Chapter Four

TEACHING AND LEARNING ALGEBRA IN KINDERGARTEN THROUGH 12TH GRADE

The way in which a mathematics curriculum is organized shapes students' opportunity to learn. A research agenda aimed at understanding and supporting the development of mathematical proficiency should examine the ways in which mathematics instruction is organized. It should do so by looking closely at the organization and presentation of particular mathematical topics and skills in the school curriculum.

Mathematics teaching and learning are probably best studied within specific mathematical domains and contexts, but there may be aspects of mathematics teaching and learning that are more general and can be studied across multiple domains and contexts. Where systematic inquiry focused on learning specific areas of mathematics has been conducted previously—for example, research on children's early learning of numbers, addition, and subtraction—the payoff for teaching and learning has been substantial.[1] This experience suggests that it would be fruitful to focus coordinated research on how students learn within other topical domains of school mathematics. This research should include studies of how understanding, skill, and the ability to use knowledge in those domains develop over time. It should also include studies of how such learning is shaped by variations in the instruction students are offered, by the ways that instruction is organized within schools, and by the broader policy and environmental contexts that affect the ways schools work.

For a number of reasons, which we discuss next, the RAND Mathematics Study Panel recommends that the initial topical choice for focused and coordinated research and development should be algebra. We define algebra broadly to include the way in which it develops throughout the kindergarten through 12th grade (K–12) curriculum and its relationship to other mathematical topics upon which algebra builds and to which it is connected.

[1]Kilpatrick, Swafford, & Findell, 2001.

ALGEBRA AS A MATHEMATICAL DOMAIN AND SCHOOL SUBJECT

We use the term "algebra" to broadly cover the mathematical ideas and tools that constitute this major branch of the discipline, including classical topics and modern extensions of the subject. Algebra is foundational in all areas of mathematics because it provides the tools (i.e., the language and structure) for representing and analyzing quantitative relationships, for modeling situations, for solving problems, and for stating and proving generalizations. An important aspect of algebra in contemporary mathematics is its capacity to provide general and unifying mathematical concepts. This capacity is a powerful resource for building coherence and connectivity in the school mathematics curriculum, across grade levels, and across mathematical settings.

Historically, algebra began with the introduction of letter symbols in arithmetic expressions to represent names of undetermined quantities. These symbols might be "unknowns" in an equation to be solved or the variables in a functional relationship. As the ideas and uses of algebra have expanded, it has come to include structural descriptions of number systems and their generalizations, and also the basic notions of functions and their use for modeling empirical phenomena—for example, as a way of encoding emergent patterns observed in data. Algebra systematizes the construction and analysis of the formulas, equations, and functions that make up much of mathematics and its applications. Algebra, both as a mathematical domain and as a school subject, has come to embrace all of these themes.

Researchers have made many recommendations about the appropriate curricular focus for school algebra, as well as what constitutes proficiency in K–12 algebra.[2] Common to most of these recommendations are the following expectations related to algebraic proficiency:

- The ability to work flexibly and meaningfully with formulas or algebraic relations—to use them to represent situations, to manipulate them, and to solve the equations they represent
- A structural understanding of the basic operations of arithmetic and of the notational representations of numbers and mathematical operations (for example, place value, fraction notation, exponentiation)
- A robust understanding of the notion of function, including representing functions (for example, tabular, analytic, and graphical forms); having a

[2]See, for example, National Council of Teachers of Mathematics, 2000; Achieve, 2002; Learning First Alliance, 1998; and various state mathematics frameworks (for example, a mathematics framework for California at www.cde.ca.gov/board/pdf/math.pdf, a mathematics framework for Georgia at www.doe.k12.ga.us/sla/ret/math-grades-1-8-edited.pdf, and a mathematics framework for Illinois at www.isbe.net/ils/math/math.html).

good repertoire of the basic functions (linear and quadratic polynomials, and exponential, rational, and trigonometric functions); and using functions to study the change of one quantity in relation to another

- Knowing how to identify and name significant variables to model quantitative contexts, recognizing patterns, and using symbols, formulas, and functions to represent those contexts.

These recommendations also call for the concepts of algebra to be coherently connected across the primary and secondary school years and for instruction that makes these connections. Consistent with both the direction of state and national frameworks and standards, and the visible trends in instructional materials used across the United States, our proposed research would examine the teaching and learning of algebra and related foundational ideas and skills beginning with the primary and extending through the secondary levels.

