A METHODOLOGY FOR EVALUATING HOUSING PROGRAMS

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1. INTRODUCTION

This paper is intended to provide a methodology for evaluating housing programs primarily intended to benefit the participants by providing them with adequate housing at below-market rentals. In other words, we are concerned with programs like Public Housing, which attempts to provide low-cost housing for low-income families, or New York City's Mitchell-Lama program, which attempts to provide moderately priced housing for middle-income households. Our methodology is not intended for programs like Urban Renewal, which is chiefly concerned with slum elimination and redevelopment to enhance the viability of central cities.[1] However, to the extent that a project built on Urban Renewal land has the goal of providing adequate housing for its tenants at subsidized rentals, then our methodology would apply to that project. Of course, for this kind of housing program, which is essentially a transfer in kind to its participants, there may nevertheless be spillover benefits, such as neighborhood upgrading and so forth. Clearly, it would be desirable to count these "nontenant" benefits (which may, of course, accrue to program participants as well as to nonparticipants) in evaluating the program.

Benefits, unfortunately, do not usually result unless resources are expended. We must, therefore, examine the resource cost of a housing program as well as the benefits. When these magnitudes -- benefits and costs -- are known, we can evaluate the program. With

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[1] A methodology for evaluating Urban Renewal has been developed and applied. See Rothenberg [32] and Messner [19].

respect to economic efficiency, we want to know whether benefits exceed costs. With respect to the transfer goals of the program, we want to know how benefits and costs are distributed by characteristics of the target population, such as income and family size.

Before we can hope to measure benefits and costs, we must have precise understanding of what these terms mean in the context of housing programs. To gain this understanding, a model of consumer choice in the rental housing market is developed in Sec. 2. This model is then used to characterize housing programs in Sec. 3. In Sec. 4 we formulate the following benefit and cost concepts. (1) Net tenant benefits: the amount of additional income the program participant needs to be as well off without the program as with it (a consumer's surplus concept). (2) Gross tenant benefits: the previous amount plus the amount he actually pays for the program unit (called project rent). (3) Total benefits: nontenant benefits plus gross tenant benefits. (4) Tenant subsidy: the difference between market rent and project rent of the program unit. (5) Total resource cost: the sum of project rent (the tenant contribution) and nontenant contributions.

In Sec. 5 estimable forms for all these concepts are developed except for nontenant benefits. However, we show that the minimum amount of nontenant benefits necessary to justify the program on benefit-cost grounds is estimable. Under certain assumptions discussed later, all these quantities can be estimated by knowing only a few variables for each participating household and its program-provided dwelling unit. These variables are (1) the actual rent of the program unit (project rent), (2) the income of the tenant, (3) the market rent of the program unit (i.e., what it would rent for on the private market), and (4) the tenant's rent-income ratio in the absence of the program (i.e., that fraction of his income he would spend on housing were he not a participant in the program). Of these, the first two could be directly observed for on-going programs, but they would have to be estimated for proposed programs. The last two would require some form of estimation, examples of which are discussed later.

Several studies, both methodological and empirical, have dealt with benefits and/or costs of housing programs. Of those that attempted to estimate direct tenant benefits or that have suggested how this might be done, most have used as such an estimate the difference between the market and project rent of the program dwelling unit, what we call the tenant subsidy.[2] This is conceptually a serious mistake, for the tenant subsidy typically overstates net tenant benefits. At best, it may be a good estimate. At worst, it may imply that gross tenant benefits equal resource cost, in which case the program would be justified on benefit-cost grounds by definition, as will be shown later. Moreover, ignoring the distinction between these concepts obscures the fact that a subsidy in kind, like a housing program, typically requires for its justification a certain level of nontenant benefits. These issues are taken up in detail later.[3]

It should be stated from the outset that the analysis presented here is in terms of the individual household and dwelling. However, in application, the individual benefits and costs would undoubtedly be added to obtain program aggregates. Friedman [10] and many economic principles textbooks warn us against the fallacy of composition. Relationships that hold at the level of the individual household may not be true of the aggregate of households. A general equilibrium approach to the development of a methodology for evaluating housing programs would probably not be subject to this problem. However, we have found this approach too difficult to make operational and have fallen back on the partial equilibrium analysis presented here. In an attempt

[2] See, for example, Ross [31] and Bish [3].

[3] Economists have long been aware, in general, of the basic distinction made here between the tenant subsidy and net tenant benefits. It is the basis for the famous argument over the welfare effects of an income tax and an excise tax. See Friedman [10]. However, this awareness has not always carried over into analysis of housing programs as indicated above. Exceptions are Smolensky [34] and Prescott [30]. The point about nontenant benefits being required to justify in-kind transfers follows from recent discussions of the necessity of transfers for efficient resource allocation. See Buchanan [6], Hochman and Rogers [14], Musgrave [20], Olsen [25] [26] [27] [28], and Pauly [29]. Smolensky [32] also appears to recognize the point in the context of housing programs.

to mitigate the possible difficulties of our approach, we assume that full competitive equilibrium prevails in all markets and that subsidized housing units have no effect on relative marginal costs of housing and nonhousing goods.

2. CONSUMER CHOICE IN THE RENTAL HOUSING MARKET

General Formulation

Consider the consumer who wishes to obtain the most satisfaction from a fixed income. He is confronted with known prices of all the commodities he may wish to purchase, and he may not typically affect these prices by buying more or less of any commodity. He also must choose how much he will spend on housing and what type of housing to rent. In the choice of housing, however, unlike in the choice of the quantities of other goods, the consumer knows that the rent he must pay will depend on the type of housing on which he decides. For example, he knows that a five-room apartment, other things equal, will rent for more than a four-room apartment. To some extent, then, the consumer can affect how much he spends on housing by his choice of housing type. He cannot typically affect the way in which rent varies with the housing type, however. In this respect housing is like all other goods. Given these conditions, the consumer decides how much to spend on housing and how much on all other goods. He also decides how much of each good to purchase and what type of housing to rent.

