DESIGNING TO PREVENT FATIGUE FAILURES

Lloyd Kaechele

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Lloyd Kaechele*

The RAND Corporation, Santa Monica, California

ABSTRACT

There are three main problem areas in preventing fatigue failures: prediction of the fluctuating stresses that cause fatigue, behavior of the structural material undergoing these stresses, and scatter in stresses encountered in service and in fatigue behavior. This paper concentrates on these major problem areas. The data most important for design, and some techniques for using these data, are presented and discussed. The designer's role in selecting the fatigue problems that should be studied is emphasized.

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INTRODUCTION

The only satisfactory way to solve fatigue problems is to prevent them by proper design. The aim of this paper is to present some thoughts and ideas that bear on solving the main problems in designing to prevent fatigue failure. This discussion will concern more the strategy than the tactics of attacking fatigue problems, without minimizing the tactics--the details of design--that are extremely important. This approach on the strategic level will also aid those associated with designers to understand better what the designer is trying to do and how to help him do it.

Fatigue refers to structural fatigue (to distinguish it from the tired feeling produced by an attempt to read all the literature that appears on the subject). A structure is any arrangement of material whose function is the transmission of forces through space. Other functions may be served as well and may determine the configuration, but if force is transmitted, the material comprises a structure. A golf club, a bed spring, and the hull of the Queen Mary all have their structural aspects and may be subject to fatigue failures.

Fatigue failure is defined more broadly than usual. The sudden snap of your watch spring or the rear axle of your automobile is the standard kind of fatigue failure--the abrupt rupture of a structure. But the appearance of a half-inch-long fatigue crack in a major panel of an airplane wing, demanding an expensive, unplanned repair or replacement, is also a fatigue failure.

The designer has the primary responsibility for prevention of fatigue failures. This means he has a lot bigger job than simply specifying materials and sizes. He must be aware of the many details of the environment and expected use that influence fatigue. He must evaluate the information he gets, decide how trustworthy it is, and sometimes arrange to get better information.

In some very important cases the designer cannot obtain adequate information at the design stage. It is then his responsibility to see that information is obtained as quickly as possible after the designed item is in operation, in order to make sure that no unforeseen factors or faulty assumptions will adversely affect fatigue life. This sums up to a lot of responsibility for the designer. It also implies an obligation on the part of management, and possibly the user of the end item, to aid and cooperate with the designer in obtaining the vital data for antifatigue design.

The Design Engineering profession has in fact performed quite responsibly in providing adequate fatigue prevention, especially in public and private transportation, where death and property destruction are consequences of inadequate design. This is not to deny the existence of any problems--the number of lives lost in the catastrophic fatigue failures of the British Comet jet transport and some fatal fatigue failures in American civil and military aircraft stand to remind us that the problem is always there, requiring constant vigilance on the part of designers.

BACKGROUND

"Fatigue" was the name given to mysterious failures of locomotive axles over a century ago. Axles that were apparently sound would suddenly and catastrophically fail after several years of satisfactory service. Solution of these early problems with railway axles was aided by testing--a technique that still qualifies as one of the most effective means of combating fatigue failures.

Another very important factor in fatigue failures was discovered at this early date, the part played by holes, notches, sharp corners, and other stress raisers, whose neglect amounts to an invitation to fatigue.

Over a hundred years ago, the possible occurrence of fatigue in the first iron bridges, due to the vibrations and repeated passages of trains, was recognized and again tests were made. Another preventive measure, and one that has probably precluded many a fatigue failure since, was used in the bridges: the incorporation of a healthy factor of safety in the design.

Today, one of the fields in which fatigue effects are most important is that of public transportation. Air transportation, especially, with the high penalty placed on excess weight, demands a large investment in design certification and testing in order to insure that attempts to reduce weight do not lead to an aircraft structure susceptible to catastrophic fatigue failure. Fatigue also remains a major factor in the design of machinery and machine tools.

Understanding of the physical process of fatigue has advanced in the past hundred years, but we still do not know exactly what happens in a material as a fatigue crack forms and grows. As a consequence, some very elementary and basic things that designers would like to know about fatigue are still unknown. Many difficult engineering problems, however, were solved long before clear and accurate knowledge about the physical processes involved was available.

One aspect of fatigue that is still obscure is the understanding and measuring of "damage." During much of the fatigue life of a test specimen (or component of a structure or machine), damage consists of changes in the material as a consequence of cyclic plastic deformation, but there is no simple way to define this damage, or to measure it. When load cycles of different magnitude will be encountered in service, the question of how much damage is caused by each cycle and how to add the damage from one cycle to the damage caused by the others becomes very important. This is one of the main problems facing the designer responsible for antifatigue design.

