A RAND NOTE

Insurance Aspects of DRG Outlier Payments

Emmett B. Keeler, Grace M. Carter,
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October 1988

40 Years 1948-1988

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This research was sponsored by Cooperative Research Agreement C-98489/9 from the U.S. Department of Health and Human Services, Washington, D.C.

Published by The RAND Corporation
1700 Main Street, P.O. Box 2138, Santa Monica, CA 90406-2138

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N-2762-HHS

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PREFACE

The work reported in this Note was begun in the fall of 1986 when the Health Care Financing Administration (HCFA) asked the RAND/UCLA Center for Health Care Financing Policy to evaluate the way outliers (extremely costly or long length of stay cases) were reimbursed under Medicare's Prospective Payment System (PPS). The evaluation undertook to describe the problems with PPS that outliers were intended to remedy. One of these problems is the risk that expensive cases pose to hospital finances. An understanding of this problem can be increased by making an analogy between reimbursement for outlier cases and insurance. This Note describes this analogy, derives results concerning outlier policies that minimize risk under various conditions, and provides empirical comparisons of these "optimal" policies with current outlier policy.

A shorter version of this Note will appear in the June 1988 issue of the *Journal of Health Economics*. This research was sponsored under the third and fourth years of Cooperative Research Agreement C-98489/9. It should be of interest to those working on refinement of the PPS.

SUMMARY

As part of the prospective payment system, the government pays "outlier" payments for especially long or expensive cases. These payments can be viewed as insurance for the hospital against excessive losses. They mitigate problems of access and underprovision of care for the sickest patients and provide additional payments to the hospitals that take care of them, thereby making payments to hospitals more equitable. This Note characterizes the outlier payment formulas that minimize risk for hospitals under any fixed constraints on the sum of outlier payments and minimum hospital coinsurance rate. We then simulate per-case payments for a policy that did not include any outlier payments, the current outlier policy, and several other policies that minimize risk subject to different coinsurance constraints. The current outlier policy achieves each of its goals to at least some extent, but more insurance could be provided without lessening attainment of the other goals. We also discuss some problems with the implementation of the current policy, such as its reliance on day outliers.

ACKNOWLEDGMENTS

This work was supported by the Health Care Financing Administration as part of the RAND/UCLA Center for Health Care Financing under cooperative research agreement C-98489/9-04. We would like to thank Stuart Guterman, our project officer at HCFA, for his support and help throughout the project. We received very helpful reviews of an earlier draft from Joseph Newhouse and Will Manning of RAND and from two anonymous referees. We thank Daniel Byrne of RAND for programming support.

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I. INTRODUCTION

To give hospitals more incentives to control costs, the federal government recently switched from paying hospitals for the costs of each aged patient to a prospective payment system (PPS). PPS essentially pays hospitals the national average costs in each diagnosis-related group (DRG) for each patient admitted to the hospital in that group. The rate of increase in costs has slowed, but the policy leaves hospitals at risk for any costs above the average.

In addition to the regular DRG payment, the government pays "outlier" payments for especially long or expensive cases.[1] Outlier payments can be viewed as insurance against excessive losses on a case. They are intended to cover the marginal costs of care beyond the outlier threshold (a deductible on losses) and are financed by a tax on reimbursement of nonoutlier patients (a per-case premium).

In addition to reducing financial risk to hospitals, outlier payments have three other main goals. First, they may make payments more equitable by giving additional money to hospitals that treat sicker and more expensive patients than average. Second, they may reduce the problems of access for patients who can be identified by hospitals as likely to need very expensive treatment. Third, conditional on admission, they reduce the incentives for hospitals to provide less care for the very sick than society would wish them to have.

These other goals explain why the government (HCFA) has made outlier payments case by case, even though hospital risk with a limit on transfers is minimized by payments that set a limit on annual hospital losses (Arrow, 1963; Ellis and McGuire, 1988). With insurance against annual losses, strong incentives to avoid very costly patients are unchanged except in the few hospitals with large enough losses to trigger the insurance payment. Moreover, some of the hospitals that lose more than the limit may be very inefficient, and it seems misguided to give no incentives to economize to the most expensive hospitals.

Arrow (1963) and later Raviv (1979) characterized optimal insurance policies that minimized risk and administrative costs, assuming no moral hazard. However, moral hazard, the wasteful overconsumption of insured goods, seems inescapable in medical insurance that bases payments on the costs of provided services (Pauly, 1968). Attempts to avoid moral hazard through contingent contracts that give an amount depending only on the

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[1]Details of current policy are given in Sec. II.

objective illness must face the costs and difficulties of determining illness. The DRG payment system is an attempt to do so, but it is not fully successful; there is substantial variation in costs within DRG categories, much of which seems due to patient sickness.

Because moral hazard is inescapable, insurance design involves trading off the benefits of insurance against the costs of moral hazard (Zeckhauser, 1970). With insurance for the patient, the additional consumption due to insurance can be valued by the area under the demand curve for that patient and illness episode, leaving triangles of welfare losses between the demand curve and the horizontal line of no insurance. With insurance for the supplying hospital, and HCFA's buying the care, the situation is more complicated.

Following Ellis and McGuire (1986), we assume that the physician-hospital team has a utility function for each patient that increases with both net revenue to the hospital, and the quantity (in $) of care supplied to the patient.[2] For patients for whom the DRG payment is close to their social optimum quantity, competition by providers should keep the value of services close to the social optimum.

For many severely ill patients, the amount supplied by hospitals paid by DRGs with no outliers may be less than optimal. Figure 1.1 illustrates the problem for a hospital serving

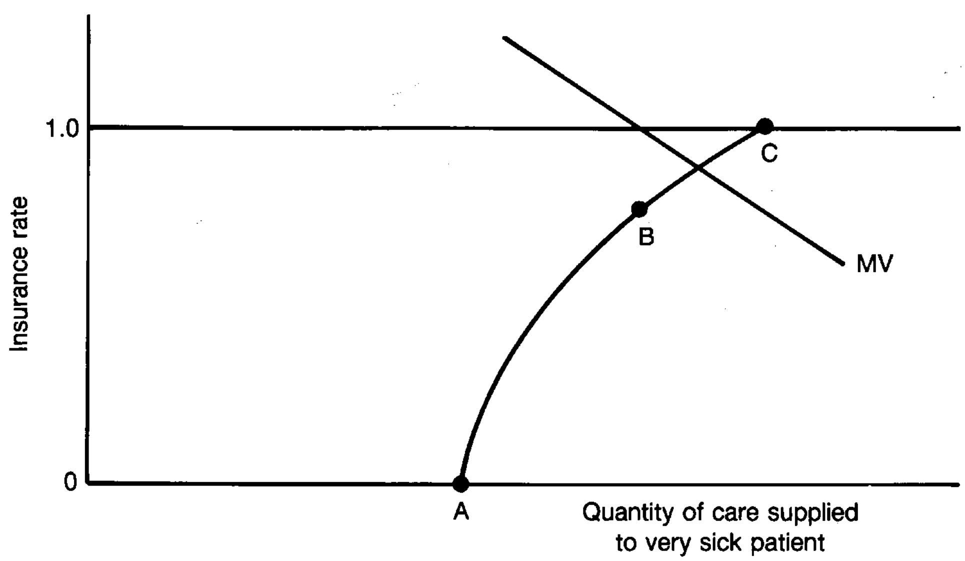

Fig. 1.1—Value and quantity of care supplied to sick patients depending on hospital reimbursement

[2]The team, and especially the nonprospectively paid physician members, are interested in more than just short-term profit—they must also give good care for their own morale and to attract future business. Thus they value increased care to the patient, and the team may increase spending even if net revenue to the hospital falls by so doing.

a very sick elderly patient. The horizontal axis measures the quantity of care supplied (in $). The vertical axis measures the fraction of marginal costs beyond the outlier threshold paid by the government (the price received by the hospital in $/total $). An insurance rate of 1.0 means that all marginal costs above the outlier threshold are paid by the government. With DRGs and no outlier payments, all marginal costs are paid by the hospital (an insurance rate of 0). For this patient, the hospital supplies care to maximize its utility, resulting in the supply curve ABC. Competition does not drive the care provided from A to the socially desirable B, because other hospitals will not compete for patients that cause heavy losses. Insurance does increase the quantity supplied, and with the hypothetical marginal social value of care curve shown, the moral hazard (change in behavior with insurance) is beneficial up to the point B.[3] This benefit is one reason Congress has mandated that 5 to 6 percent of PPS payments be paid to outliers.

