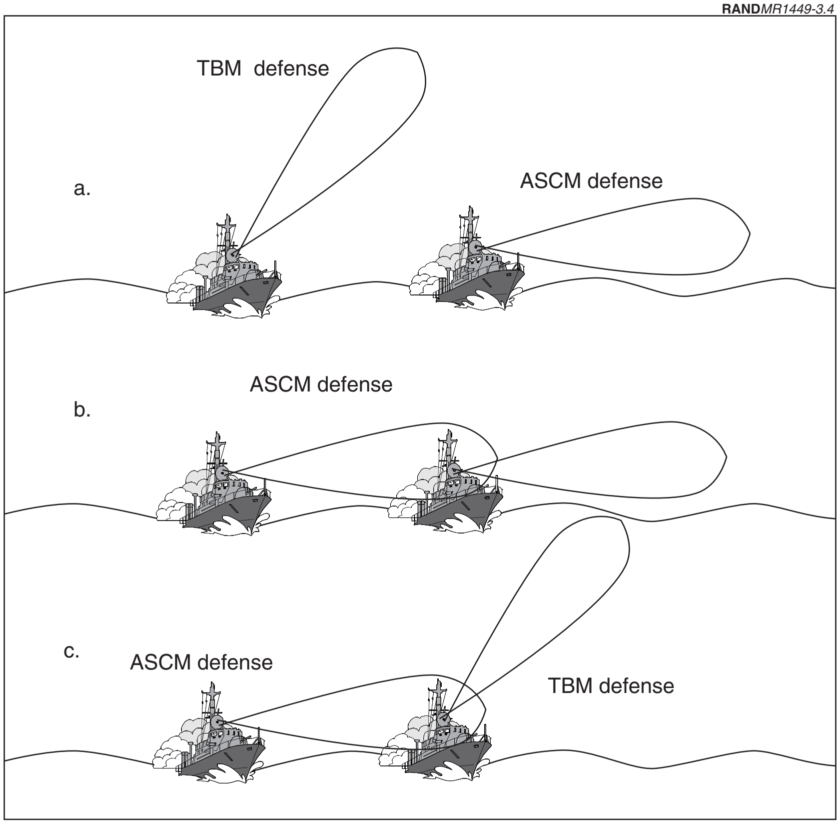

Figure 3.4—Role-Switching Operations

defense capability, meaning ballistic missile tracks built up may be lost. Once both ships have cruise missile tracks (i.e., once $A^{(b)}$ has cruise missile tracks), $A^{(c)}$ can raise the regard of its SPY-1 radar to detect ballistic missiles. Role reversal is complete when $A^{(c)}$ has built up ballistic missile tracks and can begin defending against them.

Except for a brief turnover period, divided duties operations means that only one ship can defend against cruise missiles at any time. Cruise missile saturation is relatively easy because only one ship's launchers are used to protect two ships. Cruise missiles that could be stopped by two ships become "too hard" for a single ship with a single geometry. The ballistic missile ship is unable to periodically step in to help the cruise missile ship. Similarly, ballistic missile saturation is difficult to prevent. To avoid running out a magazine the ships may have to swap roles, leaving a gap in ballistic missile defenses. After a switch, knowledge of which targets have been hit may disappear.

Network-Centric Operations—Shared COP

In the shared COP mode, both ships can see and defend against both incoming ballistic missiles and incoming cruise missiles (see Figure 3.5). We term this case network-centric because an understanding exists between the two ships concerning the nature of the attack. This implies greater connectivity than in the platform-centric case. Information on missile threat trajectories and arrival times is shared electronically, and in this sense the two ships can collaborate. As in the previous case, one ship ($A^{(b)}$) trains its SPY-1 radar to detect ballistic missiles and the other ($A^{(c)}$) to detect cruise missiles. Even though sensor information is shared, the two ships continue to operate independently: no cooperation or coordination takes place between the ships. However, both ships have cruise missile and ballistic missile defense responsibilities. As a result, poor "queue discipline" is likely in that both ships may engage the same missile or fail to engage a missile that with better coordination might have been engaged.

The decisions in this case center on how to ensure the optimal allocation of defensive weapons against both cruise and ballistic missiles

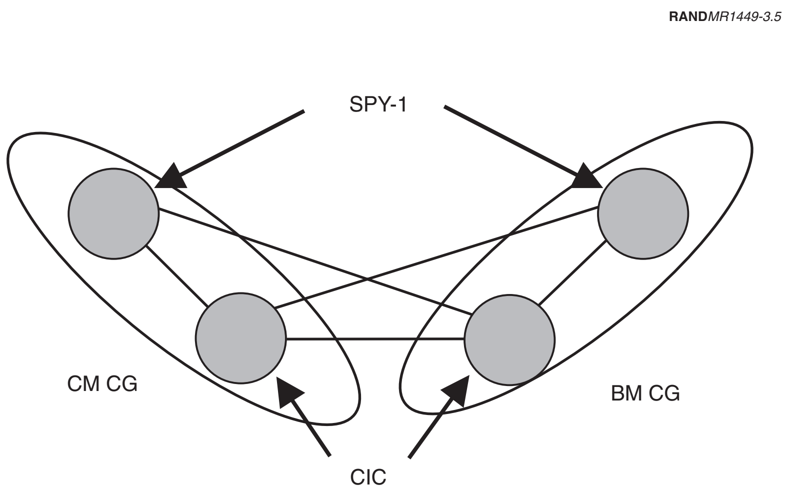

Figure 3.5—Shared COP Connectivity

given decentralized control.[4] The problem is complicated by the fact that either ship can defend against either type of threat. These decisions are described in more detail later. The decisions for both ships are:

- **Self-Protection:** The primary goal of both ships is survival. However, undue emphasis on self-protection means the other ship can be hit (possibly leaving the remaining ship more vulnerable). In addition, undue emphasis on protecting the two ships degrades defense against ballistic missiles. Each ship decides which of the incoming enemy missiles are easier for it to hit based on location and trajectory of the missile in relation to its own position. This sets up an attack priority. This can, of course, result in two ships attacking the same cruise missile target.
- **Ballistic Missile Interception:** Although the primary goal of the two cruisers is self-protection, their operational mission is to protect critical friendly infrastructure. In the absence of a cruise

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[4]The term "optimal" is used here and throughout the text in the sense that major improvements can be realized.

missile threat, the same rules applied to cruise missile defense are applied to ballistic missile defense with the same possibility that both ships attack the same incoming enemy missile.

We would expect that, under these conditions, cruise missile and ballistic missile saturation would require about twice the arrival rate of the previous case. This is based on both ships' ability to engage cruise missiles as well as ballistic missiles. Given a target difficult for one ship and relatively easy for the other, there is a natural tendency for the shot to be "assigned" appropriately. Systematically reducing incoming missiles is not possible because the ships are unable to cooperate in any organized way. Without cooperation, if one ship gets into trouble, the other generally cannot step in quickly to help. Situational awareness is increased in this case, but the Guided Missile Cruisers (CGs) cannot affect all events they can see.

Network-Centric Operations—Cooperative Engagement

As in the previous two cases, one ship has its SPY-1 trained on incoming cruise missiles and the other on incoming ballistic missiles (see Figure 3.6). Now, however, both ships have access to complete defense solutions and the allocation of ships to targets is controlled centrally—by one of the two ships engaged in the operation. We depict a separate node for this additional function for the controlling commander. Not only connectivity is required in this case, but also automated systems to assess the relevant factors that go into making the best decision. Both ships have cruise missile and ballistic missile defense responsibilities as in the previous case.

The decisions in this case center on how to ensure the optimal allocation of defensive weapons against both cruise and ballistic missiles given *centralized* control and a richer shared COP. As in the previous cases, the descriptions of how the decisions listed here are implemented in the model follow later. The decisions for both ships are:

- **Cruise Missile Defense:** Single-authority decisions to engage incoming cruise missiles are made on the basis of which ship is closer to being overwhelmed. That is, if saturation of ship defenses is imminent, defensive efforts will shift from TBMD to ASCM defense. In this case, if one ship faces an imminent threat, defensive efforts will focus to defend it. Overall, priority is given

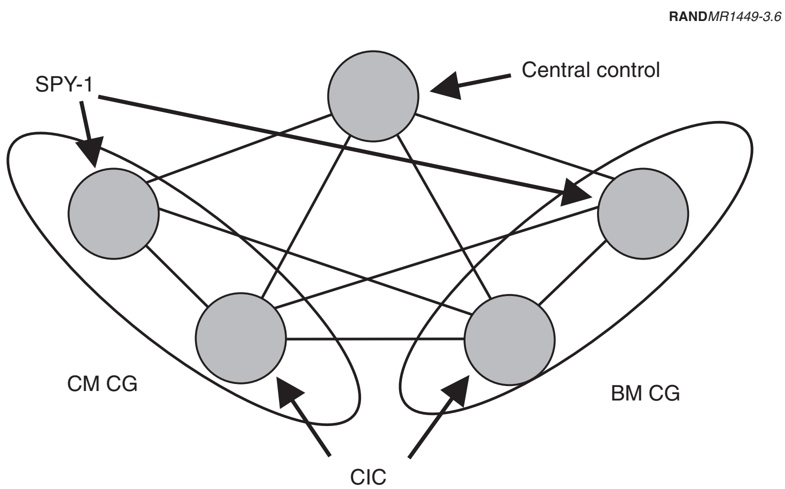

Figure 3.6—Cooperative Engagement Connectivity

to engaging cruise missiles, but now, the two ships cooperate to ensure that the most threatened ship is protected first. It may be necessary to prioritize protection of one ship over the other. A hit to the ship with the radar set for cruise missiles will leave the other ship completely unprotected for a short period, so it may be given higher priority.

- **Ballistic Missile Defense:** The single authority must prioritize the goals of self-protection, protecting the other ship, and protecting against ballistic missiles. Undue emphasis on ship protection can degrade performance against ballistic missiles. The optimal allocation of interceptors to incoming enemy ballistic missiles depends, in part, on the knowledge the central authority has concerning the distribution and projected size of the attack.

Both ships can defend against cruise missiles and against ballistic missiles, and they can cooperate and coordinate. Cruise missile saturation requires at least twice the arrival rate of the baseline case. Given a simultaneous need to engage an easy target and a hard one, an optimal decision may be made based on the relative position of the ships. Systematically reducing incoming missiles is now possible because of mutual cooperation. Similarly, ballistic missile saturation is at least twice as difficult for the reasons stated in the previous case. With cooperation, if one ship gets into trouble, the other can step in to help.

THE INITIAL ATTACK QUEUE

In any period, not all attacking cruise and ballistic missiles will be judged to be threats. Ballistic missiles not projected to damage predesignated critical infrastructure targets are not considered to be a threat—even though they may land on friendly soil. For this study, these targets are taken to be the airports of debarkation and seaports of debarkation. Cruise missiles not projected to damage the two Aegis cruisers are not considered to be threatening. The initial attack queue consists only of those missiles projected to hit critical infrastructure targets or the two Aegis cruisers.[5] The rate at which that queue is populated (the arrival rate) is therefore of interest and not the rate at which the missiles are detected by the Aegis radars.

