EFFECT OF TORSIONAL STIFFNESS REQUIREMENTS ON WING STRUCTURAL WEIGHT

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SUMMARY

Curves are obtained which show the total increase in weight of an optimum-designed sheet-stiffener compression panel when the skin thickness is increased by an arbitrary amount. These example curves are based on data from tests conducted by the NACA on 75S-T Y-stiffened panels. However, the method outlined may be applied also to the plotting of other sheet-stiffener compression panel data.

The NACA data, when put in the form outlined here, indicates the minimum amount by which the total weight of sheet and stiffeners would be increased in order to achieve a given increase in sheet thickness. If optimum design principles are followed, this increase in total weight is considerably less than the increase which would result if the amount of stiffener material is left unchanged.

The method outlined here for presenting data is of particular use in preliminary design work where the effects of aeroelasticity requirements must be evaluated in terms of increased wing weight.

INTRODUCTION

One of the problems encountered in the structural design of the wing for high speed airplanes is that of providing sufficient structure to prevent aeroelastic effects, such as loss in aileron effectiveness, from exceeding acceptable limits. Most of the aeroelastic effects experienced by the unswept wing are due to a reduction in effective torsional stiffness which accompanies an increase in flight speed. For a wing having the flexural axis located aft of the line of aerodynamic centers, there is not a great deal that can be done to prevent this apparent loss in stiffness. What the designer usually does is to provide sufficient structural stiffness so that at the speed in question the wing retains sufficient effective stiffness to keep the aeroelastic effects within specified limits.

One method of increasing torsional stiffness, without redesigning the entire wing structure, is to increase the skin thickness of the structural box. Assuming the shear flow is constant around the structural box, the expression for the section torsional stiffness, GJ (in-lb per rad/in), is

$$GJ = \frac{4\,A_e^2\,G}{\int \frac{ds}{t_s}} \tag{1}$$

where

A_e = enclosed area of the structural box

G = modulus of elasticity in shear

s = distance around the perimeter of the box

t_s = skin thickness

Writing the denominator in series form,

$$GJ = \frac{4\,A_e^2\,G}{\frac{s_1}{t_{s_1}} + \frac{s_2}{t_{s_2}} + \frac{s_3}{t_{s_3}} + \cdots + \frac{s_n}{t_{s_n}}} \tag{2}$$

It can be seen from Eq. (2) that, since the terms in the denominator are divided by skin thickness, the most efficient* way to increase the torsional stiffness at any spanwise station is to make the skin thickness uniform around the box. After this has been done the section torsional stiffness may be determined from the expression

$$GJ = \frac{4\,A_e^2\,G\,t_s}{s} \tag{3}$$

where

t_s = skin thickness (assumed to be uniform around the structural box)

s = perimeter of the structural box

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*"Efficiency" is used here in the sense of obtaining the maximum increase in stiffness for the amount of weight added.

Eq. (3) shows that for a uniform skin thickness around the box, the torsional stiffness varies directly with the skin thickness; hence, any further increase in stiffness is most efficiently obtained by a uniform increase in skin thickness.

At this point it is desirable to know how the weight of a sheet-stiffener compression panel is affected by an increase in skin thickness, the loading remaining unchanged. It is incorrect to assume that an increase in torsional stiffness would require an increase in skin thickness without any decrease in the amount of stiffener material. Actually, as the skin thickness is increased, the amount of stiffener material may be decreased, since the axial loading to be carried remains constant. This, of course, can be done only in the earlier stages of design.

By using data from tests made by the NACA, it is possible to obtain curves showing the total increase in weight caused by increasing the skin thickness by an arbitrary amount.

METHOD OF OBTAINING CURVES

The data on which the curves of this report are based were obtained from Fig. 20 of NACA Technical Note 1389, which contains data on tests of sheet-stiffener compression panels. Fig. 20 shows a plot of $q/\bar{t}$ vs q/t_s for lines of constant q/L_o, where

q = average load per inch width (lb/in) = total load/width

$\bar{t}$ = average thickness of the panel (in) = total area/width

t_s = skin thickness (in)

L_o = effective pin-end column length (in)

Since the curves were composed of a series of arcs, envelopes were drawn to represent a continuous variation. These envelope curves are shown in Fig. 1.