However, if the hospital is paid the full marginal costs of additional care for outliers, as soon as they know a case will become an outlier, they can give the highest quality care. In Fig. 1.1, the care provided will go past B to C. HCFA tries to set outlier thresholds and coinsurance to produce approximately the right amount of care for various patients, while achieving other goals. The effects of policy choices are shown in Fig. 1.2. For a given

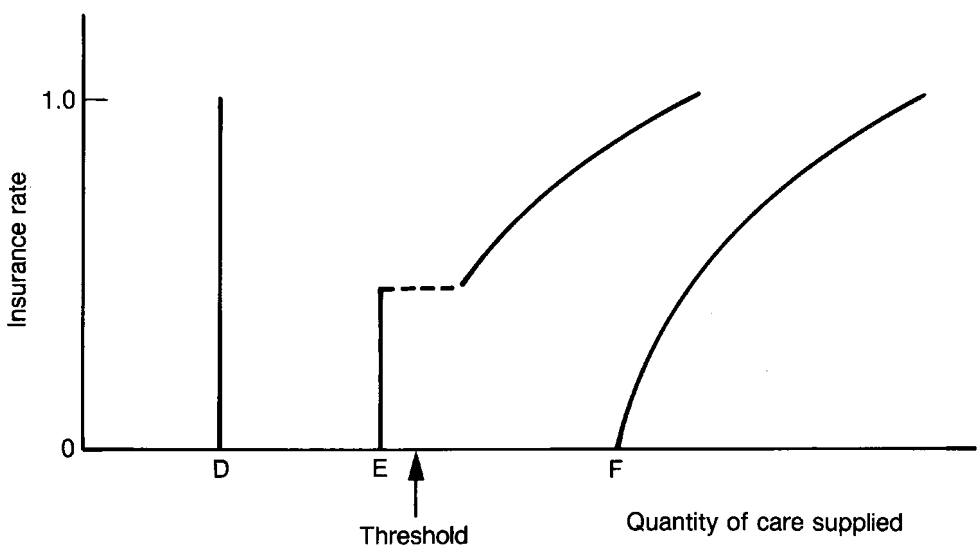

Fig. 1.2—Effects of an outlier threshold on care supplied

[3]Since society pays for the the full marginal cost of additional care, there are welfare gains to additional spending when the marginal value curve is above 1.0, and losses otherwise.

threshold, the care supplied is shown for three patients, one typical (starting at D), one fairly sick (starting at E), and one very sick (starting at F). The broken line for patient E occurs at the coinsurance rate at which it becomes desirable to switch from the inlier quantity E to the equivalently valued larger quantity, smaller net revenue position. For all coinsurance rates, the patient at D is an inlier, and the patient at F is an outlier. As the threshold is increased, the broken line for E rises and eventually F resembles E, and E becomes like D. With a given total for outlier payments, greater coinsurance makes more cases into outliers but reduces the amounts paid to the largest cases. If marginal value curves for each patient were known, we could compute the threshold and coinsurance rate that minimizes the combined welfare losses from the overall social optimum.

In this Note, we assume that Congress or HCFA has decided on case-by-case outlier payments, and has set the current target for the outlier premium and the minimum coinsurance rate by balancing goals of cost incentives for hospitals against goals of equity, access, and sufficient care for severely ill patients. Our Note deals with how payments should be made within such constraints to minimize the risk for hospitals. We do not compute the moral hazard costs and benefits of our minor variations within these constraints, because we know neither how hospitals would react to our policies, nor the value to society of spending more or less on particular patients.

After describing current outlier policy, we derive the risk-minimizing policies. We then present the results of some simulations of various policies on 1984-85 Medicare hospital stays. We show the annual financial risks to different hospitals and the expected losses from various identifiable classes of patients with a payment policy of no outliers, with the current policy, and with better (in terms of risks to hospitals) policies. We conclude with a discussion of the pros and cons of the changes in outlier policy suggested by our results.

II. DESCRIPTION OF CURRENT POLICY

Five percent of federal PPS payments are paid in outlier payments. To finance these payments, each DRG payment rate is reduced by 5 percent. Outlier payments are given for two types of cases: day outliers and cost outliers. Cases with lengths of stay (LOS) that exceed a DRG-specific threshold are classified as day outliers and paid a per diem for each day beyond the threshold in addition to the regular DRG-based payment. The per diem amount is 60 percent of the average federal payment for a day's stay in the DRG and is intended to approximate the marginal cost of each day of care beyond the outlier threshold.

Cases that have extremely high charges but are not day outliers are classified as cost outliers and are also paid a supplement. In calculating the cost outlier payment amount, the charges are transformed to "standardized costs" by multiplying them by a national cost-to-charge ratio and adjusting for teaching and disproportionate share responsibilities.[1] The cost outlier amount is then 60 percent of the difference between the standardized cost and a DRG-specific threshold that is adjusted for local wages (in some cases for the cost of living). Initially, it was intended that the outlier payments approximate the marginal cost of care beyond the threshold (i.e., that there be no coinsurance) and the 0.6 was chosen as an approximation of the ratio of marginal cost to average cost. However, it has been argued that the ratio of marginal cost to average cost is actually substantially higher than 0.6—indeed it may be as high as 1.0. If this is true, then the current policy is imposing a substantial coinsurance rate.

Although the exact thresholds have varied from year to year, the following formulas have always been used:

$$\text{day outlier threshold} = \min(\mu + a, \mu^* \exp(b^*\sigma))$$
$$\text{cost outlier threshold} = \max(c, 2.0^*\text{drgrate})$$

where

μ = geometric mean LOS for the DRG,

σ = standard deviation of the log of LOS for the DRG,

[1]Costs are increased by whatever indirect medical education and disproportionate share fractions are relevant.

drgrate = the federal payment rate for the DRG, and a, b, and c
are policy parameters that are the same for each DRG.

The FY 1986 policy that is simulated below had a = 17 days, b = 1.94, and c = \$13,500. (The parameter c is adjusted for local input prices before being compared to twice the federal payment rate.)

III. CASE-BY-CASE OUTLIER PAYMENTS THAT MINIMIZE RISK

We now establish some results on the form of outlier payments that minimize risk if there is a limit I on total outlier payments. As explained above, because we do not know how to predict or evaluate moral hazard, we assume that hospitals do not change their behavior under these minor variants of outlier policy.

In these propositions, the outlier policies are constrained to be actuarially fair at the hospital level. They ask, given that a hospital's outlier premiums on average cover the cost of outlier payments, how would the hospital like the payments to be made? Of course, actual outlier policy effects systematic transfers between hospitals. Actual outlier policies transfer resources from, for example, hospitals with a lower percentage of catastrophic cases to hospitals with a higher percentage of such cases. This is exactly what HCFA wants to reduce the problems of equity and access, but it does set up competing incentives: The high outlier hospitals will always benefit from more outlier payments, whereas the low outlier hospitals will do better with less. We discuss these transfers in the context of other payment adjustments in our concluding section. Meanwhile, the propositions apply to hospitals whose costs mirrored national average costs, or to outlier policies with premiums and conditions tailored to each hospital.

Let M be the mean of costs for a DRG, and let the nonoutlier DRG payment be the mean M. Let C be the costs to the hospital of the case, and let x be the random variable of profits = M - C. By definition, E(x) = 0.

We will only allow outlier policies that are smooth,[1] actuarially fair, and have premiums less than I,[2] the upper limit set by Congress. Formally, we require that the outlier payment function f satisfy

$$f(x) \geq x - I, \tag{3.1}$$

and

$$E(f(x)) = E(x). \tag{3.2}$$

We assume throughout that hospitals are strictly risk-averse, i.e., that their utility of profits,

[1]Policies that are not smooth would lead to unpleasant gaming of the system.

[2]The actual policy is based on total payments, but we prorate that to a premium for each case, for expositional convenience.

U(x), satisfies U''(x) < 0 and U'(x) > 0. Intuitively, the policy marked f* in Fig. 3.1 is best because the marginal value of money is higher when losses are greater.