For simplicity, we assume that only ballistic missiles are directed toward infrastructure targets and only cruise missiles are directed toward the Aegis cruisers. We further assume that Aegis cruisers are the only targets for the enemy cruise missiles. Other ships in the theater are not considered to be "in play."[6]

ARRIVAL RATES

Analysis of the value of network-centric operations begins with the average rate at which missiles arrive at the initial attack queue. This is taken to be an input. The ability of the enemy to effectively target friendly infrastructure with ballistic missiles and its ability to attack friendly ships, although important to the arrival rate as defined here, are not considered in this analysis. We assume a level of enemy capability and focus our attention on the queue itself. In a more de-

[5]This is based on the assumption that track and projection capability is sufficient to discriminate between threat and nonthreat missiles.

[6]These constraints can easily be relaxed. They are imposed here to allow for concentration on the problem of defending against the dual threat where the focus is on alternative command and control processes.

tailed simulation, the capability of the enemy to target friendly assets may be of interest. What is important here, however, is the ability of the defending Aegis cruisers to deal with a given threat.

The Attack Scenario

First, we assume that for the entire attack, the number of launched cruise missiles arriving is n_c and the number of ballistic missiles arriving is n_b. The duration of the attack is taken to be T minutes. During that interval, the arrival rate will vary. We assume that the missiles arrive in τ intervals each of which is t minutes in duration so that $T = \tau t$. We further define the average arrival rates per minute in time period i for the two types of missiles to be λ_{ci} and λ_{bi}. The total number of cruise missiles and ballistic missiles arriving in the attack (n_c and n_b) and the number of each type arriving in each time period are set parametrically. Therefore, the total number of each type of missile arriving in time period i is $\lambda_{ci} t$ and $\lambda_{bi} t$, and we have that the total attack sizes for the duration of the attack, T, are

$$n_c = \sum_{i=1}^{\tau} \lambda_{ci} t \text{ and } n_b = \sum_{i=1}^{\tau} \lambda_{bi} t .$$

Note that the average arrival rates in each time period are derived from the parametrically set number of missiles arriving in the period. Figure 3.7 illustrates a distribution of 300 cruise missiles and 50 ballistic missiles over $\tau = 5$ time periods. The shape of the distribution will depend on the enemy's attack strategy. If the objective is to saturate the allied defenses early, then the enemy might opt to coordinate the launch of a large part of its inventory to generate a large number of arrivals in the early periods. On the other hand, if the enemy's objective is to keep the cruisers busy while it pursues other offensive strategies, then a more uniform arrival rate is more likely. [7]

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[7]Clearly, an "optimal" strategy would be to fire all missiles so that they arrive simultaneously and thereby saturate the friendly defenses. However, this is nearly impossible to accomplish.

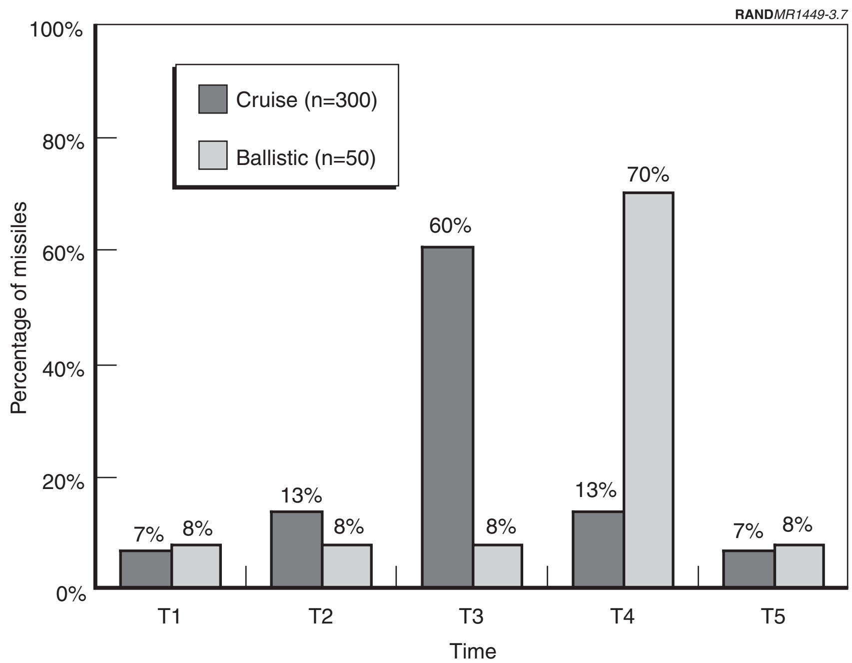

Figure 3.7—Enemy Missile Arrival Distribution

Additional Granularity

An important feature of the initial attack queue: it is "perishable." That is, service consists of destroying the missile in the queue or, in case of possibly repeated misses, absorbing its impact. A successful outcome, of course, is destruction of the missile, and therefore the sequence of arrivals is equally important. Because of this, we further subdivide each time period into subintervals with arrivals in each subinterval distributed according to a right-truncated Poisson. The right-truncated Poisson has a probability mass function of the form:

$$p[\mathbf{m}{:}\lambda] = \Phi \frac{e^{-\lambda}\lambda^{-m}}{m!}, \text{ for } m = 0,1, \dots , g,$$

where m is the number of missiles arriving in the subinterval, $g = \lambda_c t$ or $g = \lambda_b t$ is the number of cruise missiles or ballistic missiles arriving in the time period, and

$$\Phi = \frac{1}{1 - \sum_{h=g+1}^{\infty} \frac{e^{-\lambda}\lambda^h}{h!}}.$$

Note that as $g \to \infty$, $\Phi \to 1$. Therefore,

$$\lim_{g \to \infty} p[\mathrm{m}:\lambda] = \frac{e^{-\lambda}\lambda_m}{m!}, \; m = 0, 1, \ldots.$$

This is the standard Poisson distribution with mean equal to $\lambda = \lambda_c$ or $\lambda = \lambda_b$ for the given time period.

If the number of subintervals is s, then we can calculate the average number of missiles arriving in each subinterval of duration: $d = t/s$.

For example, suppose the average arrival rate for cruise missiles in a single time period is $\lambda_c = 3$ missiles per minute. If a time period is 5 minutes in duration, we can expect $g = 15$ missiles to arrive in that time. If we further assume that the period is subdivided into $s = 5$ subintervals, the average number of missiles that arrive in each of the 5 subintervals (each of which is $d = 5/5 = 1$ minute in duration) is $h = g/s = 3$. What remains is to calculate the fraction of the 15 missiles that arrive in each of the 5 subintervals.

One way to distribute the arrivals is to use the cumulative probability distribution:

$$p(\mathbf{m} \le m{:}\lambda) = \sum_{i=0}^{m} p(\mathbf{m}{:}\lambda).$$

If there are s subintervals in a time period, with g arrivals, then the number of arrivals in each subinterval, h, is calculated to be as follows:

$$h_1 = P\left(\mathbf{m} \le \frac{g}{s}\right)g,\, h_2 = p\left(\frac{g}{s} < \mathbf{m} \le \frac{2g}{s}\right)g, \ldots, h_s = p\left(\frac{(s-1)g}{s} < \mathbf{m} \le g\right)g.$$

Figure 3.8 illustrates the further distribution of the missiles in Figure 3.7 using this methodology.

The subdivision of each interval allows us to model staggered arrivals throughout the period. This means that missiles will appear as targets for the Aegis interceptors at intervals. If we assume that within each time subdivision, incoming cruise missiles are detected at the same time, then the number of opportunities for the Aegis cruiser to

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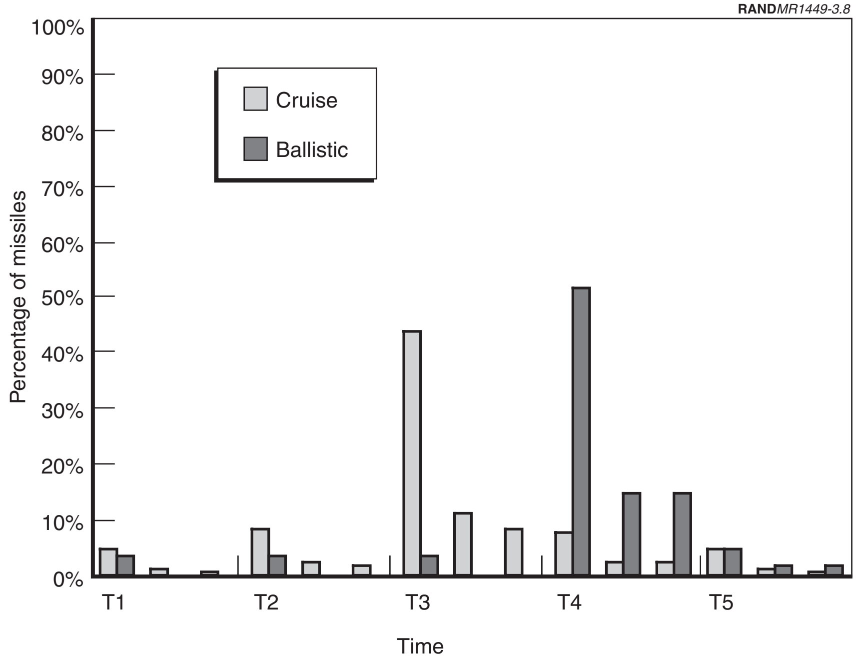

NOTE: In this example $\tau = 5$, $\tau = 5$ minutes, and therefore $T = 25$ minutes. For both cruise missiles and ballistic missiles, $s = 3$. For time period 3, for example, $g = \lambda_c \tau = 6 \times 5 = 30$ cruise missiles (approximately 60 percent of cruise missile inventory). The distribution to the subintervals is then $h_1 = 20$, $h_2 = 6$, and $h_3 = 4$.

Figure 3.8—Subinterval Enemy Missile Arrival Distribution

engage the missiles will depend on the time required to shoot a single missile and the shooting policy adopted by the cruiser.[8]

Allocation to Targets

Next, we address the distribution of arriving missiles to friendly targets. For simplicity, we assume that the distribution of missiles to targets is constant throughout all periods and subintervals. For cruise missiles, the n_c arriving missiles are allocated to the two defending Aegis cruisers, $A^{(c)}$ and $A^{(b)}$. If the proportion of incoming cruise missiles that will attack $A^{(c)}$ is $0 \le \alpha \le 1$, then the total number of incoming cruise missiles that will attack each cruiser is αn_c to $A^{(c)}$ and $(1-\alpha)n_c$ to $A^{(b)}$.[9]

The arriving ballistic missiles are allocated to infrastructure targets much in the same way, the difference being that there may be more infrastructure targets to be defended. In this case, we let ω_i be the fraction of incoming ballistic missiles, n_b, targeted against infrastructure target i, where

$$\sum_{i=1}^{\eta} \omega_i = 1$$

and η is the total number of infrastructure targets at risk. Again, we assume that the distribution of ballistic missiles to infrastructure targets is constant throughout all attack periods.

