THE INFLUENCE OF CONSTANT DISTURBING TORQUES ON THE MOTION OF GRAVITY-GRADIENT STABILIZED SATELLITES

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The conditions under which a body, subject to gravitational gradient torques, will perform stable oscillations about an equilibrium point are well known.[1,2] However in most practical cases, torques other than those due to the gravity gradient will act on the satellite.[3] The purpose of this note is to demonstrate the effect of a constant disturbing torque upon the transient response of a gravity-gradient stablized body.

It is assumed that the satellite is on a circular orbit. Thus the oblateness and other asymmetries of the earth are neglected. The only torques which act on the body are those due to the gravity gradient, and the constant disturbance.

Euler's rotational equations of motion are

$$\dot{\omega}_x - R_x \omega_y \omega_z = (M_x/I_x)_G + (M_x/I_x)_D \qquad \text{(a)}$$

$$\dot{\omega}_y - R_y \omega_z \omega_x = (M_y/I_y)_G + (M_y/I_y)_D \qquad \text{(b)} \qquad (1)$$

$$\dot{\omega}_z - R_z \omega_x \omega_y = (M_z/I_z)_G + (M_z/I_z)_D \qquad \text{(c)}$$

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where

$$R_x = (I_y - I_z)/I_x$$

$$R_y = (I_z - I_x)/I_y$$

$$R_z = (I_x - I_y)/I_z$$

The subscript G refers to the gradient torque, while D indicates a disturbing torque. From the form of Eqs. (1, a-c) it can be seen that the x y z axes are central principal axes.

The orientation of the body with respect to the local horizontal coordinates is defined by the angles α, β and φ (see Fig. 1). In terms of the orientation angles and their derivatives, the body angular rates are

$$\omega_x = \dot{\varphi} + (\dot{\alpha} + \dot{\theta}) \sin \beta \qquad \text{(a)}$$

$$\omega_y = \dot{\beta} \sin \varphi + (\dot{\alpha} + \dot{\theta}) \cos \beta \cos \varphi \qquad \text{(b)} \qquad (2)$$

$$\omega_z = \dot{\beta} \cos \varphi - (\dot{\alpha} + \dot{\theta}) \cos \beta \sin \varphi \qquad \text{(c)}$$

where $\dot{\theta}$ is the angular rate of the local horizontal axes due to the orbital motion. Finally, the gravitational gradient torques are[4]

$$(M_x/I_x)_G = -3\, \dot{\theta}^2 R_x (\sin\alpha \cos\varphi + \cos\alpha \sin\beta \sin\varphi)(\sin\alpha \sin\varphi - \cos\alpha \sin\beta \cos\varphi) \qquad \text{(a)}$$

$$(M_y/I_y)_G = -3\, \dot{\theta}^2 R_y \cos\alpha \cos\beta (\sin\alpha \cos\varphi + \cos\alpha \sin\beta \sin\varphi) \qquad \text{(b)} \qquad (3)$$

$$(M_z/I_z)_G = -3\, \dot{\theta}^2 R_z \cos\alpha \cos\beta (\sin\alpha \sin\varphi - \cos\alpha \sin\beta \cos\varphi) \qquad \text{(c)}$$