The term $q/\bar{t}$ represents the average compressive stress developed in the panel. The term q/L_o is the structural index$^{(2)}$, which is a measure of the loading intensity. The parameter q/t_s, the average load per inch width divided by the skin thickness, is a convenient means for introducing the skin thickness as a parameter.

A straight line through the origin (Fig. 1) represents a line of constant value of the ratio $t_s/\bar{t}$, the slope being equal to $t_s/\bar{t}$. By cross-plotting the curves of Fig. 1 as $q/\bar{t}$ vs q/L_o for lines of constant $t_s/\bar{t}$, we can show how changing the ratio of skin thickness to average thickness affects the allowable stress, for any value of structural index. This cross-plot is shown in Fig. 2.

It is apparent from Fig. 2 that as the value of $t_s/\bar{t}$ is increased, the allowable stress is decreased. The lower curve, for $t_s/\bar{t} = 1.0$, represents the limiting case, wherein all of the material is in the skin. This curve was calculated from Eq. (15b) in Ref. 2. This expression for the allowable stress F_c of a flat plate used as an Euler column is

$$F_c = 0.97 \, (E\tau)^{1/3} \, (q/L_o)^{2/3} \qquad (4)$$

where

E = modulus of elasticity (elastic range)
$\tau = \frac{E_t}{E}$ where E_t = tangent modulus

The position of this curve indicates that, as $t_s/\bar{t}$ approaches unity, the allowable stress drops off rapidly.

At higher values of q/L_o, it is likely that the rate of decrease of F_c with increases in $t_s/\bar{t}$ will be less pronounced. At the time of writing, however, there were no experimental data available which covered values of q/L_o above a value of 800, so this could not be shown by the curves.

It can be seen that the curve representing optimum (minimum-weight) design may be attained only by using an extremely low value of $t_s/\bar{t}$. This indicates that such a value of allowable stress is not likely to be attained in compression panels designed for use in wing structures. For this reason, it is desirable to qualify allowable-stress curves, of the type shown in Fig. 2, with values of $t_s/\bar{t}$. The ratio $t_s/\bar{t}$ is a useful parameter for estimating the relative amount of torsional stiffness which would be contributed by the panel when used in a wing structure. Its use introduces a further convenience because, according to the design charts of Ref. 1, for a specified value of t_s/t, there exists a variety of combinations of stiffener size and spacing which will yield very nearly the same allowable stress as the maximum for that value of t_s/t shown in Fig. 2.

As shown by Shanley in Ref. 2, the average thickness may be determined from curves of the type shown in Fig. 2 and approximated by a straight line as follows. If the value of q/L_o at each point on the curves is divided by the corresponding value of q/t, we can obtain curves of $\bar{t}/L_o$ vs q/L_o as shown in Fig. 3. These curves can be closely approximated by straight lines. For the set of curves in Fig. 3, the straight lines are not distinguishable from the curves except at very low values of q/L_o. The expression for the line is given by

$$\frac{\bar{t}}{L_o} = C_c + \frac{q/L_o}{F_{c_o}} \tag{5}$$

where

C_c = intercept value at $q/L_o = 0$

F_{c_o} = effective stress = $\dfrac{1}{\text{slope of the line}}$

The average thickness may be written

$$\bar{t} = C_c L_o + \frac{q}{F_{c_o}} \tag{6}$$

For any value of t_s/t, the total average thickness $\bar{t}$ required at any value of q/L_o may be found from this expression. For a value of $t_s/t = 0.6$, the expression for the average thickness is

$$\bar{t} = 0.00155\, L_o + \frac{q}{83{,}000}$$

For example, if a panel 20 inches in length and having pin-end conditions is subjected to a loading of 300 lb/in, the total average thickness required is 0.0246 inches, of which the skin thickness is 0.0148 inches. It should be remembered that these numbers refer to panels made of 75S-T alclad skin and 75S-T extruded Y-section stiffeners.