SHOOTING POLICY

The "shooting" policy affects the degree to which the weapon inventory on board the cruisers is depleted. In this work, we postulate three policies: shoot only, shoot-look-shoot, and shoot-look-salvo 2. We explain each of these next, but reserve explanation of their implications on success to later.

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[8]This is discussed later.

[9]We avoid the term "allocated" because such fine precision given the operational proximity of the two ships is not possible. This therefore is a modeling artifact.

- **Shoot Only:** This is the simplest policy. As threat missiles present themselves, one or both of the ships launch a counter missile. In case of a miss, the target missile is considered a leaker. No further attempt is made to engage it with counter missiles.
- **Shoot-Look-Shoot:** In this case, one or both of the cruisers will fire against a target missile. Next, the radar tracking the missile will determine if the engagement was a success. If not, a second shot will be fired. If the second shot misses, the target missile is again considered a leaker and no further attempt is made to engage it with counter missiles.
- **Shoot-Look-Salvo 2:** This last case begins as in shoot-look-shoot, but instead of firing just one counter missile after a miss, two are salvoed. If the salvo misses, the target missile is again considered a leaker and no further attempt is made to engage it with counter missiles.

Because there may be insufficient time to fire the "postlook" shots in the shoot-look-shoot and shoot-look-salvo 2 modes, we may also explore intermediate cases. If γ is the fraction of times that postlook shots are fired given that the "look" has been executed and it is determined that the first shot failed to intercept the incoming missile, then the total number of shots taken is $(1 - \gamma) + \gamma S$, where $S = 2$ for the shoot-look-shoot policy and 3 for shoot-look-salvo 2 policy.

SERVICE

For one cruiser, the mean rate at which attack missiles can be serviced (the firing rate) is μ missiles per minute.[10] Service, in this case, has several components. Three prominent ones are missile/launcher test, missile preparation, and flight to target. Each consists of a time delay based on other factors. The first two depend largely on the level of automation in testing, time required to load target data into the missile, and time to "warm up" subsystems. Flight to target depends on the speed of intercept and the speed of attacking missile.

[10]For simplicity, we assume the cruisers service targets at the same rate—on average.

The procedural factors affecting the service rate reflect the quality of command and control in place, and therefore modifications should be made parametrically to assess the impact on the success of the operation. Delays stemming from weapon systems operating methods are considered fixed for this analysis. However, insights might be gained concerning possible weapon systems improvements.

If we let τ_1 be the mean time to prepare a launcher, τ_2 the mean time required to launch the intercept, and τ_3 the mean time to fly out to the target, then the total mean service rate for each cruiser is:

$$\mu = \frac{1}{\tau_1 + \tau_2 + \tau_3}.$$

Service is complete when the incoming attack missile and the defending missile "meet."[11]

SURVIVABILITY

The survivability of the defending Aegis cruisers depends on their ability to engage incoming cruise missiles, their firing rate, and the ability of the terminal defenses (CIWS) to destroy terminal leakers. The firing rate can be thought of as the "service rate." When the arrival rate exceeds the service rate, then the system becomes "saturated." Whether because of intercept failure or saturation, cruise missiles not destroyed become "leakers." In both cases, the leakers join a second queue to be serviced by the CIWS. The implications of this are discussed below.

Leakers

We let p_c be the *constant* single-shot probability that an intercept from the Aegis cruiser kills an incoming cruise missile. This ignores the variations in threat weapons (such as stealthy missiles). We use

[11]As the battle proceeds, the relative length of these times will vary. Initially, all targets will be engaged at maximum range, and therefore fly-outs will be lengthy. However, later in the fight we expect that the fly-out time would be significantly reduced because targets would be engaged closer in. In any case, τ_3 will likely always be larger than τ_1 and τ_2.

this kill probability only when the threat missile is between the maximum and minimum engagement range as discussed below. As a result, p_c is independent for multiple defensive shots, and the effective kill probability will depend upon the shooting mode and p_c. For example, if p_c = 0.7, the shooting mode is "shoot-look-shoot," and only one ship engages the incoming missile, then the effective probability that a friendly interceptor will kill an enemy cruise missile is $P_K = 2p_c - p_c^2 = 0.91$, and, therefore, the probability that the attacking missile survives (becomes a "leaker") is P_L = 0.09.[12] In addition, the expected number of intercept missiles required to achieve this probability is $E_f = 2 - p_c = 2 - 0.7 = 1.3$ missiles.[13] Table 3.1 summarizes the effective kill probabilities and the expected number of missiles fired for each shooting mode.

Table 3.1

Effective Kill Probabilities

Mode	First-Shot Kill Probability	Second-Shot Kill Probability	P_K	Expected Shots Fired (E_f)
		One Ship Engaged		
Shoot	p_c	NA	p_c	1
Shoot-look-shoot	p_c	$(1 - p_c) p_c$	$1 - (1 - p_c)$	$2 - p_c$
Shoot-look-salvo 2	p_c	$(1 - p_c)[1 - (1 - p_c^2)]$	$1 - (1 - p_c)^3$	$3 - 2p_c$
		Two Ships Engaged		
Shoot	$1 - (1 - p_c)^2$	NA	$1 - (1 - p_c)^2$	2
Shoot-look-shoot	$1 - (1 - p_c)^2$	$(1 - p_c)^2[1 - (1 - p_c)^2]$	$1 - (1 - p_c)^4$	$4 - 4p_c + 2p_c^2$
Shoot-look-salvo 2	$1 - (1 - p_c)^2$	$(1 - p_c)^2[1 - (1 - p_c)^4]$	$1 - (1 - p_c)^6$	$6 - 8p_c + 4p_c^2$

[12]The probability of kill on the first shot is p_c and the probability that the target is killed on the second shot (i.e., the first shot misses and the second hits) is $(1 - p_c) p_c$. The effective probability then is $P_K = p_c + (1 - p_c)p_c = 2p_c - p_c^2$.

[13]Three possible events exist: (1) the incoming missile is destroyed on the first shot with the expectation of $1 \times p_c$ missiles fired, (2) the first shot misses and the second intercepts the missile with the expectation of $2(1 - p_c) p_c$ missiles fired, and (3) both shots miss with the expectation of $2(1 - p_c)^2$ missiles fired. The sum of these is $E_f = 2 - p_c$. This methodology is used to calculate the other expected values in Table 3.1.

In general then, the expected number of leakers in any period i is $L_i = P_L \lambda_{ci} t$, where $P_L = 1 - P_K$. This is the number that enters the terminal queue in the period. The distribution allocated to each cruiser is determined by the apportionment factor, α. This is discussed more fully below.

Saturation

Typically, in queuing models, saturation occurs whenever the arrival rate exceeds the service rate. In this application, saturation would occur whenever the rate at which enemy missiles arrive exceeds the rate at which the Aegis cruisers are able to defend against them. Although this is true in a sense, the problem is a bit more complicated because this is not a steady-state problem.

Enemy missiles are only vulnerable to friendly intercept with Standard missiles during a narrow window of opportunity as depicted by the shaded ring in Figure 3.9. If this opportunity is missed, the second line of defense is the CIWS depicted by the inner circle. The time-elevation graph in Figure 3.9 illustrates a notional flight with the times depicted as dotted lines. The times are:

- t_0: Time of enemy launch.
- t_1: Time enemy missile detected.
- t_2: Earliest time missile can be engaged with Standard missile.
- t_3: Latest time missile can be engaged with Standard missile.
- t_4: Earliest time missile can be engaged by CIWS.
- t_5: Impact time.

We assume that a Standard missile is launched when a single enemy missile heading toward the defending ship is detected—i.e., service begins at t_1. This means that the friendly force has a maximum of $t_3 - t_2$ minutes to engage the enemy missile. In general, the following condition must hold for a successful intercept:

$$t_2 \le t_1 + \frac{1}{\mu} \le t_3,$$

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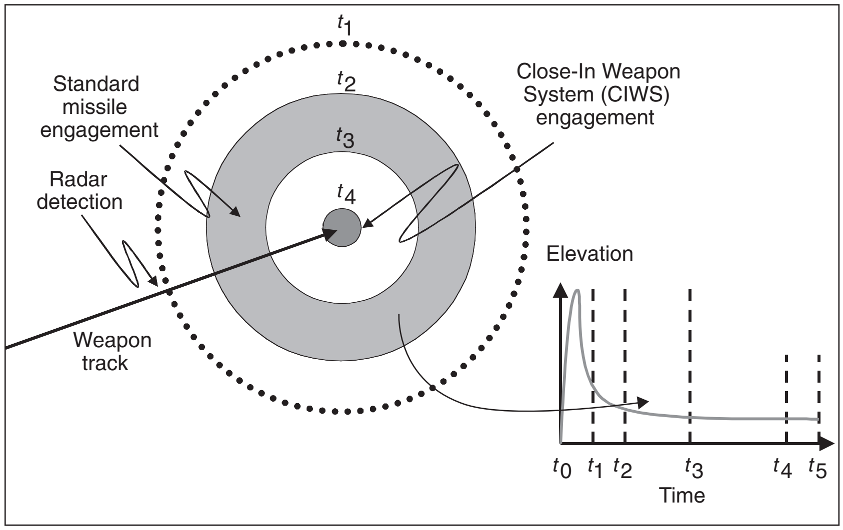

Figure 3.9—Cruise Missile Engagement Zones

where μ is the service rate as described above. For practical purposes, we can assume that $t_1 + 1/\mu \geq t_2$—i.e., it is unlikely that an intercept's launch time will be set such that it will intercept the enemy missile outside its effective range. Consequently, if h enemy missiles are detected at time t_1 (average arrival rate in a subinterval), we must have that:

$$h \leq \frac{t_3 - (t_1 + 1/\mu)}{\delta},$$

where δ is the time required to prepare the launcher for a subsequent launch: $\delta = \tau_1 + \tau_2$.[14] Hence, when the arrivals, h, exceed the service rate or when

[14]It might be argued that these times are insignificant when compared to the fly-out time and therefore in low-resolution models they may be neglected. We retain them here for completeness.

$$h > \frac{t_3 - (t_1 + 1/\mu)}{\delta},$$

we have

$$h - \frac{t_3 - (t_1 + 1/\mu)}{\delta}$$

enemy missiles that cannot be serviced in the subinterval.[15]

Because the enemy attack occurs over several of the subintervals, the average number of missiles the friendly commander must service is the average arrival rate for the subinterval plus the average not serviced in the previous subinterval. Furthermore, because the attack queue is FIFO (old targets must be serviced first), the additional enemy missiles left to be processed in the next time interval shrink its length. The implication of this is taken up next.

Recall that each subinterval is $d = t/s$ minutes long; therefore, the number of missiles that can be serviced in any subinterval is d/δ. This is based on the assumption that the second and subsequent missiles can be launched before the previous missile arrives on target. In other words, we need not consider fly-out times. Intercepts are launched at intervals of δ minutes.