THE THREE MAIN PROBLEM AREAS IN FATIGUE-PREVENTIVE DESIGN

All the various facets of the problem of designing to prevent fatigue failure are intermingled, but the division used here facilitates discussion. The main problem areas are (1) predicting service stresses, (2) evaluating fatigue behavior and cumulative damage, and (3) accounting for scatter in both areas. These will be discussed below.

The Problem of Predicting Service Stresses

Fatigue is a very strongly stress-dependent phenomenon. Many factors are important in fatigue, but the primary physical mechanism, without which there is no fatigue, is plastic deformation. Plastic deformation is produced by stress, making it imperative that the designer know as accurately as possible the nature of the stress variations that will occur during service, i.e., the number and magnitude of stress cycles.

In many cases these data can be obtained by a straightforward process. Similar structures may be in service under conditions similar to those expected for the new structure; then stress records can usually be obtained with reasonable effort. (When a similar structure is already in service, it is often decided that since the old structure was adequate, the new one will be also, and there is no need for a program to obtain stress records. This view is frequently justifiable, but it requires careful scrutiny when human life depends on its validity.)

The more interesting and difficult problems are those involving novel designs and new and unpredictable usage. One of the most important conclusions from our studies of structural-fatigue problems in the Air Force concerns service stress experience when there is this element of novelty--when the nature of cyclic loads is too different to permit certainty in extrapolation from the past and when the variety and nature of the usage that may be demanded are uncertain. In these situations, I feel very strongly that responsible design procedure must include measurement of stress variations experienced in service. The most important reason, of course, is to check the assumptions about stress variation that were made at the design stage, and to either certify the design or make modifications if necessary. Ignoring whatever might be said about hindsight and foresight, the fact remains that in almost all cases where dangerous fatigue failures have occurred, these failures could have been prevented had this action been taken.

There are two other reasons why this action is important. First, when prediction is so difficult, techniques of prediction are a major study area in themselves. It is important to evaluate and refine these techniques for the even more difficult problems that are certain to arise in the future, and records of actual service-stress experience are an absolute necessity for this purpose.

The second reason concerns scatter and will be included in the discussion of that problem. The amount of scatter in the number and magnitude of load or stress cycles among a number of items in service can be very uncertain and very important. Records of service stress are needed before investigation of this important factor can begin.

The Problem of Cumulative Damage

The designer attempts to assure fatigue-free performance by his selection of materials and configurations. He must know how materials behave under fluctuating stresses in order to prescribe the proper amount and configuration of material. The familiar S - N diagram, which presents the relationship between the magnitude of stress cycles and the number of them that produce fatigue failure, shows the basic material behavior of interest. In the simple case of antifatigue design for constant-amplitude cyclic loads, appropriate curves for the material and for the stress-concentration factor or type of joint or discontinuity involved are all that is needed. The designer can establish quite simply the proper design configuration once the magnitude and number of load cycles have been specified as design requirements. There are many additional items that might be called minor details except that neglecting them can result in premature and unexpected fatigue failures. Examples are residual stresses or surface defects produced by manufacturing processes, an unexpected corrosive atmosphere, higher humidity than expected, etc.

Problems are greatly multiplied by complicated, variable, fluctuating load patterns, such as are experienced by an airplane wing in its lifetime. A sample of this kind of loading is shown in Fig. 1, recorded from the wing of a B-47 flying through turbulence.(1) The complexity of the load variations, the uncertainty in predicting what an aircraft will experience, and the variability in the experience of various aircraft make it impossible in any practical sense to generate test data upon which to base a design.

In this situation, the problem of cumulative damage becomes important, and the question arises; What is the proper way to add up the damaging effects of many different kinds of stress cycles? Since we can't even define, let alone measure, damage in the early stages of fatigue, there just is no physically sound way to sum up the incremental damage caused by the various stress cycles in a complex pattern of stress variation. However, several theories have been offered that propose various ways for summing up these damage increments.

The simplest and most widely used theory for determining cumulative damage is known as Miner's theory, sometimes called the linear-damage theory. A paper published forty years ago in Germany by a man named Palmgren is actually the first statement of the rule that Miner offered some twenty years later. The essentials of Miner's theory are also contained in an excellent paper by Langer written several years earlier. One reason why we speak today of Miner's rather than Langer's theory is that Miner presented his work in a form that was more understandable and usable, and it was therefore widely used in aircraft design.