Fig. 2 shows how a change in the ratio $t_s/\bar{t}$ affects the allowable average stress. However, it does not show how the average thickness is affected by changes in skin thickness, the loading remaining constant. This can be done as follows.

Using Fig. 1 and choosing some value of $t_s/\bar{t}$ to be used as a reference value (designated by $t_{s_o}/\bar{t}_o$), we can obtain curves of t_s/t_{s_o} vs $\bar{t}/\bar{t}_o$ for each line of constant q/L_o, where

t_{s_o} = skin thickness of a panel having a value of $t_s/\bar{t}$ equal to the reference value

$\bar{t}_o$ = average thickness of a panel having a value of $t_s/\bar{t}$ equal to the reference value

The term t_s/t_{s_o} represents the ratio of an arbitrary skin thickness to the skin thickness of the panel having $t_s/\bar{t}$ equal to the reference value. The term $\bar{t}/\bar{t}_o$ is the corresponding ratio of the average thicknesses.

The value of $t_s/\bar{t} = 0.3$ will be chosen as the reference value because it represents a limiting case for the curves of Fig. 1 and all values of t_s/t_{s_o} and $\bar{t}/\bar{t}_o$ will be greater than unity.

For each line of constant q/L_o the intersection with the line of $t_s/\bar{t} = 0.3$ gives values of $q/\bar{t}_o$ and q/t_{s_o} for that particular value of q/L_o. Then for each line of constant q/L_o the values of $q/\bar{t}_o$ and q/t_{s_o} are divided by the ordinates and abscissas defining that line. The resulting values are plotted in Fig. 4 as $\bar{t}/\bar{t}_o$ vs t_s/t_{s_o} for lines of constant q/L_o.

For example, Fig. 4 shows that for a panel having an initial value of $t_s/\bar{t} = 0.3$ and a structural index of 300 psi, the skin thickness may be doubled at the cost of only 5 percent increase in total panel weight, if optimum design methods are employed. The dashed line shows the variation of panel weight based on the assumption that any increase in skin weight will increase the total panel weight by the same amount.

CONCLUSIONS

The curves of allowable stress F_c vs q/L_o representing "optimum" design for given types of sheet-stiffener panels show that this minimum-weight design can be attained only at very low ratios of $t_s/\bar{t}$. Since wing panels are not designed on the basis of allowable compressive stress alone, curves of this type should show how much of the panel material is made up of skin and can be counted on to resist torsion. One method of indicating the relative amount of material in the skin is illustrated by the curves of Fig. 2, which are drawn for constant values of $t_s/\bar{t}$.

The allowable stress curves of Fig. 2 can be cross-plotted as shown in Fig. 3 and closely approximated by straight lines, resulting in a simplified analytical expression for the relationships involved. Straight line approximations of this type have been used conveniently in wing weight estimation procedures(3).

Fig. 4 illustrates the manner in which panel weight must change with changes in skin thickness, and shows the over-conservatism of the assumption represented by the dashed line. For an actual design, where the values of both q and q/L_o are known, the curves of Fig. 1 could be used to calculate directly any change in panel weight for a given change in skin thickness.

As the skin becomes an increasingly greater fraction of the total panel thickness, the construction approaches the "panel" type(2) in which a lighter construction may be obtained by putting all of the material in the skin and running the separators spanwise rather than chordwise. In this type of construction, which uses all the panel material to resist torsion, the compressive strength of the panel will depend on the width rather than on the length. If the data had covered a higher range of values of q/L_o and skin thickness, the curves of sheet-stiffener construction could be extended to show a comparison of these two types of construction and would then indicate the values of t_s/t_{s_o} and q/L_o at which it would be advantageous to change from one type of construction to the other.

