FASTER-THAN-LIGHT PARTICLES: A REVIEW OF TACHYON CHARACTERISTICS

Lt. Col. Edward A. Puscher

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This Note was prepared to document the results of the first phase of an analytical effort that explored the possibility that anti-gravity might be a result of elementary (but as yet undiscovered) particles that travel faster than light. This effort falls largely outside the normal management confines of Rand's Project AIR FORCE, and is more aptly described as "exploratory research." Although it is only in the early stages of development, this information should be of special interest to particle physicists.

Lt. Colonel Edward A. Puscher is an active-duty Air Force officer assigned to The Rand Corporation.
SUMMARY

This Note documents an analytical prediction of some of the characteristics which presently undiscovered faster-than-light sub-atomic particles (called tachyons) must possess if they are to exist without violating the Theory of Special Relativity. A brief review of necessary concepts from the Special Theory is included so that the reader might more readily understand the reasoning as it is developed. Necessary, but not all, characteristics of tachyons are then identified and presented. Finally, an interesting potential relationship between tachyons and anti-gravity is discussed.
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Several articles have recently been published regarding the possible existence of faster-than-light particles, which have been named "tachyons" [1, 2, 3, 4]. Some researchers [2, 4] have attempted to present a relativistic quantum mechanical theory of tachyons, primarily with the point of view of attempting to predict how these elusive particles might be discovered in the laboratory. Considerable emphasis has been placed on the elimination of philosophical problems associated with causality (cause and effect) when particles that travel faster than light are used in thought experiments. It is sufficient for this paper simply to note that there are some good arguments [1, 2] that an apparent violation of causality is not a compelling argument against the existence of tachyons.

The intent of this paper is to enumerate some of the characteristics which tachyons must possess if they are not to violate Special Relativity Theory. After showing a brief development of thinking which culminates in a presentation of some of the necessary characteristics of tachyons, an interesting (but not compelling) possible connection of tachyons to a tentative concept of anti-gravity will be presented. Of course, anti-gravity is a very interesting topic, but one which will be discussed in the next paper on this study.

However, a desire to provide some new ideas for anti-gravity has been the driving interest of this paper. Thus, some decisions are made here which are necessary to identify particular characteristics of tachyons which would be useful in a tentative theory of anti-gravity.
(for example, tachyon energy must remain a real quantity or an effective real repelling force could not be produced.)
II. REQUIRED CONCEPTS FROM THE SPECIAL THEORY OF RELATIVITY

This section will provide the necessary concepts of the Special Theory of Relativity which are needed to develop the ideas of the following sections [5, 6].

Near the end of the 19th century, very accurate experiments were performed by Michelson and Morely which indicated that the speed of light was the same in all inertial reference frames. This led Albert Einstein to state his kinematic postulate of the Special Theory of Relativity: The speed of light in a vacuum has the same value relative to all inertial reference frames and is independent of the relative velocity of the light source and the observer. This postulate can also be used to derive the Lorentz equations, although these equations were first derived by Lorentz while attempting to reconcile the Michelson-Morely experimental results with Newtonian mechanics and the laws of electromagnetism, but without Einstein's postulate. Figure 1 contains a description of the derivation of the Lorentz equations.

Suppose, while referring to Fig. 1 that both observer 1 and observer 2 were originally somewhere to the left of the figure, while a stationary man who holds a light source (which is turned off) is at position x=0, x'=0. Both observer 1 and observer 2 are moving fast with respect to each other, along the horizontal axis, with observer 2 moving faster than observer 1, and slightly higher than observer 1 so that they do not hit each other when passing. When observer 1, observer 2, and the man holding the light source are exactly on top of each other, t=0, t'=0, and the man instantaneously turns on the light. As observer 1 and
NOTE: The speed of light is a universal constant (by experiment).

From pythagorean theorem, \[(r')^2 = (ct')^2 - (x')^2 \]
\[(r)^2 = (ct)^2 - (x)^2 \]

But \(r = r'\) since that distance is \(\perp\) to relative motion of observers and since \(c = c'\).