The preceding verbal description of consumer choice in the rental housing market may be given more precisely in mathematical form. Assume the consumer to possess an ordinal preference function defined over all commodities and over all housing characteristics that have relevance to his preferences.[4] The goods might be such things as bread, movies,

[4] The idea of characteristics of a commodity as arguments of a utility function was first used some years ago by Houthakker [15] and Theil [35] and more recently, by Lancaster [16]. Several housing studies have used utility functions with one or more housing characteristics as arguments. See, for example, Alonso [2], Harris et al. [11], Muth [21], and Smith [33]. Also, the idea has been used in some empirical work, for example, Adams et al. [1], Brigham [5], Crecine et al. [7], and Harris et al. [11].

clothing, etc., but not housing. The housing characteristics might be such things as number of rooms, condition, location, etc. His utility function may be represented as follows:

(1) $$u = u(x_1, \ldots, x_M, c_1, \ldots, c_N),$$

where x_m ($m = 1, \ldots, M$) are the goods and c_n ($n = 1, \ldots, N$) are the characteristics of housing.

It is assumed that the consumer confronts the following income constraint:

(2) $$y = \sum_m p_m x_m + r\,(c_1, \ldots, c_N)$$

where

y = income

p_m = price of the m^{th} commodity ($m = 1, \ldots, M$)

x_m = quantity of m^{th} commodity ($m = 1, \ldots, M$)

c_n = quantity of n^{th} housing characteristic ($n = 1, \ldots, N$)

$r(c_1, \ldots, c_N)$ = the rent function which gives the rent the consumer must pay for any combination of housing characteristics.

The rent function is the same for all consumers, and the form and parameters of it are fixed from the viewpoint of the individual consumer.

The decision problem facing the consumer is to choose a set of x's and c's so as to obtain the most satisfaction from his income. Mathematically: maximize (1) subject to (2). An equivalent problem is to maximize the following expression, where λ is an undetermined Lagrangean multiplier,

(3) $$L = u(x_1, \ldots, x_M, c_1, \ldots, c_N) + \lambda[y - \sum_m p_m x_m - r\,(c_1, \ldots, c_N)]$$

which gives the following first-order conditions:

(4) $$\frac{\partial u}{\partial x_m} - \lambda p_m = 0 \qquad m = 1, \ldots, M$$

(5) $$\frac{\partial u}{\partial c_n} - \lambda \frac{\partial r}{\partial c_n} = 0 \qquad n = 1, \ldots, N$$

(6) $$y = \sum_m p_m x_m + r(c_1, \ldots, c_N).$$

These M + N + 1 equations are consistent with an equilibrium in the M x's, N c's, and λ which are the variables. In addition, the solution to the constrained maximum implies not only an optimal choice of x's and c's but also an optimal distribution of expenditure between goods and housing.[5]

These conditions for consumer equilibrium may be expressed somewhat differently to provide a more meaningful interpretation. To eliminate the λ's, solve one of the first set of M equations, say the M^{th}, for λ. Using these solved values of λ, eliminate λ from all the equations, getting the following set of first-order conditions:

$$\frac{\partial u}{\partial x_m} \Big/ \frac{\partial u}{\partial x_M} = \frac{p_m}{p_M} \qquad m = 1, \ldots, M - 1 \tag{7}$$

$$\frac{\partial u}{\partial c_n} \Big/ \frac{\partial u}{\partial c_N} = \frac{\partial r}{\partial c_n} \Big/ \frac{\partial r}{\partial c_N} \qquad n = 1, \ldots, N - 1 \tag{8}$$

$$\frac{\partial u}{\partial x_M} \Big/ \frac{\partial u}{\partial c_N} = p_M \Big/ \frac{\partial r}{\partial c_N} \tag{9}$$

$$y = \sum_m p_m x_m + r(c_1, \ldots, c_N). \tag{10}$$

The first set of conditions (7) says that in equilibrium the consumer equates the marginal rate of substitution between any two goods to the ratio of their prices. In the two-good, two-characteristic case, this may be illustrated by Fig. 1.

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[5] We assume the maximum is attained for positive quantities of the x's and c's and that the second-order conditions are fulfilled.

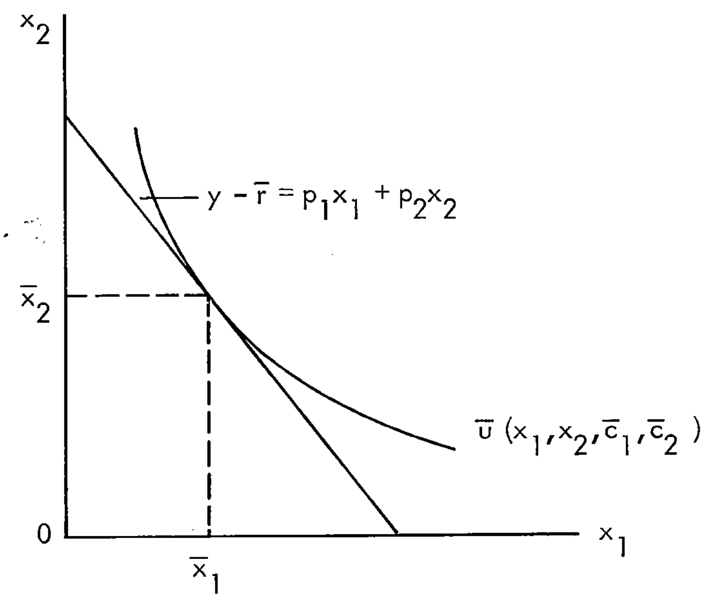

Figure 1

In the figure $\bar{u}(x_1, x_2, \bar{c}_1, \bar{c}_2)$ is the highest indifference curve that the consumer may reach with his given income, given prices, and optimal rent expenditure $\bar{r}$. The values of the x's that maximize his utility subject to his budget constraint are $\bar{x}_1$ and $\bar{x}_2$ ($\bar{c}_1$ and $\bar{c}_2$ represent the optimal quantities of the two housing characteristics).