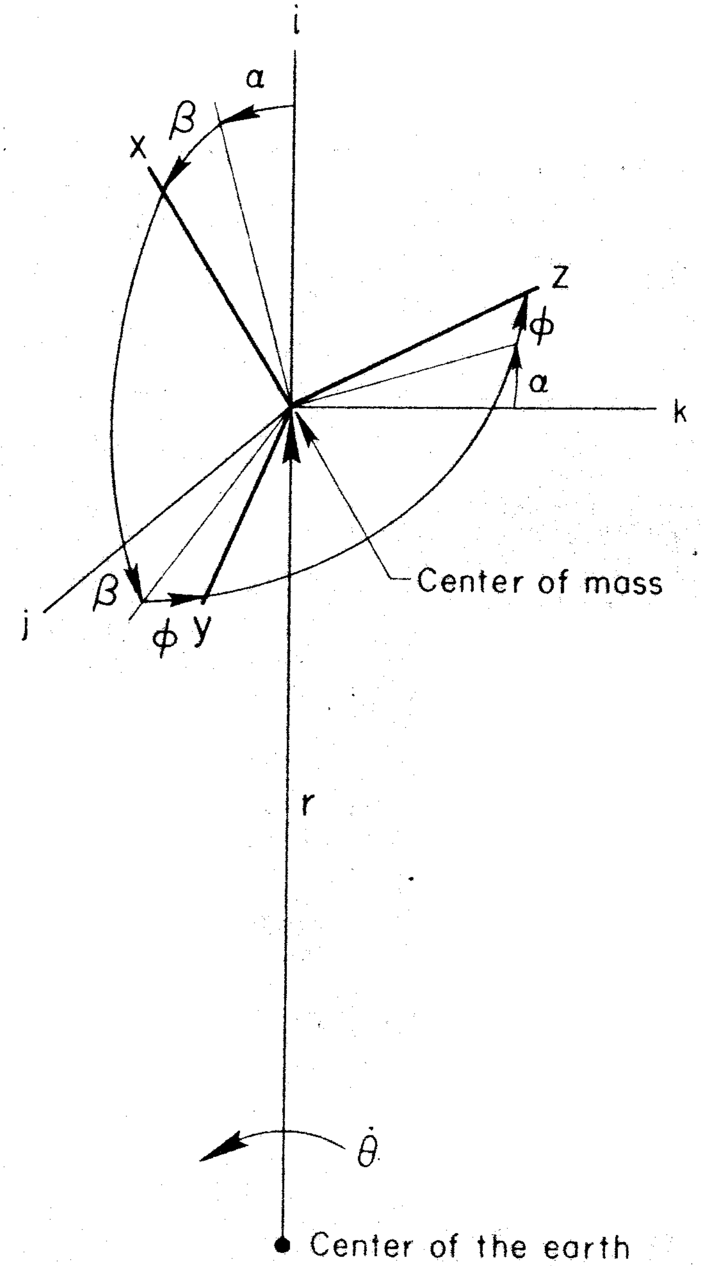

Fig.1 —Orientation of body axes with respect to orbital local horizontal coordinates

Let us consider the case in which there is a steady-state pitch angle. Such a condition might physically arise due to residual drag forces acting in conjunction with a center-of-mass, center-of-pressure, separation. Thus a steady-state value of α develops until the gradient torque is equal in magnitude to the disturbance.

If only small disturbances about the steady-state equilibrium condition are considered, then

$$\alpha = \alpha_{ss} + \delta\alpha \qquad (a)$$

$$\beta = \delta\beta \qquad (b) \qquad (4)$$

$$\varphi = \delta\varphi \qquad (c)$$

Linearizing Eqs. (1-3) about the equilibrium condition, α_{ss}, yields

$$(M_y/I_y)_D = 3\dot{\theta}^2 R_y \cos\alpha_{ss} \sin\alpha_{ss} \qquad (a)$$

$$\delta\dot{\alpha}^{\cdot} + 3\dot{\theta}^2 R_y \delta\alpha (\cos^2\alpha_{ss} - \sin^2\alpha_{ss}) = 0 \qquad (b)$$

$$\delta\dot{\beta}^{\cdot} - \delta\beta (R_z \dot{\theta}^2 + 3R_z \dot{\theta}^2 \cos^2\alpha_{ss}) - \delta\dot{\varphi} (1 + R_z)\dot{\theta} + 3\delta\varphi R_z \dot{\theta}^2 \sin\alpha_{ss} \cos\alpha_{ss} = 0 \qquad (c) \qquad (5)$$

$$\delta\dot{\varphi}^{\cdot} + \delta\varphi (R_x \dot{\theta}^2 + 3R_x \dot{\theta}^2 \sin^2\alpha_{ss}) + \delta\dot{\beta} (1 - R_x)\dot{\theta} - 3\delta\beta R_x \dot{\theta}^2 \sin\alpha_{ss} \cos\alpha_{ss} = 0 \qquad (d)$$