Then \((r')^2 - (r)^2 \rightarrow (ct')^2 - (x')^2 - (ct)^2 - (x)^2 \)
and
\[
x' = \pm \frac{(x - vt)}{\sqrt{1 - \left(\frac{v^2}{c^2}\right)}}
\]
\[
t' = \pm \frac{t - \left(\frac{vx}{c^2}\right)}{\sqrt{1 - \left(\frac{v^2}{c^2}\right)}}
\]
\[
y' = y \quad z' = z
\]

Fig. 1--Origin of mathematical reasoning

Observer 2 pass the light (which travels perpendicular to the observer's line of relative motion), they both see the light turn on. They both see the light hit a reflector at Event 1 at some later time, but even though they both see the same event, they disagree on the coordinates necessary to describe the location of the event. Because observer 2 has been going faster, he perceives that his position \(x'\) is greater than \(x\) (the position of observer 1), and he believes that Event 1 is located at a different angle from that which observer 1 believes.
Because light travels at the same speed regardless of the particular inertial reference frame chosen, it travels at the same speed for both observers, so that in the figure \( r=r' \) and \( c=c' \). After some algebra, the equations given within the box in Fig. 1 are derived. The algebraic solution method consists of assuming straight line, uniform, non-accelerated motion, where \( x' = K(x-vt) \), \( t' = M t - N x \), where \( K, M, \) and \( N \) are constants to be found, and \( v \) is the relative velocity between observer 1 and observer 2.

There are some important observations which should be made about the equations in the box of Fig. 1. The first is that there is a denominator which consists of a square root term. This means that mathematically, at least, the term could either be positive or negative and may be correctly written with either a positive or a negative sign in front of the radical. The second important observation is that the apparent position of observer 1 depends on the relative speed of the other observer with respect to himself as a reference, and vice versa for observer 1. Similarly, the time that a fast moving observer would measure depends upon the time measured by another observer and the relative velocities between observers. It should also be noted that spatial directions \( y \) and \( z \) are considered to be perpendicular to directions \( t \) and \( x \) so that there is no effect of relative motion in these directions.

Figure 2 contains the transformation formulas which relate the apparent length and the apparent time that would be measured by a moving observer with respect to that which would be measured by an observer at rest. Notice that each observer relates all experiences to his frame of reference as a stationary frame and other frames as moving with respect
to his. Therefore, each observer would notice that the other one has made incorrect measurements with respect to his own.

Figure 3 shows how mass varies as the velocity varies. Notice that the denominator contains the same radical term and that when an object is moving fast with respect to an observer, the mass of the object appears greater than it does when at rest in the observer's system. Again the velocity is the variable which controls the amount of increase. We shall return to this equation again later.

\[ l' = l \sqrt{1 - \frac{v^2}{c^2}} \]

Whenever one observer is moving with respect to another, whether approaching or separating, it appears to both observers that everything about the other has shrunk in the direction of motion. Neither observer notices any effect in his own system, however.

\[ t' = t \sqrt{1 - \frac{v^2}{c^2}} \]

Whenever two observers are moving at const velocity relative to each other, it appears to each that the other's time processes are slowed down.

Fig. 2--Length contraction and time dilatation--1
(A) 

**MASS INCREASE WITH VELOCITY**

\[ m' = \frac{m}{\sqrt{1 - \frac{v^2}{c^2}}} \]

When an object is moving with respect to an observer, the mass of the object appears greater than when at rest in the observer's system, with the amount of increase depending on the relative velocity between the object and observer.

(B) 

**VELOCITY TRANSFORMATION LAW**

\[ v' = \frac{v - w}{\sqrt{1 - \left(\frac{vw}{c^2}\right)}} \quad \text{or} \quad v' = \frac{(v-w)c^2}{c^2 - vw} \]

When an object is moving with respect to two observers (s and s'), the velocity of the object appears different than when at rest in each observer's system, with the magnitude and direction of change depending on the relative velocity between the object and s observer and that between the observers.

Fig. 3--Mass and velocity dependence upon reference systems

The velocity transformation law does not have the radical term in it (Fig. 3). Instead, the speed \( w \), which is the speed at which the Coordinate system \( S' \) moves along the x axis of coordinate system \( S \), with respect to the reference frame of \( S \), is defined. The speed \( v \) continues to be the speed along the x axis at which a particle is racing in the system \( S \). Then the velocity transformation law is derived by considering the Lorentz relationships for \( x' \) and \( t' \), realizing that \( v' \) is the speed that a particle moves along the \( x' \) axis with respect to system \( S' \), and then dividing \( x' \) by \( t' \).
Now that the necessary relationships are available for us, the very important question should be asked, "What is the maximum permissible velocity?" Figure 4 again presents the relativistic mass relationship. In 1905, Einstein used this equation to predict that the speed of light was the maximum permissible speed. This can be easily seen, for if we take the limit of the relativistic mass \( m' \) as \( v \) approaches \( c \), we see that the \( v/c \) term approaches 1, which means that the denominator approaches zero. Of course, the zero in the denominator means that the mass would have to be infinite if the velocity of a particle were speeded up to equal that of light. This equation clearly predicts that no particle which can exist at a speed slower than that of light could ever be accelerated to a speed equal to that of light.

