MINIMUM ATTRACTIVE RATE OF RETURN FOR PUBLIC INVESTMENTS

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The title of my talk for today is derived from a phrase in Principles of Engineering Economy, the text by Professors Grant and Ireson with which you are, I trust, thoroughly familiar. My talk can be regarded, in fact, as an extended commentary upon portions of the Grant-Ireson text. Let me remark here that the Grant-Ireson book has been of enormous assistance to me, as I hope it will also be for you, in clarifying both thought and practice upon the role of interest -- or, more generally, time-discount -- in investment decision. It goes without saying that I am thoroughly in agreement with the general tone and message of the book: that time must be allowed for in investment decision by the use of interest factors, and that a project is not to be regarded as economically efficient unless it shows its worth in a time-discount calculation as against the relevant alternatives.

If I am so thoroughly in agreement, you may ask, what is there left to say? Actually, I do have certain disagreements with the book -- concerning points which are, in a sense, technicalities and yet could conceivably have practical importance. In addition, when it comes to the actual choice of interest or discount rate, I will go into the matter a bit further than do Grant and Ireson. My talk will be divided into two major parts. In the first, I will consider an artificial world in which risk is absent -- so that the returns on investment are fully known in advance. My objects in examining such an unreasonably simplified case are to clarify the fundamental logic of

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interest and to resolve one or two of the technical disagreements I do have with the Grant-Ireson book. In the second part of my talk, I will examine the problem of investment decision in the real world of risk.

I. Interest in a World of Riskless Investment

Let us turn now to the artificial world in which we imagine that no uncertainty attaches to the consequences of any productive investment or of any financial transaction. We may note, first of all, that in the absence of uncertainty there would be no need for equity financing -- or perhaps we should say, no distinction between equity and debt securities, as both would promise certain specific sums of money at specific dates without any risk of default or any prospect of upward or downward fluctuation of returns. We may therefore think of all interpersonal transfers of funds in this world as being in the form of debts.

The principles underlying the determination of the rate of interest in such a riskless world have been beautifully laid out in Irving Fisher's Theory of Interest, a book that represents one of the capstones of economic reasoning. I cannot here do more than summarize one or two of the central ideas of Fisher's analysis. Fisher regards interest as the premium on current funds in terms of future repayment. What we commonly refer to as "borrowing capital" is really a misnomer; capital cannot be borrowed or lent. What we lend or borrow are current funds in exchange for future funds; in this transaction both the borrower's and the lender's capital wealth remain unchanged, as must necessarily be the case since the discounted value of the repayment equals the value of the funds borrowed. What has happened is that the borrower has shifted part of his wealth nearer in time, and the lender farther off in time; interest is the premium necessary to induce the lender to permit this. In short, one's capital -- the discounted value of one's total earning stream over time -- is unaffected by borrowing and lending. (It would be affected only by the discovery of new productive opportunities or the unexpected loss of old ones, by market revaluations of sources of income, and so forth.) Continuing with Fisher's analysis, the real rate of interest -- after adjustment for anticipated changes in the price level, a technical point I shall not take time to discuss -- is determined by the interaction of the forces of _time-preference_ and _time-productivity_. As the Grant-Ireson text puts it, time-preference makes potential lenders _demand_ interest in exchange for deferring consumption of their wealth, while time-productivity -- the technical fact that it is possible for assets to be so organized that they yield an increase in value over time -- _permits_ borrowers to offer interest.

The upshot of this discussion, and the one crucial point I will ask you to remember, is this: That in a simplified world of certainty and perfect markets, the ruling rate of interest represents an equivalence between _marginal_ time-preference and _marginal_ time-productivity. If the interest rate is 5 percent, _everywhere_ and for _every_ individual, one additional dollar of current funds is worth $1.05 of funds deferred one year -- if that were not the case, it would pay an individual to lend or borrow until the equivalence does hold for him. For example, if the interest rate were 5 percent while to some Mr. X $1 today on the margin was worth $1.10 next year, it would pay Mr. X to borrow at 5 percent -- and continue borrowing until the value to him of another $1 today were worth no more than the $1.05 he must pay in future for it. And, similarly, for _every_ investment, the marginal time-productivity of current funds in terms of funds deferred one year must be just 5 percent -- if, for example, it is 10 percent for a particular increment of investment, then the owner of the investment opportunity would borrow and make the investment, continuing to push along this margin until diminishing returns brings the time-productivity rate on the margin equal to the interest rate.