For example, when five-year-olds investigate the relationships among colored wooden rods of different lengths, they are gaining experience with the fundamental notions of proportionality and measure, an instance of using models to understand quantitative relationships. When, as six-year-olds, they represent these relationships symbolically, they are developing the mathematical sensibilities and skills that can prepare them for learning algebraic notation later on. And when seven-year-olds "skip count"—for example, count by twos starting with the number three (3, 5, 7, and so forth)—they may be gaining experience with basic ideas of linear relationships, which are foundational for understanding patterns, relations, and functions.

At the middle school level, connections of proportional reasoning with geometry and measurement appear in the following sort of analysis: If one doubles the length, width, and depth of a swimming pool, then it takes about twice the number of tiles to border the top edge of the pool, four times the amount of paint to cover the sides and bottom, and eight times the amount of water to fill the pool.

At the high school level, the following example illustrates several of the previous motifs simultaneously. Consider the temperature, T, of a container of ice cream removed from a freezer and left in a warm room. The change in T over time t (measured from the time of removal) can be modeled as a function,

$$\mathrm{T(t)} = a - b2^{-\mathrm{t}} + b$$

where a and b are constants and b is positive. By transforming this formula algebraically to the form

$$\mathrm{T(t)} = a + b(1 - 2^{-\mathrm{t}})$$

and using knowledge of the exponential function, we can see that T(t) increases from a at time t = 0 toward $a + b$ as time advances. This is because as t gets larger, 2^{-t} decreases toward zero. Many mathematicians and educators would agree that students should reach a level of proficiency that enables them to see what a and $a + b$ each represents—that is, the freezer temperature and the room temperature, respectively. This level of proficiency involves understanding what it means for this formula to model the phenomenon in question and transforming the formula algebraically to make certain features of the phenomena being modeled more visible. It also involves interpreting the terms in the formula and understanding what the formula says about the phenomena that it models.

In this case, the formula for T was given, and it was analyzed algebraically. But how does one find such formulas in the first place? This is the (usually more difficult) empirical phase of modeling a phenomenon, in which one gathers some data—say, a set of measurements of T at certain moments in time, perhaps delivered from some electronic data source—and then selects a function from his or her repertoire that best models the data. This process can be quite complex but is often feasible with the use of technology for most of the models typically included in school curricula. The education community is in the midst of a period of important changes in school algebra, with shifting and contending views about who should take it, when they should learn it, what it should cover, and how it should be taught. As recently as ten years ago, the situation was more stable: Generally, algebra was the province of college-bound students, primarily those headed for careers in the sciences. Algebra was taken as a distinct course first encountered in high school; it focused on structures and procedures and often the teaching emphasized procedural fluency and competency in manipulation of symbols.

Today's school algebra is construed by a variety of people, including mathematicians, businesspeople, mathematics educators, and policymakers, to be a broader field encompassing a wider range of subjects. Many people think it should be required of all students, not just a select few, and that it should be addressed across the grades, not only in high school. Teachers and developers of instructional materials are now committed to helping students learn algebra in such a way that it is meaningful and applicable in a wide range of contexts. In addition, the technological tools (e.g., graphing calculators and computer-based algebra tutors) available to help students understand and use algebra have changed radically. Today's school algebra is dynamic in every way.

BENEFITS OF A FOCUS ON ALGEBRA

We selected algebra as an initial area of focus for the proposed research and development program for three main reasons.

First, as we discussed earlier, algebra is fundamental for exploring most areas of mathematics, science, and engineering. Algebraic notation, thinking, and concepts are also important in a number of workplace contexts and in the interpretation of information that individuals receive in their daily lives.

A second reason for selecting algebra as an initial area of focus is its unique and formidable gatekeeper role in K–12 schooling. Without proficiency in algebra, students cannot access a full range of educational and career options, and they have limited chances of success. Failure to learn algebra is widespread, and the consequences of this failure are that far too many students are disenfranchised. This curtailment of opportunity falls most directly on groups that are already disadvantaged and exacerbates existing inequities in our society. Moses and Cobb argue forcefully that algebra should be regarded as "the new civil right" accessible to all U.S. citizens:[3]

> . . . once solely in place as the gatekeeper for higher math and the priesthood who gained access to it, [algebra] now is the gatekeeper for citizenship, and people who don't have it are like the people who couldn't read and write in the industrial age . . . . [Lack of access to algebra] has become not a barrier to college entrance, but a barrier to citizenship. That's the importance of algebra that has emerged with the new higher technology.