The reduction in the length of the current interval occasioned by unserviced missiles is summarized in Table 3.2. Both the case where unserviced missiles are present and where they are not present have implications for the number of missiles "carried over" to the next subinterval.

In the table, the quantity:

$$\frac{t_3 - (t_1 + 1/\mu)}{\delta}$$

---

[15]Note that we do not refer to these as "leakers." The term "leaker" refers to *any* enemy missile not intercepted—including misses.

Table 3.2

Missiles Carried Over

		Missiles "Carried Over"
Case I: No unserviced missiles $h_i \le \frac{t_3 - (t_1 + 1/\mu)}{\delta}$	All missiles serviced in subinterval: $h_i \le \frac{d}{\delta}$	None
	Some missiles not serviced in subinterval: $h_i > \frac{d}{\delta}$	$h_i - \frac{d}{\delta}$
Case II: unserviced missiles present $h_i > \frac{t_3 - (t_1 + 1/\mu)}{\delta}$	All missiles that can be serviced will be serviced in the subinterval: $\frac{t_3 - (t_1 + 1/\mu)}{\delta} \le \frac{d}{\delta}$	None
	Some "serviceable" missiles are not serviced in the subinterval: $\frac{t_3 - (t_1 + 1/\mu)}{\delta} > \frac{d}{\delta}$	$\frac{t_3 - (t_1 + 1/\mu) - d}{\delta}$

is the number of enemy missiles that will be serviced in the subinterval.

In addition to varying the probability that an incoming enemy missile will be destroyed, the shooting policy also affects the number of enemy missiles that can be engaged in a subinterval. The quantities depicted in Table 3.2 reflect a shoot-only policy. For the shoot-look-shoot and the shoot-look-salvo 2 policies, the time required to prepare launchers for subsequent attacks increases to 2δ and 3δ, respectively. The "look" portion of the process is considered to be instantaneous in that the incoming missile is being tracked in real time and therefore a miss will be noted immediately. In the shoot-look-shoot case, a second intercept will be fired and therefore two launch preparations are required for each missile attacked. For the shoot-look-salvo 2 case, three missiles are fired.

Although saturation can occur as a result of the ballistic missile attack against infrastructure targets, the real concern is saturation from the cruise missile attack. We assume that survivability of the Aegis cruisers takes precedence over survivability of the infrastructure targets—not because the infrastructure targets are less important, but rather because with the loss of the cruisers no further defense is possible.

THE TERMINAL DEFENSE QUEUE

As mentioned earlier, all cruise missile that are not successfully engaged using Standard missiles (as a result of misses or defense saturation) join the terminal defense queue to be serviced by the CIWS.[16] The cruise missile arrival rate during any time period for the terminal queue is equal to the average number of leakers in that period. As mentioned earlier, the number of leakers in a period, L_i, depends on the shooting policy, the engagement procedures (one or two ships firing independently), and the degree of network-centricity. The proportion of terminal leakers allocated to each of the Aegis cruisers is αL_i for cruiser $A^{(c)}$ and $(1 - \alpha) L_i$ for cruiser $A^{(b)}$.

We now let p_d be the single-shot probability that an enemy cruise missile will damage an Aegis cruiser and we let p_a be the average level of damage required to disable the cruiser. By "damage" to the cruiser, we mean that its ability to engage attacking missiles is impaired by the fraction p_d and by "disable," we mean that its ability to engage cruise missiles has been lost. When the accumulated damage exceeds p_a, we assume the ship can no longer engage attacking missiles. We let N_L represent the number of leakers required to disable an Aegis cruiser. The expected fraction of damage caused by the N_L attacking missiles follows a binomial distribution so that if $p_a = 1-(1-p_d)^{N_L}$, then the expected number of leakers required to disable an Aegis cruiser is:

$$N_L = \frac{\ln(1-p_a)}{\ln(1-p_d)}.$$ [17]

[16]We assume that there is no terminal defense queue for ballistic missiles aimed at infrastructure targets.

[17]From *SABER/SELECT (SABSEL) and Weapons Effects Data Base (WEDB) User's Manual*, U.S. Air Force.

If we let p_e be the probability that an incoming missile escapes the CIWS, then the combined probability of a leaker is $P_L = P_L\, p_e$. Suppose we consider a sequence of attacks that each result in a probability of a leaker being P_L'. The number, *X*, of enemy cruise missiles required to achieve N_L successful leakers has a negative binomial distribution:

$$f(x{:}N_L,P_L') = \binom{x-1}{N_L-1} {P_L'}^{N_L} (1-P_L')^{x-N_L} \quad x = N_L, N_L+1, N_L+2, \ldots,$$

with mean $N_L(1 - P_L')/P_L'$. For example, if we let the single-shot probability of damage be p_d = 0.6, and let the fraction of damage required to disable the cruiser be p_a = 0.25, then the expected number of leakers required to disable the cruiser is:

$$N_L = \frac{\ln(1-.25)}{\ln(1-.6)} = 0.3140.$$

This means that with a shoot-look-shoot policy, the expected number of arrivals required to yield enough leakers to disable the cruiser (given the single-shot kill probability of an interceptor p_c = 0.7 and the probability that an attacking missile escapes the CIWS is p_e = 0.3) is:

$$\frac{N_L(1-P_L')}{P_L'} = \frac{N_L(1-p_e P_L)}{p_e P_L} = \frac{(.3140)(.91)}{(.3)(1-.91)} = 10.6\,.$$ [18]

UNCERTAINTY AND KNOWLEDGE

The AN/SPY-1, the multifunction phased-array radar at the heart of the Aegis system, can detect and track incoming missiles with considerable accuracy. We therefore assume that the probability of de-

[18]The result is, of course, purely statistical. This expected value calculation does not take into account that a cruise missile cannot damage more than one ship. At least one cruise missile hit is required to disable one Aegis cruiser. Under the condition that the damage inflicted by a single cruise missile hit is expected to be disabling $(p_d)(p_a)$, N_L should be set to 1.0.

tection for any enemy missile is 1.0 and that the detecting cruiser knows the enemy missile's location, speed, and trajectory at all times. We also assume that the friendly decisionmakers have accurate information regarding their own inventories and capabilities. The only thing not known about the enemy attack, therefore, is the attack distribution—that is, when the main effort of the attack will arrive or how long it will last. We do assume that the friendly forces know the enemy's missile inventories and launch capabilities. Although this offers no information on the attack distribution, it does establish an upper bound on the attack size.

Gaining knowledge of the attack distribution means obtaining reliable estimates for the arrival rates, λ_{bi} and λ_{ci}, in Figure 3.7. These may be obtained through intelligence sources prior to the attack and from other sensors (other ships in the area; intelligence, surveillance, and reconnaissance—ISR—aircraft; etc.) during the attack. Knowing the character of the attack in a subinterval allows for a more optimal response. How this knowledge affects the cruiser and infrastructure survival rates is discussed below.

Uncertainty

For each type of missile (ballistic missiles and cruise missiles), we assume that the size, n, of the enemy missile inventories is known. Therefore, in the absence of any information, the arrival rate in each of the subintervals is estimated to be $\hat{\lambda}_i = n/T$.[19] With perfect information, the estimate is the true value, or $\hat{\lambda}_i = \lambda_i$. We assume that the fraction of remaining missiles that will arrive in period i is a random variable, x, with a beta distribution[20] as depicted in Figure 3.10.

The beta probability density has the form:

$$f(x;\alpha,\beta) = \frac{\Gamma(\alpha+\beta)}{\Gamma(\alpha)\Gamma(\beta)} x^{\alpha-1}(1-x)^{\beta-1}, \quad x \in [0,1]$$

[19]We drop the c and b subscripts. The analysis is the same for both.

[20]Although the actual missile arrivals distribution is discrete (arrival counts are integer values), we are able to use the continuous beta distribution since our analysis is performed in an expected value setting.

with mean:

$$E(x) = \frac{\alpha}{\alpha + \beta}$$

and variance:

$$V(x) = \frac{\alpha\beta}{(\alpha + \beta)^2(\alpha + \beta + 1)} .$$

For $\alpha = \beta = 1$, the well-known uniform distribution results reflecting maximum uncertainty. For $\alpha,\ \beta > 1$, the distribution has a mode:

$$\frac{\alpha - 1}{\alpha + \beta - 2}.$$

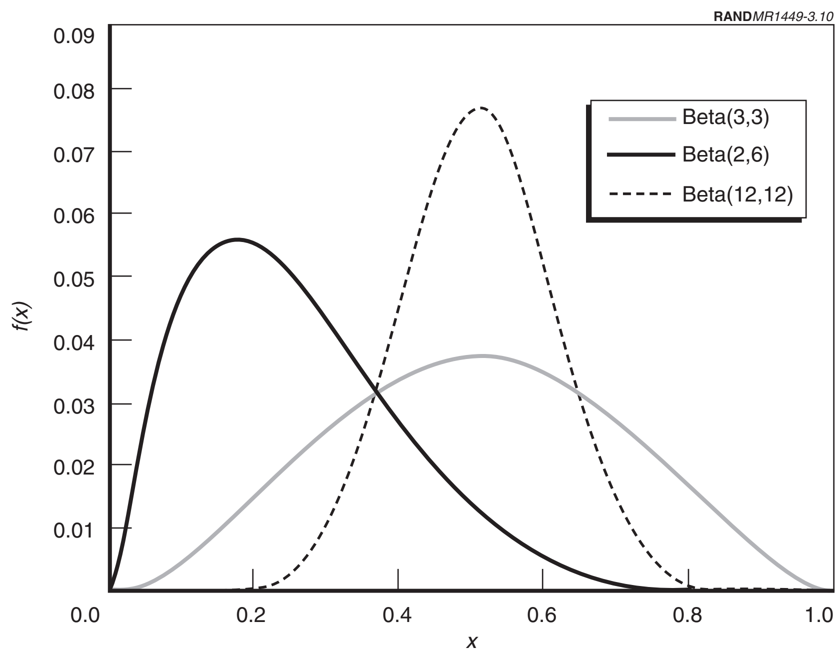

Figure 3.10—Arrival Rate Uncertainty

For purposes of this analysis, we consider only those distributions for which $\alpha, \beta \geq 1$.

The maximum number of arrivals in any time period, $i = 1,2, \ldots, \tau$, is

$$q_i = n - \sum_{j=1}^{i-1} t\lambda_j \,.$$

That is, the worst case is that all remaining missiles arrive simultaneously. Although not a practical consideration, it does provide an upper bound for the probability distribution. In the subsequent discussion, we drop the subscript *i*. It is understood that *q* refers to the remaining inventory of enemy missiles at the end of $i - 1$ subintervals.