An analogous interpretation of Eq. (8) may be given for the case where there are only two housing characteristics and two goods. In Fig. 2 the highest indifference curve that the consumer can attain is $\bar{u}(c_1, c_2, \bar{x}_1, \bar{x}_2)$. He achieves it by finding an apartment characterized by $\bar{c}_1$ and $\bar{c}_2$ with a rent of $\bar{r}$; $\bar{r}$ is also the amount of income remaining to the consumer after optimal expenditure on other goods: $\bar{r} = y - p_1\bar{x}_1 - p_2\bar{x}_2$. The iso-rent line is drawn concave to the origin, but this is not necessary. For example, if each characteristic is thought of as a good available at fixed prices, then the iso-rent line would be straight. On the other hand, if each characteristic has a rising supply price to the consumer, then the iso-rent line would be concave as drawn in Fig. 2. There are, of course, other alternatives. The iso-rent line must, however, be less convex to the origin than the indifference curve for a utility maximum to exist. Notice that the partial derivatives of the rent function serve as implicit prices of the housing characteristics, with the slope of the iso-rent line being given by the ratio of these "prices." The third first-order condition, Eq. (9), brings the first two sets together by saying that the marginal rate of substitution between the M^{th} good and the N^{th} characteristic must be equal to the ratio of their respective "prices."

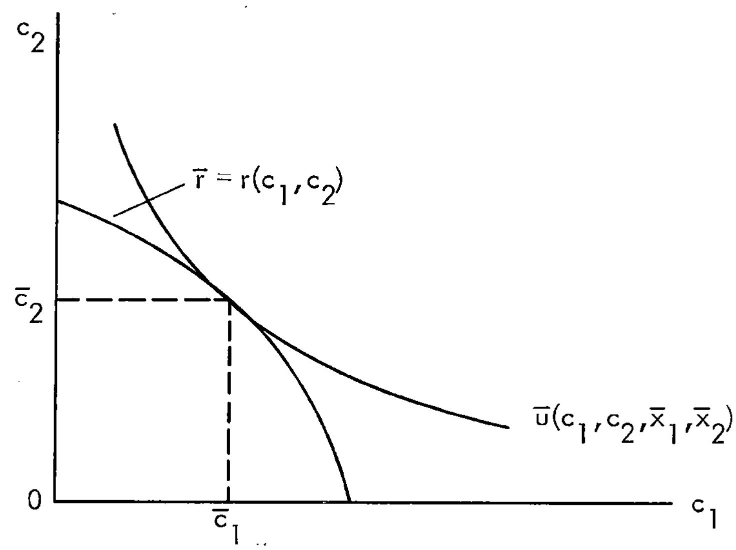

Figure 2

Specific Formulation

An alternative formulation of the model, which will be found useful in analysis of housing programs, may be presented. Treat all the x's as a Hicksian composite commodity,[6] x, whose price is p_x. That is, assume that x represents expenditures on all goods in terms of some base point set of prices and that prices change proportionately. Given these conditions, all the x's can be treated as if they were one

[6] See Hicks [13, pp. 33-34 and 312-13].

good and $p_x x$ will represent total current expenditures on all these goods; the price p_x may be thought of as the number of current dollars given up for one base-point dollar. The treatment of all nonhousing goods as a composite commodity, despite the assumptions involved, is not very restrictive from the point of view of the subsequent analysis, for price changes of these goods do not enter the analysis at all. The composite-good assumption merely serves as a convenience.

The alternative formulation of the consumer choice model treats the c's as a generalized composite commodity.[7] Call the generalized composite housing commodity h and its price p_h. Then, current rent expenditure, r, is $p_h h$. This does not require the assumption that the "prices" of the individual housing characteristics change proportionately but does require specific assumptions about the supply behavior of the c's. In particular, suppose the prices of the housing characteristics are functions of the characteristics themselves as follows:

$$(11) \qquad p_n = a_n s_n(c_1, \ldots, c_N), \qquad n = 1, \ldots, N$$

where

p_n = "price" of the n^{th} characteristic

a_n = "shift" parameter

$s_n(\ )$ = a given function of housing characteristics.

Then, if a_n are restricted to proportional changes only, i. e.,

$a_n/a_n^0 = p_h$, $n = 1, \ldots, N$, where a_n^0 is the value of a_n at some base point, and since a_n^0 can be set equal to one without loss of generality, the assumption of proportional changes can be written as

$$(12) \qquad p_n = p_h s_n(c_1, \ldots, c_N). \qquad n = 1, \ldots, N$$

Then the total current outlay on housing or rent, r, is

$$(13) \qquad r = \sum_{n=1}^{N} p_n c_n = p_h \sum_{n=1}^{N} c_n s_n \ (c_1, \ldots, c_N) = p_h h,$$

where the generalized composite housing good is defined as

$$h = \sum_{n=1}^{N} c_n s_n \ (c_1, \ldots, c_N).$$

[7] See Liviatan [18].

Housing, then, is to be treated as a generalized composite commodity. The reason for choosing this formulation over that of the Hicksian form is that it is less restrictive. It permits rent to vary as a result of the consumer's activity, rather than requiring housing characteristics to be obtained at constant unit cost and their prices to change in the same proportion. The importance of the generalized composite good theorem is that, even for a consumer whose choice of characteristics affects his rent by affecting the supply price of the characteristics, it is still possible to perform a perfect aggregation over characteristics, under the restrictions spelled out above. Of course, the use of composite goods permits two-dimensional analysis which will prove useful later in depicting relationships.

