LANCHESTER MODELS FOR PHASE II INSURGENCY

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Abstract

Deterministic forms of Lanchester's Equations applicable to Phase II insurgency are developed. Three types of military activity are identified and modeled: (1) Skirmish where maneuver is used on both sides and surprise is not a major factor, (2) Ambush where, due to the surprise element, the attackers' weapon efficiencies are undergoing rapid and significant change during the early stages of conflict, and (3) Siege, attack of a fixed perimeter fortification (typically, Strategic Hamlets). The effects of captured-in-action and troop morale as manifest in battle-stress desertions are included in each case. Numerical solutions for illustratively chosen parameters are obtained by digital computer.

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INTRODUCTION

In this paper we shall develop deterministic forms of Lanchester's Equations applicable to Phase II insurgency.* An appropriate insurgency model is invented and related to three characteristic types of military activity: skirmish, ambush, and siege. Typically, these categories account for the bulk of the ground conflict in an insurgency which has not yet escalated into traditional positional war (Phase III).

It is recognized and emphasized that military conflict models do not (and possibly cannot) properly reflect the full spectrum of the terroristic, political, and sociological factors which can be associated with insurgency. They are, therefore, not sufficient for making predictions about the overall outcome of such conflicts. Military models are useful for lesser tasks, however; for example, it appears possible with a suitable model to develop useful insights into the credibility of casualty estimates on both sides. It is clear that both the casualty levels and the estimates thereof are related to the outcome of the insurgency in important ways. Insurgency military models are also of considerable academic interest as they relate to the historical development of Lanchester and related theory.[2,3]

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*See Mao Tse-tung.[1] The first two phases of insurgency are characterized by ground-yielding operations on the part of the insurgents; in the final phase the insurgents take the strategic offensive. Phase II is generally a period of strategic stalemate. The insurgent operations become increasingly military in nature but the conflict remains localized. As necessary, the insurgents continue to yield ground for time and eventual strength.

A Military Insurgency Model

In a Phase II insurgency, it is hypothesized that the military force of either side consists of a large manpower pool from which small fighting groups are constantly being drawn for operations. The manpower pool is organizationally structured but with substantial flexibility (particularly on the side of the insurgents) and ideally permits each fighting group to be brought to a desirable strength before engaging in an offensive operation.

The respective manpower pools are composed of both voluntary belligerents and impressed neutrals. The neutrals remain as part of their pool until circumstances permit desertion. Desertions also occur both individually and as units under the stress of battle.

Operations consist of a large number of small--typically, 100-man--operations in the three general categories: skirmishes, ambushes, and sieges. It is assumed that each operation is conducted in isolation, and for simplicity no reinforcements are permitted during the course of an engagement.* The flow of manpower is illustrated in Fig. 1.

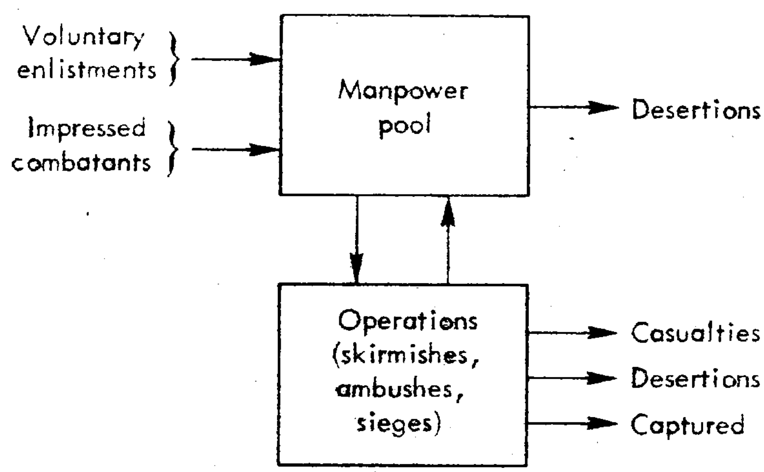

Fig. 1—Military insurgency model

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*Realistically, of course, some ground operations do include reinforcements. In addition, some include more than one category of operations during the course of a single battle. Conceptually, we handle such cases sequentially in the present paper.

The numerical strength of a manpower pool is the prime measure of the military strength of that side. The total casualty level can be obtained, of course, by summing the casualties from the individual battles. This is the area in which traditional Lanchester theory operates. In order to determine the manpower pool strength, however, one must also reflect recruitments and desertions. As a simplification (largely justified by Vietnamese experience) the weapon reserve on either side is assumed inexhaustible.