Proposition 1: The policy f* such that

$$f^*(x) = x - I \text{ for } x \geq D + I$$

$$f^*(x) = D \quad \text{for } x \leq D + I,$$

maximizes E(U(f(x)) over all smooth policies satisfying (1) and (2), for any random variable x. Moreover, applying f* to a series of random variables x(n) in a year maximizes E(U(Σf(x(n)))). The proposition says that the best insurance policy is the one that

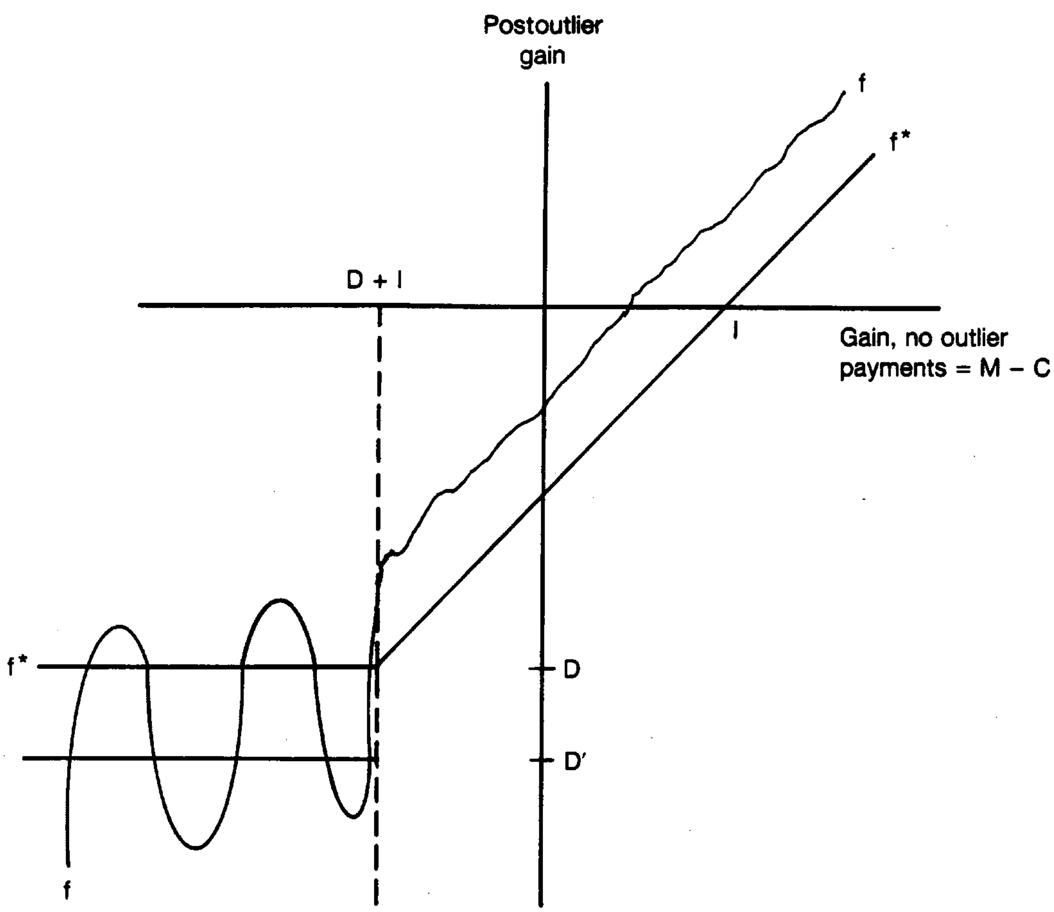

Fig. 3.1—Effects of outlier policy on gains

concentrates the available outlier payments on the cases with the greatest losses, because that is where the marginal value of money is highest.

Sketch of Proof:[3] If there is only one admission in the year, the proof is similar to Arrow's.[4] Euler's equation for variational optima leads to the necessary condition for an optimum: Any position that receives outlier payments must have the same value of (post-payment)U'(x), and hence the same value of x. (Here that value is the large negative number D.) Otherwise, in typical economic fashion there could be transfers from higher wealth to lower that improve utility. The sufficiency argument uses the concavity of U and Jensen's inequality.

The same argument is unaffected by a change from U to V(x) = U(K + x). Hence, it applies if the hospital's profit position is shifted before the admission. If it applies for any constant, then it applies for the expectation over any random variable, in particular the random variable of profits resulting from any policy over the rest of the year. Thus, the cap policy f* is optimal for all cases in the year.

POLICIES THAT MUST ALWAYS PAY COINSURANCE

We next consider outlier policies that must have coinsurance (to reduce moral hazard) of at least a minimum level on the marginal costs of expensive cases. Formally, coinsurance outlier policies must satisfy (1), (2), and the coinsurance constraint:

$$f'(x) \geq c > 0.$$

Proposition 2: The policy f* such that

$$f^*(x) = x - I \text{ for } x \geq D + I$$

$$f^*(x) = c(x - (D + I)) + D \quad \text{for } x \leq D + I,$$

maximizes E(U(f(x))) over all smooth policies satisfying (1), (2), and (3), for any random

[3]The full proofs of the four propositions are in the appendix.

[4]Arrow (1963) proved that, assuming no moral hazard, optimal insurance with a loading fee is full coverage after a deductible, i.e., our policy. The same argument applies to the case of a limit on outlier payments considered here because the shadow price of the limit acts like a loading fee.

variable x. Moreover, applying f* to a series of random variables x(n) in a year maximizes E(U(Σf(x(n)))).

Sketch of Proof: Below the deductible D, f* sets the coinsurance rate at the minimum, c (see Fig. 3.2). It is not hard to show that for each I there exists a unique D so that f* is fair. Intuitively, f* is the best allowable policy because the marginal value of money is higher when losses are greater, and any other allowable policy must start below f*, cross it once, and end up above f*. The loss from the part below f* occurs when the hospital is facing a large loss, so cannot be balanced by the gain from the part above f*.

Propositions 1 and 2 lead to a large deductible, with insurance payments over the deductible constituting as high a fraction of marginal costs as is allowed. Just as personal insurance with a deductible preserves incentives in most years, but reduces exposure to large risks, so a good outlier policy leaves most cases alone, but reduces the financial risk of catastrophic cases.

HOW SHOULD OUTLIER PAYMENTS BE VARIED ACROSS DRGS?

The optimal policy for paying more than one DRG depends on the constraints on premiums and payment. We still assume that the premium I_i for the ith DRG cannot vary within a DRG, and the outlier payments have to be made case by case. When there are many DRGs to consider, there may be a single pool of "premiums" collected from all DRGs

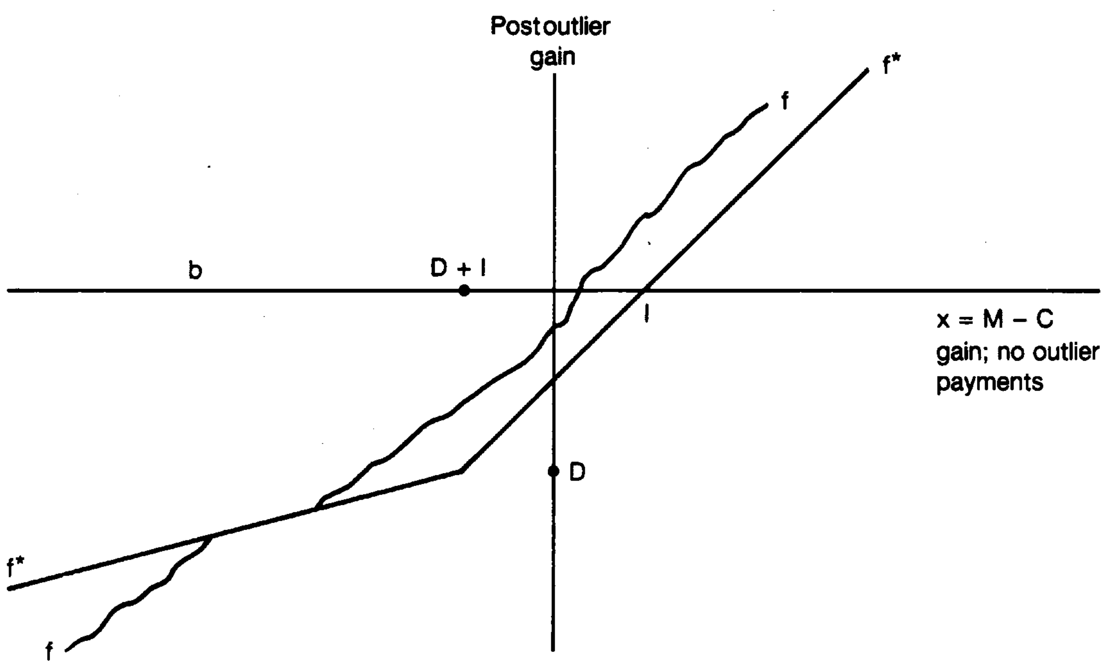

Fig. 3.2—Effects of outlier policy on gains with limit on coinsurance

that can be spent on any DRG, or we may insist that the outlier payments be individually fair in the sense that premiums from one DRG pay just for outlier payments for that DRG—i.e., no transfers between DRGs. In either case, if the hospital's utility function is quadratic, we will show the optimal policy is a stop equal average loss (SEAL) policy, i.e., we select the payment deductibles D_i by the criterion that average loss per outlier is the same in each DRG. (Under other assumptions on hospital utility, the deductibles would not satisfy this criterion.)