Finally, many U.S. high schools now require students to demonstrate substantial proficiency in algebra before they can graduate. These requirements are a result of the higher standards for mathematics that are being adopted by most states as a result of the general public pressure for higher standards and associated accountability systems. The recent "No Child Left Behind" legislation has reinforced these moves. The significant increase in performance expectations in algebra proficiency associated with these standards imposes challenges for students and teachers alike. In the near term, a lack of strong and usable research in support of instructional improvement in algebra is likely to lead to interventions and policy decisions that are fragmented and unsystematic. These interventions will be vulnerable to the polemics of a divisive political environment. In the longer term, research and development coupled with trial and evaluation are needed to create new materials, instructional skills, and programs that will enable the attainment of higher standards for mathematical proficiency.

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[3]Moses & Cobb, 2001.

Other domains of mathematics, such as probability, statistics, or geometry, might vie for focused attention, along with algebra, in our proposed research and development program. Each of these domains is important, and strong arguments could be made for why each would be a good focus for coordinated work. We expect that, over time, systematic work would be supported in these areas. Still, algebra occupies a special place among the various domains because it is more than a topical domain. It provides linguistic and representational tools for work throughout mathematics. It is a strategic choice for addressing equity issues in mathematics education, and its centrality and political prominence make it a logical choice for a first focus within a new, coordinated program of research and development.

WHAT DO WE NEED TO KNOW ABOUT TEACHING AND LEARNING ALGEBRA?

Algebra is an area in which significant educational research has already been conducted. Since the 1970s, researchers in the United States, and around the world, have systematically studied questions about student learning in algebra and have accumulated very useful knowledge about the difficulties and misunderstandings that students have in this domain. Researchers have looked at student understanding of literal terms and expressions, simplifying expressions, equations, word problems, and functions and graphs.[4] This previous work that highlights student thinking patterns and the difficulties that students typically have with algebra is invaluable as a foundation for what is needed now.

Despite the extensive research in this area, we lack research on what is happening today in algebra classrooms; how innovations in algebra teaching and learning can be designed, implemented, and assessed; and how policy decisions can shape student learning and improve equity. Because most studies have focused on algebra at the high school level, we know little about younger students' learning of algebraic ideas and skills. Little is known about what happens when algebra is viewed as a K–12 subject, is integrated with other subjects, or emphasizes a wider range of concepts and processes. Research could inform the perennial debates about what to include, emphasize, reduce, or omit. We see the proposed algebra research agenda as having three major components:

- Analyses and comparison of curriculum, instruction, and assessment
- Studies of the relationships among teaching, instructional materials, and learning
- Studies of the impact of policy contexts on equity and student learning.

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[4]Kieran, 1992.

Analyses and Comparison of Curriculum, Instruction, and Assessment

There is much debate and disagreement today over what topics, concepts, skills, and procedures should be included in school algebra. However, the debate is often based on conjecture and unsupported assumptions about what is going on in the nation's schools in the area of algebra. Our proposed research agenda includes a call for a description and analysis of the goals, areas of emphasis, topics, and sequencing of algebra as they are represented in the various curricula, instructional approaches, frameworks, and assessments currently in use. Algebra curricula today risk being categorized in oversimplified ways according to the "perspectives" on algebra they embody (e.g., "functions-based," "generalized arithmetic," or "real-world" perspectives).

Researchers could provide critically important analytic frameworks and tools for the systematic description and comparison of the curricular treatments of algebra and go on to conduct this description and comparison on a national scale. Perhaps the "pure" embodiments of these various perspectives will turn out to be relatively rare. It may be that many instructional materials integrate various perspectives into complex constructions that involve intricate decisions about sequencing and emphasis on and motivation of ideas. In short, discussions about the nature of school algebra could be much more productive if more-refined tools and analytic frameworks were available. Some tools, such as surveys and other instruments, and methodologies for such large-scale descriptive work already exist.[5] Likewise, a number of scholars and professional groups have offered ways of categorizing and describing perspectives on school algebra.[6] We need systematic, reliable information on how algebra is actually represented in contemporary elementary and secondary curriculum materials, as designed and as enacted.