REFERENCES

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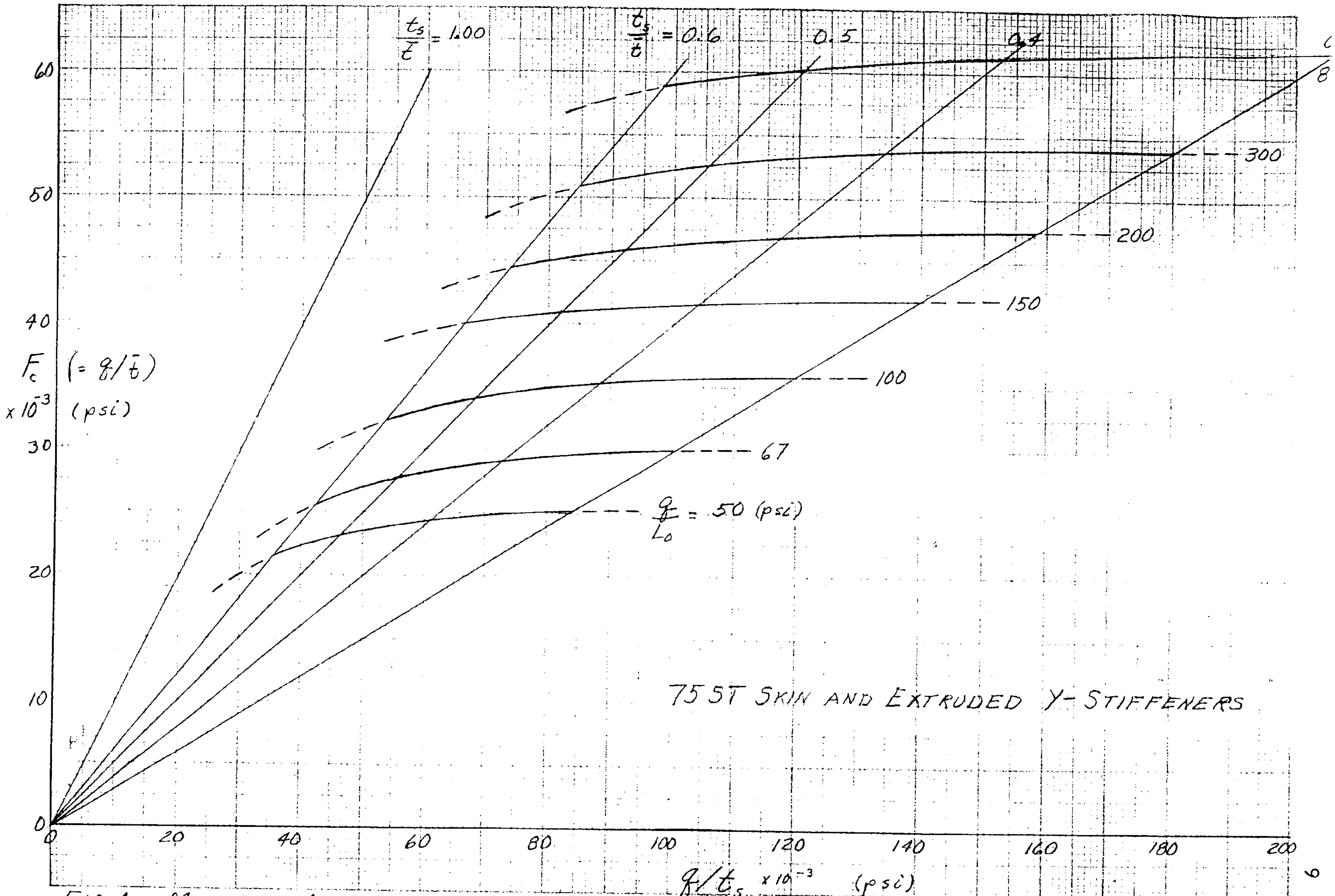

