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THE EFFECTS OF LEASE SIZE ON YIELDS
FROM OIL SHALE SURFACE MINES

R. Y. Pei, D. S. Rubenson

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This Note documents study results presented at several executive briefings to senior Department of Energy personnel, including the Assistant Secretary for Fossil Energy and the Director of the Office of Policy Planning and Analysis, and was prepared for that department under Contract No. DE AC01-80PE70271. Its purpose is to provide that office with an analytical framework to quantitatively explore the effects of lease size on oil shale mining.

Rand has developed an analytical methodology that explains the effects of lease size on yields from oil shale surface mines. That methodology was applied to the types of deposits found in the Piceance Basin in Colorado. In this study, we also explore resource recovery for open pit mining as it is affected by lease size, waste disposal, and processing facilities on the land.

The approach described here should be useful in achieving a better understanding of the relation between lease size and resource yield as it is affected by engineering and related variables, and should be useful to researchers interested in policy and program issues concerning the development of the oil shale resource.
SUMMARY

This note examines the effects of lease size and waste disposal policy on the amount of oil shale that can be recovered from an open pit mine. It also considers process facilities and waste disposal, and assesses the effects of federal leasing policy on the long-term potential of oil shale. The relation between lease size and resource recovery depends on several factors, many of which are sensitive to specific lease conditions. Nonetheless, this study shows that an understanding of that relation can be obtained through geometrical analysis of an open pit mine.

Our analysis shows that existing regulations limit the amount of recoverable oil shale to no more than 20 percent of the resource in place. This result is based on the assumption that oil shale waste products are continuously stored on site, but that they can be moved more than once (multiple handling) to increase resource recovery. If the economic constraints of single handling of wastes are considered, resource yield is limited to approximately 10 percent. Those low yields are the result of unfavorable area-to-depth ratio for the mines, which leaves large deposits of shale beneath the mine walls. Increasing lease size, and thus increasing the area-to-depth ratio, results in a higher yield.

Significantly larger yields can be obtained by allowing larger leases or by maintaining existing lease size and allowing off-site disposal of wastes. Our analysis shows, however, that the need for off-site disposal lands is negligible for leases larger than 35,000 acres. Although off-site disposal increases yield for small leases, significant amounts of land are required for waste disposal. The contiguous nature of the Piceance Basin oil shale implies that these lands may contain large quantities of oil shale. Thus, although off-site disposal may be a simple method of improving yield for small leases, it could have an adverse effect on the long-range potential of recovering oil shale.
The authors would like to thank Bill Krase for helping to formulate this analysis. We are also indebted to R. E. Horvath for reviewing this document and providing many useful comments.
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I. INTRODUCTION

U.S. deposits of oil shale represent an important potential source of liquid hydrocarbons. The Green River Formation, occupying about 34,000 square miles in northwestern Colorado, southwestern Wyoming, and northeastern Utah, contains the richest and most highly concentrated oil shale deposits in the world. One source estimates that the Green River formation contains over 8,000 billion equivalent barrels of oil shale.* About 80 percent of this resource is located on federal lands. To use these lands, private developers must obtain leases from the federal government. The Mineral Leasing Act of 1920 prevents any single, private organization from leasing more than 5,120 acres of oil shale land and prevents off-site disposal of wastes.

Most of the thickest and richest deposits of Green River Formation oil shale are concentrated in Colorado's Piceance Basin—approximately 1,200 square miles of oil shale lands. This area can accommodate about 150 leases under existing regulations. At this time, however, only two such leases have been granted. Because of difficulties encountered in developing one of those leases (Tract C-a), there is some concern about the regulations.