The equations presented in Fig. 5 illustrate that, as \( v \) approaches \( c \), the relativistic length approaches zero and that the time processes also slow down and approach zero. This is even more astounding, however, when one remembers that the mass has simultaneously approached infinite size. Again, the equations clearly indicate that accelerating a particle to the speed of light is impossible.

But what if the particle were forced to travel at the speed of light when it was produced? That is, what if the particle never existed in a state where it travelled at a speed less than that of light, and when it was produced it was produced while travelling at the speed of light. Now one might say that there is no change—the relativistic mass equation still predicts an infinite mass. But this is true only if the rest mass (the numerator) is not zero. If the rest mass or, as more
(A)  

**MASS INCREASE WITH VELOCITY**

\[ m' = \frac{m}{\sqrt{1 - \frac{v^2}{c^2}}} \]

Note: \( \lim m' = \text{infinitely large} \) as \( v \to c \)

When an object is moving with respect to an observer, the mass of the object appears greater than when at rest in the observer's system, with the amount of increase depending on the relative velocity between object and observer.

![Fig. 4--Mass and velocity dependence upon reference systems, limiting process](image)

\[ L' = L \sqrt{1 - \frac{v^2}{c^2}} \]

Note: \( \lim L' = 0 \) But Real as \( v \to c \)

Whenever one observer is moving with respect to another, whether approaching or separating, it appears to both observers that everything about the other has shrunk in the direction of motion. Neither observer notices any effect in his own system, however.

\[ t' = t \sqrt{1 - \frac{v^2}{c^2}} \]

Note: \( \lim t' = 0 \) But Real; Time is relative as \( v \to c \)

Whenever two observers are moving at constant velocity relative to each other, it appears to each that the other's time processes are slowed down.

![Fig. 5--Length contraction and time dilatation--2](image)
properly called, the "proper mass", and the denominator were both zero, then the relativistic mass would simply be undefined.

In fact, this is precisely the case for the photon. It is a particle which travels at the speed of light—never any slower or faster. It cannot exist at speeds slower than light or at speeds which are faster, and therefore has a zero proper mass. The equation which appeared to predict that the speed of light is the maximum speed has been shown to be invalid for some particles!

Suppose that some new particles exist which have as yet never been identified in a laboratory. The unique characteristic of these particles is that they exist only while travelling faster than the speed of light. (This idea could be predicted by symmetry, since known subatomic particles exist which travel below and at the speed of light. The only other regime is the one which is faster than light.) These postulated new particles are called "tachyons".

Another look at the equation in Fig. 4 will show that for tachyons, whose velocity is always greater than that of light i.e., \( v > c \), since \( v > c \) the denominator term becomes negative, so that the square root of a negative number must be taken. Of course, this would result in an imaginary denominator—clearly an impossible situation. Or is it? Suppose, as was the case for photons, the numerator were adjusted to account for the the imaginary denominator. If the numerator were made imaginary also (i.e., if the proper mass were imaginary), the imaginaries would cancel out, leaving the relativistic mass to be real. Of
course, it is the relativistic mass which enters into reactions, so the imaginary proper mass simply means that the tachyons could not exist at rest in any reference frame travelling at speeds less than that of light.
III. CONSERVATION LAWS AND SYMMETRY

A. RELATIVISTIC DEPENDENCE OF ENERGY UPON MOMENTUM

Perhaps the most powerful of the conservation laws and symmetry ideas is the conservation of relativistic energy and momentum. Figure 6 shows the pertinent three-dimensional equation. Without loss of generality, the coordinate axes can be chosen so that only the X direction

Equation: \[ E^2 - P_x^2 c^2 - P_y^2 c^2 - P_z^2 c^2 = m_0^2 c^4 \]

where
- \( E \equiv \) Energy
- \( P \equiv \) Momentum
- \( C \equiv \) Speed of light in vacuum

Each point on curve represents the \( E \) and \( P_x \) that a particle would have in a given frame of reference.