The arrival rate in the current time period therefore is $\lambda = qx$, where *q* is the known inventory of remaining missiles.[21] If $E(x) = \xi$, then the expected arrival rate in the current time period is $E(\lambda) = qE(x) = q\xi$. Since ξ is the actual fraction of the remaining inventory scheduled to be launched in the current period, we need to choose the parameters of the beta distribution, α and β, such that

$$\frac{\alpha}{\alpha + \beta} = \xi \,.$$

However, because an infinite number of combinations of α and β can yield an unbiased estimate, we choose the appropriate values under the constraint that $\min(\alpha, \beta) \geq 1$ (ensuring that a mode exists) and such that the resulting variance achieves a “target value” $V_0(x)$. The target variance reflects the information quality, *Q*, associated with the sensor suite used to produce the estimate of missile arrival rate for the current period. For purposes of this discussion, information quality is the degree to which the information is current, correct,

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[21]Note the fact that the random variable *x* is the fraction of remaining missiles that will arrive in this period implies that *x* is really a fractional arrival rate—i.e., the fraction arriving per minute.

and complete. Therefore, we set $0 \le Q \le 1$, where $Q = 1$ implies the information has maximum quality.[22]

Information Entropy

To assess the degree of knowledge present in a density, such as the one just described, we employ the concept of *Information entropy* or *Shannon entropy*. Information entropy is a measure of the average amount of information in a probability distribution and is defined as:

$$H(x) = -\int_{-\infty}^{\infty} \ln[f(x)]f(x)dx\,.^{23}$$

Information entropy is based on the notion that the amount of information in the occurrence of an event is inversely proportional to the likelihood that that event will occur. Thus there is no information in Orphan Annie's declaration that "the sun will come up tomorrow" whereas there is considerable information in the realization that an individual has won the national lottery. Because entropy is an expected value, it is also referred to as the average information in a probability density.

Knowledge

The distribution in Figure 3.10 is completely defined by the maximum time period arrival rate, q, and the mean arrival rate, $\lambda = q\xi$. From this we can calculate the information entropy, or the uncertainty in the distribution:

$$\begin{aligned} H(x) &= -\int_{x=0}^{1} f(x)\ln f(x)dx \\ &= \ln[B(\alpha,\beta)] - (\alpha-1)[\psi(\alpha)-\psi(\alpha+\beta)] - (\beta-1)[\psi(\beta)-\psi(\alpha+\beta)]\,, \end{aligned}$$

---

[22]See Appendix A for a more thorough discussion of "target variance." For a more complete discussion of information quality, see Perry, Signori, and Boon (2001).

[23]Actually, because entropy is really a statistical expectation, the quantity should be $E[\ln(f(t))]$. However, in most texts this is shortened to $H(t)$ and we adopt this convention in this report. See Shannon (1948, pp. 379–423 and 623–556).

where $\psi(c)$ is the first derivative of Euler's gamma, and

$$B(p,q) = \frac{\Gamma(p)\Gamma(q)}{\Gamma(p+q)}.$$ [24]

We can create a mapping of entropy onto a [0,1] knowledge scale by selecting an upper bound on the entropy associated with the fraction, x. Before proceeding, however, we note that the uncertainty associated with the fraction of the remaining missiles arriving in the current period is equivalent to the uncertainty associated with the estimated arrival rate, λ, or $H(\lambda) = H(x)$.

The upper bound for $H(\lambda)$, denoted $H^*(\lambda)$, occurs when $\alpha = \beta = 1$. That is, maximum entropy occurs when uncertainty is maximized. In this case, therefore, we have that $H^*(\lambda) = 0$, a natural upper bound. A lower bound is also needed. However, minimum entropy occurs when the variance is minimized or when α and β are very large so that $H(\lambda) \to -\infty$. For practical purposes, we set this to be $\alpha = \beta = 12$ for which $H(\lambda) = -32.3192$.[25] We can define knowledge, therefore, as:

$$K(\lambda) = 1 + \frac{H(\lambda) - H_{\min}(\lambda)}{H_{\min}(\lambda)} = \frac{H(\lambda)}{H_{\min}(\lambda)} = \frac{H(\lambda)}{-32.3192}.$$

Note that the maximum time period arrival rate, q, is determined by the known enemy missile inventory, n, and the attack history. A natural maximum value for q therefore is the known enemy missile inventory. This presupposes, of course, that the arrival rates for all preceding periods were 0 and that the entire inventory will arrive in the next time period.[26]

In this form, knowledge is then used to influence the decisions taken subsequently and the outcome produced, as described below. First,

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[24]See Appendix A for details concerning the computation of this quantity.

[25]See Appendix A for a more detailed treatment.

[26]This also ignores the fact that if the arrival rate up to now is 0, it is difficult to maintain that the attack has started!

however, we assess the effects of collaboration and network complexity.

COLLABORATION

Collaboration is a process in which individuals work together to achieve a common goal. It is important because it enhances the degree of shared awareness in a group focused on solving a specific problem or arriving at an agreed decision. Several reasons point to why collaboration might be expected to improve the degree of shared awareness, including the potential for increased sharing of information and experience as well as synergy of inference. However, other factors can degrade performance, such as disruptive interactions, misunderstandings, or overvaluing a particular point of view because of the persuasiveness or authoritarian role of an individual team member. For this reason, the opportunity to collaborate can both add to and detract from effective combat operations. Here, we treat the contributions only, but in varying degrees. We postpone a discussion of the negative effects for future research.[27]

In general, as the opportunity for a decision team to collaborate increases (more connections), the better the decision is—provided that the quality of the collaboration is good. The quality of human collaboration can depend on several factors, among which are the experience of the team, the amount of time they have been working together, the procedures in place to facilitate collaboration, the personalities of the individual members, and the knowledge the team members possess about the critical element(s) of the operation. [28] Here we deal primarily with collaboration among automated systems—compressed time scales might not accommodate human interactions.

Figure 3.11 collects the three operating procedures described above for reference. The degree of collaboration and the complexity of the network vary considerably among the three.

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[27]For a fuller discussion of collaboration and shared awareness, see Perry, Signori, and Boon (2001).

[28]See, for example, Wegner (1987, pp. 185–208).

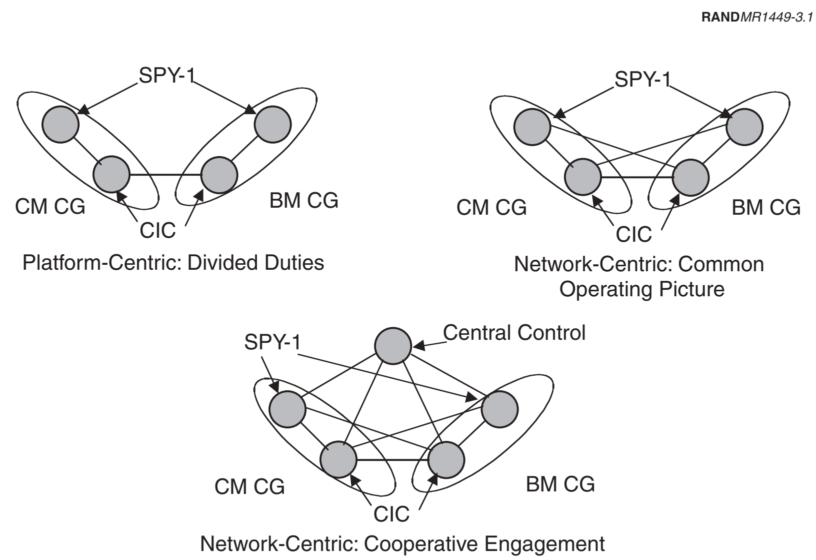

Figure 3.11—Missile Defense Operating Procedures

A Reliability Model

In each of the three diagrams in Figure 3.11, each ship is depicted as comprising two nodes: one representing the CIC and the other representing the SPY-1 radar. "Collaboration" between the two consists of passing signals to the radar screen in the CIC and receiving signals from the CIC to adjust the radar's regard. The results of collaboration here are shared information about where enemy missiles are located from the radar and where to look next from the CIC.[29] In the two network-centric cases, the SPY-1 radars are connected to the CICs in both ships. In these cases, the additional information made available by this dual connection is considered a form of positive collaboration and thus increases the reliability of the assessment of

[29]The Office of the Secretary of Defense's Information Superiority Metrics Working Group (ISMWG) defines collaboration to be a process in which two or more people actively share information while working together toward a common goal. See, for example, Alberts et al. (2001, pp. 27–28).

the enemy attack distribution. This is predicated, of course, on the assumption that the information exchanged is of high quality. Finally, in the cooperative engagement case, all shipboard nodes are connected to the central control facility, thereby increasing the opportunity for positive collaboration.

Statistical reliability appears to be an appropriate model for assessing the effects of collaboration. First, we let $c_{ij}(t) \in [0,1]$ represent the effects of collaboration between two nodes i and j, where t is the time required to complete the collaboration. The problem is to examine the nature of the functional, $c_{ij}(t)$, for each collaborative pair and for the entire network. The general form of $c_{ij}(t)$ is:

$$c_{ij}(t) = 1 - e^{-\int_0^t r(s)ds},$$

where $r(s)$ is called the failure rate function and, in this case, is dependent on the nature of the collaboration.[30] Note that for $t = 0$, $c_{ij}(t) = 0$.[31] That is, with no time to collaborate, it is impossible to share any information. With more time to collaborate, we would expect to experience increasing amounts of shared information. To the extent that that is desirable, more time to collaborate leads to improved benefits from that collaboration.

Note that $c_{ij}(t)$ can be viewed as a cumulative probability. If t is a random variable representing the time required for a successful collaboration, then $c_{ij}(t_0) = P(t \le t_0)$.

As an example, suppose we have a complete network consisting of three nodes—1, 2, and 3. The three nodes therefore generate three collaborative connections, 1-2, 1-3, and 2-3. Suppose that the collaboration probabilities for these three pairs are determined by the following failure rates: $r_{12}(s) = 1$, $r_{23}(s) = 2$, and $r_{13}(s) = 3$. From this, we discover the following collaboration probability functions:

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[30]We used t previously to represent the length of a time period. Its current use is quite different.

[31]There are several good texts on reliability engineering. See Ayyub and McCuen (1997) and Pecht (1995), for example.