Miner's rule for evaluating cumulative fatigue damage is

$$\Sigma \frac{n}{N} = \text{Fatigue-life fraction}$$

The number of stress cycles of any kind that will be applied to the article is denoted by n, and the number of these stress cycles that has been found by test to produce failure (when applied alone) is denoted by N. The quantity n/N is often called the cycle ratio.

Miner's theory states that when many different types of stress cycles are applied, fatigue failure will occur when the sum of cycle ratios, called the fatigue-life-fraction (FLF), becomes equal to one. At any other time, FLF represents the fraction of fatigue life that has been used up by applying stress cycles. (Using the FLF designation helps to keep it clear that the summation of cycle ratios does not represent a measurement of damage.) When FLF is less than one, additional stress cycles can be applied, the number depending on the severity of the additional cycles.

The one basic concept in Miner's theory is that equal values of cycle ratio result in equal amounts of damage, whatever the kind of stress cycle or stage of life.

There is great activity today in the area of cumulative-fatigue-damage theories, and several new theories have been offered in the past few years.

The big question for the designer is, Has anything better than Miner's theory come forth, and if so, what, and how do I use it? This is a valid question, because aid to the designer is a good measure of the worth of cumulative fatigue damage theories. Viewing a theory in the cold light of design use is also the quickest way to identify the significant differences between one theory and another and also to locate any deficiencies or limitations in a theory as far as design application is concerned.

Sometimes, hypothetical physical models for the fatigue process (which are important in fatigue research), serve to complicate and confuse the presentation of design-oriented cumulative-damage theories. There is an opportunity for designers to make an important contribution here by careful investigation of cumulative-fatigue-damage theories as design tools. The designer has special capabilities which qualify him, more than anyone else, to evaluate cumulative-fatigue-damage theories, and he is understandably one of the parties most interested in the development of theories that lead to simple, useful, and reliable antifatigue design methods.

Some cumulative-damage theories are constructed to incorporate the results of fatigue tests made with variable-amplitude stress cycling, which is certainly a reasonable approach. However, the particulars of the test program supporting such a theory must be carefully noted, and great care must be exercised in applying such a theory in design work unless the design requirements happen to be very similar to the test conditions. If there are significant differences, this kind of theory must be substantiated with tests that are representative of the expected service.

If it is possible to perform a test program that adequately simulates the expected service experience, a pragmatic application of Miner's theory can help to provide useful design data. For example, a test program may disclose that FLF averages 0.83 at failure for a series of variable-stress tests of a specified part. This value can then be used, instead of 1.0, in conjunction with Miner's theory. If the test program discloses that the part as designed does not have an adequate lifetime, design stress levels will have to be reduced (assuming the part is already well designed in terms of stress flow and surface condition). By using the basic S - N data and Miner's theory for determining cumulative damage (with the revision that the summation of cycle ratios equals 0.83 at failure), a new design stress can be found that assures adequate life.

But, too large a change in stress level can invalidate the experimental conclusion that FLF equals 0.83 at failure. How large a change is acceptable and the trend of change in the value of FLF at failure with changes in stress level can only be determined by further testing. This is an extremely important area in which very valuable information can be gathered from a well-planned test program. Unfortunately, most test programs in fatigue research do not provide directly applicable information in this area.

As a concluding remark on this subject, my personal conclusion from studies of cumulative-fatigue-damage theories is that Miner's theory is as good as anything available.(2) Some other theory may do a better job in specific circumstances, and if so it should certainly be used, but with great awareness of the possible pitfalls in straying too far from the specific conditions under which the theory has been shown to be valid.

The Problem of Scatter In Fatigue Life

A large amount of scatter is often observed in the life of fatigue-test specimens that seem to be identical. There may also be much scatter in the stress variations experienced among a group of manufactured items. Three conclusions are presented in this section on the subject of scatter in fatigue:

1. The appropriate way to evaluate the effects of scatter is from the design viewpoint.

2. While there is seldom opportunity for enough tests to firmly establish statistical properties of fatigue scatter, there are some sound procedures with which to proceed toward safe design in the face of scatter.

3. A proposed method which utilizes fatigue-damage calculations helps to overcome the difficulty in handling scatter in service-load experience.

1. Design View of Scatter. Scatter in life is always emphasized when fatigue is discussed because life is the only quantity that can be measured in tests. The designer should not be directly concerned with scatter in life; life should be a prescribed design requirement. The designer's proper concern is the scatter in stress level among test specimens that have the same life. (Unfortunately, there is no way to apply a life of a specified number of cycles and find the stress level that produces failure at that life.)