Despite the flurry of intense debates over algebra, we know far too little about the relevant aspects of what is happening in the schools. We know little about which instructional materials and tools are used in the nation's classrooms for the teaching of algebra, how teachers use these materials in their practice, or how student learning of algebra is assessed. How much has the algebra curriculum actually changed at the high school level? How much have the ideas and tools of algebra, from any perspective, permeated the elementary school curriculum? How do elementary and secondary teachers understand and use algebra, and what perspectives of this domain typify their knowledge? Large-

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[5]For example, The Third International Mathematics and Science Study (TIMSS) Curriculum Framework (see Mullis et al., 2001); the "Study of Instructional Improvement" (2000) and other alignment frameworks such as the framework described by Porter & Smithson, 2001.

[6]See, for example, Chazan, 2000; Bednarz, Kieran, & Lee, 1996; Kaput, 1998b; Lacampagne, Blair, & Kaput, 1995.

scale descriptive studies might examine such areas as teachers' use of textbooks, the ways in which technologies are used in algebra classrooms, what tools and approaches teachers draw on in algebra instruction, and how the ideas of algebra are integrated with other areas of mathematics.

Before recommendations for change and improvement in the teaching of algebra can be fully realized, and in order to invest most strategically in widespread intervention, educators, policymakers, funders, and researchers need to understand the current state of affairs in the nation's classrooms. In addition, we also lack knowledge of what students learn with different versions of algebra—what skills they develop, what understanding of algebra they have, and what they are able to do with algebraic ideas and tools. Yet, most assessments are built on strong assumptions about when students should study algebra and what they should learn. Analytic work can make these assumptions more explicit and clarify the consequences of misalignment between what students are being taught and what high-stakes assessments are demanding.

Studies of Relationships Among Teaching, Instructional Materials, and Learning

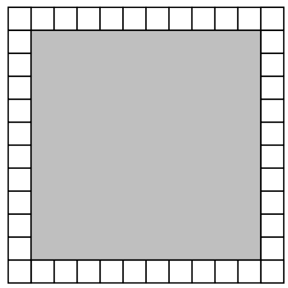
The desired outcome of this proposed agenda of research and development is for the nation's students to understand algebra and be able to use it. Achieving this outcome will mean (1) selecting key ideas of algebra and algebraic ways of thinking to be developed over the K–12 spectrum; (2) designing, testing, and adapting instructional treatments and curricular arrangements to help students learn those ideas and ways of thinking; and (3) assessing the outcomes. Each of these choices would need to be described, articulated, measured, and related to student learning, and high-quality evidence would need to be collected to study the impact of various designs. The strategy we envision involves designing particular instructional approaches and comparing them with existing regimes, as well as with one another. Such systematic work would permit the development of knowledge and tools for the teaching and learning of algebra at various levels and over time. Several considerations and areas of focus, which we discuss next, should shape the organization of work in this part of the research agenda.

Given the range of perspectives about what should constitute school algebra, there is space in this agenda for research that develops curricular and instructional approaches that play out and test the implications of particular perspectives. For example, Carpenter and his colleagues have adopted the view that the teaching of arithmetic can serve as a foundation for the learning of algebra.[7] Their research explores how developing elementary students' capacity to exam-

[7] Carpenter & Levi, 1999.

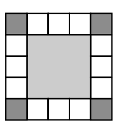
ine, test, and verify or discard conjectures can support important learning about mathematical relations, language, and representation. Building on existing work,[8] other perspectives on algebra need to be developed and studied, such as instruction that follows the historical evolution of algebra, or instruction that takes geometry, rather than arithmetic, as a starting point for instruction in algebra.[9] To illustrate how basic ideas of algebraic structure have been introduced to middle school students from a geometric perspective, consider the following example:[10]

> You are going to build a square garden and surround its border with square tiles. Each tile is 1 foot by 1 foot. For example, if the dimensions of the garden are 10 feet by 10 feet, then you will need 44 tiles for the border.

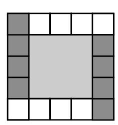


> How many tiles would you need for a garden that is n feet by n feet?