Oil shale deposits in the basin are thick, ranging from a few hundred feet at the edge of the basin to as much as 2,000 feet in the center. Open pit mining is attractive because the fraction of resource extracted may be large: without considering disposal or processing area requirements, one assessment estimates that 90 percent of the total may be recoverable.** Additional advantages of open pit mining are the relative ease of backfilling solid waste, and freedom from many of the hazards of subterranean mining. Alternative techniques for oil shale extraction do not have those advantages. In subterranean mining (often referred to as room and pillar), only a thin portion of a

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** Ibid., p. 123.
thick deposit can be extracted and backfilling of waste is extremely
difficult. Advanced techniques, such as true in situ (TIS) processing
or modified in situ (MIS), processing, are as yet unproven. On the
other hand, little question exists as to the technical feasibility of
open pit mining.
II. FACTORS AFFECTING YIELDS FROM SURFACE MINES

There are three major factors that limit yields from surface mines: the placement of process facilities on the lease, sloped mine walls, and on-site waste disposal. Yield is also affected by lease size, operational mining requirements, and the physical properties of oil shale. The significance of most of these factors can be determined only by detailed geological and topographical analysis of specific lease conditions. In this study, we concentrate on those factors that contribute to a general understanding of the relation between lease size and resource recovery. This section describes those factors and provides the values assumed for each.*

PROCESS FACILITIES

Since restrictions on lease size limit the amount of available land, shale that lies under land set aside for process facilities will be lost. Process facilities usually contain equipment for retorting, upgrading, and other functions required for the conversion of solid oil shale to a liquid hydrocarbon. Since large oil shale facilities have not been constructed, the amount of land required for them is uncertain. Conceptual design studies have estimated that such facilities will occupy about one square mile for a plant that provides a throughput of approximately 50,000 - 100,000 barrels per day.** Here we assume that an oil shale lease can be proportioned into two squares—one for the process facilities and one for the mine. The process facilities are assumed to occupy one square mile (640 acres) for each 5,120 acres leased (e.g., two square miles for a lease of 10,240 acres). This assumption

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** See, for example, Rio Blanco Oil Shale Project, Detailed Development Plan, March 1976.
allows us to decouple the process facilities from our analysis by assuming that the percentage of yield is reduced by a fixed amount for all lease sizes. The rationale for this assumption is that larger leases will produce more shale and require more process facilities. Without hard data on large oil shale facilities, no "economy of scale" has been assumed here.

MINE WALLS

The second major factor that limits the yield is sloped mine walls. To ensure structural stability, open pit mines must be constructed with gradual slopes, and the shale beneath those sloping walls cannot be extracted. The amount of shale lost is determined by wall slope and mine depth; mine depth is determined by the thickness of the shale and the overburden covering it. Gradual slopes and deep mines result in large quantities of lost shale. Steep slopes and shallow mines minimize losses. Losses associated with oil shale mines will be large for both reasons. The structural integrity of oil shale rock is weak compared to rock in most hard rock mines. In two different site-specific studies,* final mine wall slopes of 37 and 45 were estimated; our analysis uses the intermediate value of 41 degrees. The large depth of oil shale mines also implies significant losses from the mine walls. Here we assume a 1,700 ft. mine depth with layers of overburden (600 ft. deep) and shale (1,100 ft. deep) in constant thicknesses across the lease.** These depths are representative of deposits of about 20 to 25 gallons per ton and are located about halfway between the edge and the center of the Piceance Basin.

ON-SITE WASTE DISPOSAL

The third yield-limiting factor results from the requirement that all oil shale waste products be stored on site. These waste products require substantial areas with the subsequent loss of recoverable shale. The amount of shale lost depends on the area required for waste storage

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* Banks, C. E. et al., Technical and Economic Study, April 1975; Rio Blanco Oil Shale Project, Detailed Development Plan, March 1976. ** Later in this report we will use other values for shale depth.
and the area available for storage. The required area is determined by the allowed slope of the waste piles and the volume expansion of in-situ shale after processing, which is approximately 25 percent. For large piles, the slope is limited to approximately 14 degrees with no upper limit on height other than that imposed by the slope. The severity of the waste disposal problem is illustrated in Fig. 1, which shows that one cubic foot of in-situ shale will require 10 square feet of area for the resultant spoil pile.