Then treating housing as a generalized composite commodity, denoted h, and all other goods as a Hicksian composite good x, whose prices are respectively p_h and p_x, the analysis of consumer choice may be restated as follows.

(14) $$\text{Maximize} \quad u = u(x, h)$$

subject to

(15) $$y = p_x x + p_h h.$$

The first-order conditions for this problem are

(16) $$\frac{\partial u}{\partial x} \Big/ \frac{\partial u}{\partial h} = \frac{p_x}{p_h}$$

(17) $$y = p_x x + p_h h.$$

These may be represented graphically as follows:

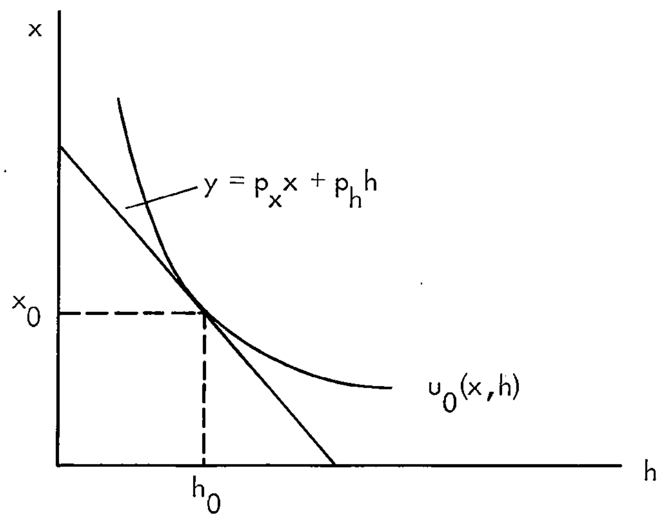

Figure 3

Here the consumer chooses to buy x_0 units of nonhousing goods and h_0 units of housing, thereby attaining the most satisfaction possible, u_0, within the limits of his income. The quantities x_0 and h_0 may be thought of as expenditures on goods and housing, respectively, in terms of some base-point set of prices and housing supply conditions. Then, x_0 and h_0 would have the dimensions base-point dollars, while p_x and p_h would have the dimensions current dollars per base-point dollar. Thus, $p_x x_0$ and $p_h h_0$ would be current expenditures on goods and current rent of housing.

This model of consumer choice will be used to characterize housing programs in the next section and to determine measures of benefits and cost in Sec. 4.

3. HOUSING PROGRAMS AND CONSUMER CHOICE

Introduction

The essential feature of most housing programs, from the point of view of the participant, is that they present him with the opportunity of obtaining housing at a lower rental outlay than if he were to purchase the same housing on the open market. Housing programs use a variety of subsidies to achieve this purpose. The question of interest here is how one might characterize these programs in terms of the theory of consumer choice developed above.

First distinguish two cases.

Type I: the participant is permitted free choice in the selection of housing, but he pays less-than-market rental on what he chooses. (He may, of course, pay nothing.)

Type II: the participant is presented with an all-or-none choice. To participate in the housing program at all, he must live in a housing unit specified by someone else, and pay less-than-market rent.

Type-I Housing Program

Utilizing the model of consumer choice presented earlier, the effects of a Type-I housing program on the participant may be represented as in Fig. 4. The consumer is confronted with the budget constraint given by $y = p_x x + p_h h$ (shown as L_1 in Fig. 4), where y is his fixed income, p_x is the fixed price of the composite commodity x, p_h is the fixed price of the composite good h. Under free market conditions, the consumer maximizes his utility by buying x_1 units of the composite commodity and h_1 of housing, which entails a rent expenditure of $r_1 = p_h h_1$.

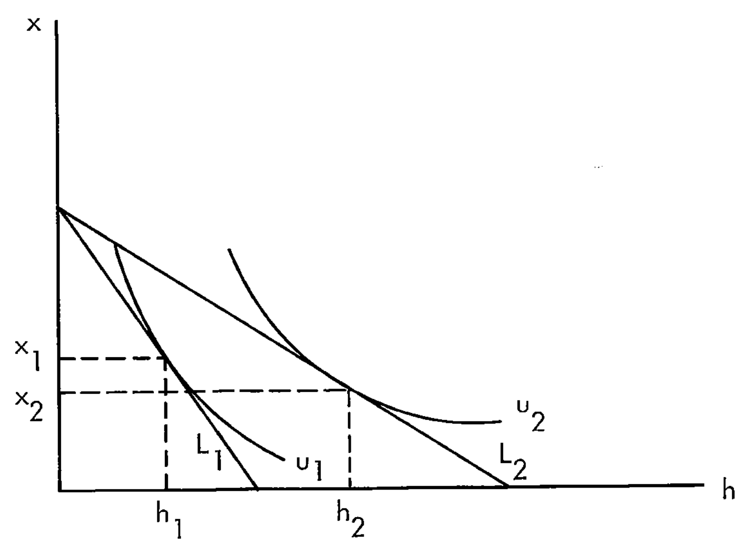

Figure 4

Suppose now that the consumer is permitted to participate in a housing program which allows him to obtain housing at less than market rental. In particular, suppose that a housing unit for which the market rent is $r = p_h h$ costs participants in the housing program αr, where $0 < \alpha < 1$. This implies a new budget constraint, $y = p_x x + \alpha p_h h$, and a new optimum consumption of the composite good and housing, indicated by x_2 and h_2 in the figure. The budget line (shown as L_2 in Fig. 4) shifts out to the right since α is less than unity. The consumer is thereby permitted to reach a higher indifference curve, indicated by u_2 in the figure. The quantity h_2 is the number of dollars' worth of housing (in terms of some base point) that the consumer obtains. On the private market he would have to pay $p_h h_2$ in current dollars, but, because of the housing program, he pays only $\alpha p_h h_2$. Concretely, if the consumer is given a 10 percent subsidy so that he pays only 90 percent of market rent for his housing, his optimum choice of housing might rent for $100 on the market but he pays only $90.