The design viewpoint of fatigue-scatter problems is best seen in a comparison with similar problems with scatter in static design (where the mechanical property exhibiting scatter may be yield strength or ultimate tensile strength). Such a comparison is shown in Fig. 2 (taken from Ref. 3). First, in the static design, yield stress must not be exceeded in the outermost fibers of a shaft. In the second case, the shaft must survive a specified number of rotating bending cycles. Shafts designed on the basis of average (median) material properties are used as a reference point, but they have only a 50 per cent chance of performing satisfactorily. Figure 2 shows the percentage weight increase that is needed to decrease the probability of failure. This is the natural consequence of scatter in material properties--scatter in yield stress in one case and scatter in the stress that permits a specified number of cycles to be applied without failure in the other case. The two curves are quite similar, which illustrates the fact that fatigue as viewed from the standpoint of design is not so different from other problems of scatter in material properties that the designer must deal with.

Most material strength properties are, as a matter of course, given in terms of minimum guaranteed values. Fatigue data, however, are almost always given in terms of average or median values, largely due to the greater expense of testing enough specimens to establish minimum guaranteed values for the wide variety of conditions that are of interest to the designer. Fatigue scatter will become less a bugaboo as we move toward viewing the problem as scatter in design-stress data rather than in life and start to incorporate minimum guaranteed values into our thinking, as we already have with other mechanical properties.

2. Handling Scatter with Confidence. The special difficulty of making enough fatigue tests to prove out a design makes techniques for assigning confidence levels to results of a smaller number of tests very useful. This section describes some ways for the designer to make use of tools that are available from the fields of probability and statistical theory.

The number of cycles to failure in fatigue tests is commonly assumed to have a log-normal distribution. That is, if test results are plotted as a histogram with equal increments of the logarithm of the number of cycles to failure, the histogram will take the bell shape of the normal distribution if enough tests are made. There are many other possible choices of a probability distribution, and the proper choice can generate much discussion and controversy.

Much of the controversy can be avoided by looking at the problem from the design viewpoint. Different distributions can lead to great differences in estimating life or failure probability. Used in design, different distributions are likely to have a very slight effect on the final cross-sectional area specified by the designer.

Here we shall use the normal distribution (of logarithm of cycles to failure) because it is one of the most familiar, its properties are well known, and tabulated values for this distribution are widely available. The following discussion of confidence and confidence levels is restricted to the normal distribution.

The essential features of confidence levels are depicted in Fig. 3. The axis extending to the right represents number of cycles to failure in a series of fatigue tests. The short vertical lines represent results for each individual test specimen. The number of cycles corresponding to the mean value of the logarithm of cycles to failure is denoted by $\overline{N}$, and the log-normal distribution is assumed to describe the scatter behavior. Confidence-level techniques allow us to determine the number of cycles denoted by N^* in Fig. 3. As indicated in that figure, the value is found from

$$\log N^* = \log \overline{N} - k\hat{s}$$

The symbol $\hat{s}$ in this equation is an estimated value of standard deviation. Both it and $\overline{N}$ are easily computed from test results. Values of k are found from tables.(4,5) (The value of k depends on what value of failure probability, P, is specified, on what value of confidence level, x, is specified, and on how many items were tested. Values of k decrease as more items are tested, as the specified value of failure probability is increased, and as the confidence level is decreased.)

An important statement can be made about the value of N^* found this way: With x per cent confidence, the probability is P or less that any item like those tested will fail in fatigue before application of N* cycles. There is a true value of N* that we cannot know without making far more tests than is possible. The procedure allows us to be fairly confident that our estimate of N* is on the safe side. For example, selecting a 95 per cent confidence level, the probability is 0.95 that the group of specimens we selected by chance to test will give us an estimate of N* which is lower than the true value of N*.

In design for constant-amplitude loading, the k-factor technique leads to an S - N curve or series of curves as illustrated in Fig. 4. These curves are simply the result of applying the k-factor procedure described above to the test data ordinarily used to plot average-value S - N curves. With these curves, the stress level that will assure a specified lifetime with a specified failure probability and confidence level can be determined and used as the proper design stress.

For variable-amplitude loading a problem concerning basic assumptions arises. There is no such thing as an S - N curve for a single specimen (in the way that there is a stress-strain curve for a single specimen). With no possible proof, it must be assumed that an S - N curve corresponding to a specified probability level is appropriate for cumulative-damage calculations at that probability level. For variable-amplitude loading, values of failure probability and confidence level can thus be specified as requirements. The appropriate curve in Fig. 4 can then be used with a cumulative-damage theory to find the design stress level that will provide the required lifetime.

