LONG-TERM COMPETITION IN A DYNAMIC GAME: THE COLD FISH WAR

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I. INTRODUCTION

In games where players move more than once, it is reasonable that their moves should depend on information about the previous history of play. In analyses of repeated games, simple forms of dependence on history have been used to characterize various kinds of equilibrium behavior. This article extends the use of history-dependent strategies to an example of a more general class of games called dynamic games.

The "Great Fish War" of Levhari and Mirman [1980](LM) and Mirman [1979] provides an interesting and tractable example of a dynamic game. In this game, several players jointly exploit a renewable resource. LM compute a particular subgame perfect equilibrium in which players' strategies depend only on the current stock of fish. Like many Nash equilibria, it is Pareto inefficient.

This "recursive equilibrium" may be defined as follows: let player i's concave utility for current consumption c_i be $U_i(c_i)$ and let his discount factor be δ_i. Current stock is S, total current consumption is c, and the stock available next period is $f(S - c)$, for a concave function f with $f(0) = 0$. LM assume that current consumption by player i is a function $c_i(S)$ of current stock, chosen to solve the recursive equation:

$$V_i(S) = \max_{c_i} U_i(c_i) + \delta_i V_i[f(S - c_i - \sum_{j \neq i} c_j)].$$

This defines the best reply of player i to the consumptions planned by the other players. In the LM example, the best reply equations have a unique simultaneous solution, which is the recursive equilibrium.[1]

In this model, the only externality linking the players is the renewal process. If the resource or commodities produced from it were to be sold on a noncompetitive market, additional externalities would be created [Levhari and Mirman, note 3]. Within the LM strategic framework such additional externalities have been analyzed by Reinganum and Stokey [1985] and Eswaran and Lewis [1984]. These articles continue to exclude strategies that depend on the previous history of play:

> ". . . threats are not allowed, so that actions based on retaliation for behavior in the past will not be considered."
>
> (Levhari and Mirman, p. 323)

In the present article, threats are explicitly introduced, and the resulting behavior analyzed.[2]

I propose to treat dynamic game strategies as cooperative agreements. Threats are required to secure performance,[3] but they are voluntarily and collectively used. I therefore regard the Pareto inferior recursive equilibrium as a security level representing the absence of cooperation.

I shall first describe the cooperative behavior supported by the simple, moderate, and credible threat that greets any defection with an immediate and irrevocable return to the recursive equilibrium. Perfectness considerations require both cooperative and threat behavior to be thus "recursively supported" (**RS**). Since players in this model avoid the unfortunate consequences of the Great Fish War by threatening to greet breaches of cooperative behavior with a reversion to that state, I call the new equilibria the "Cold Fish War." Although this choice of threat may seem *ad hoc*, it is supported by theoretical, practical, and empirical considerations. Withholding cooperation directs attention to the possibility of cooperation itself rather than to the specifics of cooperative behavior. There are informational advantages to using the *status quo ante* for punishment. As a disagreement point in an underlying bargain over the form of cooperation, it has a natural credibility. This approach is suited to situations where the players can communicate, but cannot make binding commitments.

Following the cooperative interpretation, I expect Pareto optimal cooperative behavior, if it can be supported. Failing this, cooperative behavior should be Pareto optimal relative to the set of RS outcomes. This reflects "collective rationality" in the same way that equilibrium behavior reflects individual rationality.

Although the threat of non-cooperation defines the set of possible agreements, it serves as a security level rather than a punishment of first resort. Collective rationality applies to threats as well as to cooperative behavior.

A player contemplating defection in a dynamic game might not believe that Pareto inferior punishments would be carried out. The final section of this article addresses this possibility directly. I construct a strategy which ensures a particular cooperative outcome in the face of renegotiation. Punishments must be sufficiently severe to deter defections, and must be RS to ensure that they will be voluntarily executed. If one such punishment is Pareto dominated by another, there is scope for renegotiation. I therefore require punishments to be Pareto optimal relative to the set of RS threats that would have deterred the initial defection. Again, this may seem arbitrary, since the selected behavior could be supported by immediate reversion to recursive equilibrium or stronger threats. But these other threats, being more costly, are in a sense less credible. There are many examples of mutually advantageous alternatives to non-cooperation or stronger punishment -- refinancing provisions in loan contracts, credit ratings, repeat offender laws, "point" systems for driving offenses, etc.

The following two stage game illustrates this point. The first stage is Prisoners' Dilemma, and the second is a coordination game, with payoffs and feasible strategies that are independent of the first stage outcome.

	H	G
H	(6, 6)	(0, 7)
G	(7, 0)	(1, 1)

Stage 1

	A	B	C
A	(0, 0)	(0, 0)	(4, 12)
B	(0, 0)	(6, 6)	(0, 0)
C	(12, 4)	(0, 0)	(0, 0)

Stage 2

The stage 2 game has four equilibria. Denoting mixed strategies for either player by [pr(A),pr(B)], the equilibria are given by:

Name	Strategy	Payoff	Name	Strategy	Payoff
E1	[A][C]	(4, 12)	E2	[C][A]	(12, 4)
E3	[B][B]	(6, 6)	E4	[1/6,1/3][1/6,1/3]	(2, 2)

Perfect equilibria of the two-stage game are constructed by "attaching" an equilibrium of stage 2 to each outcome of stage 1. Cooperation in the Prisoners' Dilemma of stage 1 may be supported by the reward of E3 and the threat of E4 following any defection, or alternatively by the threat of Ei if player i defects, or E3 if both defect. I regard the latter threats as "renegotiation proof" since they are Pareto optimal in the second stage game. These attachments are shown below.

Subgame perfect:

		=		
E3	E4		(12, 12)	(2, 9)
E4	E4		(9, 2)	(3, 3)

Renegotiation proof:

		=		
E3	E2		(12, 12)	(12, 11)
E1	E3		(11, 12)	(7, 7)

The social costs of E4 make it less "credible" in a collective sense than the renegotiation proof punishment.

Section II describes the model, the recursive equilibrium, and the Pareto optima. Section III defines and characterizes the recursively supported equilibria and gives conditions for their Pareto optimality. Section IV presents an appropriate definition of "renegotiation proof equilibrium" for this example and explores its properties. Compared to recursive equilibrium, all renegotiation proof cooperative behavior and almost all renegotiation proof punishments are strictly better for all players. Punishments for repeated or severe defections entail extraction paths that approach recursive equilibrium levels and are ultimately costly to all players. Farsighted players have many possibilities for renegotiation proof cooperative behavior, some of which may be fully optimal. Similarly, there may be many renegotiation proof punishments for mild and/or first offenses. However, the scope for negotiation over the terms of RNP strategies narrows dramatically if players are myopic and/or defections are repeated or severe.