Generalized Lanchester Theory

Turning our attention to the small engagements, we identify three types of force depletion: Casualties including KIA and non-walking wounded (WIA), surrender, and desertion. The total force depletion rate is the sum of the rates from each of these sources.

Both sides are permitted supporting weapons. However, the insurgent supporting weapons are light and highly portable (mortars and recoilless rifles); counterinsurgent supporting weapons include ground-attack aircraft. No supporting weapon duels are allowed. It is presumed that the insurgents disengage rather than participate in this type of activity.

Assuming that the Lanchester Square Law holds on the average, the casualty attrition equations become:

$$\left.\begin{aligned} \frac{dm_c}{dt} &= -k_n(t)n - \sum_i K_i(t)W_i \\ \frac{dn_c}{dt} &= -k_m(t)m - \sum_j K_j(t)W_j \end{aligned}\right\} \qquad (1)$$

where m and n are the number of engaged personnel on opposing sides, W_i and W_j are the supporting weapon strengths, and t is a time-like variable. It is noted that, in general, the weapon efficiency coefficients, k_n, k_m, K_i, K_j, are explicit functions of time. (They are all positive quantities such that dm_c/dt, $dn_c/dt \leq 0$.

Two assumptions are made concerning the surrender and desertion rates:

1. First, their expected values can be predicted by similar deterministic laws. The two variables will, therefore, be combined into a single force depletion term designated sub s + d.

2. The incidence (or rate) of friendly surrenders and desertions depends on both the friendly casualties and the difference between the friendly force ratio and unity.*

Expressing these assumptions as power series functions:

$$\left.\begin{aligned}
\frac{dm_{s+d}}{dt} &= a_m - \left[b_{m_1}\frac{dm_c}{dt} + b_{m_2}\left(\frac{dm_c}{dt}\right)^2 + \ldots\right] - \left[c_{m_1}\left(\frac{n}{m} - 1\right) + c_{m_2}\left(\frac{n}{m} - 1\right)^2 + \ldots\right] \\
\frac{dn_{s+d}}{dt} &= a_n - \left[b_{n_1}\frac{dn_c}{dt} + b_{n_2}\left(\frac{dn_c}{dt}\right)^2 + \ldots\right] - \left[c_{m_1}\left(\frac{m}{n} - 1\right) + c_{m_2}\left(\frac{m}{n} - 1\right)^2 + \ldots\right]
\end{aligned}\right\} \quad (2)$$

where $\frac{dm_{s+d}}{dt}, \frac{dn_{s+d}}{dt} \leq 0$

and $\left.\begin{matrix} c_m > 0 \text{ for } n/m > 1 \\ c_n > 0 \text{ for } m/n > 1 \end{matrix}\right\}$ and zero otherwise.

It is reasonable to expect the terms a_m, a_n to be equal to or greater than zero. The larger the value of a_m, a_m, the better would be the training and motivation of the troops on that side. For poorly motivated peoples, $a = 0$ can be assumed. This implies surrenders and/or defections as soon as a superior enemy force is encountered and/or as soon as casualties are sustained.

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*cf. Fig. 1 where it is noted that defections can occur directly from the manpower pool at any force level.

As an approximation for data fitting purposes, only the first order terms in dm_c/dt, dn_c/dt, and the first and second order terms in $(n/m - 1)$, $(m/n - 1)$ will be retained. The resulting generalized attrition equations are:

$$\left.\begin{aligned} \frac{dm}{dt} &= a_m - \left(b_m + 1\right) k_n(t)n - c_{m_1}\left(\frac{n}{m} - 1\right) - c_{m_2}\left(\frac{n}{m} - 1\right)^2 - \left(b_m + 1\right)\sum^{i} E_i(t)W_i \\ \frac{dn}{dt} &= a_n - \left(b_n + 1\right) k_m(t)m - c_{n_1}\left(\frac{m}{n} - 1\right) - c_{n_2}\left(\frac{m}{n} - 1\right)^2 - \left(b_n + 1\right)\sum^{j} E_j(t)W_j \end{aligned}\right\} \quad (3)$$

$$\frac{dm}{dt}, \frac{dn}{dt} \leq 0$$

$$\left.\begin{aligned} c_m &> 0 \text{ for } n/m > 1 \\ c_n &> 0 \text{ for } m/n > 1 \end{aligned}\right\} \text{ and zero otherwise}$$

Equations (3) are sufficiently general to represent a wide variety of insurgency situations. The balance of this paper will consider various special cases.