If Transfers Between DRGs' Premiums Are Allowed

Suppose that the total outlier premiums collected from all DRGs can be spent on any DRG. Let M_i be mean cost for DRG_i, C_i be the costs of a particular case, and $x_i = M_i - C_i$ be the profit on that case. Let w_i be the proportion of all cases that are in DRG_i.

Proposition 3: Suppose there is an overall limit I on $\Sigma w_i I_i$, and each DRG must satisfy

$$f_i(x_i) \geq x_i - I_i, \tag{3.1'}$$

and

$$\Sigma w_i E(f_i(x_i)) = \Sigma w_i E(x_i). \tag{3.2'}$$

A. The optimal outlier policy is a set of cap policies as in Proposition 1, with the same deductible D for each DRG.

B. If each DRG must also satisfy

$$f_i(x_i) \geq c > 0, \tag{3.3'}$$

and hospital utility is quadratic, then the optimal outlier policy is a set of policies as in Proposition 2 with thresholds set so that the average loss per outlier is the same for each DRG.

Individually Fair DRG Payments

If all hospitals had about the same mix of DRGs, the exact value of the I_i would not matter as long as they raised I. Thus, we could have equal payments per case, the current system $I_i = 0.05\ M_i$, or any other set of $\{I_i\}$.

Since hospitals have very different mixes of DRGs, it matters what the $\{I_i\}$ are. In the current system some DRGs are moneymakers and some are losers, because outlier payments do not equal the outlier premiums. If we want each DRG to be individually fair, the base DRG payment $M_i - I_i$ should be calculated taking outlier policy into account. Moreover, implementing the equal loss rule proved below requires computing the premiums for each DRG. (These premiums will no longer be an equal proportion of mean expenditures for the DRG.)

Let the individually optimal policy for DRG_i be parameterized by D_i. There is a unique I_i associated with each D_i. Individual fairness together with the current proportional $I_i = kM_i$ means that the loss thresholds D_i on low-risk DRGs will be much smaller (in absolute value) than on the high-risk DRGs. From the insurance point of view, it is better to vary the I_is than the D_is.

Proposition 4: Suppose that each DRG must satisfy (3.1'), and

$$E(f_i(x_i)) = E(x_i). \tag{3.2''}$$

If the hospital's utility function is quadratic, then:

A. If there is no coinsurance restriction, the optimal caps D_i are the same for all DRGs.

B. Under the coinsurance restriction (3.3'), the optimal thresholds D_i are set so that the average loss to each DRG per outlier is the same.

From the point of view of insurance, the risk depends on the absolute level of the loss, not on the loss relative to the revenue of the case. It is just as bad to lose $10,000 on an expensive DRG as on a cheap one.

IV. SIMULATION METHODOLOGY

We wished to examine how much insurance is provided by the current outlier policy and how well it attains the goals of compensating for systematic risk to hospitals and patients. We simulated the current outlier policy and three stop loss policies with the same deductible in each DRG and different coinsurance rates (0, 0.2, and 0.4).[1] The 0.4 coinsurance corresponds to the rates used for current outlier policy, under the assumption that the ratio of marginal cost to average cost is approximately 1.0. The no coinsurance policy provides the maximum possible insurance. The 0.2 policy provides an intermediate position that may appeal to policymakers. The deductibles for the simulations were set so that total outlier payment would be approximately the same in each run. For comparison, we also simulated a policy that had no outlier payments. The payment rates used for the "no outlier" simulation were increased to maintain budget neutrality.

The simulation takes a 20 percent sample of bills for discharges from short stay hospitals that occurred between July 1, 1984, and June 30, 1985, and determines what the payment would be using the fiscal year 1985 national rates and each outlier policy.[2] No attempt has been made to model possible changes in hospital behavior that may be induced by changes in the outlier policy, because data are not available.

To simulate profits and losses of each policy, we had to estimate the cost of each case (our data listed only charges). Ratios of costs to charges (RCC) were calculated for each hospital using data from the cost report for the first year that the hospital was on PPS (PPS I) and a 20 percent sample of bills from the PPS I year. Charges for each simulated case were then multiplied by the hospital's RCC to estimate the cost for the case.[3] The outlier payment for a stop loss policy is then calculated as:

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[1]Setting equal deductibles is equivalent to equal average loss thresholds when there is no coinsurance and a good approximation in the other two cases.

[2]The rates were adjusted for the wage index and Cost of Living Adjustment (COLA), where necessary, and took account of the special payment provisions for transfer cases, sole community providers, teaching hospitals, and regional referral centers. No extra payments were provided for disproportionate share providers. The DRGs were grouped using the FY 1986 version of the grouper. The simulation is of a fully implemented PPS, rather than of the actual point in the transition period where the case occurred. The sample excludes hospitals in the states that were not operating under PPS during FY 1985: Maryland, Massachusetts, New Jersey, and New York.

[3]Charges for cases discharged between July 1, 1984, and September 30, 1984, were inflated by one year so that costs would correspond to the same time period as payments.

$$\text{MAX}[c \cdot (\text{cost - DRG payment - deductible}), 0],$$

where 1 - c is the coinsurance rate. The deductibles used were $5,500 for the 0.40 coinsurance policy, $7,900 for the 0.20 coinsurance policy, and $10,000 for the policy with no coinsurance.[4] The estimated cost was subtracted from the DRG payment plus outlier payment to estimate the accounting profit for the case.

Since we used actual rates and DRG weights, the simulations are not "fair" in the sense of having payments equal costs. Although the initial rate-setting process aimed at setting fair rates, it did not anticipate the substantial changes in hospital practices and in medical coding that occurred with the onset of PPS.[5] Thus, on average, hospitals earned substantial profits in the first years of PPS, and this is reflected in our simulations. Also, the amount of profit within each DRG varies partly because the actual rates were reduced uniformly across DRGs independent of the amount of outlier payment that would be received for cases in the DRG.

To obtain a rough measure of the risk that each hospital faces, we will assume that each hospital has its own population of cases that might appear for admission and that the actual cases are drawn independently from this population. For a particular hospital, the population of cases is associated with a distribution of profit; and the expected profit and risk to the hospital from a particular case depends on the mean and dispersion of that distribution, respectively. The expected profit per case may be due either to errors in setting the payment rates (e.g., unmeasured severity, errors in measuring input prices) or to the efficiency of the hospital in producing care. In either case, it is not part of the risk that is addressed by the insurance aspects of the policy. The risk associated with the case depends only on the dispersion of profit within the hospital's population (under the assumption that cases are drawn independently and at random from the population).

The insurance goal of outlier policy is to reduce the (random) risk to a hospital's profitability. Since losses on some cases can be offset by profits on other cases, we are not so much interested in the loss from a single case as in the loss during some fixed period of time. Also, since larger hospitals usually have greater borrowing power, the importance to a hospital of a loss of any given dollar amount should decline with the size of the hospital.

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[4]The outlier thresholds were adjusted for input prices as in the current policy (but teaching hospitals were treated just as nonteaching hospitals).

[5]See Guterman and Dobson (1986) and Carter and Ginsberg (1985).