For expositional convenience, I limit attention to the case of two players who discount the future at the same rate. All results generalize readily to the n-player non-symmetric case.[4]

II. THE EXAMPLE

In this section, I review the assumptions underlying the LM model, compute their recursive equilibrium, and describe Pareto optimal behavior. Then I extend the model to include threat strategies.

Assumptions

The initial stock of fish is S_0. Player i's utility for current consumption is $U_i(c_i) = \ln(c_i)$. The common discount factor is δ.

LM show by backward induction that recursive equilibrium strategies are stationary: player i consumes a constant fraction h_i of the stock in each period.[5] With threat strategies, these fractions differ between the cooperative and punishment phases of the game.[6]

Suppose the players select a pair $h = [h_1, h_2]$ of extraction rates. Then, following LM, the stock available at the beginning of period t is:

(1) $$S_t = [(1 - H)S_{t-1}]^{\alpha}, \text{ where } H = h_1 + h_2, \text{ and } \alpha \in (0,1).$$

Recursive Equilibrium

Once the equilibrium decision rules for the LM game are known to be stationary and linear, they can be found by simple calculus. To begin, I solve recursively for the stock at time t as a function of the initial stock:

$$(2) \qquad S_t(h,S_0) = (1 - H)^{\lambda(t)} S_0^{\alpha^t}, \text{ where } \lambda(t) = \alpha(1 - \alpha^t)/(1 - \alpha).$$

The present discounted value to player i of playing his part of the vector h when the initial stock is S_0 is:

$$(3) \qquad V_i(h,S_0) = \sum_{t=0}^{\infty} \delta^t \ln[h_i S_t(h,S_0)].$$

Using (2), I can suppress the subscript "0" and write (3) as:

$$(4) \qquad V_i(h,S) = [\Psi_i(h) + (1 - \delta)\ln(S)]/[(1 - \delta)(1 - \alpha\delta)], \text{ where}$$
$$\Psi_i(h) = (1 - \alpha\delta)\ln(h_i) + \alpha\delta\ln(1 - H).$$

This holds for any stationary strategy and will be used repeatedly. For comparison purposes, I suppress the dependence on initial stock, since the "S" term is separable from the "h" terms. Maximizing $\Psi_i(h)$ with respect to h_i for each i gives the best reply equations:[7]

$$(5) \qquad h_i = (1 - \alpha\delta)(1 - h_j).$$

Solving equations (5) simultaneously gives the recursive equilibrium:[8]

$$(6) \qquad h_i^{re} = (1 - \alpha\delta)/(2 - \alpha\delta).$$

Using (4) and (6), the discounted present value of recursive equilibrium extraction as a function of the initial stock S is:

$$(7) \qquad V_i^{re}(S) = \frac{[(1-\alpha\delta)\ln(1-\alpha\delta) + \alpha\delta\ln(\alpha\delta) - \ln(2-\alpha\delta) + (1-\delta)\ln(S)]}{[(1 - \delta)(1 - \alpha\delta)]}.$$

Pareto Optimality

In the symmetric model, Pareto optimality entails consuming in aggregate $(1 - \alpha\delta)$ times the total stock in each period.[9] This is clearly less than in recursive equilibrium where $2(1 - \alpha\delta)/(2 - \alpha\delta)$ of the stock is consumed.

Threat Strategies

The set of feasible extraction rate vectors is denoted:

$$(8) \qquad X = \{h \varepsilon R^2_+ : \ \Sigma\, h_i \le 1\}.$$

A pair of *strategies* is written $f = [f^0, \ldots, f^t, \ldots]$, where $f^0 \ \varepsilon \ X$ and for all $t \ge 1$, $f^t : X^{t-1} \to X$. Here, for $t > 1$, X^t is the t-fold product of X, with generic member z^t. The set of strategies is denoted F. The *behavior associated with f* includes the sequences of extraction rates $x^t(f)$, stocks $S^t(f)$, and histories $z^t(f)$ defined by:

$$(9) \qquad \begin{array}{ll} x^0(f) = f^0 = z^0(f) & S^0(f) = S_0 \\ x^t(f) = f^t[z^{t-1}(f)] & S^t(f) = [S^{t-1}(f)]^\alpha [1 - \Sigma x_i^{t-1}(f)]^\alpha \\ z^t(f) = [x^t(f), z^{t-1}(f)]. & \end{array}$$

Player i's payoff when the strategies f are used is denoted $U_i(f)$:

$$(10) \qquad U_i(f) = \sum_{t=0}^{\infty} \delta^t \ln[x_i^t(f) S^t(f)].$$

The strategies f are *stationary* if and only if $x^t(f) = x^*(f)$ for all t. In this game the strategy space for each player depends on the other players'choices. [See fn. 6] If C is a coalition (a subset of the set of players), the C-strategy g_C is *feasible for C given f*, or $g_C \ \varepsilon \ \Phi_C(f)$ if and only if, for all t and all $z \ \varepsilon \ X^{t-1}$,

$$(11) \qquad \sum_{j \varepsilon C} f_j^t(z) + \sum_{i \varepsilon C} g_{C,i}^t(z) \le 1.$$

The strategy vector f with the strategies of members i of C replaced by their g_C strategies, $g_{C,i}$, is denoted $(f|g_C)$.

The strategies f are *equilibrium strategies* if and only if for all i, and all $f_i \varepsilon \Phi_i(f)$:

(12) $$U_i(f) \geq U_i(f|f_i).$$

The strategies f form a *subgame perfect equilibrium* in the game with initial stock S if and only if for all τ and all partial histories $z \varepsilon X^{\tau-1}$, the continuation strategies $g(\tau,z)$ defined by:

(13)
$$x^0(g) = g^0 = f^\tau(z)$$
$$x^t(g) = g^t(x^{t-1}(g),\ldots,x^0(g)) = f^{t+\tau}(x^{t-1}(g),\ldots,x^0(g),z)$$

form an equilibrium in the game with "initial stock"

$$S(\tau,z) = [S^{\alpha^\tau}][\prod_{s=0}^{\tau-1}(1 - \sum_i z_i^s)^{\alpha^{\tau-s}}].$$

Finally, the strategies f are *subgame stationary* if and only if, for all t and all $z \varepsilon X^{t-1}$, the continuation strategy $g(t,z)$ is stationary. Much of what follows uses subgame stationary strategies, so I suppress the strategic description and work directly with the associated behavior. Where necessary, general definitions are given in footnotes.