SKIRMISH

A skirmish, as used herein, involves a relatively limited commitment of resources where the engagement conditions during the course of the battle are (on the average) independent of time. In particular, this implies that the weapon effectiveness coefficients are constant, and

$$\left.\begin{aligned} \frac{dm}{dt} &= A_m - B_m n - C_{m_1}\frac{n}{m} - C_{m_2}\frac{n^2}{m^2} - D_m\sum^{i} E_iW_i \\ \frac{dn}{dt} &= A_n - B_n m - C_{n_1}\frac{m}{n} - C_{n_2}\frac{m^2}{n^2} - D_n\sum^{j} E_jW_j \end{aligned}\right\} \quad (4)$$

$$A_m = a_m + c_{m_1} - c_{m_2} ; A_n = a_n + c_{n_1} - c_{n_2}$$

$$B_m = \left(b_m + 1\right) k_n \quad ; B_n = \left(b_n + 1\right) k_m$$

$$C_{m_1} = c_{m_1} - 2c_{m_2} \quad ; C_{n_1} = c_{n_1} - 2c_{n_2}$$

$$C_{m_2} = c_{m_2} \quad ; C_{n_2} = c_{n_2}$$

$$D_m = b_m + 1 \quad ; D_n = b_n + 1$$

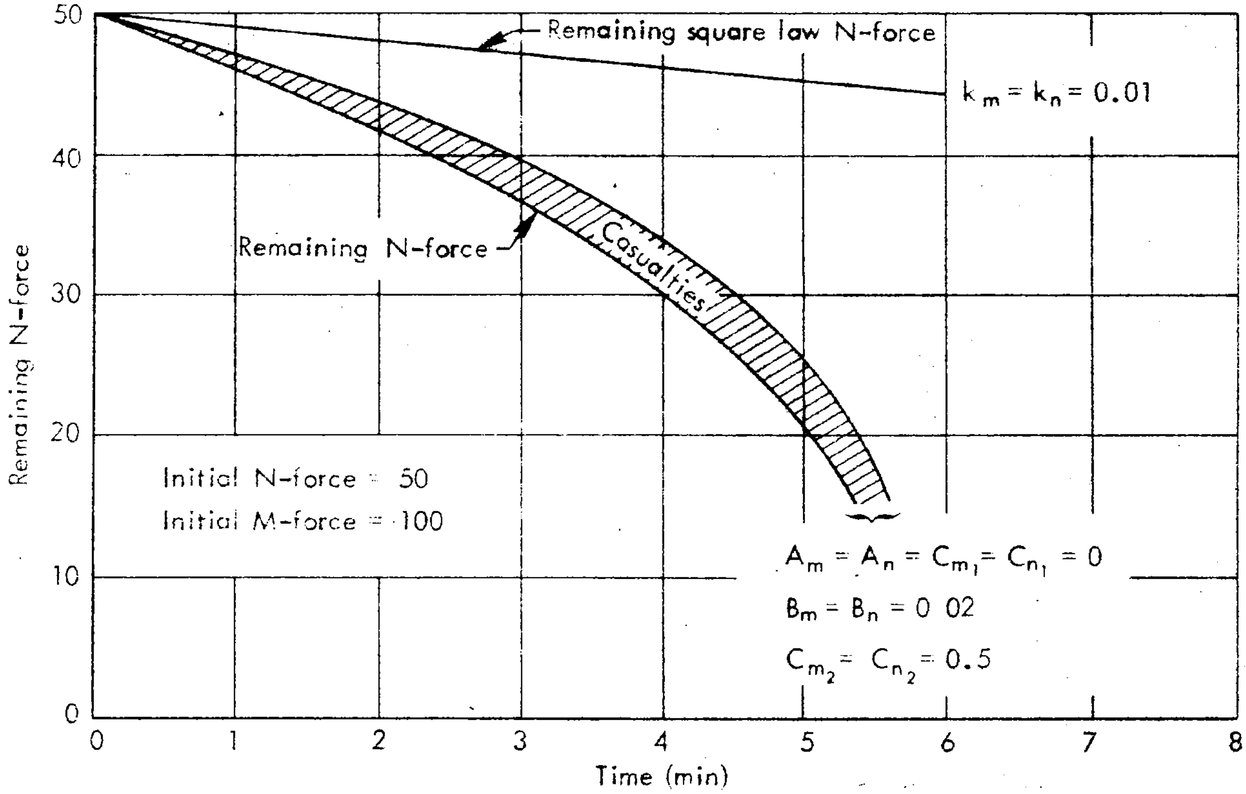
$$\left.\begin{array}{l} C_m > 0 \text{ for } n/m > 1 \\ C_n > 0 \text{ for } m/n > 1 \end{array}\right\} \text{and zero otherwise}$$