Because of the great expense of testing, the analysis of the results of full- or large-scale variable-amplitude tests for the purpose of certifying a design or establishing the need for design revisions is very important. From the curves in Fig. 4 for various probability levels, it is clear that FLF at failure in these tests can only be equal to unity on the average. In practice there will be some distribution of the value of FLF at failure.

If scatter as measured by standard deviation does not vary with stress level (or if cumulative damage under variable-amplitude loading behaves as though scatter were independent of stress level), the value of the logarithm of FLF at failure may have the normal distribution. The above discussion of the k-factor can then be applied directly to the results of variable-amplitude tests, to determine whether the design provides low enough failure probability for the required lifetime with the required confidence level. There is some evidence for the validity of this approach, but its validity in specific case needs to be established.

In another approach, S - N curves for various probability values (as shown in Fig. 4) provide a cumulative probability distribution of lifetime under a specified load spectrum. The result is illustrated in Fig. 5. The theoretical distribution obtained in this manner can be adjusted to conform with experimental results to some extent. For example, if the experimental average of FLF at failure is not equal to unity, the theoretical probability curve can be shifted by multiplying all values of FLF by the experimental value, as shown by the dashed curve in Fig. 5. (This is equivalent to shifting the probability distribution of the logarithm of FLF to make the experimental and theoretical median values coincide.) The importance of having test data available that reasonably simulate service conditions when making these uncertain excursions into new design areas must again be emphasized.

3. Canonical Damage Rates. The combination of scatter and uncertainty in fatigue behavior and in service-load experience causes difficulty. Simplified handling of this problem is possible through use of a concept I shall call "canonical damage rate". This terminology simply means that a standard method of computing cumulative fatigue damage has been selected. The values computed

in this standard way can be used to evaluate scatter in service-load spectra without the additional complication of scatter in fatigue behavior. Scatter in canonical damage rates can be studied and analyzed to establish design criteria for attaining satisfactory lifetimes with specified failure probabilities.

The general procedure is simply outlined. An appropriate S - N curve is obtained by using the method described above. (Prescribed values of failure probability and confidence level are associated with the selected curve.) Stress-experience records are then established as best they can be. Equipment already in use that is very like the equipment being designed is the ideal source of stress-experience records. In some cases, the best way available for establishing service-stress experience may be a purely analytical approach.

The next step is to select a standard increment of time (100 hours, one year, etc.) and find FLF for each stress record by using Miner's theory with the selected S - N curve.

The succeeding steps are not as clearly defined, since, to my knowledge, scatter in load spectra has not been investigated in this manner. There are several possibilities. The probability-distribution aspects of the canonical damage rate would be of interest in determining the importance of scatter in load spectra. Discovery of a way to apply the k-factor approach to this scatter would establish confidence levels on lifetime that include load-spectra scatter effects. Study of changes in canonical-damage-rate distribution with different S - N curves (reflecting the choice of probability and confidence levels) would also be interesting. Evaluation of joint probability distributions encompassing both fatigue behavior scatter and scatter in load spectra might then be possible.

Finally, the evaluation of scatter in canonical damage rate for a selected probability and confidence-level curve (based on Miner's theory) would provide a rational basis for accounting for scatter in any design based on average load spectra, even if some other cumulative-damage theory or empirical data were the basis for design.

The level of effort required to go this far and the value of doing so depend on the nature of the structure being designed and the conditions of its use. This or a comparable effort should be required for structures for which public safety depends on proper design to prevent fatigue failure.

SUMMARY AND CONCLUSIONS

The thread that runs through all the preceding discussion is the importance of viewing structural fatigue from the design viewpoint. The best solution to fatigue problems is prevention, and this can be done effectively only during the design stage.

Difficulty in antifatigue design can arise from inadequate design data in three major areas, the service-load experience, the fatigue behavior of the material of construction in the design configuration, and finally the scatter and probability considerations needed for both the load experience and fatigue behavior. By investigating the effect of uncertain data on design configurations, the significant data can be identified and efficient ways to obtain the needed data can be determined.

Emphasis on post-design data in the form of stress records is a special responsibility for the designer of structures that represent advances into unknown usage and loading enivornments.

A useful weapon against fatigue is the presence of "fatigue consciousness" in designers. This is an awareness of the possibility of fatigue failures and more important, of the many small details of design, manufacturing, use, and environment that can have such a great effect on fatigue behavior. An experienced designer, acquainted with these areas, is worth more than his weight in texts and reports on fatigue.