III. RECURSIVELY SUPPORTED EQUILIBRIA

Behavior associated with history dependent strategies can be divided into *normal behavior* -- what happens if all players follow the strategies -- and *off-path* or *punishment behavior* -- what should happen after a defection. I concentrate on "normal" behavior induced by history dependent strategies with certain reasonable properties.

From the non-cooperative (individual) point of view, no player should wish to deliberately defect from either normal behavior (Nash equilibrium) or the threats implicit in off-path behavior (perfect equilibrium). From the cooperative point of view, reasonable strategies represent self-enforcing contracts voluntarily agreed to by self-interested agents. Hence, individual rationality should be augmented by collective rationality considerations.

Although it is customary to invoke Pareto optimality as the most basic aspect of collective rationality, I believe the voluntary aspect of cooperative behavior is more important. Even players unable to agree on Pareto optimal behavior may be able to reach some form of agreement. The recursive equilibrium, representing non-cooperative behavior, serves as a threat point or security level for the players. Therefore, I concentrate on RS normal behavior: that which can be policed by the threat of recursive equilibrium.

When the recursive equilibrium is unique, players should regard it as a good prediction of the consequences of disagreement, especially in an ongoing game where it is the *status quo ante*. It plays a role analogous to the fixed threat of Nash bargaining theory.[10]

After characterizing RS normal behavior, I give necessary and sufficient conditions for its Pareto optimality. Further implications of collective rationality are deferred to the next section.

Let $b = [b^1, \ldots, b^t, \ldots]$ be an infinite sequence of extraction rates representing normal behavior. A recursively supported equilibrium strategy supporting b is:

$$(14) \qquad f_i^t(z^t) = \begin{cases} b_i^t & \text{if and only if } z^t = [b^1, \ldots, b^{t-1}] \\ h_i^{re} & \text{otherwise,} \end{cases}$$

where z^t is the history of play during the initial t-1 periods, and h_i^{re} is player i's recursive equilibrium strategy. In this example, I may[11] limit attention to *subgame stationary strategies*: $b_i^t = h_i$ for all t.

Let h be cooperative extraction rates, and H the total extraction. Using (14), h can be *recursively supported for player i* if and only if:

$$(15) \qquad V_i(h,S) \geq \max_d \{\ln(dS) + \delta V_i^{re}[(1 - h_j - d)^\alpha S^\alpha]\}.$$

If h can be recursively supported for all players, $h \in RS$.[12] Using (4) and (7), $h \in RS$ if and only if:

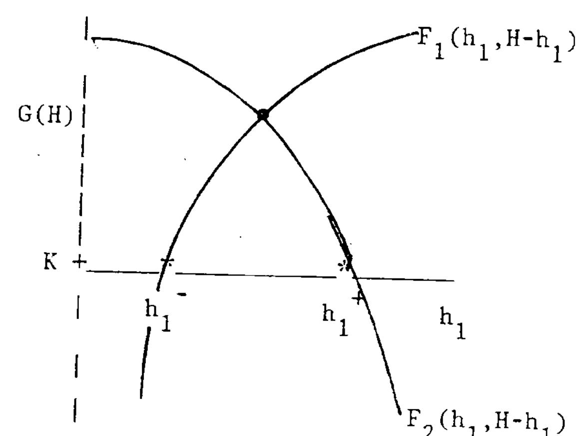
$$(16) \qquad F_i(h) \equiv \Psi_i(h) - (1 - \delta)\ln(1 - h_j) \geq K \equiv F_i(h^{re}).$$

The set of h such that $F_i(h) = F_i(h^{re})$ is called the *i-boundary* of RS, denoted ∂_i. (16) leads immediately to:

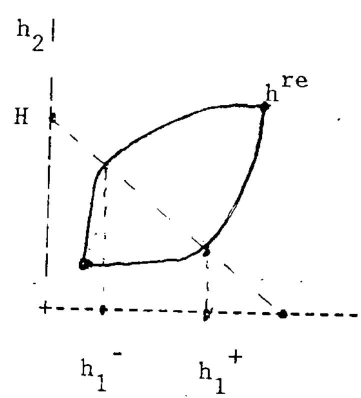
Proposition 1: h^{re} solves the problem: $\max \Sigma h_i$ s.t. $h \in RS$ (proof omitted).

Now I determine the RS shares of a given total extraction and the set of RS total extractions. Fixing total extraction at H, note that $F_1(h_1, H - h_1)$ $[F_2(h_1, H - h_1)]$ is monotone increasing [decreasing] in h_1. Therefore, the smallest h_1 consistent with recursively supported extraction of H is the unique solution h_1^- to $F_1(h_1, H - h_1) = K$, and the largest is the unique solution h_1^+ to $F_2(h_1, H - h_1) = K$ (Fig. 1a).

The height of the intersection of the $F_i(h)$ is shown as G(H) in Fig. 1a. H is recursively supported if and only if $G(H) \geq K$. This condition is always satisfied by the recursive equilibrium. More generally, $F_1(h_1, h_2) = F_2(h_2, h_1)$, so G(H) is achieved at $h_1 = h_2 = H/2$. Thus:



a) Max/min shares of RS total



b) A typical RS set

Fig. 1 -- Recursively Supported Equilibrium Extractions

(17) $G(H) = (1-\alpha\delta)\ln(H) + \alpha\delta\ln(1-H) - (1-\delta)\ln(2-H) - \delta(1-\alpha)\ln(2)$.

The set RS of recursively supported extraction vectors is obtained by varying H in (16), as in Fig. 1b. G is a concave function, strictly decreasing near recursive equilibrium, so it is straightforward to show that RS is a convex, compact, symmetric set with a nonempty interior.

The recursive equilibrium always has the largest RS total extraction, and Pareto optimality implies a strictly lower total extraction.[13] RS has a Pareto optimal member if and only if $G(1 - \alpha\delta) \geq K$, i.e., $\delta\ln(2 - \alpha\delta) - \delta(1 - \alpha)\ln(2) - (1 - \delta)\ln(1+\alpha\delta) \geq 0$. This Pareto optimum is unique if and only if $G(1 - \alpha\delta) = K$.

In what follows, equilibrium strategies using only RS behavior are called *recursively supported equilibrium* (**RSE**) strategies.

IV. RENEGOTIATION PROOF EQUILIBRIA

There are always feasible agreements (members of RS) that Pareto dominate the recursive equilibrium [RE]. Defection from such an agreement can almost always be prevented by RS threats that dominate RE. This section presents a simple refinement of equilibrium that captures these collective rationality ideas.