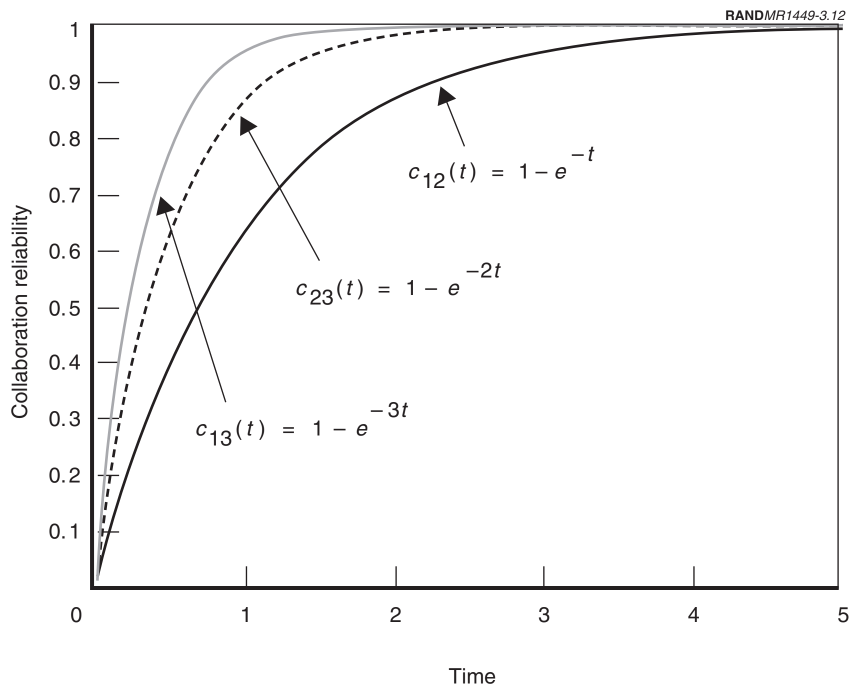

Figure 3.12—Collaboration Reliability Curves

$c_{12}(t) = 1 - e^{-t}$, $c_{23}(t) = 1 - e^{-2t}$, and $c_{13}(t) = 1 - e^{-3t}$. Figure 3.12 depicts varying collaboration probabilities for these three pairs.

Note that the time at which successful collaboration between two nodes occurs depends on the form of its failure rate function, $r(s)$. In this case, a constant was selected because we can model earlier successful collaboration by simply increasing the constant value. In general, we select a form:

$$c_{ij}(t) = 1 - e^{-\theta t} \text{ for } t \geq 0.$$

The next step is to combine the collaboration effects before assessing its effect on knowledge and subsequently the success of the defensive missions. Before we do this, however, we take advantage of the fact that $c_{ij}(t)$ is a cumulative probability and calculate its density function:

$$f_{ij}(t) = \frac{dc_{ij}(t)}{dt} = \theta e^{-\theta t} .$$

Applying this to the three collaboration estimates we get:

$$f_{12}(t) = e^{-t}$$
$$f_{23}(t) = 2e^{-2t}$$
$$f_{23}(t) = 3e^{-3t} .$$

These are exponential distributions with $1/\theta$ being the mean time available for nodes i and j to collaborate with a variance of $(1/\theta)^2$.

The entropy calculation for the exponential distribution with parameter θ is:

$$H(t) = -\int_{t=0}^{\infty} \ln[\theta e^{-\theta t}]\theta e^{-\theta t} dt = 1 + \ln\left(\frac{1}{\theta}\right) = \ln\left(\frac{e}{\theta}\right).$$

Note that entropy varies with the variance of the distribution as should be expected. As $1/\theta$ increases (θ decreases), $H(t) = \ln(e/\theta)$ also increases. Note also that entropy is unbounded for this distribution.[32]

We can use the entropy function to develop a measure of knowledge by assessing the "certainty" in the density function. This requires an approximate upper bound to be assigned to $H(t)$, the equivalent to assigning a maximum expected time to complete a collaboration. If we let $(1/\theta)_{max} = \theta_{min}$ represent the maximum expected time, then a measure of certainty or knowledge can be written as:

[32]This is true for all continuous distributions.

$$K(t) = \ln\left(\frac{e}{\theta_{\min}}\right) - \ln\left(\frac{e}{\theta}\right) = \ln\left(\frac{\theta}{\theta_{\min}}\right).^{33}$$

Note that this quantity is dimensionless and therefore can be used directly to influence combat MOEs. It is desirable, however, for the measure of knowledge to be normalized. This can be accomplished by noting that when $\theta = \theta_{\min}$, $K(t) = \ln(1) = 0$ and when $\theta / \theta_{\min} = e$, $K(t) = \ln(e) = 1$. This suggests the following definition for the knowledge gained from the collaboration between nodes i and j:

$$K_{ij}(t) = \begin{cases} 0 & \text{if } \theta < \theta_{\min} \\ \ln(\theta / \theta_{\min}) & \text{if } \theta_{\min} \le \theta < e\,\theta_{\min} \\ 1 & \text{if } \theta \ge e\,\theta_{\min} \end{cases}.^{34}$$

Note that for small values of θ, the mean and variance are large, thus implying great uncertainty and therefore little knowledge. For large values of θ, the opposite is true and therefore considerable knowledge is gained. $K_{ij}(t)$, therefore, models the positive effects of having more time, on average, to collaborate.

The next step is to develop a total system collaboration factor that accounts for all pairs of collaborating nodes. The goal is to discern how the enemy proposes to distribute the cruise and ballistic missiles over the τ time periods. This means that the knowledge function, $K(\lambda)$, calculated earlier, should be modified by incorporating the effects of collaboration. Although collaboration between nodes in each of the three cases studied can occur simultaneously, it is necessary that all node pairs collaborate. This makes the overall system collaboration model sequential, and, therefore, using an inverse reliability model we get that system collaboration is:[35]

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[33]Because the term $K(t)$ is derivative of information entropy, we extend the convention of using t as the functional argument.

[34]For additional information on the use of information entropy as a measure of knowledge, see Perry and Moffat (1997, pp. 965–972).

[35]It is sequential as opposed to parallel. The latter would occur if the system required that collaboration occur between nodes a and b or nodes c and d, but not both. We assume that collaboration occurs between all connected nodes.

$$c_M(t) = 1 - \prod_{[i,j]} c_{ij}(t) .$$

Therefore, $c_M(t)$ is large for systems with several collaborating pairs. Using the knowledge factor for each collaborating pair derived from the collaboration function instead of the collaboration function itself, we get:

$$K_M(t) = 1 - \prod_{[i,j]} K_{ij}(t) .$$

This assumes that the collaboration effect from each collaborating pair is equal in value. The effects of collaboration then can be represented using a linear model:

$$K_c(\lambda) = K_M(t)[1 - K(\lambda)] + K(\lambda) ,$$

where $K_c(\lambda)$ is the knowledge about the attack distribution that accounts for collaboration effects. If $K_M(t) \approx 0$, there is little or no collaboration effect. On the other hand, when $K_M(t) \approx 1$, the collaboration effect is "complete" and $K_c(\lambda) \approx 1$.

COMPLEXITY

A well-connected network is necessary for effective command and control, but it is not sufficient. For this reason, we refer to the network as the *potential energy* in a command and control system. The sufficient condition that must be added is the command and control process that operates over the network. This is the *kinetic energy* of the command and control system and to be effective, it must produce quality information and allow for creative command and control arrangements that are reflected in good combat outcomes. It is always possible to misuse a well-connected network and to effectively use one that is not well connected.

As the network in a network-centric operation increases in size, it also increases in complexity. This stems from the actual and potential connections possible. Unless properly managed, this complexity could detract from the mission rather than support it. However, there are cases in which the added richness in connectivity accompanying increases in network size can enhance operations. Hence, complexity as collaboration can have both positive and negative effects.

In this example, collaboration is limited to the two CICs and, in the cooperative engagement case, with the central controlling authority as well. Complexity is generally not a factor in these simple cases. Consequently, we defer the detailed discussion of developing the complexity metric to Chapter Four, where its impact is considerably greater. However, with the addition of other ships in the area participating in the conflict or in the context of other operations in progress affecting the two cruisers attending to missile defense, complexity might become an important factor. For that reason, we include it in the formulation. Unlike the TCT case discussed in the next chapter, we measure the effects of collaboration and complexity on the knowledge function, $K(\lambda)$. Although indirect, the effect of this process on combat outcomes is still measurable.

A Logistics Model

In this work we only examined the degradation in performance stemming from complexity. Although, as will be shown, we can minimize the negative effects through the selection of appropriate parameters, we have not addressed the potential synergies that may derive from a complex network except through collaboration. This remains for future research.

For the divided-duties and shared COP cases, there are four nodes with a maximum of six connections, and, for the cooperative engagement case, there are five nodes with a maximum of 10 connections. Complexity is a function of the number of connections in a network, and, therefore, we let this be the independent variable in calculating its effects. We let C_ν represent the total number of network connections for each of the three cases: $\nu = 1$, divided duties, $\nu = 2$, shared COP, and $\nu = 3$, cooperative engagement. Consequently, from Figure 3.11 we have $C_1 = 3$, $C_2 = 5$, and $C_3 = 9$. The general complexity function is:

$$g(C_v) = \frac{e^{a+bC_v}}{1+e^{a+bC_v}},$$ [36]

where a and b determine both the region of minimal impact and the size of the region of rapidly increasing impact. For example, for the three cases described in this analysis, if we let $a = -7$ and $b = 0.9$, we get the curve depicted in Figure 3.13. The values for each case are identified in the figure.[37]

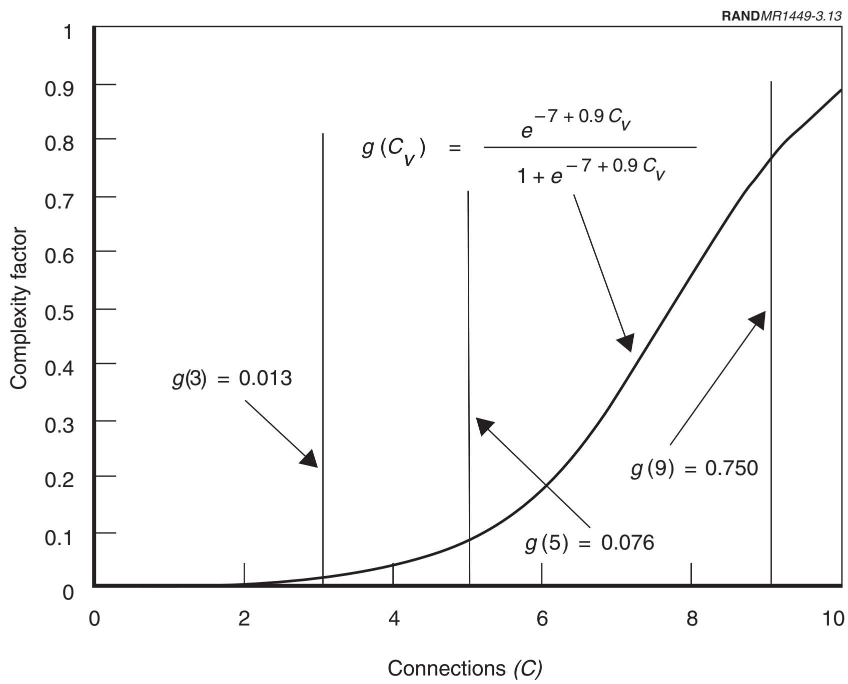

Figure 3.13—Complexity

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[36]This curve is sometimes referred to as the *logistics response function* or the *growth curve*. See Neter and Wasserman (1974). See Chapter Four for a more detailed development.

[37]The value, $b = 0.9$, in this example is significantly higher than we might recommend in practice.