FIG 1. MAXIMUM ALLOWABLE STRESS FOR 75ST SHEET-STIFFENER PANELS FOR DIFFERENT VALUES OF q/t_s AND STRUCTURAL INDEX

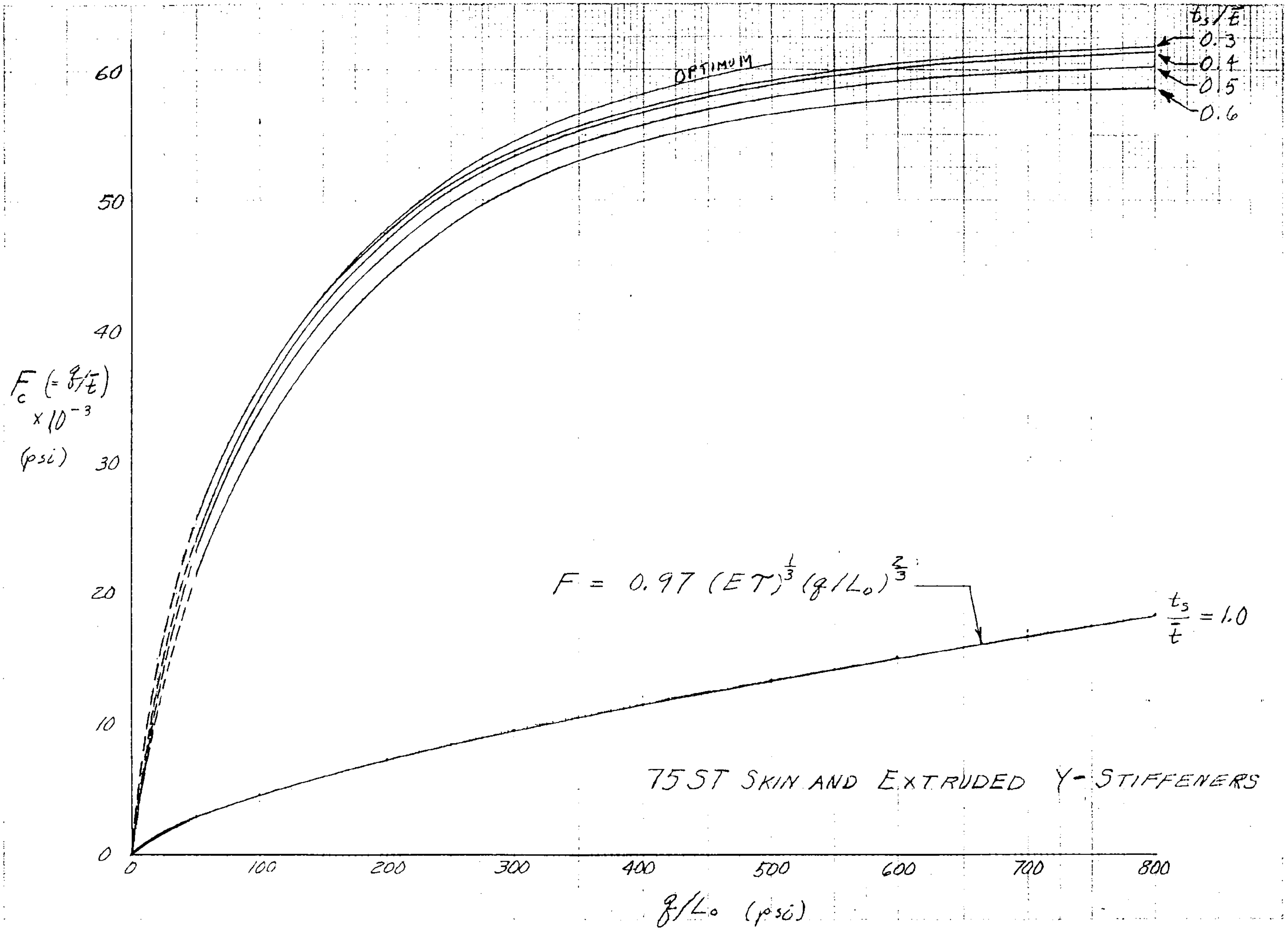