Now, with this view of the significance of the market interest rate in the background, let us turn to the problem of investment decision. Here we come to my first technical disagreement with Grant and Ireson. On page 76 of the fourth edition, they refer to "three good ways" for comparing the series of disbursements and receipts associated with possible investment opportunities. These ways are: (1) calculate present worth using a stipulated minimum attractive rate of return as an interest rate; (2) calculate annual net benefit -- Grant and Ireson say "annual cost," but "annual net benefit" or "annual net receipt" are better expressions since both disbursements and receipts may be involved -- using a stipulated minimum attractive rate of return as an interest rate; and (3) compare prospective rate of return with a stipulated minimum attractive rate. Of these, I regard the present-worth criterion as fundamental; it directly comports with Fisher's demonstration that the individual should rationally maximize his wealth or capital value -- which is nothing but the present worth of his anticipated income stream. The annual-net-benefit criterion I will not discuss; while posing some algebraic complications, it is mathematically equivalent to the present-worth criterion. This leaves the prospective-rate-of-return criterion -- you really should never use this one, as it is not consistently correct and you cannot always tell in advance whether you will get correct answers with it or not.

Let me illustrate with some algebraic formulas and numerical examples. In the table you will see the general formulas, in which we solve for PW to find present worth and solve for ρ to find prospective rate of return. In the formulas, the s_i represent the annual terms (any of which can, in principle, be positive or negative) of a receipts-disbursements stream. (The s_0 term will usually be negative, representing an initial current outlay.) The interest rate is signified by i. The corresponding rules are to adopt a project if $PW > 0$, for the present-worth criterion -- or if $\rho > i$, for the prospective-rate-of-return criterion. In Example 1 we have an investment with the very nice annual receipts-disbursement stream (-1,2,1). Using an interest rate (or "minimum attractive rate of return") of 10 percent, we see that both rules tell us to adopt the project. So far, so good.

Now let us turn to a more complex case, in which during one of the later years a negative term appears -- this might correspond to closing-down costs for a mine. The example used is (-1,5,-6), still with a 10 percent interest rate. The present worth is negative, and the present-worth rule tells us simply not to adopt the project. But what of the prospective-rate-of-return rule? Two things can be said about it: First, the rule is ambiguous, since it yields two solutions -- 100 percent and 200 percent. But second and more important, both solutions are wrong; both solutions indicate adoption of the investment (ρ_1 and ρ_2 both exceed 10 percent), but the more fundamental present-worth rule tells us to reject. What justifies us in accepting the answer of the present-worth rule as more fundamental? Well, let me just assure you that if you go about adopting projects like Example 2 you will end up in the poorhouse. I do not have time here to explain why the prospective-rate-of-return rule fails in such cases and why the present-worth rule will always work; let me refer you instead to my article, "On the Theory of Optimal Investment Decision," published in the Journal of Political Economy for August 1958 and reprinted in the book Management of Corporate Capital edited by Ezra Solomon.

I will also mention one additional point. Suppose the interest rate is not constant over time -- which in a world of certainty, means that there are known future changes in the rate of interest. There is no easy way of coping with this under the prospective-rate-of-return rule -- just another symptom of the fatal defect of that rule -- but it is simple to generalize the present-worth formula to allow different interest rates between now (time 0) and time 1, time 1 and time 2, etc.:

$$PW = s_0 + \frac{s_1}{1+i_1} + \frac{s_2}{(1+i_1)(1+i_2)} + \ldots + \frac{s_n}{(1+i_1)(1+i_2)\ldots(1+i_n)}$$

This now brings me to a second topic where I have to express disagreement with the Grant-Ireson approach -- namely, "capital rationing." This is a term I dislike intensely. It has caused a great deal of confusion, and I very much regret the shift in emphasis of the 4th edition that places this "capital rationing" idea in a very central role. What this idea is intended to suggest, I believe, is that in a certain decision situation you might have just so much funds at your disposal and, for one reason or another, cannot acquire more. Note that the term itself is a poor one -- it is current funds, not "capital," which is rationed. Secondly, such an arbitrary limitation on current funds is characteristically a short-run difficulty, and I will quote Grant and Ireson against themselves here since they say, and I agree, that investment decisions should normally be made on grounds of long-run economy. Thirdly, the concept of capital rationing is liable to abuse in that planners will often or even typically think that their "ration" of current funds is something that they must spend no matter what. In other words, planners get into the habit of thinking that projects are ratholes down which to dump the rationed funds, when I would prefer them to seek an allocation of funds by a showing of good projects available. (Of course, Grant and Ireson would not by any means endorse such an abuse, but many people who talk of "capital rationing" think in this way.) Finally, the Grant-Ireson solution for the capital rationing situation is technically incorrect, since it depends upon using as "the minimum attractive rate of return" the computed "prospective rate of return" of the marginal project in the set of available alternatives -- but as the prospective rate of return of the marginal project may be ambiguous or may lead to incorrect choices (as in Example 2 in the table), it obviously cannot provide a reliable guide. I will mention here that, if you ever really face a situation of capital rationing, several correct technical solutions are provided in an Appendix to Chapter 7 of the Hirshleifer-DeHaven-Milliman book, Water Supply. Of these, the simplest is to calculate the present worth, and then choose among projects not on the basis of present worth exceeding zero(presumably the rationing means that more projects exceed zero than can be currently financed) but on the basis of highest present worth per dollar of the current (rationed) funds.