Consequently, we will use as our measure of risk the standard deviation of annual profit due to a random draw of cases expressed as a percentage of the annual Medicare reimbursement.[6]

We calculate the risk for a particular hospital as follows. Let:

r = average reimbursement per Medicare case for the hospital,
s = within hospital standard deviation of profit per Medicare case, and
n = number of Medicare cases per year.

Then

$$\text{risk} = 100*(s/(\text{sqrt}(n)*r)).$$

[6]It might be more appropriate to use total revenue than Medicare revenue as the scale factor. However, insofar as other payors hold hospitals at risk for losses, revenues from other sources may not be available to offset Medicare losses. In any case, data on total revenues are not available.

V. SIMULATION RESULTS

HOW CLOSE IS CURRENT POLICY TO A STOP EQUAL AVERAGE LOSS POLICY?

If current outlier policy acts as a stop loss policy, it should target its payment on the most expensive cases. To check this, we have categorized each case by the magnitude of its accounting loss under the no outlier policy. Table 5.1 shows that the current outlier policy puts a nonnegligible fraction of its payments on cases with losses that are too small to be paid under a stop loss policy. The current policy pays a correspondingly smaller fraction of funds to the cases with the greatest loss. The cases in the category with the largest losses in Table 5.1 average a loss of $28,350 under the current policy compared to only $10,990 for the policy with no coinsurance, $15,400 for the 20 percent coinsurance policy, and $20,500 for the 40 percent coinsurance policy.

The difference in the distribution of outlier payments under the current policy and the stop loss policies is due primarily to the rule giving precedence to day outlier payments over cost outlier payments. In the current policy, 78.5 percent of outlier cases are day outliers and are paid using the per diem. Among long stay cases, the relationship between total costs and

Table 5.1

DISTRIBUTION OF OUTLIER PAYMENTS BY AMOUNT OF LOSS UNDER NO OUTLIER POLICY

		Percent of Outlier Payments			
				Stop Loss	
Amount of Loss Under No Outlier Policy ($000)	Thousands of Cases	Current Policy	COINS = 0.4	COINS = 0.2	COINS = 0.0
Profitable	1208	1	0	0	0
0 - 5	356	13	0	0	0
5 - 10	31	20	13	3	0
10 - 15	10	16	17	14	8
15 - 20	4	12	14	14	13
20 - 25	2	8	10	12	12
> 25	5	29	46	57	66
Total	1617	100	100	100	100

LOS is weak. For day outliers, the percentage of the within DRG variation in cost that can be explained by LOS has a median value of 37 across DRGs, and the range from the 10th to the 90th percentile of DRGs (when ranked by percentage of variation explained) is 10 to 68.

Another factor contributing to the difference between current policy and a stop loss policy is the way thresholds are defined. Two-thirds of the cases are in DRGs where the day outlier threshold is 17 days above the geometric mean LOS. These cases thus face a constant deductible measured in days before they receive day outlier payments. The remaining cases face a smaller number of deductible days before receiving a day outlier payment. This formula might be justified as providing greater protection to hospitals with a lower case-mix index, because those hospitals tend to be smaller hospitals and may need greater protection from risk. It is, however, very difficult to rationalize the formula for the cost outlier threshold. The deductibles implied by the thresholds[1] decrease linearly with DRG weight until the point where the federal payment rate equals $6,750 (equivalent to a DRG weight of about 2.2) at which point they start to increase. This exposes the hospitals with the smallest case-mix indices (typically the smallest hospitals) and the largest case-mix indices (which we will see have the most variable cases) to the largest risk. The reason this peculiar formula has only a modest effect on the total outcome is that the day outlier rule preempts the cost outlier threshold in almost 80 percent of the cases.

The current policy is not actuarially fair at the hospital level because the rates for all DRGs are reduced by a uniform percentage, but the outlier payments are not distributed proportionately. Although simulated outlier payments total 4.18 percent of all simulated federal payments,[2] almost one-quarter of the cases were in DRGs that received less than 1.5 percent of payments and 8 percent of cases were in DRGs that received more than 7.5 percent of payments in outliers.

This variation among DRGs represents potential transfers among hospitals that specialize in DRGs at opposite ends of this spectrum. The DRGs that receive higher outlier payments are typically those with a greater variance in charges and length of stay and also

[1]The deductible is the cost outlier threshold minus the DRG payment amount. The cost outlier threshold (before adjusting for wages and COLA) is the maximum of $13,500 and twice the federal DRG payment. For small weight DRGs, the threshold is $13,500, and thus the deductible decreases linearly with DRG weight. When twice the federal payment rate for the DRG is greater than $13,500, the deductible equals the federal payment rate.

[2]The rate-setting process assumed that outlier payments based on these cutoffs would total 5 percent of federal payments. Our number would be higher if our data included New York and Massachusetts, but we do not know how much higher.

longer lengths of stay and higher weights.[3] This has the effect of concentrating outlier payments on hospitals that have higher case-mix indices and thus, because these hospitals also have higher within DRG costs (Thorpe, Cretin, and Keeler, forthcoming), it helps to compensate for systematic errors in the payment system.

The data in Table 5.1 have shown that the current outlier policy does not provide the maximum amount of insurance possible. To evaluate this finding, we next assess the extent of the risk that exists and determine quantitatively how well the current policy and stop loss policies achieve all three goals of reduction from random risk, amelioration of systematic risk to the hospital, and reduction of the incentive to discriminate against patient groups.

RISK WITH NO OUTLIER PAYMENTS

Table 5.2 shows the risk that would be faced by different types of hospitals under a PPS system that did not have outlier payments along with the average number of cases annually and the within-hospital standard deviation of profits per case. The relative risk is greatest for the smallest hospitals, as expected. However, risk also increases with the variability of cases, and because larger hospitals typically have a more heterogeneous case mix than small hospitals, they face more risk than their size would otherwise indicate. (Other things equal, risk would be proportional to the square root of annual cases.)

In the absence of outlier payments, the typical hospital would face a standard deviation of annual profit (from its Medicare caseload) that is equal to 2.89 percent of Medicare revenues, and Table 5.2 shows the risk faced by typical hospitals in various groups. Are these risks high? One way to judge is to ask how frequently profits will be substantially lower than expected due solely to a random draw of cases. The normal probability distribution shows that if the hospital has a standard deviation of 2.5 percent, then in 11.5 percent of years it would find its Medicare profit to be below the expected value by more than 3.0 percent of Medicare revenues. This may be acceptable, although it makes it difficult to judge management performance and thus interferes with the PPS incentive to make hospitals more efficient. The 11.4 percent of hospitals with standard deviation of 4.5 percent or greater (594 hospitals in our sample) face much more serious risks. In 1.4 percent of years, Medicare profits will be below the expected value by more than 10.0 percent of Medicare revenues.

[3]The correlation coefficients between the percentage of payments from outliers in a DRG and various characteristics of the DRG are as follows: (1) 0.69 for the standard deviation of charges per case, (2) 0.73 for the standard deviation of LOS, (3) 0.44 for average DRG weight, and (4) 0.44 for average LOS.

Table 5.2

EFFECT OF RANDOM DISPERSION OF COSTS WHEN NO OUTLIERS ARE PAID, BY HOSPITAL CHARACTERISTICS

Hospital Group	Number of Hospitals	Per Case Standard Deviation of Profits	Annual Cases per Hospital	Standard Deviation Annual Profits [a] ($000)	Percent of Reimbursement
Bed size[b]					
1-99	2616	1862	565	378	3.68
100-199	1109	2712	1568	1105	2.46
200-299	586	3378	2772	1909	1.82
300-499	502	3948	4189	2766	1.55
500+	169	4492	6626	4726	1.22
Urban	2434	3771	2419	2481	2.42
Rural	2557	1895	859	697	3.33
Nonteaching	4248	2734	1269	1312	3.06
Some teaching	630	3991	3654	3204	1.86
Major teaching	113	6197	3479	4094	2.13
Total	4991	3261	1620	1928	2.89

[a] Data are hospital weighted rather than case weighted.
[b] Data on bedsize are missing for nine hospitals.

RISK REDUCTION

As we expected, stop loss policies dramatically decrease the random risk posed by an individual case (Table 5.3). The case weighted average within-hospital standard deviation of profit under a no outlier policy is $3,261. Under the stop loss policy with no coinsurance the standard deviation is reduced by 28 percent to $2,345, assuming 4.2 percent of total payments are made in the form of outlier payments. With 20 and 40 percent coinsurance rate, the reductions are almost as large as the no insurance case. The current outlier policy achieves a standard deviation of $2,695 per case—a 17.4 percent improvement over the no outlier policy or about 60 percent of the maximum possible improvement that occurs with the no coinsurance case.