The area available for waste storage is limited by three factors. The working slope of the mine (the face of the mine being developed) must accommodate equipment and roads and therefore must have a gradual slope. For our analysis, the working slope of the mine will be limited to no more than 19 degrees with the horizontal, although it can be increased to 41 degrees at the end of the process. The area available for waste storage is also limited by the need to maintain a clear width between the edge of the backfilled waste and the working slope to be maintained. We have assumed this width to be 500 ft. The third factor that limits the waste storage area is the possibility that economic and environmental considerations may preclude any multiple handling of waste. During the mining process, areas initially used for mining can

\[
\begin{align*}
1 \text{ ft.}^3 & \quad \text{In-situ shale} \\
1.25 \text{ ft.}^3 & \quad \text{Loose shale} \\
1.25 \text{ ft.}^3 & \quad \text{Spoil pile}
\end{align*}
\]

**Factor of 10 growth in area**

*Fig. 1 - Shale waste occupies large areas*
later be used for waste storage. Conversely, areas initially set aside for waste storage can be mined. Such flexibility, however, requires multiple handling of waste. Single handling of waste means that waste initially placed in a disposal area cannot be transferred; thus, the effective area for waste storage is decreased.

The assumptions underlying our analysis are summarized in Fig. 2, which shows one additional assumption not discussed above: we assume that a 500-ft. perimeter will be required around the mine.* Fig. 2 also illustrates the assumed mining strategy. An initial cut is made in the mine, and waste is stored on the lease. As the mine expands, waste is backfilled into the mine at a rate that maintains the 500-ft. working distance at the bottom of the pit. For small leases, the required opening occupies a large portion of the lease, leaving little room for waste storage. This factor can play a significant role in limiting the amount of shale that can be extracted.

* Other assumptions about mine development can be found in the Appendix.

Fig. 2 - An oil shale surface mine
III. RESULTS

Figure 3 illustrates resource yields obtained from the existing lease size of 5,120 acres.* The third frame in Fig. 3 shows that placing the process facilities on the lease reduces yield by about 10 percent. The fourth frame shows that the mine perimeter and walls reduce yield by another 40 percent, the yield that can be obtained if off-site disposal of waste is allowed. As shown in the fifth frame, requiring that waste be stored on site lowers the yield to about 20 percent—the yield that can be obtained under existing regulations. The last frame shows the effects of on-site disposal coupled with the requirement for single handling of wastes. This constraint, which is an economic rather than a regulatory constraint, reduces potential yield to about 10 percent.

The low yield for a 5,120 acre lease is a result of an unfavorable area-to-depth ratio, which in turn results in large deposits of shale left beneath mine walls. This unfavorable ratio also implies that the minimum opening required for mining (see Fig. 2) will occupy a significant portion of the mine, limiting the capacity for waste storage and hence total amount of shale extracted. Increasing area-to-depth ratio by increasing lease size lessens the effects of these problems. Figure 4 shows how the same constraints affect a lease of 15,360 acres (three times 5,120 acres). A smaller percentage of shale is trapped beneath the mine walls, and the minimum opening required for mining is a smaller fraction of the total mine volume. The improvement in area-to-depth ratio means a higher yield for a larger lease.

Figure 5 shows the effect of lease size on yield as a percentage of the total in-situ shale. We assume that as lease size increases, more process facilities will be required and no economy of scale will be realized. The result of this is indicated by the horizontal line, showing that the yield is limited to 88 percent for all lease sizes. The effect of mine wall slopes results in the yields represented by the

*Details about the methodology and algorithms can be found in the Appendix.
Fig. 3 - What limits a 5,120 acre lease?
Fig. 4 - What is the effect of larger leases?
(3 X 5,120 acres)
curve labeled "mine walls." The value of this curve for a lease of 5,120 acres indicates the yield that can be obtained when off-site disposal of waste is allowed without any increase in lease size.

Limitations brought about by existing regulations require on-site disposal. Existing policy is represented in Fig. 5 by the point above a lease of 5,120 acres on the branch labeled "on-site disposal." This curve shows that the effect of on-site disposal becomes insignificant for lease sizes greater than 35,000 acres. The bottom curve, marked "single handling" is the result of requiring on-site disposal and observing the economic constraint described in Section II. It is interesting to note that plans for developing one of the two existing 5,120 acres lease tracts with open pit mining took into consideration the single-handling constraint. * The government's decision not to allow off-site disposal forced developers to abandon plans to use open

pit mining on that tract. The results presented in Fig. 5 provide some insight into the economic factors leading to that decision.