FIG 2. MAXIMUM ALLOWABLE STRESS FOR 75ST SHEET-STIFFENER PANELS FOR CONSTANT VALUES OF $t_s/\bar{t}$

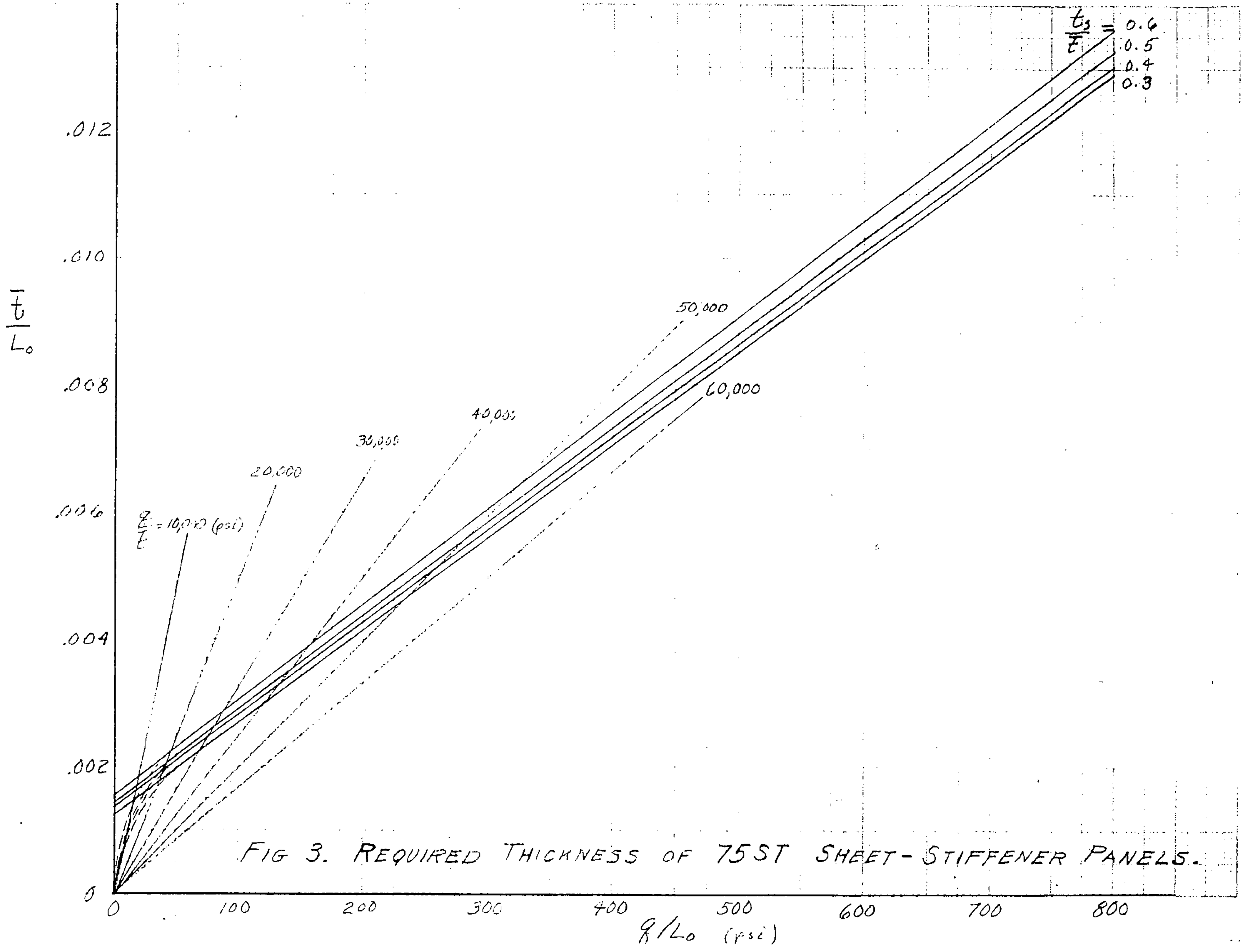

FIG 3. REQUIRED THICKNESS OF 75ST SHEET-STIFFENER PANELS.