The risks being limited occur in the cases that would cause the largest losses in the absence of outlier payments. Consequently, the greatest improvement in risk position occurs for the hospitals with the most variable case loads. Table 5.4 shows that the largest

Table 5.3

REDUCTION IN RISK PER CASE ACHIEVED
BY VARIOUS OUTLIER POLICIES

Policy	Within-Hospital Standard Deviation of Profit per Case
No outliers	$3,261
Current policy	$2,695
Coins = 0.4	$2,424
Coins = 0.2	$2,365
Coins = 0.0	$2,345

Table 5.4

REDUCTION IN RISK ACHIEVED BY VARIOUS OUTLIER POLICIES

Hospital Group	Standard Deviation of Annual Profits as Percent of Reimbursement				
			Stop Loss Policies		
	No Outliers	Current Policy	COINS = 0.4	COINS = 0.2	COINS = 0
Bed size					
1-99	3.68	3.49	3.30	3.34	3.37
100-199	2.46	2.12	1.91	1.89	1.89
200-299	1.82	1.52	1.36	1.34	1.33
300-499	1.55	1.25	1.11	1.07	1.05
500+	1.22	0.95	0.86	0.83	0.82
Urban	2.42	2.08	1.88	1.86	1.85
Rural	3.33	3.17	3.01	3.05	3.07
Nonteaching	3.06	2.82	2.64	2.66	2.67
Some teaching	1.86	1.55	1.41	1.38	1.37
Major teaching	2.13	1.69	1.46	1.37	1.34
Total	2.89	2.64	2.46	2.47	2.48

NOTE: Data are hospital weighted rather than case weighted.

hospitals, urban hospitals, and teaching hospitals receive the most reduction in random risk from the current policy. Although a stop loss policy would also provide the greatest benefits to the same groups, it would provide more protection than the current policy to every class of hospital.[4] We next assess how well the policies perform on other goals.

DISTRIBUTION OF FUNDS

Table 5.5 shows that all the outlier policies result in very similar distributions of outlier payments among most classes of hospitals.[5] Urban and large hospitals get substantially higher outlier payments, although the extent to which this is due to sicker patients rather than to inefficiency is currently unknown. The smallest category of hospitals in the table pay a "premium" in rate reduction of $89 per case, and thus, under the current policy, they receive $56 ($89 - $33) less than they would in the absence of outlier policy. These funds are transferred to larger hospitals; hospitals with 500 or more beds pay $177 premium per case but receive substantially more than this in outlier payments.

Our stop loss policies do not provide any special treatment for teaching hospitals, yet they receive almost the same amount as under the current policy.[6] If major teaching hospitals were paid as nonteaching hospitals under the current policy, they would receive roughly 17 percent less outlier funds than they actually do receive. Hospitals that are currently entitled to disproportionate share payments[7] also receive about the same amount of outlier payments under all policies. The last hospital group in the table singles out the 4 percent of hospitals (with 8 percent of the cases) that have the highest percentage of admissions in the form of transfers from other hospitals. These hospitals also receive similar payments under all the policies.

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[4]The comparison within the SEAL policies may seem surprising. The smallest hospitals have very few of the most expensive cases and thus get a larger share of the outlier payments with a larger coinsurance rate. There are so many small hospitals that they cause the total line to show greater risk reduction with more coinsurance. (If the data were case weighted rather than hospital weighted, the relative performance of the various SEAL policies would be reversed and look more like their performance for larger hospitals.)

[5]There are swings in the distribution of outlier payments across geographical regions. Going from the current policy to a stop loss policy would shift outlier funds from the East Coast to the West Coast.

[6]Teaching hospitals are paid indirectly for medical education. Their DRG payments are multiplied by a factor that is a function of the ratio of interns to beds.

[7]These payments are designed to help hospitals that serve a disproportionately large share of poor patients. They are based on the percentage of Medicare beneficiaries entitled to SSI, and of non-Medicare beneficiaries eligible for Medicaid.

Table 5.5

OUTLIER PAYMENTS PER CASE

Hospital Group	Current Policy	Stop Loss Policy COINS = 0.4	COINS = 0.2	COINS = 0
Bed size[a]				
1-99	33	46	41	39
100-199	83	97	93	91
200-299	132	139	137	137
300-499	181	179	182	186
500+	257	214	221	227
Urban	168	166	168	170
Rural	41	47	42	39
Nonteaching	87	97	94	93
Some teaching	195	176	178	181
Major teaching	410	382	411	435
Not disproportionate share	108	110	108	108
Disproportionate share	208	203	209	214
Receives fewer transfers	122	123	122	122
Receives more transfers	261	251	266	278
Total	134	134	133	133

[a] Bedsize is missing for nine hospitals.

PATIENT GROUPS

In other research, we examined the clinical and demographic characteristics of patients for whom the regular PPS payment was substantially below actual costs to the hospital. We found that patients who had experienced an expensive stay within six months before the current stay were more likely to generate large losses.[8] Individuals with end-stage renal disease or diabetes as secondary complications to their current stay also were more apt to generate large losses. Being black also predicts large losses, but this effect is

[8]This finding is consistent with the earlier work of Zook and Moore, 1980.

substantially reduced when we control for the hospital. These characteristics could easily be known to the hospital at admission time, and thus the hospitals could discriminate against these patients if they choose to do so.

In Table 5.6, we show the average profit for each case under each policy by several of the characteristics that predict large losses. We also show the standard deviation of profit per case, since admission decisions may be predicated on the fear of a large loss as much as on the expectation of a small loss. All of the outlier policies improve the expected profit position for these vulnerable groups of patients compared to a no outlier policy. The outlier policies also result in a substantial shrinking of the standard deviation of profit thus reducing the likelihood of a large loss to the admitting hospital. The stop loss policies perform at least as well as the current policy—actually providing substantially more protection against the likelihood of a large loss for patients with end-stage renal disease and patients who had previously had an expensive hospitalization.

Table 5.6

MEAN AND STANDARD DEVIATION OF HOSPITAL PROFITS BY PATIENT CHARACTERISTICS

Patient Groups	Number of Cases	No Outlier Policy	Current Policy	Average Profit per Case Stop Loss Policy COINS = 0.4	COINS = 0.2	COINS = 0	Standard Deviation of Profit Outlier Policy	No Current Policy	Stop Loss Policy COINS = 0.4	COINS = 0.2	COINS = 0
Black	131,945	346	440	397	399	401	4,385	3,343	2,956	2,817	2,768
End-stage renal disease	20,332	420	505	561	567	572	5,573	4,567	3,757	3,577	3,513
Diabetic complication	46,931	277	332	298	291	287	4,272	3,136	2,931	2,815	2,787
Prior excessive loss	48,942	-706	-486	-438	-439	-438	5,905	4,617	3,822	3,631	3,571
Total	1,620,469	567	561	561	561	562	3,701	3,008	2,634	2,547	2,517

NOTE: Data are restricted to admissions occurring between April 1, 1984, and March 31, 1985.

VI. DISCUSSION

Because the costs of medical treatment are so variable, the people or agencies who pay for that care are necessarily at financial risk. Different insurance schemes or payment methods can shift the risk from patients to hospitals or the government, but there is an unavoidable tradeoff between the reduction of risk to those deciding on the level of treatment and the incentives to provide treatment that costs more to society than it is worth to the beneficiary. The decision to switch from cost payments to a PPS has transferred risk from the government to providers and reduced incentives for inefficient treatment; but it has created potential problems in risk and equity to the hospitals, and in access for potentially expensive patients.

Outlier payments mitigate these problems by reducing the risk to hospitals from especially costly cases, and thereby reducing the problems of access and equity, while only rarely affecting the incentives to economize that were the goal of PPS. Current policy is much better than none. Our goal in this Note is to suggest some improvements, while retaining the case-by-case outlier payment method.