Fig. 6--Conservation of relativistic momentum and energy
is important, as is shown in the second equation of the figure. From the analytic geometry, this equation can be plotted as in the plot shown. Note that since the equation consists of terms which are squared, the plot must have two halves, and in fact, is the plot of a hyperbola which breaks upward and downward. It is interesting to note that only the top half of the hyperbola is considered to be acceptable according to most physicists today, since we do not have a good perception of the physical interpretation of the negative energy which is described by the bottom half of the hyperbola. Nevertheless, in 1928 Dirac predicted that there was an anti-particle for every particle of physics. Soon thereafter the positron (the anti-particle of the electron) was discovered, thus lending credibility to the Dirac theory, and allowing many physicists to believe that there was indeed an explanation to the bottom half of the hyperbola. Notice the symmetry of the plot. Each point of the curves represents the energy and the momentum that a particle would have, as perceived in a given Lorentzian frame of reference, when undergoing some experiment. In other words, although the experiment is the same, different observers would "see" different amounts of energy and momentum during the experiment, with the amounts of each based only upon the particular frame of reference being used (but these parameters must, of course, be read from points on the curves). The asymptotes for the curves are the lines which represent particles of light (luxons).

The actual equations which were used to produce the sketch are also shown on the left side of Fig. 6. It should be noted that the top curve is simply the positive term taken from the square root of the denomina-
tor and that the bottom curve is a representation of the energy and momentum associated with the negative sign in front of the radical term.

Notice that this plot shows that as positive energy increases, so does momentum, and vice-versa. Of course, the momentum could be taken as positive or negative, depending upon the direction of the coordinate system chosen. The plot also shows that as negative energy increases in a negative direction, so does momentum increase (in either a positive or a negative direction, depending upon the direction of the coordinates chosen.)

B. RELATIVISTIC DEPENDENCE OF ENERGY UPON VELOCITY

The equation which describes the relativistic relationship between energy and velocity is given in Fig. 7. An assumption must now be made. One can see by looking at the equation that whenever \( v \) becomes greater than \( c \) (\( c \) = the speed of light), the energy would become imaginary. But imaginary energy can have no meaning to people on earth and therefore can have nothing worthwhile to do with any usable theory of anti-gravity. We must force the energy to remain real at all times. The assumption is that since the speed of light is real, the relativistic mass must be real or the energy could not be measured (or, perhaps more correctly for those who believe that energy itself is an abstract term and cannot be measured directly, that the capacity to do work could not be inferred).

There are only three possibilities regarding the particle velocities:
Equation
\[ E = \frac{m_v c^2}{\sqrt{1 - \left(\frac{v}{c}\right)^2}} \]
\[ m_0 \equiv \text{proper (rest) mass} \]
\[ m_v \equiv \text{relativistic mass} \]
\[ v \equiv \text{particle velocity in this reference frame} \]

Assumption
E is real
Since \( c^2 \) is real, \( m_v \) must be real or \( E \) could not be measured.

Possibilities
(a) \( v < c \)
\[ E \] would be real
(b) \( v = c \)
\[ E = \frac{\pm m_0 c^2}{0} \]
which is impossible unless \( m_v \) (rest mass) = 0. Therefore, rest mass of photon must be zero and experiment verifies this.
(c) \( v > c \)
\[ E = \frac{\pm m_0 c^2}{\sqrt{\left(\frac{v}{c}\right)^2 - 1}} \]
which is complex and could not be measured unless \( m_0 \) is imaginary.

**Note:** Relativistic mass remains real even though rest mass is imaginary.

---

Fig. 7--Relativistic dependence of energy on velocity--equations

- The particle is travelling (with respect to a particular reference frame) at a speed less than that of light. This would result directly in a real energy (as seen from the equation of Fig. 7) and is the common case for everyday physics.
- The particle travels at a speed equal to that of light. In this case, the rest mass must equal zero and we have the case of the photon, whose zero rest mass has been verified by experiment. The energy, although undefined, is still real and presents no obstacles to our thinking.
-16-

- The particle travels at a speed greater than that of light. In this case, the numerator, i.e., the proper mass, must become imaginary in order to make the energy remain real (and measurable). Again note that the relativistic mass remains real and presents no contradiction to verifiable physical experiments.