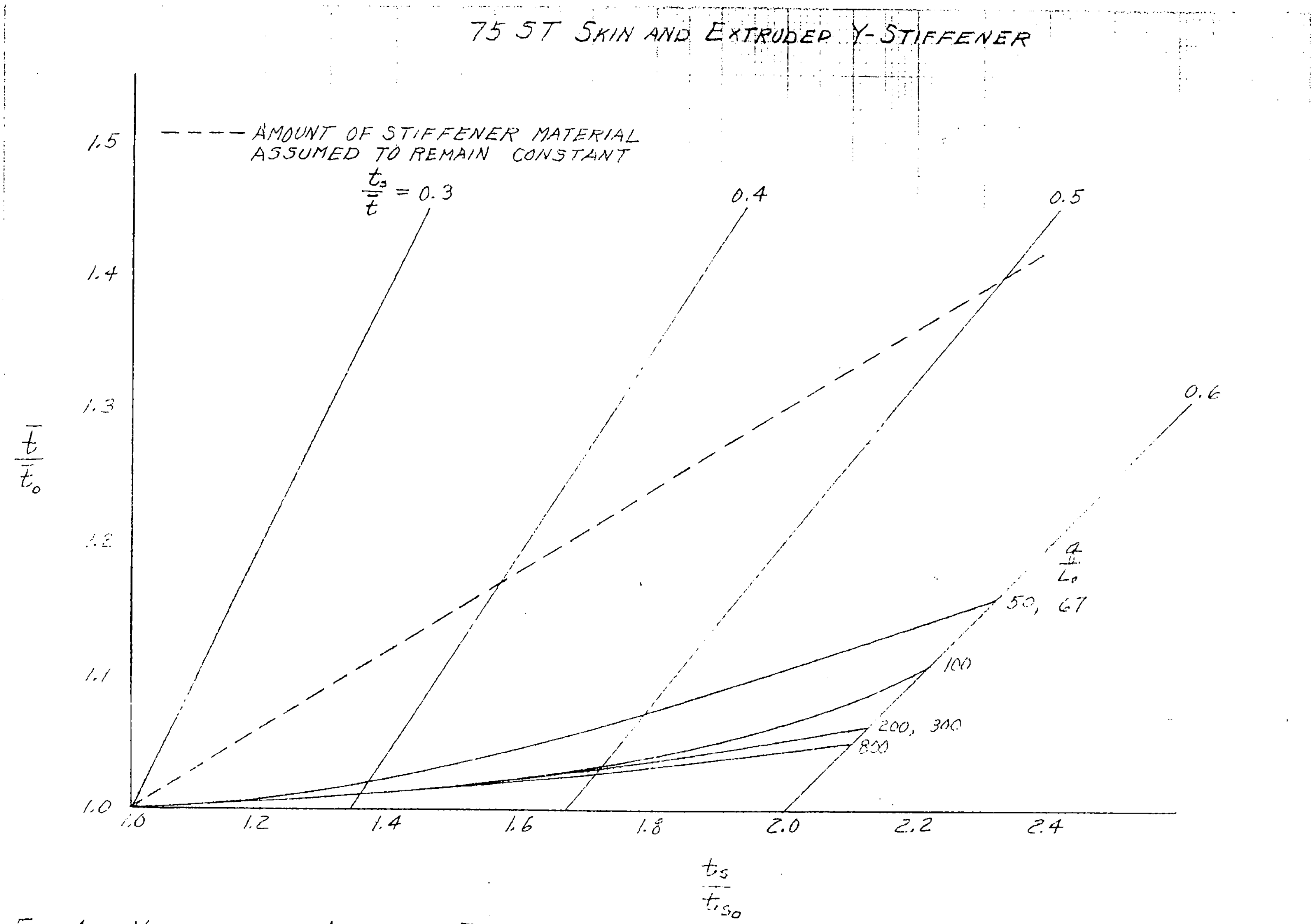

Fig 4. Variation of Average Panel Thickness with Changes in Skin Thickness.