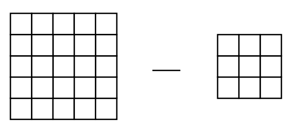
Teachers have found that students will generate many expressions in response to this question,[11] often with strong and clear connections to the actual physical representations of the situation. For example, one correct answer is $4n + 4$, which students will explain by noting that there are four sides, each of which is n feet in length—the "4n" counts the tiles needed along each of the four sides, and the "+ 4" picks up the corners. This is illustrated by the diagram at the left below (when $n = 3$). Two other representations that would be correct (when $n = 3$) are also shown below.



$4n + 4$



$4(n + 1)$



$(n + 2)^2 - n^2$

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[8]See, for example, Chazan, 2000; Gallardo, 2001; and Heid, 1996.

[9]Wheeler, 1996, p. 318.

[10]Adapted from Lappan et al., 1998, p. 20.

[11]See Phillips & Lappan, 1998, and Ferrini-Mundy, Lappan, & Phillips, 1996.

Because these different algebraic expressions represent the same physical quantity (the number of tiles needed), students can use the geometry to establish their equivalence. Here, the geometric perspective introduces students to early ideas of algebraic structure.

Language plays a crucial role in algebra, and so a program of research in this area should include work on language. Words used in algebra—*distribute*, *factor*, *model*, and even *plus* and *minus*—are familiar to students from other contexts. In commenting on algebra research, Wheeler[12] asks, "What happens to one's interpretation of the plus sign . . . when it is placed between two symbols which *cannot* be combined and replaced by another symbol?" That is, what do students make of the algebraic *expression* $a + b$ after years of being able to compress expressions such as 3 + 5 into the single number 8?[13] Algebra may be a key site for the development of students' mathematical language, where the translation of everyday experiences into abstract representations is essential. Problems with language may affect English-language learners in different ways than it affects students for whom English is a second language.[14]

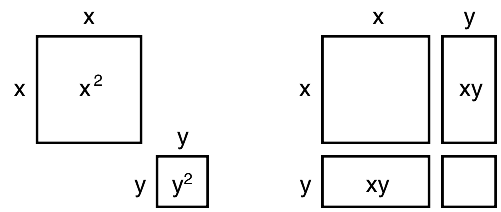
It will also be important for researchers to solicit projects designed to examine the connections among significant ideas within different treatments of algebra. For instance, there is a base of research about students' understanding of function[15] that reveals difficulties that students have in distinguishing functions from other relations and in interpreting graphical representations of functions. Yet, we know little about the relationship between a student's understanding of how functions relate and ideas such as correlation and curve fitting in data analysis. How can teachers and instructional materials effectively make links between related mathematical ideas so that students' knowledge builds systematically over time? Algebra is replete with instances where connections are likely to help build students' understanding. Consider the fact that many students may learn to manipulate x's and y's and never realize that x^2 has a geometric representation—a square with a side length of x. They do not recognize that they can visualize the difference between $x^2 + y^2$ and $(x + y)^2$ quite simply with a diagram, such as the following:

[12] Wheeler, 1996, p. 324.

[13] See Collis, 1975.

[14] See Moschkovich, 1999; Khisty, 1997; Secada, 1990; Gutiérrez, 2002a.

[15] See Harel & Dubinsky, 1992, and Leinhardt, Zaslavsky, & Stein, 1990.



As the example illustrations in this chapter suggest, connections between different areas of mathematics—algebra and arithmetic, algebra and geometry, or algebra and statistics—can be fruitfully examined using algebra as a domain for research.

Research and development should focus on how algebra can be taught and learned effectively across the elementary and secondary years. This will involve substantial longitudinal and cross-sectional comparative work. Certain ideas can be introduced early and come to play key roles in more-advanced algebra learning.

We know, for instance, both from research and from the experience of teachers, that the notion of "equal" is complex and difficult for students to comprehend, and it is also a central mathematical idea within algebra. The equals sign (=) is used to indicate the equality of the values of two expressions. When a variable x is involved, the equals sign may denote the equivalence of two functions (equal values for all values of x), or it may indicate an equation to be solved—that is, finding all values of x for which the functions take the same value. Many studies of students' understanding and use of equality and equation solving[16] have shown that students come to high school algebra with confused notions of equality. For instance, some students think of an equals sign not as a statement of equivalence but as a signal to perform an operation, presumably based on experience in the elementary school years with problems such as 8 + 4 = _____. In fact, some secondary students will, at the beginning of their algebra studies, fill in the blank in "8 + 4 = ____ + 3" with 12. Researchers have suggested that this tendency comes as a result of children's experience in executing arithmetic operations and writing down an answer immediately to the right of an equals sign.