We would expect excessive complexity to have a negative effect on our ability to “know” the enemy attack distribution. Therefore, the combined effects of collaboration and complexity on knowledge is:

$$K_{cC_v}(\lambda) = [1 - g(C_v)]\{K_M(t)[1 - K(\lambda)] + K(\lambda)\}.$$

This is now used to modify the arrival rate estimates at each time period, $\hat{\lambda}_{ci}$ and $\hat{\lambda}_{bi}$. Recall that there are τ arrival time periods, each of which is t minutes in duration. The total number of cruise missiles and ballistic missiles in the enemy's inventory is n_c and n_b, respectively, and the attack horizon is $T = \tau t$ minutes. With no information about the attack distribution, the best estimate is that $\hat{\lambda}_{ci} = n_c / T$ and $\hat{\lambda}_{bi} = n_b / T$. However, with perfect information we have that $\hat{\lambda}_{ci} = \lambda_{ci}$ and $\hat{\lambda}_{bi} = \lambda_{bi}$, the true arrival rates. The following functions incorporate the collaboration and complexity modified knowledge to produce the desired estimated arrival rates:

$$\begin{aligned} \hat{\lambda}_c &= \left(1 - K_{cC_v}(\lambda)\right)\frac{n_c}{T} + K_{cC_v}(\lambda)\lambda_c \\ \hat{\lambda}_b &= \left(1 - K_{cC_v}(\lambda)\right)\frac{n_b}{T} + K_{cC_v}(\lambda)\lambda_b \end{aligned}.$$

DECISIONS

The implementations of the decisions for the three cases described are addressed next. For all three cases, one of the ships, $A^{(c)}$, searches for cruise missiles and the other, $A^{(b)}$, for ballistic missiles. The decisions center on the allocation policy that best protects both of the cruisers and the critical infrastructure targets.

Decisions are made at the beginning of a period or subinterval depending on the case.

Remaining inventories of Standard missiles on each ship are critical to the decision process. The subinterval consumption rate for these missiles depends on the length of the subinterval, firing rate, shooting policy, and number of ships engaging each missile. The number of Standard missiles (ACM and ABM) fired at each enemy missile is reported in Table 3.1 and repeated for convenience as Table 3.3. The

Table 3.3

Expected Shots Fired

Mode	Expected Shots Fired ($E_f : f = \{c,b\}$)
One Ship Engaged	
Shoot	1
Shoot-look-shoot	$2 - p_c$
Shoot-look-salvo 2	$3 - 2p_c$
Two Ships Engaged	
Shoot	2
Shoot-look-shoot	$4 - 4p_c + 2p_c^2$
Shoot-look-salvo 2	$6 - 8p_c + 4p_c^2$

NOTE: In this table, p_c is the single-shot probability that the enemy cruise missile is destroyed. These values are reported in Table 3.1.

number of Standard missiles required to destroy an enemy missile is denoted by the term, E_f, where the subscript denotes either ACMs ($f = c$) or ABMs ($f = b$). E_f is an expected value and is therefore consistent with an expected value analysis, such as this one.

Platform-Centric: Divided Duties

Only the role-switching decision is modeled for this case. The decision is made at the beginning of each interval, and it is based on the remaining inventories of ACMs and ABMs on board each ship and the estimated remaining attack distribution. For a ship to be eligible to assume an ACM or an ABM defense role, it must first have a minimum number of interceptors of the appropriate type in its remaining inventory. Next, the estimated arrival of missiles in this period must be enough to justify the switch.

We let δ_c be the minimum number of ACMs and δ_b be the number of ABMs needed to assume those defense roles, respectively. In addition, we let $I_c^{(c)}$ and $I_b^{(c)}$ represent the remaining inventories of ACMs and ABMs on board $A^{(c)}$. Similarly, $I_c^{(b)}$ and $I_b^{(b)}$ represent the remaining inventories of ACMs and ABMs on board $A^{(b)}$. Inventories of both missile types on board both ships are expected to be reduced by E_f per incoming missile of type *f* as the attack proceeds. The following rules then apply:

1. **When to Switch Roles:** The need to switch roles is based on remaining inventories of Standard missiles on either of the defending cruisers. Consumption of ACMs on the ship designated to defend against cruise missiles and consumption of the ABM inventories on the ship designated to defend against ballistic missiles are factors. A need to switch occurs therefore when either $I_c^{(c)} < \delta_c$ on $A^{(c)}$ or $I_b^{(b)} < \delta_b$ on $A^{(b)}$ or both.
2. **Designated Ballistic Missile Ship,** $A^{(b)}$: If a need to switch has been established (rule 1. above), and if the number of ACMs on board exceeds the threshold ($I_c^{(b)} > \delta_c$), and if the inventory of ACMs exceeds the estimated number of cruise missiles arriving in the next interval ($I_c^{(b)} > t\hat{\lambda}_{ci}E_c$), then the ballistic missile ship will assume a cruise missile defense role.[38]
3. **Designated Cruise Missile Ship,** $A^{(c)}$**:** If a need to switch has been established (rule 1. above), and if the number of ABMs on board exceeds the threshold ($I_b^{(c)} > \delta_b$), and if the inventory of ABM missiles exceeds the estimated number of ballistic missiles arriving in the next interval ($I_b^{(c)} > t\hat{\lambda}_{bi}E_b$), then the cruise missile ship will assume a ballistic missile defense role.

Network-Centric: Shared COP

In this configuration, both ships perform ACM and ABM roles. Without coordination of fires, both ships act independently—but with shared information. Each ship engages the targets it feels it can best intercept. Consequently, it is possible that both ships respond to a given enemy cruise or ballistic missile. It is also possible that both ships decide not to respond to a given threat. Several factors combine to determine how often either of these cases will occur. However, we do not treat them explicitly here. Instead, we assume that the fraction of enemy missiles that will be attacked by both ships is given by a constant, γ (we allow γ to vary parametrically).

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[38]Note that $t\hat{\lambda}_{ci} = \sum_{j=1}^{s} \frac{t}{s}\hat{\lambda}_{cj}$ where *s* is the number of subintervals within an interval.

In this case, decisions are made at the beginning of each subinterval. The added fidelity is needed to adequately assess the effects of the decisions taken.

The decision to engage an enemy missile is based primarily on the relative geometry (including positions and heading) of the ship and the enemy missile. However, in addition to the relative geometry, consideration is also given to remaining inventories of missiles and the anticipated attack arrival rate for the next subinterval.

In general, it is desirable to engage a threat missile as early as possible. This means that, unconstrained, each ship desires to attack incoming missiles in the order of their proximity to their own ship. The constraints on this behavior is the need to husband inventories. The better each ship is able to anticipate what comes next, the better it can deal with the present threat.

The following decision rule takes location, inventories, and estimates of the future attack distribution into consideration. We focus on the cruise missile threat because this is critical to the survivability of the two defending ships. The same analysis is applied to ballistic missiles.

- **Location:** Although the representations in this analysis do not include relative geometry, we can infer location through the fraction of enemy missiles directed at each of the cruisers (α for $A^{(c)}$ and $1 - \alpha$ for $A^{(b)}$). The assumption is that if an enemy cruise missile is directed against either ship, it is likely that that ship will decide it is in the best position to engage it. However, γ percent of the total missile arrivals have an expected impact point that threatens both ships—i.e., in γ percent of the cases, both ships feel that they are the impact point. For these arrivals, decisions are based on both ships attempting to intercept. The enemy intent is to aim α percent of the cruise missiles at $A^{(c)}$ and $1 - \alpha$ percent of the cruise missiles at $A^{(b)}$. The ship $A^{(c)}$ perceives that $\alpha - \alpha\gamma + \gamma$ percent of the arrivals are aimed toward it, and $A^{(b)}$ perceives that $(1 - \alpha)(1 - \gamma) + \gamma$ percent of the arrivals are aimed toward it. An individual ship's decisions are based on this expected arrival allocation.

- **Inventories:** As in all cases, inventories are constraining. If the cruise missile attack in the current subinterval, *j*,

$$\frac{t}{s}\lambda_{cj} \text{ enemy missiles}$$

absorbs the entire inventory of ACM ($I_c^{(c)}$ or $I_c^{(b)}$) weapons, then nothing will be left to defend against subsequent attacks.[39] Consequently, current inventories of ACM weapons must be weighed against the current attack and estimates of future attacks:

$$F_c = \sum_{i=j+1}^{s\tau} \frac{t}{s}\hat{\lambda}_{ci}.$$

For ballistic missile attacks, a similar estimate of future attacks is:

$$F_b = \sum_{i=j+1}^{s\tau} \frac{t}{s}\hat{\lambda}_{bi}.$$

- **Future attack distribution estimate:** The nature of future cruise missile attacks against the two cruisers affects the decision to engage current threats. The anticipation of a more intense attack materializing in the future forces both ships to husband their ACM assets. The decision would be to let the CIWS take care of incoming missiles more likely to hit their targets. We use the parameter, $0 \le w \le 1$, to indicate how much weight we should assign to the future attack in our allocation decision.

The decision rule for $A^{(c)}$ based on these three considerations therefore is to engage a maximum of $d_f^{(c)}$ enemy missiles of type f ($f = \{c, b\}$), where $d_f^{(c)}$ is defined as follows:

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[39]Note that there are $s\tau$ subintervals in the attack horizon, where s is the number of subintervals in an interval and τ is the number of intervals.

$$d_f^{(c)} = \begin{cases} \beta \frac{t}{s} \hat{\lambda}_{fj} & \text{if } I_f^{(c)} \geq \beta \left(\frac{t}{s} \hat{\lambda}_{fj} + wF_f \right) E_f \\ \beta \dfrac{\frac{t}{s} \hat{\lambda}_{fj}}{\left(wF_f + \frac{t}{s} \hat{\lambda}_{fj} \right)} & \text{otherwise,} \end{cases}$$

where $\beta = \alpha - \alpha\gamma + \gamma$ when $f = c$ and $\beta = 1$ when $f = b$. Similarly, for ship $A^{(b)}$, we have $\beta = (1-\alpha)(1-\gamma) + \gamma$ when $f = c$ and $\beta = 1$ when $f = b$.

Network-Centric—Cooperative Engagement

In this network configuration as in the shared COP case, the decision to be made is which ship(s) ($A^{(c)}$, $A^{(b)}$, or both) should attempt to intercept incoming cruise missiles in a subinterval and how many missiles each can "safely" engage in a subinterval. The differences between this decision and the shared COP case are as follows: in this case, a central control authority makes the decision based on shared information from both ships, and, in the cooperative engagement case, future attack distributions are considered separately for each subinterval rather than lumped into a single sum of future arrivals (i.e., the computation of F_c). In the COP case, instances occur where both ships attempt to intercept the same cruise missile since defensive fire is not coordinated. In the cooperative engagement case, however, there are also instances where both ships attempt to intercept the same cruise missile. This occurs because the controlling authority has determined that the expected remaining ACM operating life for the remaining Aegis cruisers is longer if both attempt to intercept in the current subinterval. The controlling authority considers the current attack scenario, future expected attacks, and the expected life of each Aegis cruiser that will result under different defensive options. In the cooperative engagement case, the objective of the controlling authority is to use all available information to determine the cruise missile defensive strategy that will keep the ACM capabilities of the remaining Aegis cruiser(s) in operation for the longest period (in terms of subintervals).