There is a procedure that I feel would help to insure freedom from fatigue failure in large new structures in which fatigue is a design consideration and in which failure would be extremely costly in dollars and life. The essential feature would be a Fatigue Advisory Panel, composed mainly of the best designers, those most experienced in antifatigue design, from industries with experience in constructing or fabricating structures similar to the novel structure under consideration. The task of this panel would be a complete review of the design (concepts and details) and fabrication procedures applied to the new structure, for the express purpose of identifying any areas of potential fatigue trouble. Many administrative details would need to be worked out to make this procedure a reality, but the concept of utilizing the experience and perception of the designers best qualified to help prevent fatigue failures in advanced structures deserves consideration. The concept need not be limited to antifatigue design, and its extension to other areas of design would probably prove useful in avoiding other failures, such as that of the submarine Thresher.

The final remark I wish to make is on the role of research in designing to prevent fatigue failures. I have two words of advice for the designer who is contemplating the current flow of literature on fatigue research, "Don't panic". The designer can easily be inundated by statistical analyses, disloacation theories, and stacking-fault energies, but he can keep his head above water by evaluating the outpouring with the yardstick of use in design. Not only is this possible, but it is an opportunity, and in some sense the designer has an obligation to do this, since one of the most valuable inputs into decisions about directions for fatigue research should be the measure of value to the designer.

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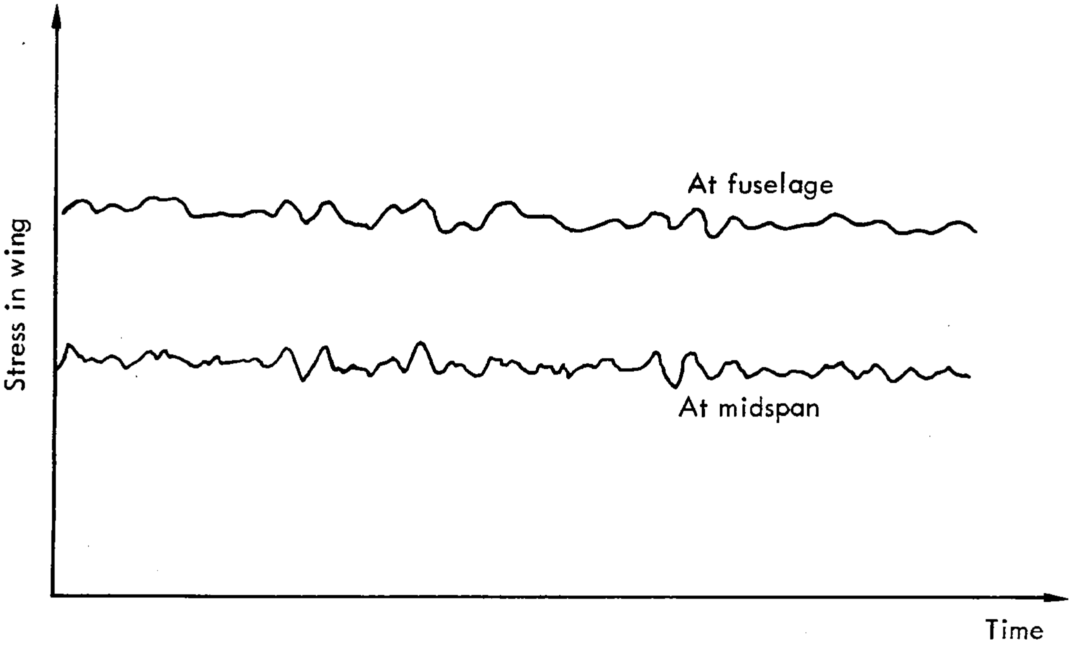


Fig. 1—Stress variation in the wing of a B-47 flying through turbulent air (from Ref. 1)