The results of the previous section characterize the set of feasible agreements. This section begins by describing the feasible punishments sufficient to deter a given defection. Then "renegotiation proof" [RNP] strategies are defined. They are recursively supported equilibrium strategies with two additional properties:[14]

1 normal behavior is undominated among feasible agreements; and
2 punishments are members of RS that are not dominated by other feasible sufficient punishments.

These conditions are motivated by the following ideas. First, the voluntarism argument for RS behavior applies to punishments as well as normal behavior. Second, renegotiation threatens an agreement if it *would* benefit all players. Thus behavior should be Pareto optimal relative to "viable" alternatives. Viable agreements are members of RS. Viable punishments are members of RS that deny the defector any net profit. A punishment that does not meet these conditions will lead to further defection, so the other player(s) should not realistically expect to enjoy the benefits of cooperation.

For this two-player game, RNP strategies resemble strong perfect[15] equilibrium strategies. The main difference is that strong perfect equilibrium requires Pareto optimal normal and punishment behavior, while RNP only requires optimality relative to a restricted set of possibilities. Proposition 2 shows that the stronger optimality requirement is not fruitful: the game has no strong perfect recursively supported equilibria.

A series of propositions establishes the relationship between RNP strategies and recursive equilibrium. RNP normal [punishment] behavior is always [almost always] a strict Pareto improvement over RE, but myopia and/or severe or repeated defection will force the players onto extraction paths which are close to recursive equilibrium levels.

Proposition 3 establishes that the boundaries ∂_i of RS are steeper than the level sets of the payoff functions Ψ_i [which bound the set of sufficient punishments]. It follows that recursive equilibrium is the unique minimizer of each player's payoff over RS. Immediate consequences are: i) that RE is never the normal behavior for RNP strategies; ii) that RE is the unique Pareto minimal feasible sufficient punishment for any defection from an RS agreement; and iii) that RNP strategies never employ RE as a punishment, unless the initial behavior lies on the boundary of RS [w.r.t. the defecting player] and the defection is a best reply in the static sense of eq. (5).

A defection is "ε-sensible" if it would improve the player's payoff by at least $\varepsilon > 0$ in the absence of punishment. Proposition 4 shows that a finite number of ε-sensible defections by a single player will force the players into a neighborhood of RE, increasing total consumption and reducing all payoffs. By the same token, defections which are not 0-sensible lead to Pareto improvements.

The section concludes with a discussion of the possibility for bargaining between the players. If players are sufficiently farsighted, the candidates for RNP normal behavior may include a range of fully Pareto optimal extraction vectors. Even if there are no Pareto optima in RS, there may yet be many potential agreements, which are fully described in proposition 5. Finally, the possibility of negotiation over punishment is considered. Repeated or severe defections dramatically restrict the feasible punishments. Proposition 6 describes the conditions under which the RNP response to defection is uniquely determined, and characterizes the punishment.

Sufficient Punishments

Let $h \in RS$ and $d \in (0,1 - h_j)$. The RS punishments k that would keep player i from defecting to d when current stock is S and current stationary behavior is h are:

$$(18) \quad R^i(S,h,d) = \{k \in RS: V_i(h,S) \geq \ln(dS) + \delta V_i[k,(1 - h_j - d)^{\alpha}(S^{\alpha})]\}$$

In this example, (18) is independent of the current stock so $R^i(S,h,d) \equiv R^i(h,d)$: the set of $k \in RS$ such that:

$$(19) \quad \Psi_i(h) \geq (1 - \delta)\Psi_i(d,h_j) + \delta\Psi_i(k),$$

where (d,h_j) is h with its i^{th} component replaced by d.[16] $R^i(h,d)$ always contains h^{re}. The boundaries of $R^i(h,d)$ are the boundaries of RS and a level curve of Ψ_i. Notice that (19) reduces to the condition for recursive equilibrium as the players become more myopic $[\delta \to 0]$. Any $h \in RS$ can be supported by a subgame stationary perfect equilibrium strategy f with associated behavior $x^*(f) = h$ that always uses RS punishments. One is the "collapsing" strategy defined in eq. (14), but others dominate it for every course of play.

Renegotiation Proof Strategies

If all parties to an agreement would prefer another feasible course of action, the agreement is not collectively rational. Punishments which can be improved upon without losing their effectiveness are not collectively credible. The renegotiation proof [RNP] strategies defined here ensure collective rationality and credibility by requiring all behavior to be relatively Pareto optimal.

Formally, if B is a subset of the set X of feasible stationary extractions its *relative Pareto set* P(B) consists of those members undominated by other members of B [members of P(X) are *Pareto optimal*]:

(20) $$P(B) = \{h \varepsilon B: \text{ for all } h' \varepsilon B, \text{ there is } i \text{ s.t. } V_i(h') < V_i(h)\}.$$

A subgame stationary equilibrium f with $x^*(f) = h$ is *renegotiation proof* [RNP] if it satisfies two conditions. The first is collective rationality: $h \varepsilon P(RS)$. The second is collective credibility. Let y be the move at date t and $z \varepsilon X^{t-1}$ the history of play prior to t. If

- $y = f^t(z)$, then $x^*(g[t+1,(y,z)]) = y$;
- $y_i \neq f_i^t(z)$, but $y_j = f_j^t(z)$, then $x^*(g[t+1,(y,z)]) \varepsilon P(R^i[f^t(z),y_i])$;
- $y_i \neq f_i^t(z)$, both i, then $x^*(g[t+1,(y,z)]) = h^{re}$[17]

Here, g(t,z) is the "continuation strategy" defined in (13). Defection by a single player is deterred by a relatively Pareto optimal sufficient punishment. If both players defect the agreement collapses, reflecting the power of precedent.

If possible, rational players will commit to RNP punishments. Even if commitment is impossible, any Pareto superior RS alternative to RNP punishment leaves the defector with a residual profit. This precedent ensures continued defection, so offers of renegotiation beyond the RNP limits should be regarded with skepticism.

Every $h \in P(RS)$ is supported by a renegotiation proof strategy, since RS and the $R^i(h,d)$ are compact and nonempty. There are, however, no RSE strategies that support (fully) Pareto optimal behavior by optimal threats, which are in turn supported by optimal counter threats *ad infinitum*.

This can be inferred from the fact that the Fish War has no strong perfect equilibria. The idea of the proof is quite simple. Optimality entails a total extraction rate of $(1 - \alpha\delta)$. Strong perfect equilibrium total consumption is fixed along all possible paths. The incentive to defect increases as a player's share of total extraction decreases. Repeated defections must be punished with ever greater reductions in consumption. But consumption by other players is bounded above, which places a finite lower bound on the utility a defector can be forced to accept. This ultimately precludes effective punishment. I shall, however, demonstrate the proposition directly.