The results presented in Fig. 5 are based on a mine with a seam depth of 1,100 ft. of oil shale and 600 ft. of overburden. Similar calculations for a thin seam (500 ft. of shale and 300 ft. of overburden) are shown in Fig. 6. It is clear that current leasing restrictions also limit the yield from thinner deposits and that substantial benefits can be obtained with larger leases. Results differ from the case cited earlier in that larger yields can be obtained with smaller leases. Thus, the exact form of the relation between lease size and resource yield depends on the particular lease.

Figure 7 shows the effect of lease size on off-site disposal requirements for our analysis of a 1,700 feet deep mine. This area represents the amount of land required for off-site disposal to obtain the "mine walls" curve shown in Fig. 5. For lease sizes greater than 45,000 acres, there will be sufficient area available for waste so that no additional off-site area is required. In other words, the waste from an area currently being mined can be held on site in adjacent areas and later removed and backfilled to permit mining of the areas previously covered by waste (this would violate the economic constraint of single handling for a small portion of the waste generated). The two data points shown in Fig. 5 indicate approximate values for off-site disposal lands from two site-specific studies.*

Since precise values of the parameters depend on specific conditions of the lease, the values used here can only represent the authors' best attempt to generalize the results. To better understand the possible effect of those values, sensitivity analyses were conducted. The magnitude of the change in yield resulting from a change in input parameters depends on the size of the lease under consideration and the particular constraint being considered (e.g., single handling, mine walls, etc.). Figure 8 shows the results of our sensitivity analysis.

*The values used in Fig. 5 were estimated from C. E. Banks, Technical and Economic Study, April 1975; and from the Rio Blanco Oil Shale Project, Detailed Development Plan, March 1976. The values calculated for this study are consistent with those values.
Fig. 6 - Yields from thin seams
(300 ft. overburden, 500 ft. shale)

Fig. 7 - Off-site disposal requirements
decrease with lease size
conducted for the specific case of a 15,360 acre (3 X 5,120 acres) lease when on-site disposal and single handling are required. The results show the changes in resource yield associated with 10 percent changes in the values of the input parameters. For example, a 10 percent change in the shale expansion coefficient, from 1.25 to 1.275 (25 percent expansion to 27.5 percent), changes the resource yield by 8 percent of its base case value; from 30 percent to 32.4 percent recovery of the total oil in the ground. Although the specific values only apply to the case outlined above, the results illustrate several important effects that can apply to almost all cases. Fig. 8 shows that resource yields are only mildly sensitive to changes in values of the input parameters. A 10 percent change in any single input parameter results in less than a 10 percent change in yield. Figure 8 also shows that the two most sensitive parameters are the waste pile slopes and the shale expansion coefficient; both are characteristic of shale waste.
IV. LONG-TERM RESOURCE UTILIZATION

The results presented in Fig. 5 show that only a small fraction of shale can be recovered under the existing constraints. Significant improvements can be realized with larger leases or off-site disposal of wastes. Although off-site disposal dramatically increases yield, Fig. 7 shows that significant amounts of land will be required for the waste. In a region such as the Piceance Basin, where the shale resource is essentially contiguous, disposal lands may contain large deposits of shale. Figure 9 considers the effects of allowing off-site disposal and four other leasing policies on the long-term utilization of 102,000 acres that produce 1.5 million barrels of oil per day. The bar labeled "Size 1" assumes that existing regulations will be maintained, and the land divided into twenty 5,120-acre leases. The figure also shows that dividing the land into two leases (each of 10 X 5,120) acres or one lease (of 20 X 5,120 acres) can significantly enhance the

![Bar chart showing years of production at 1.5 million BPD for different leasing policy options.](image)

Fig. 9 - Long-term implications of leasing policy
(total area = 20 X 5,120 acres)
long-term potential of oil shale. The option of maintaining existing lease size and allowing off-site disposal is not as effective because shale beneath the off-site disposal piles is not extracted. Thus, off-site disposal is a simple method of improving yield for small leases, but it may have an adverse effect on the long-range potential of shale.