It is likely that the enemy will adopt a strategy that focuses on destroying the Aegis cruisers early, thus clearing the way for a concentrated ballistic missile attack. It is also likely that the friendly cruisers will be positioned to provide maximum ballistic missile defense. A longer expected period of ACM operation implies that there is more time to intercept ballistic missile arrivals and to protect the sister ship from arriving cruise missiles. So, the cruise missile defense option that provides the longest expected ACM operating life for the surviving cruisers is then pursued in the current period. As in the shared COP case, our focus during decisionmaking is on defending against cruise missile attack on the defending ships. The ballistic missile defense decisions are similar but of secondary importance. Whatever time and resources are left over in the subinterval will be devoted to the ABM attack mission. Assuming, as usual, that the decision is to be made at the start of subinterval j, we first calculate the number of threatening cruise missiles expected to arrive in each future subinterval

$$\frac{t}{s}\hat{\lambda}_{ci}, (i = j, \ldots, s\tau).$$ [40]

With this and the information assumed available to the decisionmaker, the following decision inputs are used:

- **Inventories:** As in the shared COP case, inventories of ACMs must be carefully allocated to ensure future operation. When a ship's cruise missile inventory is depleted, it may no longer operate in an ACM role. The expected attack size in the current subinterval is

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[40] In the absence of any additional information, the expected number arriving in each of the remaining subintervals is

$$\frac{n_c - \sum_{i=1}^{j-1} \frac{t}{s} \lambda_{ci}}{s\tau - j},$$

where the summation on the numerator is the total number of enemy cruise missiles that have already been launched in the preceding $j - 1$ subintervals, n_c is the total number of enemy cruise missiles to be launched, and $s\tau - j$ is the number of subintervals remaining.

$$\frac{t}{s}\hat{\lambda}_{fj} \,,$$

and the remaining inventories for the $A^{(c)}$ and $A^{(b)}$ ships are $I_c^{(c)}$ and $I_c^{(b)}$ ACMs and $I_b^{(c)}$ and $I_b^{(b)}$ ABMs, respectively. For an expected attack distribution, if $A^{(c)}$ is solely responsible for ACM operations, its inventory of ACMs will be depleted in subinterval $v_c^{(c)}$. This occurs when:

$$I_c^{(c)} = \sum_{i=j}^{s\tau} \frac{t}{s}\hat{\lambda}_{ci} E_c^{(c)} \,.$$

In the formulation of $v_c^{(c)}$, the subscript c indicates the ship is defending against cruise missiles, the superscript, (c), is the ship whose inventory we are assessing ($A^{(c)}$ in this case), and the argument, (c), is the ship defending against the incoming missiles.

We can define similar measures for the ship $A^{(b)}$ and for the case where both ships are performing ACM duties. In the latter case, we assign the ACM duties to the ship with the largest remaining inventory, or when $v(b,c) = \max\{v_c^{(c)}(b,c), v_c^{(b)}(b,c)\}$. Note that the expected number of ACMs fired, $(E_c(c), E_c(b)$, or $E_c(b,c))$ depends on the number of ships performing ACM duties.

- **Expected Survivability:** The survivability of each ship depends on the number of leakers, (L), the single-shot probability that an enemy cruise missile will damage an Aegis cruiser assuming it strikes the cruiser (p_d), and the fraction of damage required to disable the cruiser (p_a). We can then compute the number of leakers required to force the ship out of commission (OOC) and, therefore, render it unable to take on ACM or ABM duties. The number of Red leakers that force a single ship to be OOC is N_L.[41] To take both ships OOC (given that α percent of the cruise missiles are aimed at $A^{(c)}$ and $1 - \alpha$ percent are aimed at $A^{(b)}$), a total of

[41]The formula for the number of leakers required to disable an Aegis cruiser is discussed above under the "Terminal Defense Queue" subhead.

$$N_L^{(b,c)} = \frac{N_L}{\min(\alpha, 1-\alpha)}$$

leakers are needed to disable both ships. At the beginning of subinterval *j*, we compute the quantity:

$$M_j(r) = \frac{N_L^{(b,c)} - L_j}{(1-P_K(r))(1-P_{CIWS})},$$

the additional threatening ACMs that must be launched by the enemy in order to force both ships to be taken OOC, where:

1. L_j is the number of leakers up to the beginning of time period *j*.
2. $P_K(r)$ is the effective probability of kill that can be achieved by the ACMs with *r* defending ships given a fixed shooting policy.
3. P_{CIWS} is the kill probability for the CIWS.

Because the probability of kill for their ACMs and CIWS is the same for both ships, $M_j(1)$ is the same for both $A^{(b)}$ and $A^{(c)}$. Also, we know that $M_j(1) \le M_j(2)$. For an expected attack distribution, the period when both ships are OOC, assuming only one performs cruise missile defense and has an adequate inventory and firing rate to attempt intercept of all missiles, is the subinterval, $\kappa(1)$. This occurs if the following equality is satisfied:

$$M_j(1) = \sum_{i=j}^{s\tau} \frac{t}{s} \hat{\lambda}_{ci} \ ,$$

When both ships defend, then the period when both ships are OOC is the subinterval, $\kappa(2)$, that satisfies

$$M_j(2) = \sum_{i=j}^{s\tau} \frac{t}{s} \hat{\lambda}_{ci} .$$

Given these values at the beginning of period j and the objective of maximizing the duration of ACM operating life, a decision regarding the number of ACM defenders is made. It is assumed that, once assigned to an ACM defense role, the ship will attempt to intercept every incoming cruise missile in the current subinterval without regard to future attacks (future attacks have already been incorporated in the decision). The resulting decision rules are depicted in Table 3.4.

This decision matrix implies that the priority is to extend the ACM operating life of both ships. Where all cruise missile defense possibilities lead to the same expected ACM operating life, both ships are assigned ACM roles to achieve higher kill probabilities. Finally, when it is decided that only one ship should perform ACM defense, the role is assigned to the ship with the largest ACM inventory.

The assignment rules for ballistic missile defense do not account for survivability of the two ships. In this simple assessment, we neither track the relative survivability of the infrastructure targets nor attempt to prioritize them. Therefore, we assume that all infrastructure targets are to be defended equally. We construct the simple

Table 3.4

Ship Assignment Decision Rules for Cruise Missile Defense

Comparison	Assignment
$\min\{\upsilon(b,c),\kappa(2)\} \geq \min\{\upsilon_c^{(c)}(c),\kappa(1)\}$ and $\min\{\upsilon(b,c),\kappa(2)\} \geq \min\{\upsilon_c^{(b)}(b),\kappa(1)\}$	Assign mission to both ships
$\min\{\upsilon(b,c),\kappa(2)\} < \min\{\upsilon_c^{(c)}(c),\kappa(1)\}$ and $\min\{\upsilon_c^{(b)}(b),\kappa(1)\} < \min\{\upsilon_c^{(c)}(c),\kappa(1)\}$	Assign mission to $A^{(c)}$
$\min\{\upsilon(b,c),\kappa(2)\} < \min\{\upsilon_c^{(b)}(b),\kappa(1)\}$ and $\min\{\upsilon_c^{(c)}(c),\kappa(1)\} < \min\{\upsilon_c^{(b)}(b),\kappa(1)\}$	Assign mission to $A^{(b)}$
$\min\{\upsilon(b,c),\kappa(2)\} < \min\{\upsilon_c^{(b)}(b),\kappa(1)\}$ and $\min\{\upsilon(b,c),\kappa(2)\} < \min\{\upsilon_c^{(c)}(c),\kappa(1)\}$ and $\min\{\upsilon_c^{(b)}(b),\kappa(1)\} = \min\{\upsilon_c^{(c)}(c),\kappa(1)\}$	Assign mission to $A^{(c)}$ if $I_c^{(c)} \geq I_c^{(b)}$ or assign mission to $A^{(b)}$ if $I_c^{(c)} < I_c^{(b)}$

decision rule that both ships will attempt to intercept all incoming TBMs as long as they survive and have not depleted their inventory of ABMs.

SUMMING UP

In this chapter, we have linked the effectiveness of the two Aegis cruisers in defending against both the cruise missile and ballistic missile threat to alternative command and control processes and to alternative operational networks. To do this, it was first necessary to establish adequate measures of effectiveness and performance. Next, we developed mathematical models of collaboration and network complexity to assess the performance of the alternative command and control procedures. Finally, these models were used in an allocation decision process that directly influenced the survivability of the cruisers and the infrastructure targets they were defending.

The Measures

In this vignette, the Aegis cruisers are given two missions: defend against cruise missile attacks against themselves and prevent enemy ballistic missiles from destroying key allied infrastructure targets. For both missions, the measure of success or MOE is survivability—that is, *the fraction of the critical infrastructure targets that survive the attack and the "fraction" of the cruisers that survive the attack.*

The defending ships can detect, identify, and track attacking enemy missiles. What they are less able to do is predict how the enemy will distribute these missiles over time. Knowing the attack distribution contributes directly to the allocation of missile interceptors and therefore to the survivability of both the cruisers and the friendly infrastructure targets. A measure of how well the alternative command and control procedures and networks perform (MOP) therefore is *the degree to which the friendly commander "knows" the enemy's attack distribution.*

The Metrics

Network complexity and collaboration are combined to provide an estimate of the number of cruise and ballistic missiles expected to arrive in the next and subsequent time periods. These estimates are then used in the allocation decision rules for each of the three alternative command and control processes and network configurations. The estimates are:

$$\hat{\lambda}_c = \left(1 - K_{cC_v}(\lambda)\right)\frac{n_c}{T} + K_{cC_v}(\lambda)\lambda_c$$
$$\hat{\lambda}_b = \left(1 - K_{cC_v}(\lambda)\right)\frac{n_b}{T} + K_{cC_v}(\lambda)\lambda_b .$$

The terms $\hat{\lambda}_c$ and $\hat{\lambda}_b$ are the current estimates of attacking cruise and ballistic missiles respectively. $K_{cC_v}(\lambda)$ represents the knowledge about the attack distribution informed by the complexity of the network and the collaboration that has taken place. The subscript v refers to the case being examined ($v = 1$: platform-centric, $v = 2$: COP, and $v = 3$: cooperative engagement), λ_c and λ_b are the true attack distributions, and n_c and n_b are the attack sizes for cruise and ballistic missiles to be launched over a total of T minutes. The knowledge function is bounded between 0 and 1 with $K_{cC_v}(\lambda) = 1$ representing perfect knowledge. When this occurs, $\hat{\lambda}_{ci} = \lambda_{ci}$ and $\hat{\lambda}_{bi} = \lambda_{bi}$ and when knowledge is poor ($K_{cC_v}(\lambda) = 0$) we have that $\hat{\lambda}_{ci} = n_c / T$ and $\hat{\lambda}_{bi} = n_b / T$. This last case means that our estimate is that the missiles are uniformly distributed over the attack horizon, T. These equations are the measure of performance.