Figure 8 is a sketch of the equation shown on that page. Since this equation has a radical term for the denominator, it also can be written with both a positive and a negative sign in front of the right hand side. Therefore, the equation will plot as upper and lower curves which represent the positive energy and the negative energy portions. Note that in these curves, the asymptote is again the speed of light. The center part of the curves represents the normal case where particle velocity is less than that of light. The tachyon portions of the curves would be a plot of imaginary energy unless the ordinate were changed to represent the normalized energy which is associated with the imaginary proper mass. Notice how the curves as plotted in Fig. 8 provide both left-right and top-bottom symmetry.

Figure 8 shows that for ordinary particles (tardyons), the energy increases as the velocity of the particle increases, up to an infinite energy associated with particle speeds equal to light. Notice that the addition of even infinite amounts of energy could not accelerate the particle to or past the speed of light. The plot also shows that for the negative energy tardyon section, the subtraction of energy (or the addition of negative energy) also would accelerate the particle, but never to speeds equal to or greater than light.
The case for the tachyons is different. Figure 8 demonstrates that as energy is added to tachyons, the tachyon velocity would decrease. Even infinite amounts of additional energy could not succeed in slowing the particles down to the speed of light. Subtracting positive energy is seen to increase the tachyon velocity, thus leading to the unfamiliar possibility of infinite speeds with zero energy at the limit. Since the curves are symmetric, the figure shows that as negative energy is
reduced, the tachyon velocity is also increased without limit. Although these are strange characteristics when compared to our everyday experience, they are certainly not impossible to accept.

C. DEPENDENCE OF ENERGY UPON MOMENTUM FOR TACHYONS

Since tachyons must have imaginary proper masses, the imaginary term must be written on the right side of the energy-momentum conservation equation of Fig. 6. But the square of the imaginary part simply turns out to be the negative of a real term, as can be seen from the equation in Fig. 9. The negative term forces the hyperbola to break left and right, rather than up and down as it did in Fig. 6. Again, this plot completes the symmetrical representation of the conservation equation.

However, a new complication appears to arise. The curves now can be seen to cross the abscissa, which means that a single event could be noticed by two different Lorentzian observers, differing only by the relative speeds by which they travel past the event, and one observer would detect a negative energy while the other observer could detect a positive energy. Figure 6 shows that this never occurs with tardyons. Since our physical concept of "negative energy" is somewhat less than "well based in experiment," this at first appears to be a major stumbling block to the existence of tachyons. I shall return to this point later.
D. Dependence of Energy Upon Coordinate System Speed for Tachyons

With a considerable amount of algebra, the equation stated in Fig. 10 can be reduced to an easily plotted equation also given in Fig. 10. Of course, this equation again provides a symmetric plot. The plot shows that any event that had a positive energy would appear to decrease rapidly in energy, and in fact, would pass through zero energy and enter the negative energy region, with the rapidity of change being a function of the relative speeds of the coordinate systems. It is important to
Equation:
\[ E^2 - p_x c^2 = m_0^2 c^4 \]
or
\[ E = \frac{m_0^2 c^2}{\left(\frac{v'}{c}\right)^2 - 1}^{1/2} \]
where

or

\[ \frac{E}{m_0 c^2} = \pm \frac{\left(c^2 - w^2\right)}{-\left(c^2 - v^2\right)\left(c^2 - w^2\right)^{1/2}} \]
Assumption: \( E \) must remain real

\[ E = \frac{m_0^2 c^2}{w^2 - c^2} \]

\[ \frac{w}{c} = 1 \]

Crossover occurs at \( w = \frac{c^2}{v} \)

Fig. 10--Dependence of energy upon coordinate system speed for tachyons

note that on this plot, the relative velocity of the coordinate systems always remains less than that of light, and the crossover occurs whenever the relative speed of the coordinate systems \( (w) \) times the velocity of the particle as viewed from an observer in one of the coordinate systems \( (v) \) is equal to the square of the speed of light. In other words, a fast traveller could view a tachyonic event as having negative energy, while a slower observer could view the same event as having a positive energy, or vice-versa. Again, the cross-over of energy from positive to
negative or from negative to positive which we first observed in Fig. 9 could occur.
IV. THE VALUE OF SPACE-TIME CONSIDERATIONS