Assuming no moral hazard, we have proved that among case-by-case outlier payment schemes that are actuarially fair, and have limits on total outlier payments, risk-averse hospitals with quadratic utility functions would prefer a SEAL policy. Such a policy provides more insurance against random risk, by focusing payments where they are needed most, in the cases with biggest losses. With no coinsurance required, it sets an equal deductible on losses per case and reimburses all expenses above the deductible. If, for reasons of moral hazard, it seems wise to make hospitals pay some coinsurance for any care, the optimal policy pays the minimum required coinsurance after the deductible and the deductibles for different DRGs are set so that average losses per outlier case are equal across different DRGs. This policy is similar to the current cost outliers except that it has different thresholds.[1]

Current outlier policy most frequently reimburses outliers based on the length of stay for the case rather than on cost. After controlling for DRG, cost is only weakly related to LOS among very long stay cases. These facts result in the current policy paying roughly 20 percent of its funds to cases that would not qualify for any outlier payments under a SEAL

[1]Also, hospital-specific RCCs are used to estimate costs in our policy.

policy with the same 40 percent coinsurance rate. In addition, the current policy allows a small number of cases to incur catastrophic losses which would not occur under any SEAL policy. Thus the current emphasis on day outliers is an inefficient way of providing either insurance or increased incentives to provide higher quality care to the most expensive patients. Because the payments to some day outlier cases may exceed marginal cost, the current policy may be encouraging too much care in these cases.

Our simulations show that changing to a SEAL policy would reduce the standard deviation of profits for an individual case by 10 to 13 percent from current values. This change would have no substantial effect on the distribution of funds among classes of hospitals defined by surrogates for tertiary care facilities. There would be somewhat improved incentives to care for the most vulnerable patient groups. Thus the SEAL policy would increase the attainment of all the goals of outlier policy over the amount attained under current policy.

We found that small hospitals (under 100 beds) have a much more homogeneous case mix than larger hospitals. This affected our results in several ways. First, small hospitals are less at risk from PPS than one would expect solely from their size, although as a group they still experience substantially greater risk than larger size hospitals. Second, the current policy and the SEAL policies are targeted at the most expensive cases and thus provide the least protection to the smallest hospitals who have the fewest such cases. Nonetheless, under the current policy, many of these hospitals experience large swings in profits from purely random occurrences. The SEAL policies would provide only slightly improved protection to small hospitals.[2] To do substantially better for these hospitals, it would be necessary to provide insurance policies with different thresholds from those given to larger hospitals.

In the SEAL policy and the current policy, outlier premiums are a certain fraction of mean expenses. Because outliers are much more common in some DRGs, these DRGs are getting transfers from other DRGs. It would be better to set the premiums for each DRG based on the expected payments, so that hospitals that specialize in certain DRGs would not gain or lose from outlier payments.

In addition to its pure effects on risk (transfers from bad years to good years within the same hospital), outlier policy currently transfers money from hospitals that care for less expensive patients within DRGs to hospitals that care for more expensive patients. This is desirable, assuming that expense is an attribute of the patient rather than of the hospital, but

[2]It may be desirable to let substantial risk push smaller hospitals away from the care of extremely sick patients who might receive better care if they were transferred to bigger facilities.

there are other mechanisms that do the same thing. These include "severity" adjustments, and other payments for factors associated with increased costs, such as indirect medical education, and disproportionate share. As the package of policies to make transfers is formulated, the parts of the package must be adjusted to take account of other parts. This has not in general been done. For example, the indirect costs of teaching were initially calculated without accounting for the fact that teaching hospitals get more than their share of outlier payments and thus HCFA was essentially paying teaching hospitals twice for the same expenses.

The costs and benefits of outlier policy depend on the other incentives facing the physician-hospital team. If physician payments also are paid prospectively, the likelihood of and associated costs of overspending (relative to some societal judgment of how much care is desirable for a given patient) become smaller, and the costs of underspending or denying access become greater. Then outlier policy has to shoulder more of the burden of protecting the sicker patients and those who treat them.

We have identified patient groups whose care results in hospital losses. In our simulation of the current policy, patients who had an unusually expensive hospitalization within six months before admission resulted in an average loss of $486 with a standard deviation of $4,617. Other patient groups showed significantly lower profits than average and will show an average loss as the slowing of rate increases transfers PPS profits from hospitals to the government. The groups analyzed here are all identifiable at admission, and indeed we expect that hospitals could easily identify subgroups of persons who will cause much larger losses. Although it would be possible to increase payments for some of these groups so that profits reached the average level, there would still be a large number of patients in the group who produce extraordinary losses to the hospital.

Instead of providing the outlier insurance policy to hospitals, HCFA might allow hospitals a choice of alternative outlier policies or even leave the entire insurance problem to the private sector.[3] However, if hospitals could choose less insurance, such "solutions" would provide smaller incentives for increased care for costly patients and access for vulnerable groups. Also such solutions cannot achieve the goal of transferring funds to compensate for systematic errors in the payment of tertiary care hospitals. The hospitals that know they will receive a lower-than-average return in outlier payments would be foolish to pay current premiums.

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[3]Tichon (1986) states that hospitals can buy both case-by-case and year-end aggregate private outlier insurance.

Appendix

PROOFS OF THE PROPOSITIONS

PROOF OF PROPOSITION 1

Proof: Using the notation in the text, we can rewrite f(x) as $x - I + g^2(x)$, where (3.1) is satisfied automatically since $g^2(x) \geq 0$, and (3.2) becomes $E(g^2(x)) = I$. In insurance terms, I is the outlier "premium" and $g^2(x)$ the payment. It is squared to insure that it is nonnegative. Let U be the hospital's utility function. We assume that the hospital is strictly risk-averse, i.e., that $U'' < 0$ and $U' > 0$.

Then the choice of the optimal outlier policy (from the point of view of the hospital) is the following simple calculus of variations problem:

Find g(x) to maximize

$$E(U(x - I + g^2(x)))$$

subject to

$$E(g^2(x)) = I. \quad \text{(A.1)}$$

Let p(x) be the pdf for X. The Euler's equation that defines a necessary condition for optimal g for this problem is:

$$F_g + \lambda G_g = 0, \quad \text{(A.2)}$$

for F the maximand, G the auxiliary constraint, since g' doesn't enter into F or G. (See Gelfand and Fomin, 1963, p. 43.) In this case (A.2) is:

$$U'2g\,p + \lambda 2g\,p = 0. \quad \text{(A.3)}$$

Factoring out 2g p, we see that for g to be extremal, either

$$p\,g = 0 \text{ or } U' = -\lambda, \text{ a constant.}$$

In other words, a necessary condition for f^* to be optimal is that all positions where outlier payments are positive must have the same value of U', and hence the same value of $x - I + g^2(x)$. This is the same necessary condition found in Arrow (1963). The only smooth policy that satisfies this is the cap policy.

It is not hard to show that for each I there exists a unique cap policy. Let $P(x)$ be the cdf of x. As we cap losses at a point D that becomes smaller in absolute value, the part collected is $(1 - P(D + I))I$, which becomes smaller, and the part paid = $P(D + I)\, E(x - D)$ over the range $x < D + I$, which becomes larger. Hence, there is a unique D where these two terms are equal.

We now show that the cap policy shown as f^* is indeed optimal.[1] Split the initial gains into parts above and below $D + I$. Let f be any other allowable policy. Let $D' = E(f)$ in the range $x \leq D + I$. Comparing f to f^* in the range $x \geq D + I$, where $f \geq f^*$, we have $0 \leq U(f) - U(f^*) \leq (f - f^*)U'(f^*) \leq (f - f^*)U'(D)$ by concavity of U (twice). Let $K = \int(f - f^*)\ p(x)\, dx$ from $x = D + I$ up. Then since $E(f) = E(f^*)$ overall, we have $K = (D - D')/P(D + I)$. In the part below $D + I$, $E(U(f)) \leq U(E(f)) = U(D')$ by Jensen's inequality. But by concavity $U(D') - U(D) \leq (D' - D)U'(D) = - KU'(D)/P(D + I)$. Combining the parts above and below $D + I$, we have $E(U(f) - U(f^*)) \leq KU'(D) - KU'(D) = 0$. QED

A similar argument shows that the cap policy f^* is the one that minimizes the variance of postpayment situations among the class of allowable policies. If the cases are independent, this means that this policy for each case minimizes the variance of year-end profits, since the variance of the sum of independent variables is the sum of the variances. It is reasonable to assume that cases are independent (this does not rule out the possibility that each hospital has a different distribution, only that cases at a hospital have no effect on the probability of subsequent cases). However we do not need independence to show that only the cap policy can be optimal.