The powerful abstract concepts and notation of algebra allow the expression of ideas and generalized relationships. Equally central to the value of algebra is the set of rules for manipulating these ideas and relationships. These concepts, notation, and rules for manipulation are invaluable for solving a wide range of problems. Learning to make sense of and operate meaningfully and effectively

[16]See, for example, Kieran, 1981, and Wagner, 1981.

with algebraic procedures presents formidable challenges to learning and teaching.

It is especially challenging for teachers to motivate student interest and to foster persistence in this work on symbolic fluency that is so central to algebraic proficiency. Researchers, developers, and practitioners alike ask how such capacity is effectively fostered over time. For example, what kinds of meaning can be attached to symbols and manipulations to support the learning of their use and significance? Although some algebraic relations can be modeled experientially, others are essentially abstract or formal in character. In cases in which the relations are more abstract, meaning can often be located in the very patterns and structure of the formulas and operations themselves. And what levels of skill or fluency are appropriate for various grades or courses? For instance, the proficiency in manipulation of algebraic symbols that should be expected of a beginning calculus student is probably more elaborate and developed than what would be expected of an eighth-grade student. Compelling arguments can be made that procedural fluency is enhanced by intense use. But such intense use could also be designed as part of conceptual explorations of mathematical problems or as part of carrying out mathematical projects, and indeed has been addressed this way in many recent curricular treatments of algebra.

Another current issue that is closely related to the development of symbolic fluency is how different instructional uses of technology interact with the development of algebra skills and algebraic concepts. The increased availability of technology raises new questions about what is meant by "symbolic fluency." Research suggests that graphing calculators and computer algebra systems are promising tools for supporting certain kinds of understanding in algebra, including understanding of algebraic representations.[17] At the same time, important questions remain about the role of paper-and-pencil computation in developing understanding as well as skill. These are questions that appear at every level of school mathematics. Empirical investigation and evidence are essential for practitioners who need stronger evidence for making wise instructional decisions.

Research about algebra has focused more closely on student learning issues than on algebra teaching issues. As Kieran (1992) notes, "The research community knows very little about how algebra teachers teach algebra and what their conceptions are of their own students' learning."[18] For the ambitious changes in algebra instruction and curriculum that are underway nationally to be effective, teachers, teacher educators, and developers of instructional materials need

[17]See Heid, 1997, and Kilpatrick, Swafford & Findell, 2001.

[18]Kieran, 1992, p. 395.

research-based information about different models for algebra teaching at different levels and the impact of those models on student learning of different aspects of algebra. Moreover, research could uncover ways in which teachers work, how they use particular opportunities to learn, and how they use instructional materials, and the like, as they plan and teach lessons. For example, although elementary teachers' use of texts has been investigated in various studies, less is known about how algebra teachers use textbooks, tools, technology, and other instructional materials. Yet, such knowledge would be critical to any large-scale improvement of algebra learning for U.S. students in that it would guide the design and implementation of instructional programs.

In summary, the changing algebra education landscape demands that we direct collective research energies toward solving some of the most pressing problems that are emerging as a result of these change. Research into algebra teaching, learning, and instructional materials should be at the forefront of efforts to improve outcomes for all students in learning algebra in the nation's K–12 classrooms.

IMPACT OF POLICY CONTEXTS ON STUDENT LEARNING

A focus on algebra also brings us squarely to issues related to the organization of the curriculum in U.S. schools, to the requirements for course taking and high school graduation, and to the uses of assessments for purposes of accountability that have far-reaching consequences. All of these policy-context issues relate in crucial ways to matters of equity, students' opportunities to learn, and the prospects for all students in U.S. schools to have a wide range of choices in their professional and personal lives. Thus, research is crucial for better understanding the implications and results of various policy choices and the range of curricular and structural choices (when algebra is taken and by whom, for example) made by schools and districts at a time when the pressures and demands on teachers, administrators, and state and local policymakers are considerable and conflicting.