Before returning to the positive-to-negative energy transitions, I shall introduce some necessary concepts of space-time. The left side of Fig. 11 consists of my attempt to sketch the four-dimensional space of space-time on a two-dimensional surface. All four axes are orthogonal to each other. All light pulses which are emitted at the event which is at the origin of the figure will travel along the outside surface of the

Fig. 11--Visualization of time
light cone, and the light cone always makes an angle of 45 degrees with the time axis. Notice that the time axis is really a distance axis for purposes of plotting space-time, and is equal to "ct." This definition allows reasonable plots to be made. All tardyon particles which are emitted at the origin of space-time coordinates must travel at speeds slower than light, and must, therefore, have paths (or "world lines") which are constrained to lie within the light cone. All world lines which extend toward the positive time axis are travelling into the future, and those which travel down would appear to travel into the past.

The right side of Fig. 11 is a two dimensional representation of the light cone and the space-time coordinates.

If another coordinate system, which travels along the X axis of the first, is superimposed on the first frame, the plot would appear as in Fig. 12. Notice that the second frame does not have orthogonal coordinates, that the skewness of it is a function only of its speed with respect to the orthogonal frame, and that the relative velocities of the frames remain less than light.

Suppose that a tachyon were emitted at the origin of the space-time curve represented in Fig. 13. Further suppose that the tachyon were sent out in a straight path along the X axis. Since the speed of a tachyon is constrained to be greater than that of light, it must be constrained to exist only outside of the light cone. Since the light cone can be represented by the same line to both observers, the only question is, "Should the event which signifies the disappearance of the tachyon be represented by a point as at "E" (which is above the X' axis), or should it be represented by a point below the X' axis?"
\[ \theta = \tan^{-1}\left(\frac{W}{c}\right) \]

- Eqn of light cone in s coordinates is \( x = \pm ct \)
- Eqn of light cone in s coordinates is \( x' = \pm (ct)' \)
- Plot of light cone in both s and s systems is blue line

Fig. 12--Space-time for 2 observers in relative motion along X axis

Suppose that the event signifying tachyon "birth" was at the origin in Figure 13 and that signifying "destruction" were at point E. Then, according to the observer in the orthogonal frame, the particle would appear to be emitted at the origin, travel along the X axis, and at some later time \( ct_A \) it would appear to be at position \( X_A \). According to the faster observer who is also travelling along the X axis, at some different (from \( ct_A \)) time \( ct'_B \), the particle would appear to be at position
(1) Equation
\[ v' = \frac{(v-w)c^2}{c^2-wv} \]
Integrating \( x' = \int v' \, dt' \) gives \( x' = \left[ \frac{(v-w)c}{c^2-wv} \right] (ct') \)

(2) Sketch

(3) Analysis
- For tachyons, \( v > c \). This places \( E \) outside light cone.
- \( v > w \) always since \( v \) is for tachyons and \( w < c \) for Lorentz transformation
  Numerator is always positive

Fig. 13--Dependence of time upon velocity of particle and relative velocity of reference system \( S' \) WRT system \( S \)--event end point above \( X' \) axis

\( X'_B \). There is nothing strange about this, since, for each observer, the particle appears to have been shot along his position axis, and at some later time from the initial event it appears to have moved.

Now suppose that the event representing tachyon destruction occurred below the \( X' \) axis as at point \( E_A \) in Fig. 14. Further suppose that initial conditions remain as were those for point \( E \). Now the observer who sits on his orthogonal frame would notice that at some time which he calls zero his tachyon is emitted and at some later time it is
(1) Equation  \[ v' = \frac{(v-w)c^2}{c^2-wv} \]

Integrating \( x' = \int v' \, dt' \) gives \( x' = \frac{(v-w)c}{c^2-wv} \, (ct') \)

(2) Sketch

(3) Analysis  
For tachyons, \( v > c \). This places \( E \) or \( E_A \) outside light cone. 
\( v > w \) always since \( v \) is for tachyons and \( w < c \) for Lorentz transformation 
Numerator is always positive 
\( X' \) is negative if \( wv > c^2 \) unless \( (ct') \) is negative 
Whenever \( wv > c^2 \), time reversal must occur 
Note that \( E/|m_0^*| c^2 \) becomes negative when \( wv > c^2 \)

Fig. 14--Dependence of time upon velocity of particle and relative velocity of reference system \( S' \) WRT system \( S \)--event end point below \( X' \) axis

absorbed. Again, there is nothing strange about this. But the other observer notices that at some time, which he calls zero', he sees the emission of the tachyon and at some EARLIER time (\( ct_B' \)) he sees that the tachyon has been absorbed! For this observer, a strange thing has happened. He thinks that the tachyon has been absorbed before it was emitted (and that "causality" has been violated)!