A general solution of the skirmish equations is not available. However, accurate numerical solutions are easily obtainable by digital computer. Figure 2 depicts two representative solutions (assuming no supporting weapons) using an Adams-Moulton predictor-corrector integration technique.[4] Also shown are the corresponding square law solutions assuming no defections or captures. Characteristically, the major effect of desertions or captures is to shorten the duration of the conflict. A secondary effect (not shown) is to reduce casualties on the stronger side.

AMBUSH

In contrast to the skirmish, the ambush represents a case where the time dependence of the weapon efficiency coefficients are important, and perhaps dominant. This time dependency results from the changing cover (shielding) available to the individuals of the defensive side.

In an ambush, due to the surprise element, defensive cover is initially minimum. As the engagement progresses, however, the defense



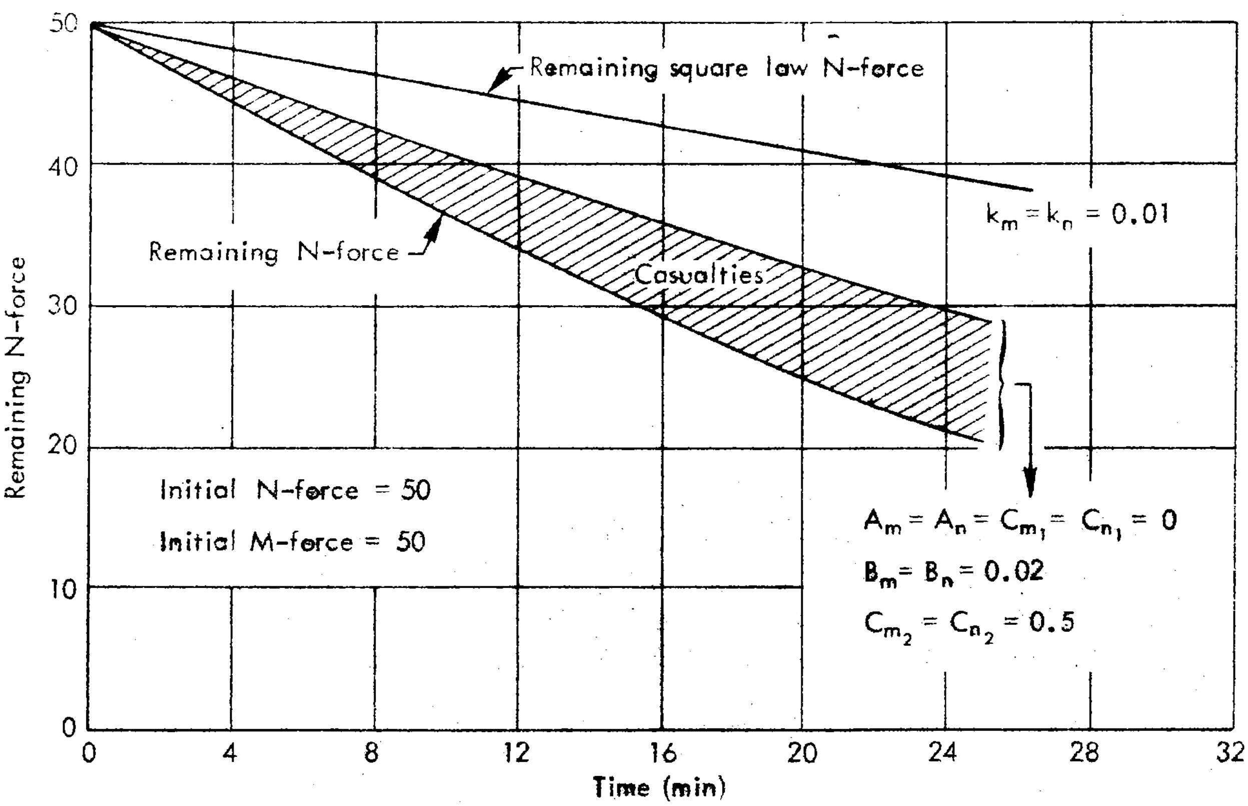


Fig. 2—Skirmish illustrations (no support weapons)

seeks whatever cover is available and gradually improves this situation. The attackers, on the other hand, have a relatively secure position which remains constant until they choose to break off the engagement.