The numbers in Fig. 9 are based on single handling of wastes. If we assume multiple handling, and if lease regulations could be altered to extract shale on disposal lands, the results would still strongly favor large leases over small leases with off-site disposal. Dividing the basin into several small leases would scatter shale process facilities throughout the basin, making it difficult to connect the many small mines. At best, they could only be joined to form a highly irregular shape, which would lead to significantly lower yields. If the comparison were based on multiple handling, the quantity of waste requiring such handling would be significantly larger for the case of off-site disposal. Although this would not alter resource yields, it would affect the economics of the process.

The results presented in Fig. 9 can be interpreted differently by fixing the number of years of production and determining the amount of land required. Figure 10 indicates that even if shale production levels are not intended to fully use the entire resource, the most efficient use of land is achieved by a small number of centralized development projects.
Fig. 10 - Long-term land use implications
(1.5 million BPD for 95 years)
V. CONCLUSIONS

We have shown that existing federal lease restrictions severely limit the amount of oil shale that can be extracted. Larger leases offer higher resource yield with more efficient use of lands. Off-site disposal of waste increases the potential yield of individual leases, but that waste may cover land with extensive oil shale deposits.

Besides explaining the need for larger leases, the results point up two issues to be considered when developing an oil shale leasing policy. First, the nature of the oil shale resource implies that large-scale mining operations will offer greater efficiency. Potential problems with competition and monopolies must be addressed if large leases are allowed, but the Piceance Basin appears large enough to accommodate several large mines. Second, the results indicate the highly interactive nature of individual oil shale projects. Placing a few small leases in the middle of the Basin could severely inhibit plans for large leases. Conversely, waste disposal on low-grade deposits at the edge of the basin may facilitate large-scale mining operations. Leasing policy, therefore, should be formed on a basin-wide level, considering basin topography, geography, and long-range resource production goals.
APPENDIX

This Appendix presents the method we used to obtain the results reported in this Note.

LEASE LAYOUT

We assumed that the lease could be divided into two squares—one for the process facilities and one for the mine. The process facilities occupy one square mile per 5,120 acres. Figure A illustrates this layout. Figure A also shows a 500 ft. perimeter surrounding the mine. This perimeter is 500 ft. thick for all lease sizes. The symbol "W" in Fig. A represents the width (or length) of the mine.

![Diagram of lease layout]

Fig. A — Lease layout
As stated in the main text of this Note, our assumed base case lease has an overburden thickness of 600 ft. and a shale thickness of 1,100 ft. Figure B shows these dimensions on a cross-sectional view of the mine.

![Fig. B - Lease cross section]

RESOURCE YIELDS

Figure 5 shows resource yields that can be obtained under four different constraints. All results are based on a comparison of the volume of shale extracted and the total volume of shale in the lease. The yield can be expressed mathematically by the following equation:

\[
\text{Resource yield} = \frac{\text{(Volume of shale extracted)}}{\text{(Total volume of in-situ shale)}} \quad (1)
\]

In all cases the total volume of in-situ shale is assumed to consist of an 1,100 ft. thick seam underneath the entire lease. This volume (in cubic feet) is given by:

\[
\text{Total volume of in-situ shale} = 2.4 \times 10^{11} N \quad (2)
\]

where \( N \times 5,120 \) acres is the lease size.
The next four sections describe the methodology used to compute the amount of shale extracted for each constraint illustrated in Fig. 5.

**Process facilities**

The process facilities are assumed to occupy about 12 percent of a lease for all lease sizes. Here we assume that all shale underneath these facilities is lost, which results in the horizontal line shown in Fig. 5 limiting all yields to no more than 88 percent.