What are the conditions under which this seemingly impossible situation arises? Note from the equation in Fig. 14 that the denomina-
tor turns negative whenever the product of the relative speeds of the frames and the velocity of the tachyon exceeds the square of the speed of light.

By referring to Fig. 10, one can see that these are the same conditions under which the energy of a tachyon turns from positive to negative or from negative to positive. Figure 15 is another look at Fig. 9 which shows that time reversal also occurs whenever a particular observer senses that the tachyonic energy turns negative.

\[ E^2 - p_x^2 c^2 = (m_0^*)^2 c^4 \]

but \( (m_0^*)^2 = -m_0^2 \)

which is real, but negative

![Diagram of energy-momentum](image)

TIME REVERSAL OCCURS FOR NEGATIVE ENERGY TACHYONS

Fig. 15--Plot of energy-momentum for faster-than-light particles
This has led some researchers [1] to identify a "Reinterpretation Principle": The proper interpretation of negative energy particles that travel backward in time is that they can be considered to be positive energy particles which travel forward in time. This reinterpretation invalidates causality objections to the existence of faster-than-light signals and permits construction of a consistent theory of tachionic behavior. Although this "reinterpretation principle" seems at first exposure to be a hastily conceived artifice for removing objections to tachyon existence, further consideration demonstrates that the "artifice" may have considerable merit and should not be taken lightly.
V. TACHYON CHARACTERISTICS

The foregoing analysis has provided information concerning the characteristics of these new particles, tachyons, if they really exist. If they do exist, then tachyons must have at least those characteristics (but not only those characteristics) which have been presented in this paper. Because of their importance to the remaining research, these characteristics are summarized in Fig. 16.
(1) Must have imaginary rest mass
(2) Always travel faster than light
(3) Could have either $\equiv$ or $\equiv$ relativistic mass \(m'_v\)
(4) Will have real energy
(5) Energy may be either $\equiv$ or $\equiv$
(6) Energy could be either $\equiv$ or $\equiv$ and still have $\equiv$ momentum
   or
   Energy could be either $\equiv$ or $\equiv$ and still have $\equiv$ momentum
(7) Must have imaginary "proper length"
(8) Must have imaginary "proper lifetime"
(9) Adding $\equiv$ real energy would slow down the $\equiv$ energy tachyons
(10) Adding $\equiv$ real energy would slow down the $\equiv$ energy tachyons
(11) May have infinite speeds
(12) Infinitely fast tachyons have zero energy
(13) Infinitely fast tachyons still have momentum
(14) Real energy could be measured (either $\equiv$ or $\equiv$) by an observer in an s' frame moving fast WRT s frame, but at $w < c$.
(15) There are many s' frames where an energy which appears $\equiv$ to an observer in the rest frame s would appear as $\equiv$ to the observer in s'.
(16) Same as (15) except for appears as $\equiv$ to s observer would appear $\equiv$ to s' observer.
(17) Time reversal will occur whenever (15) or (16) occurs. (whenever $ww > c^2$)
(18) Tachyons which have $\equiv$ (or $\equiv$) energy can also appear to SAME observer to be going backward in time. (but not necessary)
(19) Objects made of tachyons would appear to have $\equiv$ length unless $\equiv \sqrt{\ }$ is used.
(20) Objects made of tachyons would appear to have time reversal unless $\equiv \sqrt{\ }$ is used.
(21) Since entropy must increase, tachyons would appear to violate 2nd Law of Thermo except for time reversal occurs at same time, ie, entropy decreases are accompanied by time reversal. (natural occurrences tend to go from higher ordered systems to lower ordered systems - low ordered systems have low entropy).

Fig. 16--Tachyon characteristics
VI. FIRST CONSIDERATION OF AN ANTI-GRAVITY THEORY BASED ON TACHYONS:

NEWTON'S LAW

Since Newton's Law has predicted the correct results of all non-relativistic experiments which have been performed using "ordinary" masses, it should be a good starting equation to help with anti-gravity research. The purpose of the anti-gravity research would be to identify an equation similar to Newton's, which would correctly predict measurable and therefore useful anti-gravity forces. These anti-gravity forces might then be produced and applied in our everyday lives.