Type-II Housing Program

In certain housing programs the participant is not given the option of determining the type of housing he wishes. Rather, he is given an all-or-none choice. He either accepts a particular apartment (at the subsidized rental) or does not participate in the program at all.[8] Fig. 5 is used to illustrate this type of program.

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[8] The consumer may be given more choice than this. For example, in Federal Public Housing, one is allowed to reject the first two apartments offered before dropping to the bottom of the waiting list. Of course, these apartments are supposed to be equivalent but in fact are different because of age, location, etc.

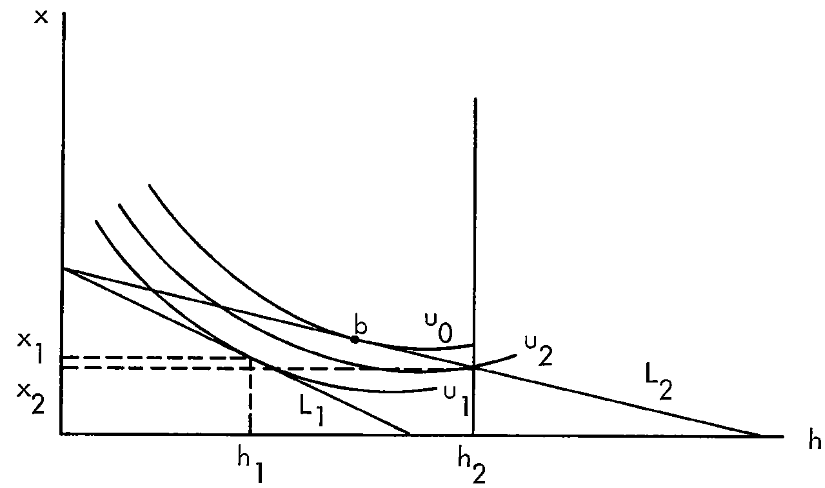

Figure 5

The vertical line at h_2 represents the fixed amount of housing the tenant must purchase under the housing program. He will pay $\alpha p_h h_2$ for it, where, as before, $0 < \alpha < 1$. If he were to buy it on the competitive housing market, the consumer would have to pay $p_h h_2$. His pre-program position is indicated by (x_1, h_1) in Fig. 5 which has a budget constraint L_1 and a utility level u_1 associated with it. Faced with a fixed income, a fixed price for the composite commodity, and the requirement that $\alpha p_h h_2$ be paid for h_2, the amount of x is determined at x_2. The tenant's "choice" is, therefore (h_2, x_2) in Fig. 5 and is associated with a utility level u_2. Notice that if the participant had been given freedom of choice along budget line L_2, he would have chosen point b and achieved a higher level of satisfaction u_0 by trading off some housing for some of the composite good x.

It is, of course, possible that h_2 be chosen such that the participant would be at point b on the indifference curve u_0. The Type-II program would correspond to the Type-I program in that h_2 is the level of housing he would have chosen anyway. It might also be noted that h_2 could be chosen low enough so that the consumer would prefer to trade off some of the composite good for more housing. Fig. 5 illustrates what is probably the more realistic case, i.e., where the participant is required to purchase more housing than he would prefer.

4. BENEFITS AND COSTS OF HOUSING PROGRAMS

Tenant Benefits

How might one assess the direct tenant benefits of housing programs? One way is to find the dollar amount that will make the consumer as well off without the program as with it. There are several ways to do this, all classed under the rubric "consumer's surplus."[9] Consumer's surplus as used here is the additional money that would have to be given to the consumer to make him as well off as under the program, given that he must pay market prices for the goods he buys and market rental for the housing he consumes. The meaning of this statement may become clearer in the context of the model of consumer choice developed above.

Referring to Fig. 6, assume that an individual who is not a participant in a housing program has a given income and faces given prices of all goods and housing.

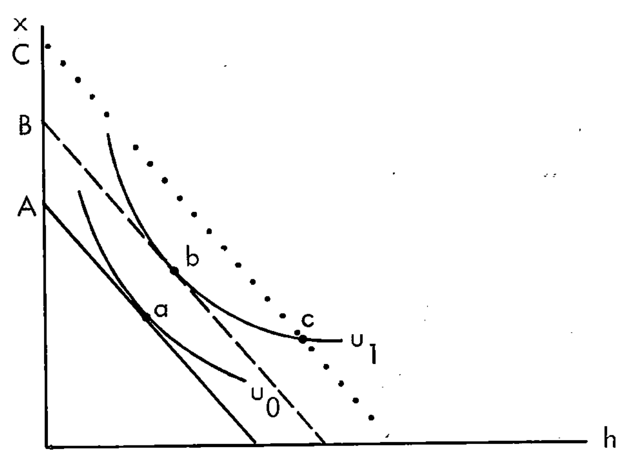

Figure 6

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[9] A good discussion of consumer's surplus may be found in Blaug [4, pp. 335-345].

The solid line in Fig. 6 represents the level of his income and relative prices of all goods and housing. The individual would achieve the most satisfaction, under these circumstances, by consuming quantities of the composite nonhousing good x and the generalized composite housing good h indicated by the point a on the indifference curve u_0.

Now, assume that the individual becomes a participant in a housing program and that the quantity of housing and all other goods enjoyed under the program are indicated by the point c on the indifference curve u_1.[10] Of course, he now pays less-than-market rental for his dwelling unit and enjoys a higher level of satisfaction under the program than without it. If he were to obtain the same level of utility as u_1 but be required to pay market rental for his housing, he would need an income represented by the dashed line in Fig. 6 which is tangent to the u_1 indifference curve at the point b. This line is drawn parallel to the initial budget constraint, indicated by the solid line, to reflect the fact that the prices of x and h are the same in both situations. If the income associated with point b is represented by y and the individual's actual income by y_0, then the net tenant benefits of the program are

$$B_t^n = y - y_0. \tag{18}$$

This is the amount of money which, if added to his income, would lead the consumer to attain a level of satisfaction u_1 equal to that attained under the housing program. This amount can be represented in terms of the nonhousing good by the distance AB in Fig. 6.[11]

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[10] From here on no distinction will be made between the Type-I and Type-II housing programs. All results will hold for either type.