A *strong perfect RS equilibrium* is a subgame stationary strategy f with the following properties:

- $x^*(f) \in P(X)$;
- f is a recursively supported perfect equilibrium (see fn. 12); and
- for any t and any $z \in X^{t-1}$, $g(t,z) \in P(X)$.

Proposition 2: There are no strong perfect RS equilibria.

Even the modest condition that punishments belong to P(RS) leads to nonexistence [Proposition 4].

RNP Strategies and Recursive Equilibrium

The recursive equilibrium can be shown to be the unique Pareto minimal member of RS, and thus of any $R^i(h,d)$. We begin by showing that the level sets of F_i [eq. (16)], which define the boundaries ∂_i of RS, are steeper than the level sets of Ψ_i [eq. (4)], which define player i's payoff and the boundary of $R^i(h,d)$.

The boundaries differ because the implicit punishment defining RS is insensitive to the actual defection, while punishments in $R^i(h,d)$ are chosen to "fit the crime." The best defection against any constant punishment is $d^* = (1 - \alpha\delta)(1 - h_j)$ [see eq. (5)], which accounts for the extra $(1 - h_j)$ term in (16). The slopes of the level sets are:

$$\text{(21)} \qquad \frac{dh_j}{dh_i}\Big|F_i = \frac{(1 - h_j)}{h_i}\left[\frac{(1 - \alpha\delta)(1 - h_j) - h_i}{(1 - \delta)h_i - (1 - \delta - \alpha\delta)(1 - h_j)}\right] \equiv S2.$$

$$\text{(22)} \qquad \frac{dh_j}{dh_i}\Big|\Psi_i = \frac{[(1 - \alpha\delta)(1 - h_j) - h_i]}{\alpha\delta h_i} \equiv S1$$

Proposition 3: $h \in RS$ implies $0 \le S1 \le S2$.

This proposition has several immediate corollaries:

Corollary A: h^{re} uniquely minimizes Ψ_i over RS all i.

Corollary B: h^{re} never belongs to P(RS).

Corollary C: h^{re} uniquely minimizes Ψ_i over $R^j(h,d)$ all i,j,h,d.

Corollary D: h^{re} does not belong to $P(R^i(h,d)$ unless $F_i(h) = F_i(h^{re})$ and $d = (1 - \alpha\delta)(1 - h_j)$ [compare eq. (5)].

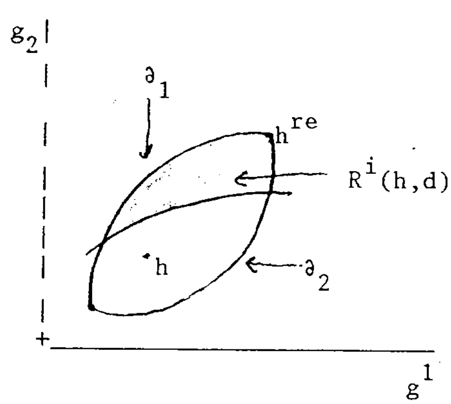


Fig. 2 -- RS and a typical $R^i(h,d)$ set

Fig. 2 shows the set RS and a typical $R^i(h,d)$. The position of $R^i(h,d)$ depends on d as well as h. A defection d is ε-sensible if $\Psi_i(h) + \varepsilon \le \Psi_i(d,h_j)$.[18] $R^i(h,d)$ includes h unless d is 0-sensible. The following proposition describes the effect of repeated and/or severe ε-sensible defections on RNP behavior.

Proposition 4: Let $[h^t,d^t]$ be a sequence of extraction rates and ε-sensible defections by player i such that $h^{t+1} \in R^i(h^t,d^t)$ for all t. Then if:

$$(23) \qquad t \ge t^* = [\delta\Psi_i(h^0) + (1 - \delta)\ln(2 - \alpha\delta) - K]/[\varepsilon(1 - \delta)]$$

player i will have no ε-sensible defections, and:

$$(24a) \qquad \Psi_i(h^{re}) \le \Psi_i(h^{t^*}) \le \Psi_i(h^{re}) + [\varepsilon(1 - \delta)/\delta]$$

$$(24b) \qquad h_j^{re} \ge h_j^{t^*} \ge h_j^{re} - [(e^{(\varepsilon/\delta)} - 1)/(2 - \alpha\delta)] \text{ for } j = 1,2.$$

As $\varepsilon \to 0$, the neighborhood defined by (24) shrinks to the recursive equilibrium, although the extent of defection required to reach this neighborhood increases. Repeated (large t) or serious (large ε) defections force the players onto an extraction path that is only slightly better (slower) than recursive equilibrium. The rate of convergence is greater the closer h^0 is to h^{re}. After t* ε-sensible defections, the players reach an "ε-recursive equilibrium" in the sense of Radner [1981] -- each player almost maximizes his payoff w.r.t. the other's current behavior.

Punishment for serious or repeated defections must eventually be costly to both players. Reversing the argument leads to the corollary observation that following repeated or severe ε-sensible defections, a defection which is not 0-sensible will evoke a punishment which is profitable to the punisher and entails a slower overall rate of extraction.

Bargaining

RNP normal behavior must come from P(RS). Proposition 5 establishes that this set is an interval bounded by the "ideal points" r^i of the players. Formally, r^i is the unique maximizer of Ψ_i over RS. It always belongs to ∂_j.

Proposition 5: P(RS) is the union of P_1, P_2, and P_+, where:

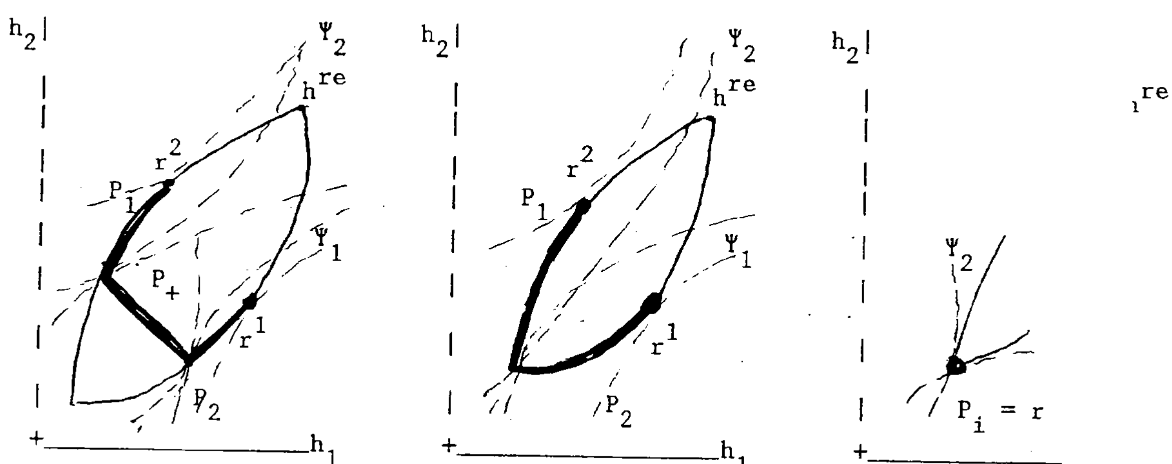
P_i = {h ε RS: a) $h \in \partial_j$; b) $h \leq r^i$; and c) $h_1 + h_2 \geq 1 - \alpha\delta$}; and