Let me conclude this discussion as follows. In a hypothetical world of certainty or absence or risk, the investment decision rule to use is simple. You cannot go wrong if you always judge on the basis of present worth, adopting any project for which present worth is positive while using the market interest rate or rates in the calculation. If you must choose between two projects which are alternatives, the obvious and correct choice is the one with greater present worth. (This is another place where the prospective-rate-of-return rule goes wrong, as indicated in the table.) And, finally, if current funds really are rationed, and if the projects showing positive present worth more than exhaust this rationed amount, the simplest rule is to choose among projects on the basis of highest present worth per dollar of the current rationed funds. But this rationing difficulty ought not arise, since if you have a project of positive present worth you should be able to acquire the necessary current funds to finance it -- certainly so in a riskless world.

II. Investment Decision in a Risky World

Let us now leave the riskless world, and do our best to analyze the problem of investment decision where the consequences of investment are not known with certainty. I will assume, however, that the decision-maker will always be able to conceive of a subjective probability distribution for the possible consequences of investments. Also, I will simplify my problem at first by ignoring the all-too-important role played by tax considerations.

In this world of risk, we now make the distinction between debt and equity financing -- ignoring complex intermediate cases like preferred stock, income bonds, etc. All of these securities represent different ways of partitioning the risk of fluctuations of return among investors. The net rates of interest on bonds and the current yields on stocks will incorporate certain adjustments or allowances for risk.

I would like first to clear up an elementary confusion about "risk," a word which is unfortunately used in two quite different senses. Consider a one-year bond promising a yield of 5 percent. We sometimes think of the riskiness of such an instrument as probability of default, or more generally probability of some unfavorable contingency. Suppose that our subjective probability distribution is such that we believe with 80 percent probability the bond will be redeemed with interest, but with 20 percent probability it will be defaulted as to interest, though principal will certainly be repaid. Then the yield of the nominal 5 percent bond is 4 percent on the average -- we have adjusted for risk by finding the mathematical expectation of return and using that instead of the nominal quoted rate of 5 percent. Obviously, anyone who is at all rational will make some such calculation in his evaluation of investment opportunities -- this is not a matter of taste, but of mathematics.

But you have probably heard that, while some people display risk-aversion, others may have risk-preference. You would be quite mistaken, however, if you thought that having risk-preference means preferring a 5 percent bond with some risk of default to a 5 percent bond with no risk of default -- that would not be risk-preference, but plain foolishness. The term risk-preference refers, rather, to taste as to variability of return, as in the following situation. Suppose in comparing two equity securities you consider that, on the average, they will both yield you 10 percent -- but security A may with equal probability yield 9 percent or 11 percent while security B may fluctuate between 0 percent and 20 percent. To prefer "risk" in this sense -- to choose the return with greater variability -- would not be necessarily irrational, but might be regarded as an expression of a valid personal preference. When I refer to "risk" I will try to be careful to distinguish between risk in the sense of probability of an unfavorable contingency and risk in the neutral sense of variability of return.

This distinction has considerable importance for the following reason. As you know, the law of large numbers tends to eliminate variability -- if I toss a fair coin a million times I can be very sure that the overall proportion of heads will be close to one-half. Some people have argued, therefore, that since the government undertakes many projects it can as a practical matter ignore "risk." Whether this statement has any validity depends upon which conception of risk is meant. If the government has many projects, if the projects are independent, and if each has a mathematical expectation of yielding 10 percent, the government can be reasonably sure that the overall return will be close to 10 percent -- the law of large numbers will truly tend to eliminate variability risk. But suppose a government planner is considering a project that will yield 10 percent only if every possible contingency turns out favorably -- passing over the possibility of unfavorable developments on the ground that the government can ignore risk. This would of course be an error; the calculated "if-all-goes-well" return is a kind of nominal or optimistic yield which ought to be adjusted to a mathematical expectation of yield by taking into account the probabilities of the unfavorable developments. You might think the argument for ignoring this kind of expected-value "risk" is a very crude fallacy, but it would not be hard to find examples in both theoretical and practical analyses.