Reviewing the argument for the optimal policy for an individual case, we see that I does not enter into Euler's equations (A.3), so that the condition remains necessary when we try to optimize $x - K + g^2(x)$ for $K \neq I$. If the policy is optimal for all K, then it is optimal for the expectation over any random variable Y. Since any policy for the rest of the cases in the year leads to a random variable of resulting profits for the rest of the year, the cap policy is optimal for the first case given any policy for the other cases. The same argument shows that the cap policy is optimal for the second (or any other) case, given any policy for the first and subsequent cases. Hence, the cap policy is optimal for all cases. QED

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[1]Thanks to Robert Bell for help with this proof.

PROOF OF PROPOSITION 2

Proof: We first show that for each I there exists a unique D so that f* is fair. Let P(x) be the cdf of x. As the point D increases toward zero, the premium collected, $(1 - P(D + I))I$, becomes smaller, and the part paid, $P(D + I)\, E[-(1 - c)(x - D - I) - I \mid x \leq D + I]$, becomes larger. Hence, there is a unique D where these two terms are equal.

Let f be any allowable policy. We first show that if f(x) satisfies $f(y) > f^*(y)$ for some $y < D + I$, then $f(x) > f^*(x)$ for all $y \leq x \leq D + I$. For if $f(x) = f^*(x)$, by the mean value theorem, there exists z such that $f'(z) = (f(x) - f(y)/(x - y)) < c$, contradicting the coinsurance restriction (A.1). If $f(x) \leq f^*(x)$ for all $x \leq D + I$, then $f(D + I) = f^*(D + I) = D$ by continuity of f, since by Eq. (1), $f \geq f^*$ when $x \geq D + I$. Thus, we can split f into up to three parts, a lower part where $f \leq f^*$, the part where f runs along f* in the range $x \leq D + I$, and the part where $f \geq f^*$. Now pick any point $b \leq D + I$ where $f(b) = f^*(b)$.

Let U be the strictly risk-averse hospital's utility function. Split the initial gains into parts above and below b. Comparing f to f* in the range $x \geq b$, where $f \geq f^*$, we have

$$0 \leq U(f) - U(f^*) \leq (f - f^*)U'(f^*) \leq (f - f^*)U'(b),$$

by concavity of U (twice). Let $K = \int_b^\infty (f - f^*)p(x)\, dx$. Then, since $E(f) = E(f^*)$ overall, we have $K = \int_{-\infty}^b (f^* - f)p(x)dx$. In the part below b, $U(f) - U(f^*) \leq (f - f^*)U'(f^*) \leq (f - f^*)U'(b)$. Combining the parts above and below b, we have

$$E(U(f) - U(f^*)) \leq KU'(b) - KU'(b) = 0.$$

Hence, f* is optimal. The argument that showed that the optimal policy for an individual case was optimal for all cases in the year for Proposition 1 remains valid here. QED[2]

PROOF OF PROPOSITION 3

Proof: Given the amount of outlier payments allocated to DRG_i, Propositions 1 and 2 show that it is best to set a threshold D_i where payments start and pay the maximum insurance fraction allowed past that point. Since the premium pool can be spent on any DRG, the optimum set of $\{D_i\}$ is defined by the condition that the additional benefit of adding dollars to DRG_i's outlier pool must be the same for each DRG that has outlier

[2]This proof is not a generalization of the proof of Proposition 1, and in fact, the coinsurance constraint makes it a little easier.

payments. In case A, this means simply that $U'(D_i)$ must be the same for all i, which implies that D_i must be the same for each DRG.

In the coinsurance outlier case B, if the current profit position is K, the utility of postpayment profits in DRG_i (with $_i$'s omitted),

$$= \int_{-\infty}^{D+I} U(K + c(x - (D + I)) + D)dP(x) + \int_{D+I}^{M} U(K + x - I)dP(x),$$

and the reimbursements by the insurance company R,

$$R = \int_{-\infty}^{D+I} (1 - c)(D - x) - cIdP(x) + \int_{D+I}^{M} - IdP(x).$$

Hence, $dR/dD = P(D + I)(1 - c)$, since both integrands = -I at $x = D + I$. Similarly, because the utility gains and losses caused by shifts in the boundary $D + I$ cancel, and we are hypothesizing that I_i is not affected by the changes in D_i,

$$dU/dR = dU/dD\ dD/dR = E(U'(K + c(x - (D + I)) + D) \mid x \leq D + I), \tag{A.4}$$

after dividing dU/dD by $P(D + I)\ (1 - c)$.

Thus, at the optimum, the average of the marginal utility of gains over all outliers in DRG_i as given in (A.2) must be the same for each DRG. If utility is quadratic, then the derivative of utility is linear, and (A.2) will be the same for each DRG if and only if the average loss per outlier in each DRG is the same. QED

PROOF OF PROPOSITION 4

Proof: Again we look at the optimum policy for the last case of the year, assuming that all previous cases have led to a profit K. Assume year-end utility $U(K + x)$ is quadratic with

$$U'(K + x) = a - x, \tag{A.5}$$

for x the profits on the last case, and a bigger than any possible value of x.[3] For simplicity, assume there are just two DRGs (the formulas generalize easily).

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[3]Assuming that patients do not make supplemental payments, gains per case in DRG_i are bounded by the payment M_i.

The limit on expected total payments is

$$w_1 I_1 + w_2 I_2 = I(D). \qquad (A.6)$$

Let $P_i(x)$ be the cdf of gains in DRG_i. In what follows, we will denote $P_i(D_i + I_i)$ by $O_i = Prob(x \le D_i + I_i)$ for x in DRG_i. Then for individual fairness, we must have:[4]

$$(1 - O_i)I_i = \int_{-\infty}^{D+I} (1 - c)(D_i + I_i - x) - I_i dP_i(x), \qquad (A.7)$$

Equations (A.6) and (A.7) define unique values for I/tp3/c/martha/N2762/text2, D_1 and D_2 for each choice of I_1. Ultimately, we will compute the value of I_1 for which $dU/dI_1 = 0$ by substituting in the relationships derived from (A.6) and (A.7) by differentiating with respect to I_1, namely,

$$dI_2/dI_1 = - w_1/w_2, \qquad (A.6')$$

$$(1 - O_i) = O_i[(1 - c)dD_i/dI_i - c], \qquad (A.7')$$

where (A.7') has no terms in P'_i since each side of (A.7) = - I at D + I.

Now the expected utility can be computed as (with some $_i$'s omitted)

$$U = \Sigma_i w_i[\int_{D+I}^{M} U(K + x - I)dP(x)$$

$$+ \int_{-\infty}^{D+I} U(K + c(x - (D + I)) + D)dP(x)],$$

$$dU/dI_1 = w_1\{\int - U'(K + x - I)dP(x) +$$

$$\int U'(K + c(x - (D + I)) + D)[(1 - c)D' - c]dP(x)\},$$

$$+ w_2 dI_2/dI_1 \ \{\text{same terms in } I_2, D_2\}.$$

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[4]We give here the argument for case B. The argument for case A is similar, with the right-hand integrand in (A.7) replaced by $(D_i - x)dP_i(x)$.

Substituting in (A.5), (A.6'), and (A.7'), we obtain

$$= w_1[\int - (a - x + I)dP + \int(a - c(x - (D + I)) - D)(1 - O)/OdP(x)] \qquad \text{(A.8)}$$

$$- w_1 \text{ [same terms in } I_2, D_2].$$

Define $N_i = E(x|x \geq D + I)$, $B_i = E(x|x \leq D + I)$. Since outlier premiums are fair, and gains sum to zero, the postpayments gains must sum to zero, too. In symbols, for each DRG,

$$(1 - O)[N - I] + O[D + c(B - (D + I))] = 0. \qquad \text{(A.9)}$$

Substituting (A.9) in (A.8), we obtain dU/dI_1,

$$= w_1\{[(- O - (1 - O))(D_1 + c(B_1 - (D_1 + I_1)))]$$

$$- \text{[same terms in } DRG_2]\}.$$

But this is

$$w_1\{- \text{[average loss per outlier in 1]} + \text{[average loss per outlier in 2]}\}.$$

Thus, $dU/dI_1 = 0$ requires the average loss per outlier in both DRGs to be the same. QED

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