P_+ = {h ε RS: $h_1 + h_2 = 1 - \alpha\delta$}.

Although P_+ may be empty, the P_i are not. Figure 3 shows possibilities corresponding to various parameter values.

Similar considerations apply to RNP punishments. If $r^j \in R^i(h,d)$, $P[R^i(h,d)]$ consists of all members of ∂_i between its intersection with the boundary of $R^i(h,d)$ and r^j (Fig. 4b).

Following severe or repeated ε-sensible defections, $R^i(h,d)$ will not include r^j. In this case there is no scope for negotiation.



a) Pareto optimal agreements (high δ)

b) Relatively optimal agreements (moderate δ)

c) Unique agreem (low δ)

Fig. 3 -- Optimality of Agreements

Proposition 6: If r^j does not belong to $R^i(h,d)$, $P[R^i(h,d)]$ is the unique minimizer of total consumption over $R^i(h,d)$. [See Fig. 4c]

The nature of punishment for repeated or severe defections is further clarified by the observation that renegotiation proof punishment for defection from a suboptimal agreement must lie on ∂_i, which is upward-sloping by proposition 3. Therefore, the RNP response to an ε-sensible defection from a suboptimal agreement increases consumption for both players, as in Figs. 4b and 4c. This is stated formally in the following corollary to proposition 3.

Corollary E: If $h \in \partial_i$, d is an ε-sensible defection from h, and $g \in P[R^i(h,d)]$, then $g > h$ (proof omitted).

In Fig. 4a, player 1 has defected from a Pareto optimal agreement. Renegotiation proof punishment decreases player 1's consumption while increasing player 2's. But RNP punishment for repeated or severe defections increases consumption for both players no matter what the original agreement.

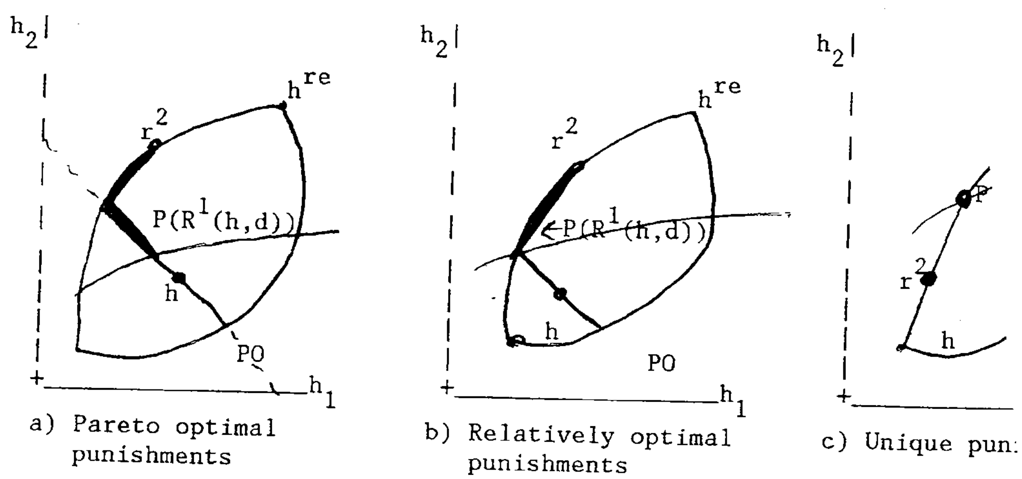


Fig. 4 -- Optimality of Punishments

V. CONCLUDING REMARKS

This analysis has several applications. With this definition of renegotiation proof equilibrium, Section IV sheds light on the reciprocal punishments embedded in institutions created by self-interested individuals in ongoing dynamic games. Although propositions 4 and 6 imply that repeated or severe defection reduces the possibility of cooperative behavior, the structure of renegotiation proof equilibria allows players to "return from the brink" by defections that are not "0-sensible:" punishments for such defections may be Pareto improving. To preserve the equilibrium, this improvement must be limited to the player who did not defect. To induce a return to cooperation requires changing the definition of equilibrium to encourage altruistic activity. One way is to extend each player's payoff function to include the other's utility lexicographically -- this does not upset existing equilibria.

Regression to noncooperative behavior may be a general feature of renegotiation proof strategies in dynamic games with unique Pareto inferior recursive equilibria. It would be interesting to extend the methods developed here to games possessing multiple recursive equilibria.

Finally, one might interpret this analysis as an argument for a more sophisticated concept of "renegotiation." One might adapt Farrell's [1984] approach and call a set of extraction rates renegotiation proof if every member is secured by a threat to play another member and no member of the set Pareto-dominates another. In general, one would expect many such sets and seek a Pareto-superior one. For the game considered here, propositions 1, 4 and 6 can be used to show that the only such set is the recursive equilibrium itself.

FOOTNOTES

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1. This is essentially the procedure followed in Mirman [1979]. Levhari and Mirman [1980] use backwards induction to achieve the same result. In LM, it is initially assumed that after a finite number of periods, the remaining stock will be divided in some fixed proportions, p_i. Consumption by player i, c_i^1, is a fraction h_i^1 of current stock S. In the penultimate period, player i will choose h_i^1 to maximize:

$$\text{(N1)} \qquad U_i(h_i^1 S) + \delta U_i(p_i f[(1 - \Sigma h_j^1)S]).$$

Simultaneous solution of these "one-period best replies" determines the one-period recursive equilibrium shares and value function $V_i^1(S)$ found by substituting the shares into (N1).

In the previous period, players choose shares h_i^2 to maximize:

$$\text{(N2)} \qquad U_i(h_i^2 S) + \delta V_i^1(f[(1 - \Sigma h_j^2)S]).$$

Simultaneous solution of these "two-period best replies" leads to the "two-period" recursive equilibrium shares and value functions. LM show that the "T-period" shares and value converge as $T \to \infty$. Mirman [1979] shows that they converge to the shares and value given in (6) and (7).