Since there is little motivation for defections or surrenders on the attacking side, and since the defense cannot bring its supporting weapons into play in the early stages of the fight, the ambush equations become*

$$\left.\begin{aligned} \frac{dm}{dt} &= a_m - \left(b_m + 1\right) k_n(t) - c_{m_1}\left(\frac{n}{m} - 1\right) - c_{m_2}\left(\frac{n}{m} - 1\right)^2 - \left(b_m + 1\right)\sum^{i} E_i(t)W_i \\ \frac{dn}{dt} &= -k_m m \end{aligned}\right\} \quad (5)$$

The attacker small arms weapon efficiency coefficients can be expressed as

$$k_n(t) = \frac{r_n A_T(t) P_{HK}}{2\pi\sigma^2} \qquad (6)$$

where r_n is the rate of fire, $A_T(t)$ is the presented area of the defensive targets, P_{HK} is the single-shot kill probability and σ is the single-shot radial dispersion of fire.

Assuming the supporting weapons have fragmentation warheads

$$E_i(t) = \int_A D_T(t)\left\{1 - e^{-A_T(t)\sum^{k} \rho_{H_k} P_{HK_k}}\right\} dA \qquad (7)$$

where ρ_{H_k} is the areal density of fragments of mass group k, P_{HK_k} is the single-hit kill probability of these fragments, A is the area

*The assumption regarding the inability of the defense to use supporting weapons in early stages may not be valid beyond, say, 15 minutes.

affected by the weapon, and $D_T(t)$ is the ground density of defending troops.*

A reasonable representation of $A_T(t)$ is:

$$A_T(t) = \frac{A_{T_\infty}}{1 - e^{-\alpha t - \beta}} \qquad (8)$$

where A_{T_∞} is the steady-state value of the defensive cover function and α and β are constants reflecting the severity of the ambush. Typical values for A_{T_∞} are 0.1 ft^2 for prone troops against rifle fire and 0.5 ft^2 against high-explosive fragmentation weapons.

Again, representative numerical solutions of the ambush equations have been carried out by digital computer assuming no supporting weapons. (See Fig. 3.) Values of α and β have been assumed at 6.2 and 0.1 (units of per minute and minutes, respectively); this implies that within 1/2 minute, the targets achieve approximately 95 percent of their eventual cover.

The equal force case illustrated eventually evolves in favor of the attacker due to his initial cover advantage. In contrast, when a vastly superior force is ambushed, the contest is prolonged although the outcome is not changed. The latter result suggests that a useful tactic for the smaller force is to initiate an ambush with the intention of disengaging when the defenders have reached near-parity in cover.

SIEGE

It is reasonable to divide sieges into two stages: (1) an initial "softening-up" phase where support weapons are primary (the riflemen are generally out of range), and (2) an assault stage where the offensive artillery barrage must of necessity be lifted.

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*The density of defending troops is in general a function of time. A large variety of defensive strategies are possible. Brackney[5] has described two of these analytically: the constant density defense and the constant area defense.