[11] Some readers will recognize this consumer's surplus measure as the "price equivalent variation." There are at least four other consumer's surplus measures that could have been used. These may be classified as the Hicksian measures (which are in terms of indifference curves) and the Marshallian measure (which is in terms of the demand curve). In principle the Hicksian measures are preferred to the Marshallian in that the former do not require the assumption of a constant marginal utility of income. These measures do differ among themselves and from the Marshallian measure; in particular, the price equivalent variation tends toward the high end of the spectrum. See Hicks [12]; also Blaug [4].

We define gross tenant benefits to be the sum of net tenant benefits and the project rent, R_p, paid by the tenant, i.e.,

$$B_t^g = B_t^n + R_p. \tag{19}$$

This is the dollar value of the housing program for the participant whose alternative is housing on the private market.

Thus, conceptual measures of net tenant benefits and gross tenant benefits have been derived from the model of consumer choice presented earlier. It may be noted that B_t^n is not measurable from observable data since the income necessary to achieve the level of satisfaction u_1 is not directly observable. So, in the general case presented above, net tenant benefits cannot be measured. However, a method of estimating B_t^n from observable data will be presented in the next section.

Cash Grants and Housing Programs

Referring again to Fig. 6, let the quantities associated with point c be x^c units of the nonhousing good and h^c units of housing. The market value of this combination is given by $p_x x^c + p_h h^c$, which is represented in the figure by the dotted line passing through the point c. Now since the consumer participates in a housing program, he does not pay this amount for the combination (x^c, h^c); rather, he pays $p_x x^c + \alpha\, p_h h^c$. The difference between these two amounts is also the difference between the market rent, R_m, and the project rent, R_p, of the subsidized dwelling unit, as follows:

$$(p_x x^c + p_h h^c) - (p_x x^c + \alpha\, p_h h^c) = p_h h^c - \alpha\, p_h h^c = R_m - R_p = S. \tag{20}$$

This is obtained by noting that $p_h h^c$ is merely the market rent of h^c units of housing and $\alpha\, p_h h^c$ is merely the project rent of h^c units of housing. We call this difference the tenant subsidy. In terms of the nonhousing composite good, S is equal to the distance AC in Fig. 6.

It is interesting to investigate the relative magnitudes of the net tenant benefits and the tenant subsidy. Refer again to Fig. 6. Note that the dotted line lies above the dashed line through point b.

Or, in terms of the nonhousing composite commodity, the distance AC is greater than the distance AB. This implies the tenant's valuation of the program unit is less than the market's valuation. Moreover, if market prices equal average costs, it would be cheaper to give the consumer a cash transfer equal to net tenant benefits rather than an in-kind transfer equal to the tenant subsidy. Thus, to justify an in-kind transfer, other benefits at least equal to the difference between the tenant subsidy and net tenant benefits must exist.

Nontenant Benefits

In general, then, it is more appealing from the point of view of the recipient and may be cheaper for society to give cash grants for purchase of housing and other goods at currently prevailing market prices. Why, then, does society choose to provide subsidized housing in one form or another rather than a cash grant? One reason is that those who provide the benefits are not solely concerned with the satisfactions of the recipients but with their own satisfactions as well. It raises the utility of the provider to see the provided better off, where "better off" means the recipient lives in "standard" as opposed to "nonstandard" housing. The recipient of a cash grant might not choose to live in "standard" housing. The housing program is, therefore, a way of imposing "our" will on the participant. Another reason for housing programs is that they are believed to have ramifications on the community well-being beyond those experienced by the participants. These are typically called secondary benefits or spillover effects and encompass such things as elimination of blight and slums, mitigation of poverty, revival of downtown business areas of the central cities, achievement and/or maintenance of an adequate middle-income household component in the central city, etc. Some attempts have been made to incorporate these kinds of nontenant benefits into an economic evaluation of housing programs.[12] However, the conceptually satisfactory schemes are, at this early stage of their development,

[12] See, for examples relevant to housing, Nourse [23], Adams et al. [1], Rothenberg [32], and Olsen [27].

practically impossible to apply. At the other extreme are ad hoc methods tailored to specific problems. Our analysis provides no conceptual measurement of these benefits.

Costs

With respect to costs, we only wish to observe here that the resource cost, C, of providing subsidized housing can be divided into two parts: (1) that part which is contributed by the program participant himself, which we have called project rent, R_p, and (2) that part contributed by nontenants of program units, which we denote C_{nt}. The identity

$$C = R_p + C_{nt} \tag{21}$$

is an alternative to regarding costs as the sum of component expenditures such as fuel, utilities, payroll, taxes, and so forth. Eq. (21) highlights the size of subsidy involved in providing subsidized housing. Moreover, as will be shown later, under certain circumstances the nontenant contribution and resource cost may be estimated with the same data used to estimate tenant benefits.

5. ESTIMATING BENEFITS AND COSTS

Estimating Tenant Benefits

One way of estimating net tenant benefits is to estimate the tenant subsidy instead. This has been done before in the evaluation of housing programs,[13] but usually it was not observed, perhaps because it was not realized, that the tenant subsidy and net tenant benefits were not the same thing. In fact, it was shown above that the subsidy will typically be larger than the benefit. Nevertheless, it may be that they are quite close, in which case the tenant subsidy would be a good estimate of net tenant benefits. Usually, of course, one does not know whether these two measures will be close, and treating them as if they are begs the question.