2. Several other papers use threat strategies in the dynamic game context. Lewis and Cowans [1984] uses the threat of recursive equilibrium to police a symmetric Pareto optimal agreement. My approach differs in several respects: I describe all behavior supported by this threat; my analysis of cooperative behavior includes both nonsymmetric and suboptimal behavior; and most importantly, I address the possibility of renegotiation.

Tolwinski [1982,1983] and Tolwinski and Haurie [1984] use threat strategies to model bargaining in a dynamic game. Strategies depend on previous states, but not separately on previous actions. In addition, payoffs depend only on the current state, and not on current actions. Recursive equilibrium is taken to reflect the absence of either communication or binding contracts, and cooperation is the result of communication without binding commitments. My interpretation explicitly concerns the creation of binding contracts within the game. Moreover, without communication Nash equilibrium is inappropriate, since it requires the strategies of all players to be common knowledge. Tolwinski's solution resembles my idea of recursively supported equilibrium, although its conception owes more to Nash bargaining theory than to Nash equilibrium. His players declare threats to be used if cooperation breaks down. Best-reply payoffs to these threats determine the set of "enforceable" agreements and also the "threat point" to be used in bargaining. Tolwinski (1982) uses as threats the recursive equilibrium and minmax strategies in a zero-sum game played over the difference between the players' payoffs. The bargaining theories used are the Nash bargaining solution and a version of the Raiffa (1953) interpersonal-comparison bargaining solution.

3. Arbitrary threat strategies give results similar to those for the discounted supergame [Cave, 1979]. The example used in this article has unbounded security levels, leading to a kind of "Folk Theorem:" every feasible and individually rational vector of payoffs is supported by an equilibrium. The same outcomes can be supported by subgame perfect equilibria. Consider an arbitrary feasible sequence of extraction

vectors that all players should follow unless one or more of them defects, in which case each player is required to consume the entire remaining stock. Any defector's payoff is $-\infty$, so the strategies are in equilibrium. It is subgame perfect, since it induces equilibrium in subgames where a player has defected. Each player expects the others to consume all the remaining fish. Carrying out the punishment thus gives the same payoff as failing to carry out the punishment: $-\infty$! This result is peculiar to games with unbounded security levels, but the collective implausibility of threats used to support perfect equilibrium is more general; once such a threat is activated, every party to the original agreement may prefer some outcome achievable as a Nash equilibrium by collective change of strategy.

4. A more general and detailed version is available from the author.

5. Players with general concave utility and renewal functions will use stationary, but not necessarily linear, consumption functions $c_i(S)$.

6. Two limitations of this example deserve comment. As noted earlier, the unbounded utility functions produce peculiar results. But this aspect of the example is inessential to the analysis in this article. The results remain the same if utility is bounded below, or equivalently if there is a (small) "subsistence consumption" σ so that i's utility becomes $\ln(c_i + \sigma)$, although either alternative considerably complicates the discussion. The other limitation is that players can choose any rate of consumption without regard to the choices of other players: infeasible choices can arise. There are several ways of dealing with this. One is to observe that in equilibrium players will never choose infeasible consumptions; at worst (if $U'(0) < +\infty$) they will be indifferent between an infeasible consumption and a feasible one that exhausts the stock. Another is to use a rationing scheme to associate to every infeasible h a feasible one; e.g., $h/\|h\|$. Again, this complicates the exposition without affecting the analysis.

I regard the example as expository rather than realistic. Its limitations are only important if the results cannot be extended to less limited situations. Since they can, and since these special features simplify the exposition, I use the original example.

7. Compare Levhari and Mirman, note 9.

8. Compare Levhari and Mirman, eqs. 14-16.

9. In general Pareto optimal total extraction is not unique, but recursive equilibrium always entails a strictly larger total extraction.

10. For a summary of this theory, see Roth [1979].

11. Amir [1984].

12. More generally, a stationary strategy n-tuple f is a *recursively supported equilibrium*, written $f \in RSE$, if and only if $x^*(f) \in RS$. We also say that f is a *recursively supported perfect equilibrium* if and only if, for all t and $z \in X^{t-1}$, the continuation strategy $g(t,z) \in RSE$.

13. In the nonsymmetric case all Pareto optimal total extractions may be smaller than the smallest recursively supportable total extraction.

14. Farrell [1984] has written casually on a related solution concept.

15. Moulin [1982] contains a general discussion of strong equilibrium. Here, f is a *strong equilibrium* if and only if, for all coalitions C, and feasible strategies $g_C \in \Phi_C(f)$, there is $i \in C$ s.t. $U_i(f|g_C) < U_i(f)$. Cave [1979] and Rubinstein [1980] analyze strong perfect equilibria of repeated games. Here, f is a *strong perfect equilibrium* if and only if, for all t and all $z \in X^{t-1}$, the continuation strategies $g(t,z)$ are strong equilibria. In this game, all optimal outcomes are strong equilibrium outcomes, but there are no strong perfect equilibria.

16. Compare this with the analogous condition for stationary equilibrium in discounted supergames [Cave, 1979]: stationary behavior can be sustained as an equilibrium if and only if the payoff exceeds $(1 - \delta)$ times the best reply payoff plus δ times the player's one-shot maxmin payoff.

17. Alternatively, we could require $x^*(g[t+1,(y,z)])$ to belong to the nonempty set $P(R^1[f^t(z),y]\ R^2[f^t(z),y])$, which punishes both defectors.

18. Alternatively, we could state this in terms of the strategic model as $U_i(f^h) \leq U_i(f^h|f_i^d) + \varepsilon$, where f_i^h is the memoryless strategy defined for all t,z by $f_i^{h,t}(z) = h_i$.

APPENDIX: PROOFS OF PROPOSITIONS

This Appendix contains proofs of propositions 2-6. Proofs of other results are available on request.

Proposition 2

proof: Assume not, fix t and $z \varepsilon X^{t-1}$, and let the next move be y s.t.: $y_i \neq f_i^t(z)$, but $y_j = f_j^t(z)$. Since f is perfect, $x^*(g[t+1,(y,z)]) \varepsilon R^i[f^t(z),y_i]$. In addition, $f^t(z) \varepsilon P(X)$ and $x^*(g[t+1,(y,z)]) \varepsilon P(X)$. h is Pareto optimal so (20) is:

(A1) $\delta\Psi_i(g) + (1 - \delta)\Psi_i(d,h_j) \leq (1 - \alpha\delta)\ln(h_i) + \alpha\delta\ln(\alpha\delta)$.