**Mine walls**

The curve labeled "mine walls" in Fig. 5 shows the limitations of 41-degree mine walls without considering the effects of storing shale waste on site. The volume of shale extracted is a four-sided, flat-top pyramid extending from a depth of 600 ft. to a depth of 1,700 ft. The total volume extracted (shale plus overburden) is a similarly shaped pyramid extending from the surface to a depth of 1,700 ft. Figure C shows a cross section of this volume. The total volume extracted is given by:

\[
V = x^2 h - \frac{2h^2 x}{\tan \theta} + \frac{4}{3} \frac{h^3}{\tan^2 \theta}
\]  

(3)

where  
\( \theta \) = wall angle  
\( h \) = vertical height  
\( x \) = width of the large base

![Fig. C — Cross section volume mined for "mine walls" case](image-url)
The volume of shale removed is obtained by using (Eq. 3) with the following values:

\[ x = W - 2(600 \text{ ft.})/\tan \theta = W - 1,380 \text{ ft.} \]

\[ \theta = 41\text{-degrees} \]

\[ h = 1,100 \text{ ft.} \]

Using these values, the volume of shale (in cubic feet) extracted is:

\[ V = \frac{(W-1380)^2}{1100} - 2.8 \times 10^6(W-1380) + 2.8 \times 10^9 \quad (4) \]

\( W \) is the width of the mine; its value can be expressed in terms of lease size by using the geometry shown in Fig. A. This relationship can be shown to be:

\[ W = 13,959 \sqrt{N} - 1,000 \quad (5) \]

where \( N \times 5,120 \text{ acres} \) is the lease size

\((W \text{ is in feet})\)

This formula shows that for \( N=1 \) the mine width is approximately 13,000 ft. Using Eqs. (4) and (5), the volume of shale extracted can be computed as a function of lease size. This can be inserted into the numerator of Eq. (1) to obtain the resource yield. For \( N=2 \) (a 10,240 acre lease), the volume of shale mined is \( 2.9 \times 10^{11} \), whereas the total volume of shale on the lease is \( 4.8 \times 10^{11} \), for a yield of about 60 percent, as shown in Fig. 5.
Single Handling

This section considers yields obtained when on-site disposal and single handling of waste are required. The losses imposed by these constraints are illustrated in Fig. D, which shows a cross-sectional view of the mine at three different stages of development.

The first frame of Fig. D shows the stage at which the final depth and the required working distance of 500 ft. are first reached. The second frame shows an intermediate stage at which the backfilled waste is wedge-shaped. The third frame shows the final stage for single handling of waste. The 500-ft. distance separating the backfilled waste pile and the 19-degree working slope are maintained throughout the process. The working slope can be increased to the angle of the final walls (41 degrees) at the end of the process. The mine cut is assumed to run in and out, across the entire width of the mine.

The mine is allowed to progress through its various development stages as long as the following constraint is obeyed for all mine states:

\[
\text{Volume available for waste storage} > (K) \times (\text{Volume extracted}) \quad (6)
\]

where \( K \) = the spent shale expansion coefficient equal to 1.25.

The volume available for waste storage in the pit is not large enough to satisfy this constraint for all stages (consider the first frame of Fig. D, where no volume is available in the pit but substantial mining has taken place). To satisfy the constraint, an area must be set aside for waste storage, here represented by the area above \( X_L \) on Fig. D. Since (Eq. 6) must be maintained for all stages, in the absence of double handling of waste, the mine stage that requires the greatest

*This policy can severely limit the yield from smaller leases. Alternative policies are discussed in the last section of this Appendix.
\( \theta_w \) = Waste pile angle of repose = 14°
\( \theta_m \) = Mine wall angle = 41°
\( \theta_{ws} \) = Working slope angle = 19°

D = Clear distance \( \geq 500 \) ft

Fig. D — Mine development with single handling
value of $X_L$ determines the size of $X_L$, which also determines the amount of shale not recovered. In general, the configuration shown in the second frame of Fig. D will determine the required value of $X_L$.

$X_L$ determines the amount of shale lost because of the single handling constraint. The size of $X_L$ is the largest value needed to ensure that (Eq. 6) holds for all mine stages. Once this is determined, the total volume of shale extracted can be calculated. The geometry in the third frame of Fig. D shows that the volume of shale extracted is a four-sided pyramid with a rectangular base. Using standard geometric formulas, this quantity can be calculated and inserted into (Eq. 1).