[13] See, for example, Bish [3].

For these reasons, it is desirable to have a measure of net tenant benefits which is not identically equal to the tenant subsidy. Such a measure was developed above; it is the amount of additional income the tenant would need in order to be as well off not participating in the program as participating in the program.

However, this measure of net tenant benefits is unobservable and must, therefore, be estimated. In order to obtain an estimate of net tenant benefits and consequently gross tenant benefits, we must assume a particular form of the utility function. Let us assume that an individual's utility function is

$$u = h^{\beta} x^{1-\beta} \tag{22}$$

and that his budget constraint is

$$y = p_x x + \alpha p_h h, \tag{23}$$

where the variables are defined as before and β, the consumer's rent-income ratio, is a parameter. The assumed utility function implies that the consumer will spend 100β percent of his income on housing and the remaining amount on all other goods (all other goods being represented here by a composite commodity). The following demand relations emerge from the maximization of the utility function subject to the budget constraint:

$$h = \frac{\beta y}{\alpha p_h} \tag{24}$$

$$x = \frac{(1-\beta) y}{p_x} . \tag{25}$$

Now suppose the consumer participates in a housing program (i.e., $\alpha < 1$) and obtains h_1 units of housing and x_1 units of other goods with an income of y_0. He may obtain these under a Type-I or Type-II housing program; either way the analysis is not affected. Now, the utility level associated with these amounts is, of course,

$$u_1 = h_1^{\beta} x_1^{1-\beta}. \tag{26}$$

How much income is necessary for the consumer to obtain u_1 when required to pay market rental? This is found by substituting the demand equations (with $\alpha = 1$) into the utility function, setting $u = u_1$, and solving for y. The resulting expression is

$$y = \left[\frac{y_0 - p_x x_1}{\alpha\beta}\right]^{\beta} \left[\frac{y_0 - \alpha p_h h_1}{1-\beta}\right]^{1-\beta}. \tag{27}$$

This may be simplified in a manner that makes the expression more amenable to estimation. Note the following

$$\alpha = R_p / R_m \tag{28}$$

$$y_0 - p_x x_1 = R_p \tag{29}$$

$$\alpha p_h h_1 = R_p \tag{30}$$

so that

$$y = \left(\frac{R_m}{\beta}\right)^{\beta} \left(\frac{y_0 - R_p}{1-\beta}\right)^{1-\beta}. \tag{31}$$

It can also be shown that this expression is invariant to any positive monotonic transformation of the utility function.[14]

Then, net tenant benefits are $B_t^n = y - y_0$, where y is given by Eq. (31). Gross tenant benefits are, of course, $B_t^g = B_t^n + R_p$.

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[14] Eq. (31) may also be found by minimizing $p_x x + p_h h$ subject to $u_1 = h_1^{\beta} x_1^{1-\beta}$. The author is indebted to E. O. Olsen for this point.

The assumed utility function implies certain things about consumer behavior. Specifically, it implies unitary price and income elasticities for housing and nonhousing. That is, for a given income, expenditures on housing and all other goods remain unchanged as prices change. Also, for given prices, expenditures on both goods increase and decrease in proportion to increases and decreases in income. It is really the restrictions on housing demand and expenditures that are important here, for the nonhousing good is meant to represent all nonhousing consumption. Thus, if it were true that the demand for housing had unitary price and income elasticities, then it would necessarily be true that the same conditions held for the composite nonhousing good. Thus, evidence in favor of unitary price and income elasticities for housing would be evidence in favor of the particular utility function used above (or any other utility function that had the same implications).

There are housing demand and household expenditure studies which may be used to get evidence on this issue. The results of these studies differ depending, for example, on the data used, the estimation technique, the income definition, and so on. However, several of these studies have produced findings consistent with the assumed utility function, i.e., unitary price and income elasticities.[15] Others have not.[16] However, on balance the evidence favors unitary price and income elasticities.[17]

The assumed utility function recommends itself for two reasons. It is consistent with some empirical evidence on the demand for housing and family expenditures, and it is simple enough to permit the kind of manipulation necessary to obtain a measure of tenant benefits.

In order to obtain an estimate of gross or net tenant benefits, observations on or estimates of the following variables are required: (1) the market rent of the unit, (2) the project rent of the unit,

[15] See, for example, Muth [22].

[16] See, for example, Lee [17].

[17] See deLeeuw [8].

(3) the income of the tenant, and (4) the parameter β . For an ongoing program, the tenant's income and his project rent are observable, but market rent and β are not and must be estimated. For a proposed program or project, we would also need estimates of (2) and (3).

Market rent is the amount the program unit would rent for on the private market. Since the unit is not offered on the private market, its market rent is not observable. There are several ways one could go about obtaining an estimate of market rent. Perhaps the easiest way is simply to ask someone with intimate knowledge of the housing market to estimate the market rentals of program units.[18] Another approach would be to estimate a rent function (i.e., rent as a function of housing characteristics) and to use it to estimate market rents of program units.[19] There are undoubtedly other approaches as well.

The other unobservable magnitude is the parameter β of the utility function. As was noted above, this parameter represents the proportion of income spent on housing in the absence of the program. The currently observed ratio cannot be used because the tenant will typically be spending differently on housing than he normally would (i.e., than he would as a nonparticipant). A couple of ways suggest themselves for estimating this parameter. One would be to find the average rent-income ratio of people in the private rental housing market and to use that as an estimate of β. A better way would be to estimate a relation between the rent-income ratio and family characteristics such as income, family size, etc. Then, this relation could be used to estimate the β for a particular tenant.[20]

Several ways of obtaining the unobservable magnitudes necessary for estimating benefits of housing programs have been briefly discussed. Which of these, or other, methods are used will depend in part on the nature of the analysis and on the nature of available data.