The Pareto optimal punishments for the defection d from h ε P(X) are:

(A2) $PR^i(h,d) = \{k \varepsilon RS\text{: } k \varepsilon P(X) \text{ and } k_i \leq \max[k_i(h,d),1]\}$ where $\delta\ln[k_i(h,d)] = \ln(h_i) - (1-\delta)\ln(d) + \alpha(1-\delta)[\ln(\alpha\delta) - \ln(1-h_j-d)]/(1-\alpha\delta)$.

Note that $k_i(h,h_i) = h_i$. We show that there are defections near h_i for which $k_i(h,d)$ strictly reduces player i's consumption. The derivative of $k_i(h,d)$ w.r.t. d is:

(A3) $$\frac{dk_i(h,d)}{dd} = \left[\frac{\alpha}{(1 - \alpha\delta)(1 - h_j - d)} - \frac{1}{\delta d}\right](1 - \delta)k_i(h,d).$$

$k_i(h,d)$ is decreasing in d near h_i, and reaches a minimum at:

(A4) $d^* = (1 - \alpha\delta)(1-h_j)$.

At d^*, $1 - h_j - d^* = \alpha\delta(1 - h_j)$, and $1 - h_j = \alpha\delta + h_i$, so we get:

(A5) $\delta\ln(k_i(h,d^*)) = \ln(h_i) - (1-\delta)\ln(\alpha\delta+h_i)/(1-\alpha\delta) - (1-\delta)\ln(1-\alpha\delta)$.

Hence $k_i(h,d^*) < h_i$ follows from $h_i \leq 1 - \alpha\delta$. Player i's Pareto optimal RS payoffs are a closed interval bounded away from 0 [by (16)], hence

$k_i < h_i$. Thus, near the minimal Pareto optimal h_i in RS, k_i is not in RS. **QED**

Proposition 3

proof: Let $h \varepsilon$ RS. By prop. 1, $h_i \leq (1 - \alpha\delta)(1 - h_j)$; hence $S1 \geq 0$. S2 can be written:

$$\text{(A6)}\quad S2 = \frac{\alpha\delta(1 - h_j)S1}{[(1 - \delta)h_i - (1 - \delta - \alpha\delta)(1 - h_j)]},$$

so it suffices to show that

$$\text{(A7)}\quad (1 - \delta)h_i \geq (1 - \delta - \alpha\delta)(1 - h_j),$$

since by (A6), $S2 \geq S1$ if $S2 \geq 0$ and $1 \geq h_1 + h_2$. The RHS of (A7) is monotone decreasing in h_j, so it suffices to show that h^* defined by equality in (A7) is not in RS. Solving for h^*, we get $h_i^* = (1 - \delta - \alpha\delta)/[2(1 - \delta)]$, whence the proposition follows by (16) and concavity of $\ln(\cdot)$. **QED**

Proposition 4

proof: choose any term $h = h^t$ of the sequence, and for simplicity let $d = d^t$ and $k = h^{t+1}$. The conditions that a) $h \varepsilon$ RS; b) $k \varepsilon$ RS; c) d is an ε-sensible defection for i from h; and d) $k \varepsilon R^i(h,d)$ may be written:

$$\text{(A9a)}\quad \Psi_i(h) - (1 - \delta)\ln(1 - h_j) \geq K;$$

$$\text{(A9b)}\quad \Psi_i(k) - (1 - \delta)\ln(1 - k_j) \geq K;$$

$$\text{(A9c)}\quad \Psi_i(d,h_j) \geq \Psi_i(h) + \varepsilon; \text{ and}$$

$$\text{(A9d)}\quad \Psi_i(h) \geq (1 - \delta)\Psi_i(d,h_j) + \delta\Psi_i(k).$$

For given h,d, the furthest k from h^{re} [in any metric] comes at the point $k^*(h,d)$ where (A9b) and (A9d) hold with equality; this follows from proposition 3. For a given h, the ε-sensible defection d that maximizes the distance between $k^*(h,d)$ and h^{re} can be seen from (A9d) to be the one with the smallest value of $\Psi_i(d,h_j)$; thus (A9c) must hold with equality. Combining (A9c) and (A9d), we conclude that along the slowest path to recursive equilibrium, $\Psi_i(h) - [\varepsilon(1 - \delta)/\delta] = \Psi_i(k)$. Substituting into (A9b), we conclude that:

(A10) $$\ln(1 - k_j) = [\Psi_i(k) - K]/(1-\delta) = [\Psi_i(h) - K]/(1-\delta) - \varepsilon/\delta.$$

Maximizing $\Psi_i(d,h_j)$ w.r.t. d, we conclude from (A9c) that player i has no ε-sensible defections if $\ln(1 - h_j) > \varepsilon/\delta - \ln(2 - \alpha\delta)$. The proposition follows by routine computation. **QED**

Proposition 5

That P_+ belongs to P(RS) is obvious. V_j is concave on a neighborhood of RS. Therefore the slope of V_j indifference curves through points h of ∂_i is less [greater] than the slope of ∂_i according as h <[>] r^j. The proposition follows from proposition 3: for h ε P_j, the indifference curves of player j are steeper than those of player i, so the preferred directions of movement along ∂_i are opposed. Points along ∂_i for which $h > r^j$ are worse for both players than r^j. Points interior to RS are dominated by points on RS. Finally, if P_+ is nonempty, points on ∂_i for which $h_1 + h_2 > 1 - \alpha\delta$ are dominated by the intersection of ∂_i and P_j.
QED

Proposition 6

proof: Proposition 3 and eqs. (23) and (24) imply that r^j lies on ∂_i between h^{re} and h^{min} as shown in Fig. 3: at h^{min} [h^{re}] the slope of the $V_j(h)$ indifference curve is less [greater] than the slope of ∂_i, and $V_j(h)$ is C^1.

If r^j ε $R^i(h,d)$, the slope of $V_j(h)$ though any point in $R^i(h,d)$ exceeds that of $V_i(h)$, so k ε $P[R^i(h,d)]$ implies k ε ∂_i. Elementary geometry shows k to be the intersection of the highest $V_i(h)$ indifference curve in $R^i(h,d)$ with ∂_i. Moreover, the direction of decrease of G is a convex combination of the directions of increase of $V_i(h)$ and $V_j(h)$, showing that the Pareto superior punishment minimizes total consumption over $R^i(h,d)$. **QED**

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LONG-TERM COMPETITION IN A DYNAMIC GAME: THE COLD FISH WAR

Cave