How does the securities market adjust for risk? This is a very big question which has not yet been successfully analyzed. I will assume here, and ask you to accept the proposition that the market does not equate the mathematical expectations of yields everywhere but, rather, displays risk-aversion. The main evidence for this is the fact that equity values seem to be consistently discounted relative to equity earnings more heavily than bond values are discounted relative to bond interest -- in other words, equities on the average yield more than bonds. (I am thinking of earnings rather than dividends as the yield on equity.)

To examine first the problem of corporate investment decision, let us imagine a corporation with a financial structure consisting only of debt and common stock, with a management faithfully serving the interests of stockholders, and -- for the moment -- let us assume no corporate profits tax. Then the management's goal at any moment of time should be to maximize the wealth of the existing stockholders, which implies the use of a present-worth criterion in evaluating investments. But what interest rate should be used? Suppose that our firm has equal values of bonds and stocks outstanding, that the market quotes the bonds to provide a net yield of 4 percent while the stock has an earnings yield of 6 percent (with expectations of a level trend of earnings). If the proposed investment leaves the risk picture in terms of overall variability of return substantially unchanged, the question is whether to use 4 percent, 5 percent, or 6 percent as the discount rate for evaluation purposes. In this case I think it is pretty clear that 5 percent -- the average of the debt and equity yields required by the market for firms of this risk-type -- is the correct rate to use.

The most common error made would be to argue that, since bonds can be floated at 4 percent, the project could be incrementally financed on a pure debt basis, and then would yield a profit on the average if calculations showed positive present worth at 4 percent. The mistake here is failing to appreciate that the variability of return of the new investment would then be loaded entirely on the existing equity. Since the existing equity requires a 6 percent return (because of the previously existing variability), and since the new investment by hypothesis does not change the variability picture, the equity yield of the new investment must also be 6 percent if stock values are not to fall. With the debt yield of 4 percent balanced equally against the 6 percent required equity yield, the indicated rate of discount is 5 percent. In general, the weighted average of the stock and bond yields in the existing capitalization should be used as the discount rate, for investments which leave the firm in the same risk-class as before. The 4 percent rate would be appropriate only if the new investment were itself riskless.

The corporate income tax affects decisions in rather complex ways which I cannot fully go into here. For a good investment the situation must be that, after adoption of the proposed new investment and the requisite debt-equity financing arrangements, the firm's existing stockholders must be at least as well off. Again, if the new investment does not change the risk picture, we may assume that the pre-existing weighted average yield on current funds is still appropriate for judging new investments. In the example discussed, corporate income tax could be allowed for by discounting <u>after-tax</u> yields at

5 percent (assuming the same debt-equity ratio after adoption of the new investment as before). Discounting after-tax yield would be the practical way of allowing for the various complications of depreciation formulas and other peculiarities of the tax laws. In simple cases, it can be shown that a 5 percent discount on after-tax yield is equivalent in the case discussed to using an 8 1/4 percent discount rate on pre-tax yield.

Finally, with this in mind we may turn to the problem of public investment decision. We will suppose that public investments are to be evaluated from the point of view of economic efficiency in competition with other uses of scarce resources by our society; considerations other than economic efficiency may also be relevant, but they are not part of this analysis. The problem here may be expressed as follows. On the one hand, the rate on riskless investments seems to be around 4 percent. On the other hand, the interacting tax and risk effects work out so as to penalize risky investments: in the example cited, in which I purposely used figures roughly representative of private utility financing, investments had to yield on the average 8 1/4 percent pre-tax in the private sphere in order to be adopted. Now, if the government has a riskless project I would not question that, say, 4 percent is the appropriate discount rate. Even in the private sphere, an investment which was perfectly riskless could be financed on an all-debt basis without increasing variability risk on the equity, and so would be justified if a positive present worth were found at the 4 percent discount rate. But suppose that the project under consideration by a public agency is, in terms of variability risk, more like those adopted in the private sphere only if they show positive present worth at 8 1/4 percent. Should the government use 4 percent anyway, or 8 1/4 percent, or something in between?

One argument for the 4 percent rate is the risk-pooling idea. Since the government will engage in many projects, it can assume that variability will practically disappear in the overall totals, so that a 4 percent rate will be justified. As a partial counter-argument, it might be that the project risks are highly correlated so that the law of large numbers may not fully apply; for example, cheap sea-water conversion is a contingency that would unfavorably affect a whole spectrum of conventional dam-aqueduct projects. But a more fundamental counter-argument goes as follows. If we can imagine government projects as pooled, we can also imagine all private projects as pooled. On this basis, we might be practically certain that private projects like those in our example will yield 8 1/4 percent overall to society. Consequently, we should if anything subsidize private investment (or, better, eliminate the tax on private investment) rather than adopt less efficient government projects. Even failing this, we should recognize that the taxes required to finance a 4 percent government project come at least in part from resources that would otherwise be used in private projects yielding 8 1/4 percent.