Once the stage represented by the second frame of Fig. D has been reached, the mine opening generally moves faster than the waste pile. This increases the working distance (the distance between the end of the backfilled waste pile and the beginning of the working slope) to a value greater than the required minimum of 500 ft. This is indicated in the last frame of Fig. D.

These illustrations also provide some insight into the sensitivity of yield in relation to lease size. The amount of shale lost is determined by an intermediate mine stage (the second frame of Fig. D), so the total quantity of shale lost is only mildly sensitive to the size of the lease. In other words, the absolute value of $X_L$ does not vary rapidly with lease size. Thus, the larger the lease, the smaller $X_L$ is as a percentage of the total mine width.

**Double handling**

Double handling of waste allows for the possibility of extracting the shale lost, as shown in the third frame of Fig. D. Since the mine usually expands faster than the waste pile, the working distance at

---

*This can be shown by evaluating the rate of growth of the volume extracted and the resultant rate of growth of the volume available for waste storage. For most mine geometries, this occurs when the backfilled volume is in the shape of the wedge shown on the second frame of Fig. D. Exceptions are possible; they have been considered in our analysis.

**Increased yields due to handling the waste more than twice were investigated and found to be insignificant.
the bottom of the mine is greater than the minimum requirement of 500 ft. This extra distance may allow for the waste to be transferred, as shown in Fig. E; however, two factors may prevent this transfer. First, the transfer divides the waste pile in two. This separation is a less efficient way to store waste, since more area is required, and a smaller working distance at the bottom of the mine results. Second, the mine wall on the left now becomes the working slope with an angle of 19 degrees (compared to 41 degrees before waste transfer). This process creates additional waste without increasing waste storage volume; it also reduces the working distance at the bottom of the mine. If these two factors reduce the working distance to below 500 ft., waste transfer cannot occur as shown. In such cases, yield can be increased by mining to depths less than the 1,700 ft. maximum (from the beginning of the process). This approach, however, may lower the yield that can be obtained when obeying the single handling constraint. The results shown in Fig. 5 give the largest value obtainable under each constraint, even though both results may not be consistent with one mining operation.

Once waste transfer is completed, the mine can be expanded to the final configuration shown in Fig. F. If the full depth is mined, the volume of shale extracted is equal to that of the case labeled "mine walls." Figure 5 shows that this occurs for leases greater than about 40,000 acres. The mining depth for smaller leases was limited to less than the maximum of 1,700 feet to increase the yield obtained with double handling.

Fig. E — Waste transfer
Alternative mining strategies

The mining policy in this analysis assumes a mine cut across its entire width (in and out of the page on the cross-sectional views), which reduces available volume for waste storage. For a lease of 5,120 acres, the total mine width is approximately 13,000 ft. The initial cut, shown in the first frame of Fig. D, has an opening of 7,400 feet. The remaining 5,600 ft. x 13,000 ft. area is not large enough to accommodate the waste. Thus, the total mine depth cannot even be reached for a lease this size.

To attempt to increase the yield from small leases, several alternative mining policies were explored. One possibility is to mine only a thin portion of the 1,100 ft. shale deposit. This approach is not as favorable as those that limit the width of the cut. Figure G shows an example of such an approach for a lease size of 5,120 acres. In this configuration, the strip marked "1" is mined and the areas marked "2" and "3" are used for waste storage. If double handling is allowed, the waste in strip 3 can be transferred into the open pit created from mining strip 1. Strip 3 can then be mined, with excess waste stored in strips 1 and 2. After strip 3 is mined, waste from strip 2 can then be stored on strips 1 and 3 and part of strip 2 can be mined. This procedure provides more yield than can be obtained with a single cut across the entire lease. The practicality of mining a lease in this manner has not been investigated, but even if it is practical, the
yield will not increase significantly. Using this method, we found that the yield obtained with double handling is less than 20 percent for a lease of 5,120 acres.

![Diagram showing mining in thin strips](image)

Fig. G — Mining in thin strips
BIBLIOGRAPHY