Figure 17 provides Newton's Law and identifies all terms used. Because the force $F$ has been found to be an attractive force in all experiments that have been done to date, and because the force $F$ represents the gravitational force (which has been defined as positive), one would expect the anti-gravity force to show up as a negative force in the same equation. But how does one make the force show up as negative?

For all ordinary gravitational phenomena, the masses have been defined as positive, and since the distance between the masses is a squared term, the denominator is always positive. The constant is simply a device which we use to make the equation give results which agree with experiments, and therefore has been defined as positive. If we were to assign a negative value to the constant, we would predict a negative force. However, the result would contradict all experiments that have already been performed. The equation terms themselves, rather than an arbitrary choice of sign for the constant, must dictate the sign of the force.
\[ F = G \frac{m_1 m_2}{r^2} \]

\[ m_1 = \text{mass of object 1} \]
\[ m_2 = \text{mass of object 2} \]
\[ r = \text{distance between } m_1 \text{ and } m_2 \]
\[ G = \text{constant} \]
\[ F = \text{Force between } m_1 \text{ and } m_2 \]

For anti-gravity, \( F \) must be negative

For masses composed of tachyons,
\[ F = G \frac{m_1^* m_2^*}{r^2} \]

\( F \) remains real

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Fig. 17--Newton's law of gravitation for tachyons

If the constant were always taken as positive, the force would always be positive (attraction), provided the masses were either both positive or both negative (composed of sometimes hypothesized anti-particles). So even the negative mass particles (if they should exist), would not provide an anti-gravity (negative) force. If one mass were positive and the other were negative (if negative masses exist), then a negative or anti-gravitational force would be predicted by the equation.
However, this possibility seems to violate the results of experiments, since all masses appear to be composed of (different) quantities of the same particles, and therefore all macroscopic masses which undergo gravitational forces are expected to be of the same sign (either positive or negative).

If imaginary terms were considered in Newton's equation, the distance \( r \) could be imaginary. Then the denominator would be negative and we would achieve a negative force. However, this possibility would contradict the desired goal—to find an expression which could be useful for actual anti-gravity (e.g., to have a force of repulsion experienced by two masses which are a specified real distance apart). If the gravitational constant were imaginary, the force experienced by the masses would also be imaginary, providing an uninteresting result.

However, if the proper masses were imaginary (i.e., if the masses consisted of tachyons), then the force would be a real, but negative, force. An unchanged Newton's Law, using the premises that tachyons actually exist and that some masses are composed of tachyons (under some conditions), could then be used to predict anti-gravity without violating the results of any experiments that have already been accomplished.
VII. TENTATIVE CONCLUSIONS

The foregoing analysis is not meant to be convincing. I retain considerable skepticism myself, particularly questioning the legitimacy of utilizing parts of a particle theory (which is unproved) for investigating macroscopic phenomena and applying the concept of rest masses to test a gravitational formula which has been proved inapplicable for relativistic velocities. I have not intended to provide a complete rationale for saying that "tachyons" really exist, nor do I mean to imply that the only theory that makes sense for anti-gravity is a tachyon theory. Admittedly, much "arm waving" has been done. However, I did intend to point out that

a. Other researchers [1, 2, 3] have shown that the existence of tachyons is not inconsistent with special relativity or relativistic quantum mechanics.

b. Other researchers [1, 3] have shown that the causality arguments against the possible existence of tachyons are not compelling.

c. Energy/momentum conservation and particularly symmetry arguments seem to predict that tachyons could exist.

d. If tachyons actually do exist, they must have at least the characteristics listed in Fig. 16.

e. A connection between particles that travel faster than light and anti-gravity could exist.
Much more work must be done to identify additional characteristics of tachyons, particularly whether or not they should have electrical charges and their expected spin. An investigation should be conducted into the way a tachyon would react if placed into a force field. A solid theoretical model of the relativistic quantum mechanical description of tachyons must be developed if tachyons are soon to be found in the laboratory. But if the characteristics of tachyons are to be used for postulating new ideas while simply tentatively assuming that tachyons exist, perhaps enough characteristics are already known to make legitimate guesses at the framework of suitable theories. This will be the topic of the next paper.
REFERENCES