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[18] DeSalvo [9] has used this method in his analysis of New York City's Mitchell-Lama program.

[19] Olsen [24] has used this approach in his analysis of New York City rent control.

[20] DeSalvo [9] and Olsen [24] have used this approach.

A method for estimating net and gross tenant benefits has been presented. If an estimate of nontenant benefits were also available, then the benefit side of the evaluation of a housing program would be complete. However, until theoretical approaches more amenable to empirical use are developed, one must fall back on ad hoc approaches to measuring nontenant benefits of housing programs. Of course, if one has estimated total resource cost and gross tenant benefits, then the minimum amount of nontenant benefits necessary to justify the program may be obtained.

Estimating Costs

Resource cost has been defined as the value of resources necessary to provide the flow of services from a housing unit. The direct estimation of the resource cost of a housing program would require extensive cost data on development expenses and subsequent operating expenses. However, under certain circumstances, a simpler method of obtaining an estimate of resource cost may be used.

Economic theory tells us that in the long run under competitive conditions in a market, the market price of a unit of a commodity will equal the resource cost (including normal profit) of providing that unit. Thus, for competitive housing markets, one should expect the market rent of a housing unit to approximate the resource cost of that unit.

It is tempting, therefore, to contend that the market rent of a unit provided by a housing program is equal to the resource cost of that unit. However, this is not true unless an additional condition is met. This condition is that the housing program must provide housing as efficiently as does the private housing market. If the housing is provided less efficiently than that provided by the private market, then the resource cost of a unit is greater than the market rent of that unit.

The condition of efficiency is presumed to be fulfilled in competitive markets. However, since publicly assisted housing is not forced by competition to operate efficiently, one cannot make the same presumption. It has been estimated that the inefficiency associated with one housing program is not insignificant.[21] Nevertheless, if one

[21] See Olsen [27] and Smolensky [34].

could argue that a housing program provided housing as efficiently as the private market, then the market rent could be used as an estimate of the resource cost. Moreover, even when it cannot be convincingly argued that the program is efficient, using market rent as an estimate of resource cost would provide a lower bound to the true resource cost of program-provided housing and could be used as a check on any alternative cost estimating procedure.

Estimating Nontenant Contributions

This same line of reasoning may be extended to provide an estimate of all nontenant contributions where it is impossible or inconvenient directly to measure these contributions. If we assume that the program provides housing as efficiently as the private competitive housing market and we have an estimate of the market rent of program housing, then the tenant subsidy is identical to nontenant contributions.

To see this, recall that resource cost is the sum of project rent (the tenant's contribution) and nontenant contributions, or

$$C = R_p + C_{nt}. \tag{32}$$

But, resource cost equals market rent because of the assumption that the program is as efficient as the private market, so.

$$R_m = R_p + C_{nt}. \tag{33}$$

Therefore, nontenant contributions may be measured by the difference between market rent and project rent, i.e.,

$$C_{nt} = R_m - R_p, \tag{34}$$

but this is a quantity we have already defined and discussed as the tenant subsidy. It is the amount the tenant would have to pay in excess of his project rent in order to occupy a unit identical to his program unit on the private market. This makes sense because if the program were efficient, this is exactly the amount which must be contributed by others in order to permit the participant to pay only R_p.

Now it is easy to see why we are concerned if the tenant subsidy is used to estimate net tenant benefits. Under the assumptions that the housing program is as efficient as private housing and that market rent equals resource cost, gross tenant benefits would exactly equal resource cost, if the tenant subsidy were used to estimate net tenant benefits. That is, gross tenant benefits and resource cost would both equal the sum of project rent and tenant subsidy and, hence, would be equal by definition.

Since we have shown that net tenant benefits will, in general, be less than the tenant subsidy, gross tenant benefits will consequently be less than resource cost. The difference must be made up by non-tenant benefits if the program is to be justified on benefit-cost grounds.

Estimating Minimum Required Nontenant Benefits

If the analyst has an estimate of total resource cost and an estimate of gross tenant benefits, he can obtain an estimate of the minimum amount of nontenant benefits necessary to make total benefits equal total cost. Of course, it would be preferable to have a direct estimate of these nontenant benefits, but an estimate of the minimum necessary may be illuminating to policymakers. If the required benefits are small relative to resource cost or in absolute amount, the housing program would be very much like an unrestricted cash grant (equal to the tenant subsidy) and little indication of nontenant benefits would be required to justify the program. On the other hand, if required nontenant benefits are large, then the justification of the program may be questionable unless some indication of nontenant benefits is found. In any event, the magnitude of minimum required nontenant benefits should give policymakers a good idea of the "price" paid for a housing program versus an unrestricted cash grant. They are then face-to-face with the issue of whether the housing program is worth the price.

6. SUMMARY

Net tenant benefits may be estimated by using the following expression

(35) $$B_t^n = \left(\frac{R_m}{\beta}\right)^{\beta}\left(\frac{y_0 - R_p}{1-\beta}\right)^{1-\beta} - y_0 \; .$$

Gross tenant benefits may be estimated by using the following expression

(36) $$B_t^g = B_t^n + R_p .$$

The tenant subsidy (and, under certain conditions, the nontenant contribution) may be estimated from the following expression

(37) $$S = R_m - R_p .$$

Under certain conditions resource cost may be estimated by the market rent, i.e.,

(38) $$C = R_m .$$

Finally, an estimate of minimum required nontenant benefits is

(39) $$B_{nt}^{min} = C - B_t^g \; .$$

The variables have the following definitions:

B_t^n = net tenant benefits

B_t^g = gross tenant benefits

B_{nt}^{min} = minimum required nontenant benefits

R_m = market rent

R_p = project rent

y_0 = tenant income

β = tenant rent-income ratio

S = tenant subsidy

C = resource cost

With the use of these relationships, economic evaluation of housing programs can be carried out.

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