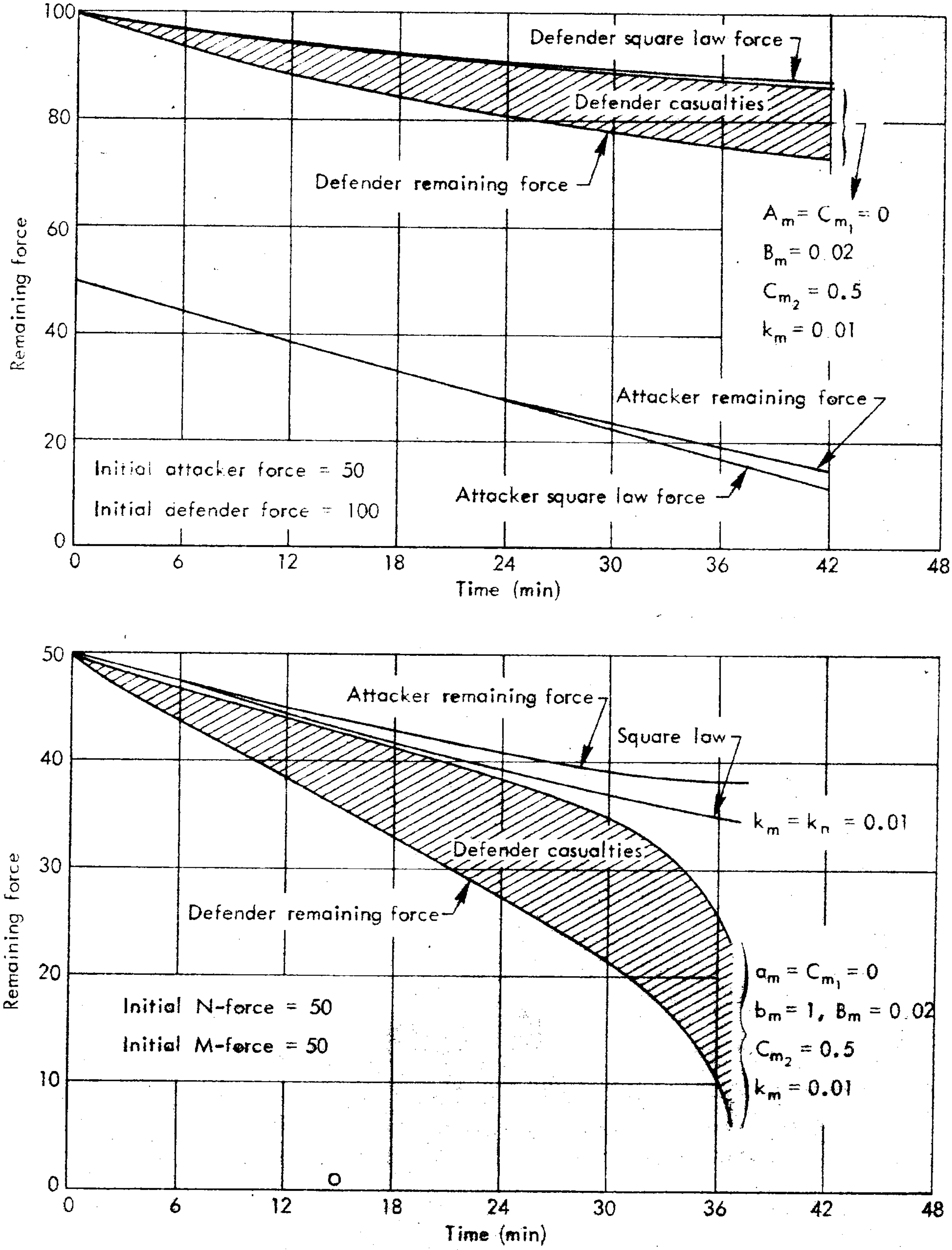

Fig. 3—Ambush illustrations (no support weapons)

In the softening-up phase, a form of Lanchester's Linear Law applies:

$$\left.\begin{aligned} m_o - m_1 &= \left(t_1 - t_o\right)\sum^{i} \bar{E}_i W_i \\ n_o - n_1 &= \left(t_1 - t_o\right)\sum^{j} \bar{E}_j W_j \end{aligned}\right\} \quad (9)$$

where conditions at the beginning and end of the interval are designated sub-zero and one, respectively. It is noted that time-averaged values of the weapon efficiency coefficients are used.

In the assault stage:

$$\left.\begin{aligned} \frac{dm}{dt} &= -P_n nm/\gamma_n A_{m_o} \\ \frac{dn}{dt} &= -P_m m/\lambda_m - \sum^{j} \bar{E}_j W_j \end{aligned}\right\} \quad (10)$$

where $\gamma_n A_{m_o}/m$ is the average time for an assault troop to acquire a defensive target, and λ_m is the average time for the defense to fire given an acquisition. Equations (10) have the time-independent solution

$$\frac{P_m}{\lambda_m}\left(m_1 - m\right) + \left(\ln \frac{m_1}{m}\right)\sum^{j} \bar{E}_j W_j = \frac{P_n}{\gamma_n A_{m_o}}\left(n_1^{\,2} - n^2\right) \quad (11)$$

It is noted that in the absence of defensive artillery support the assault is identical with Brackney's characterization for a fixed area defense.(5) Deitchman(6) has developed a time-dependent solution for this case which he uses, however, to describe an ambush situation.

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