My own view on this risk-aversion matter would be something of a compromise. That is, I might be inclined to use in the public sphere something like, say, a 6 percent discount rate for projects whose riskiness would require 8 1/4 percent in the private sphere. I am not prepared to defend this number, really. It is based upon making some partial allowance for the risk-pooling effect and for the fact that not all the funds required for government projects would displace private investment projects; some would come from consumption, and this would counteract the anti-investment effect of the corporate profits tax.

However, there is one very important factor that has been left entirely out of account so far, and that is bias. The market acts as an independent judge of the success of corporate managements in correctly evaluating benefits and costs, so that no systematic bias is evident in the yields expected by private investors. On the other hand, it is very well established that government evaluations of prospective projects are systematically over-optimistic, for reasons I will not go into here. Use of a high interest rate would be a way of correcting for this bias. It is not an ideal way, but a rough practical rule. Unfortunately, this argument does not justify any particular number. My own instinct tells me that, for projects comparable to those engaged in by private utilities, the range 8 percent-10 percent would certainly not be too high, overall.

I wish to emphasize that I have used specific numbers only to illustrate general principles that I believe should be applied. With this proviso, let me sum up by saying that if there is no bias and no variability risk, the riskless rate of discount -- say, 4 percent -- should be used for public projects. For variability risks comparable to those of projects engaged in by private utilities, but assuming no optimistic bias, I would be inclined to compromise between the riskless 4 percent and the private-sector 8 percent or 9 percent by using some figure like 6 percent. Allowing for bias throws everything open again; to correct for this factor I would use something like 8 percent-10 percent as my discount rate for this same type of utility investment.

INVESTMENT DECISION FORMULAS AND RULES

	PRESENT WORTH	PROSPECTIVE RATE OF RETURN
	Interest Rate Constant over Time	
FORMULA	$PW = s_0 + \frac{s_1}{1+i} + \frac{s_2}{(1+i)^2} + \ldots + \frac{s_n}{(1+i)^n}$	$0 = s_0 + \frac{s_1}{1+\rho} + \frac{s_2}{(1+\rho)^2} + \ldots + \frac{s_n}{(1+\rho)^n}$
RULE	Adopt project if PW > 0.	Adopt project if $\rho > i$.
Example 1 (1-1,2,1) i=10%	$PW = -1 + \frac{2}{1.1} + \frac{1}{(1.1)^2}$ $= 1.65$ Since 1.65 > 0, adopt project.	$0 = -1 + \frac{2}{1+\rho} + \frac{1}{(1+\rho)^2}$ $\rho = 141.4\%$ Since 141.4% > 10%, adopt project.
Example 2 (-1,5,-6) i=10%	$PW = -1 + \frac{5}{1.1} - \frac{6}{(1.1)^2}$ $= -1.41$ Since -1.41 < 0, reject project.	$0 = -1 + \frac{5}{1+\rho} - \frac{6}{(1+\rho)^2}$ $\rho_1 = 100\%$, $\rho_2 = 200\%$ Adopt project (?)
	Interest Rate Changing over Time	
FORMULA	$PW = s_0 + \frac{s_1}{1+i_1} + \frac{s_2}{(1+i_2)(1+i_1)} + \ldots$ $\ldots + \frac{s_n}{(1+i_n)\ldots(1+i_2)(1+i_1)}$	
RULE	Adopt project if PW > 0.	
	Mutually Exclusive Projects	
RULE	If $PW_1 > PW_2$, Adopt Project 1.	If $\rho_1 > \rho_2$, Adopt Project 1.
Example: Project 1: (-1,2,1) Project 2: (-1,0,4) i=10%	$PW_1 = -1 + \frac{2}{1.1} + \frac{1}{1.21} = 1.65$ $PW_2 = -1 + \frac{0}{1.1} + \frac{4}{1.21} = 2.31$ Since $PW_2 > PW_1$, adopt Project 2.	$0 = -1 + \frac{2}{1+\rho_1} + \frac{1}{(1+\rho_1)^2}$, $\rho_1 = 141.4\%$ $0 = -1 + \frac{0}{1+\rho_2} + \frac{4}{(1+\rho_2)^2}$, $\rho_2 = 100\%$ Since $\rho_1 > \rho_2$, adopt project 1.