In the high school and middle school curriculum of U.S. schools, algebra is typically treated as a separate course, and currently most of the material in that course is new to students. In contrast, mathematics in the elementary schools typically combines student experiences with several different mathematical domains. These traditions have recently been challenged by analyses showing that the secondary curricula in most other countries do not isolate algebra within a course apart from other topic areas.[19] The elementary and middle school curricula in most other countries treat algebra more extensively than do

[19]Schmidt et al., 1997.

the curricula in the United States. Many of the instructional materials developed in the United States in the past decade include greater integration of content areas and topics at the secondary school level and greater attention being paid to algebra at the elementary and middle school levels.

Research can address how these various curricular arrangements influence students' learning and their decisions to participate in subsequent courses. If algebra begins to permeate the elementary curriculum in the coming decade, how will its curricular trajectory in the middle and secondary schools change? Such changes will have important implications for the assessment of algebraic proficiency. Algebra's curricular scope—whether located in the traditional high school course sequence or expanded across the grades—presents important questions about the mathematical education opportunities available to diverse populations of students whose prior success with school mathematics has varied dramatically.

Because algebra has been identified as a critical gatekeeper experience, schools and districts struggle with questions about whether algebra should be required of all students and whether it should be offered in the eighth grade. There is some research to indicate that early access to algebra may improve both achievement and disposition toward taking advanced mathematics.[20] Yet, there is no robust body of work to support decisionmakers in school districts on this matter, and the issues are quite complex. For instance, school districts that have adopted a policy that all ninth-graders take algebra typically have eliminated general mathematics, consumer mathematics, and pre-algebra courses. This seems like a positive step toward raising standards for all students, and a direction that should lead to greater equity for students who have traditionally (and disproportionately) occupied the lower-level courses. Some research suggests this has indeed been the case.[21]

However, a program of research aimed at better understanding the issues surrounding algebra education should address the more subtle aspects of such policy shifts and the range of interpretations and implementations due to this shift in policy. For instance, some schools have responded with first-year algebra courses that span two years and that fulfill the high school mathematics requirement, getting students no further than if they had taken algebra in grade ten. And teachers faced with the challenge of heterogeneous classes of algebra students coming from a wide range of pre-algebra instruction and experiences, and possibly unconvinced of the wisdom that all students should study algebra, may need considerable support and professional development to deliver a

[20]See Smith, 1996.

[21]See, for instance, Gamoran et al., 1997, and Lee & Smith, forthcoming.

course that meets the high standards for the subject. Thus, new research work is needed to help illuminate the nature and range of trends in the implementation of certain policies, as well as the consequences for student learning and continuing successful participation in mathematics. Algebra has assumed a critical political and social position in the curriculum; research can help explain the implications of this position.

Research has demonstrated that taking algebra in the ninth grade significantly increases students' chances of continuing on with mathematics study and succeeding in higher levels of mathematics in high school and college.[22] The role of algebra as a gatekeeper has divided students into classes with significantly different opportunities to learn. Currently, disproportionately high numbers of students of color are inadequately prepared in algebra and do not have access to serious mathematics beyond algebra in high school. Research on tracking[23] indicates that the reduced learning opportunities that characterize low-track mathematics classes often align with socioeconomic status and race. Little is known about the impact of policy decisions, such as requiring algebra of all students or including algebra in significant ways on high school exit examinations, on students from different backgrounds and on students of color. Even without answers to such crucial questions, policy decisions that have a direct impact on students' futures are being made daily.

The United States needs to take a close look at the issue of algebra learning in those segments of the population whose success rate in learning algebra has not been high. There are promising routes to algebra proficiency that seem effective within the social context of inner-city schools or schools that serve students of color—most notably, the efforts of Robert Moses and the Algebra Project. Research is needed to clarify how mathematics instruction can capitalize on the strengths that students from different cultural and linguistic groups bring to the classroom in order to enhance the learning of algebra. We know that education is resource dependent, and that communities of poverty often suffer from a lack of well-trained teachers, efficient administrators, and equipment that might support instruction. Some communities have developed strategies intended to address these problems so that their negative effect on students' learning can be reduced or eliminated; we need to examine these strategies through research that enables generalization and refinement of such strategies. The nation also needs a far better understanding of the ways in which policies, curriculum, and professional development opportunities lead teachers toward a heightened sense of accountability for the learning of algebra by all students.

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[22] See Usiskin, 1995, and National Center for Education Statistics, 1994a, 1994b.

[23] Oakes, 1985, and Oakes, Gamoran & Page, 1992.