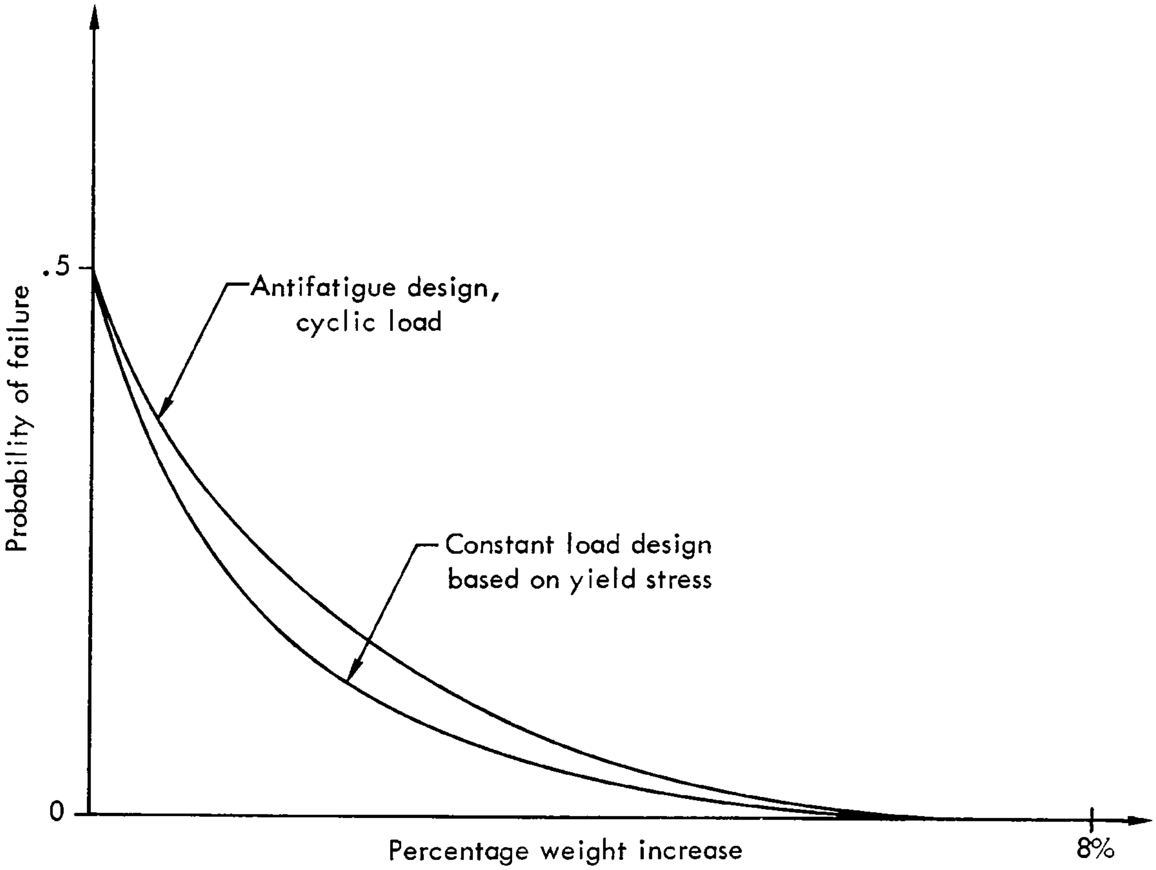
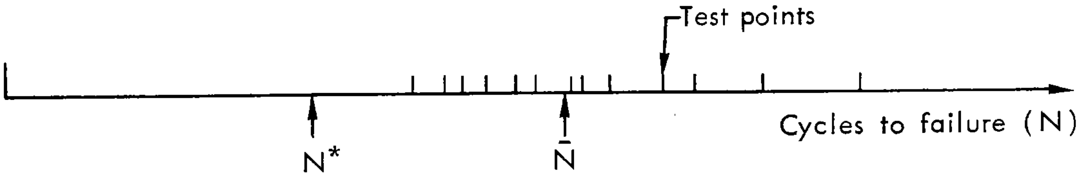


Fig. 2—Comparison of weight increase required to reduce failure probabity (to allow for scatter in material properties)

$$\log N^{*} = \log \bar{N} - k\hat{s}$$

$$\mathcal{P}\,(N<N^{*}) \leq P, \text{ with } X\% \text{ confidence}$$



Example:
With 99% confidence, the probability is less than one chance in a thousand (.001), that an item like those tested will fail in fewer than N* cycles.