In the absence of a disturbing torque, R_x and R_y are normally positive, and R_z negative for stable behavior.* Thus

*Another conditionally stable configuration is discussed in Ref. 2.

$$I_y \geq I_z > I_x \tag{6}$$

When a disturbing torque is present, the pitch response is essentially unaltered if α_{ss} is small. However, Eqs. (5c) and (5d) now have a $\delta\varphi$ and a $\delta\beta$ term, respectively, if α_{ss} is different from zero. The characteristic equation for the coupled roll-yaw motion is

$$\lambda^4 + \lambda^2\dot{\theta}^2(1 + 3R_x \sin^2\alpha_{ss} - 3R_z \cos^2\alpha_{ss} - R_zR_x)$$
$$-3\lambda\dot{\theta}^3 \sin\alpha_{ss} \cos\alpha_{ss}(R_x + R_z) - 4R_xR_z\dot{\theta}^4 = 0 \tag{7}$$

If α_{ss} is small, the four roots are, approximately,

$$\lambda_1 = \dot{\theta}\sqrt{-a + \frac{1}{2}\sqrt{b}} + \frac{1.5\,\dot{\theta}\,(R_x + R_z)}{\sqrt{b}} \sin\alpha_{ss}$$

$$\lambda_2 = -\dot{\theta}\sqrt{-a + \frac{1}{2}\sqrt{b}} + \frac{1.5\,\dot{\theta}\,(R_x + R_z)}{\sqrt{b}} \sin\alpha_{ss} \qquad \text{(a)}$$

$$\lambda_3 = \dot{\theta}\sqrt{-a - \frac{1}{2}\sqrt{b}} - \frac{1.5\,\dot{\theta}\,(R_x + R_z)}{\sqrt{b}} \sin\alpha_{ss} \qquad (8)$$

$$\lambda_4 = -\dot{\theta}\sqrt{-a - \frac{1}{2}\sqrt{b}} - \frac{1.5\,\dot{\theta}\,(R_x + R_z)}{\sqrt{b}} \sin\alpha_{ss} \qquad \text{(b)}$$

where

$$a = (1/2)(1 - 3R_z - R_zR_x) \quad ; \quad b = (1 - 3R_z - R_zR_x)^2 + 16\, R_zR_x)$$

For the moment-of-inertia distribution given by Eq. (6), λ_1 and λ_2 are complex conjugates with negative real parts, while λ_3 and λ_4 are conjugates with positive real parts.* Thus the roll-yaw motion is unstable. If the sum of R_x and R_z is zero, the real parts of Eqs. (8a) and (8b) are zero, but the pitch restoring torque is also zero! Digital solutions of the nonlinear differential equations have verified this instability.

A disturbing torque which causes a steady-state value of β is also possible. In this case the three perturbation equations are coupled and the characteristic equation is of the sixth degree. For small values of β_{ss} the roots do not have any positive real parts and the solutions are stable. However, the digital solutions of the nonlinear equations diverged for values of β_{ss} greater than about 10^o.

An examination of Eq. (3a) indicates that a constant torque about the x-axis requires that at least two of the three orientation angles have steady-state values. This case has not been examined extensively, but a limited number of digital solutions were stable for small values of $(M_x)_D$.

The fact that a bias angle in pitch leads to instability in the roll-yaw motion indicates the highly non-linear nature of the problem.

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*This assumes α_{ss} to be positive.

A similar roll-yaw behavior is caused by the forced pitch motion due to orbital eccentricity.[5] Thus it is apparent that the principle of superposition is of limited utility in stability analyses of gravity-gradient stabilized satellites.

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