Fig. 3—Illustration of confidence level applications in fatigue

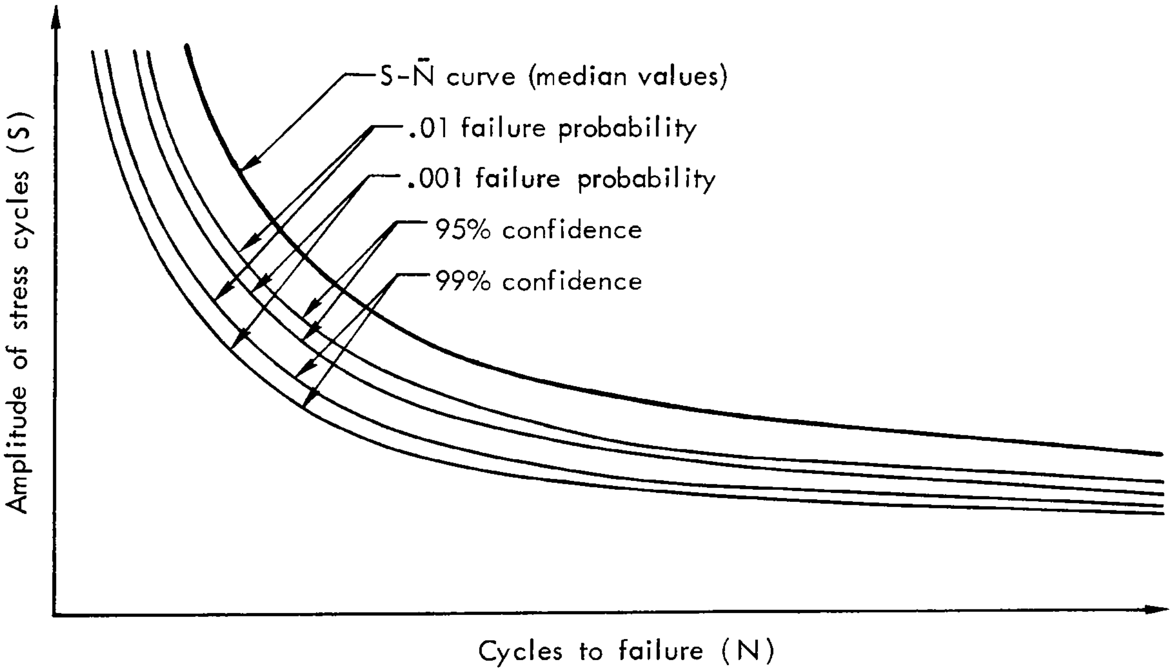


Fig.4—Illustrative S-N-P-C curves--the addition of failure probability and confidence levels to standard fatigue data

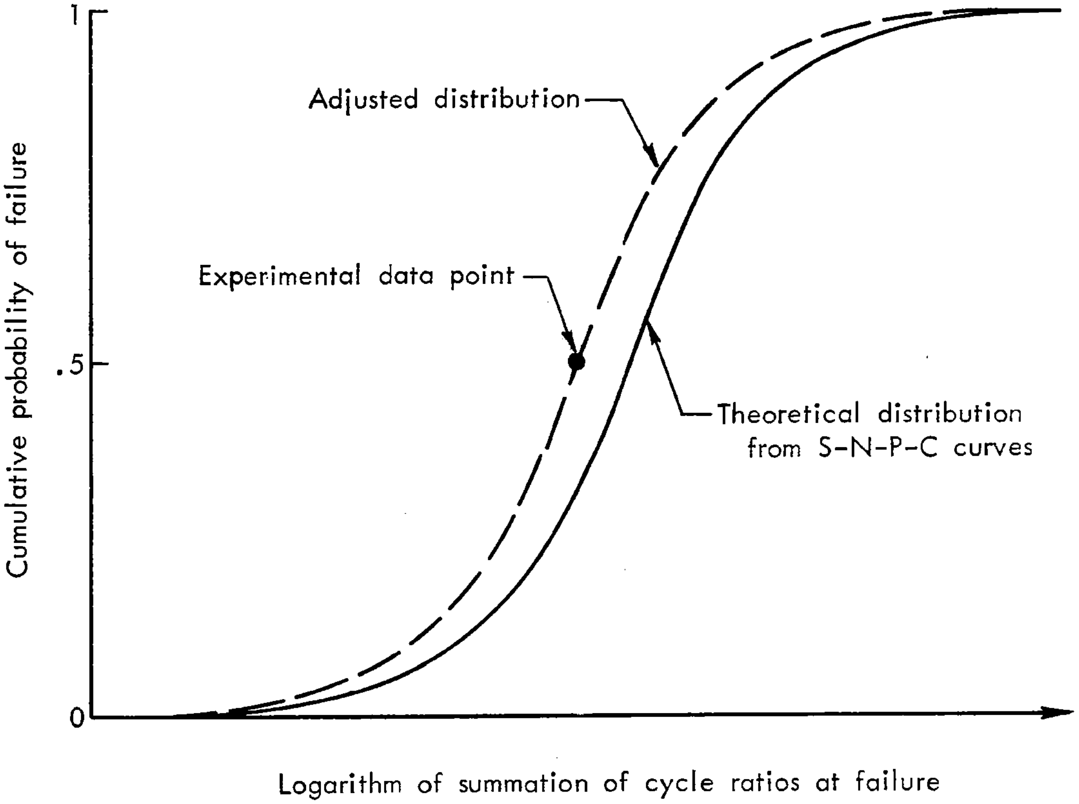


Fig.5—Combination of experiments and cumulative damage calculations with S-N-P-C data to estimate failure probabilities under spectrum loading