A NUMERICAL METHOD FOR CALCULATING INTERIOR BALLISTICS

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September 1970

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We shall describe a procedure which calculates the interior ballistic properties of guns by numerically integrating the partial differential equations appropriate to the dynamics of the propellant gas.

The equations which describe the interior ballistics are one-dimensional along the gun barrel, assuming a uniform cross section. The equation of motion of the gas in Lagrangian form is

$$\frac{\partial u}{\partial t} = - \frac{\partial}{\partial m} (P + Q), \tag{1}$$

where u is velocity, t is time, m is mass per unit area, P is pressure and Q is the artificial viscosity for handling shock waves.[1] The equation of energy conservation is

$$\frac{\partial E}{\partial t} = - (P + Q) \frac{\partial V}{\partial t} + B , \tag{2}$$

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where E is the internal energy per unit mass of the gas, $V = 1/\rho$ is the specific volume or reciprocal density, and B is the rate at which energy is added to the gas by the burning of the propellant. The equation of motion of the shell as it moves down the barrel is

$$M_{sh} \frac{du_{sh}}{dt} = P_{sh} A \ , \tag{3}$$

where M_{sh} is the mass of the shell, u_{sh} and P_{sh} are the velocity and pressure of the gas at the shell boundary, and A is the barrel cross-sectional area.

For our example we shall use a large 5-inch/54 caliber naval gun with the following dimensions:[2] chamber length, L_o=100 cm; cross-sectional area, A=130 cm^2; shell mass, $M_{sh}=3.8 \times 10^5$ gm; and total shell travel distance, L=600 cm. The propellant used is 90% nitrocellulose and 10% other volatiles, and it is in the form of small cylinders with diameter, D=.69 cm; solid density, ρ_o=1.5 gm/cm^3; and total weight, $W=9.0 \times 10^3$ gm. The propellant contains small diameter webbing which we shall neglect.

The equations of state of the gas products resulting from the burning of nitrocellulose, mainly, CO_2, CO, H_2O, H_2, and N_2, have been determined previously.[3,4] The thermal equation of state is

$$P(V-b) = nRT \ , \tag{4}$$

where the covolume correction, $b \simeq .95$ cm^3/gm; the amount of gas formed from a gram of nitrocellulose, $n \simeq .04$ gm-mole/gm; and the gas constant, R=83.15 bar·cm^3/(gm-mole·°K). The caloric equation of state is

$$E = P(V-b)/(\gamma-1) \quad , \tag{5}$$

where the ratio of specific heats, $\gamma \simeq 1.25$, is essentially independent of density. Assuming a gas ignition temperature (adiabatic flame temperature) of $T_o = 3000°K$, we obtain that the internal energy of the propellant is increased by about $E_o = 800$ cal/gm upon conversion from a solid to a gas.[4]

The burning law for a cylindrical propellant of diameter fD, where $0 \leq f \leq 1$ as the propellant burns in the surrounding gas, is empirically

$$D \frac{df}{dt} = - \beta P \quad , \tag{6}$$

where β is a burn rate constant.[4] Neglecting the webbing and the ends of the cylinders, the fraction of propellant burnt is $\phi = 1 - f^2$, and it follows that

$$\frac{d\phi}{dt} = \frac{2\beta P}{D} (1-\phi)^{1/2} \quad . \tag{7}$$

Thus the rate at which energy is added to the gas is

$$B = E_o \frac{d\phi}{dt} = E_o \frac{2\beta P}{D} (1-\phi)^{1/2} \quad . \tag{8}$$

The density of the gas as a function of ϕ is

$$\rho = W \phi / [A(L+x) - (1-\phi) W/\rho_o] \quad , \tag{9}$$

where $x = x(\phi)$ is the distance which the shell has moved down the barrel.

The solution is obtained by numerical integration as a function of position and time of the finite difference equations derived from the differential equations given above.[5,6] We assume the shell begins to move as soon as it overcomes the resistance of the engraving band at a gas pressure of $P_i \simeq 400$ bars.[4] This determines the initial gas conditions at which we start our calculation as $\rho_i = 0.1$ gm/cm^3, $E_i = 350$ cal/gm, and $\phi_i = .07$. Finally, we obtain a best fit solution by specifying the burn rate as B=.025 cm/(sec·bar), which happens to equal the experimental value.[4] The results which we have are the peak breech pressure, P_m, the shell muzzle velocity, u_m, and the shell ejection time, τ. These values are presented in Table I along with corresponding experimental values.[2] Also a history of breech pressure vs time is shown in Fig. 1.

For comparison with previous work, we carry out the same ballistics calculations using the representative "isothermal solution" described by Corner.[3] He gives expressions for P, u, and x as a function of the variable f and the "central ballistic parameter," M, the "form factor," θ, the "force constant," λ, and the burn rate, β. The time variable is introduced through integration of the time dependent equations of motion. We obtain a best fit, but not unique, solution by chosing the four parameters to give the correct value of P_m and a good pressure-time history: M=.93, θ=.60, λ=240 cal/gm, and β=.050 cm/(sec·bar). Note that β is not in agreement with the experimental value above, but must be used for consistency with the other parameters. The results are shown in Table I and Fig. 1.

The advantage of our method is that it gives the details of the gas dynamics along the gun barrel and it leads directly to a good reproduction of the experimental results with only the small adjustment of the burn rate parameter and it is capable of being readily modified to account for small corrections, such as, heat losses and friction, which can lead to improved results.[6] The earlier, analytic methods,[3,4] such as the one listed above, are often detailed but are usually difficult to modify directly and usually depend for accuracy upon the somewhat arbitrary adjustment of several parameters.

Table I. Summary of Experimental and Theoretical Ballistics Results.

	P_m	u_m	τ
Experiment	.37±.02	.81±.02	13.±1.
Present Theory	.37±.04	.94±.04	12.±1.
Corner Theory	.37±.02	.88±.02	13.±1.

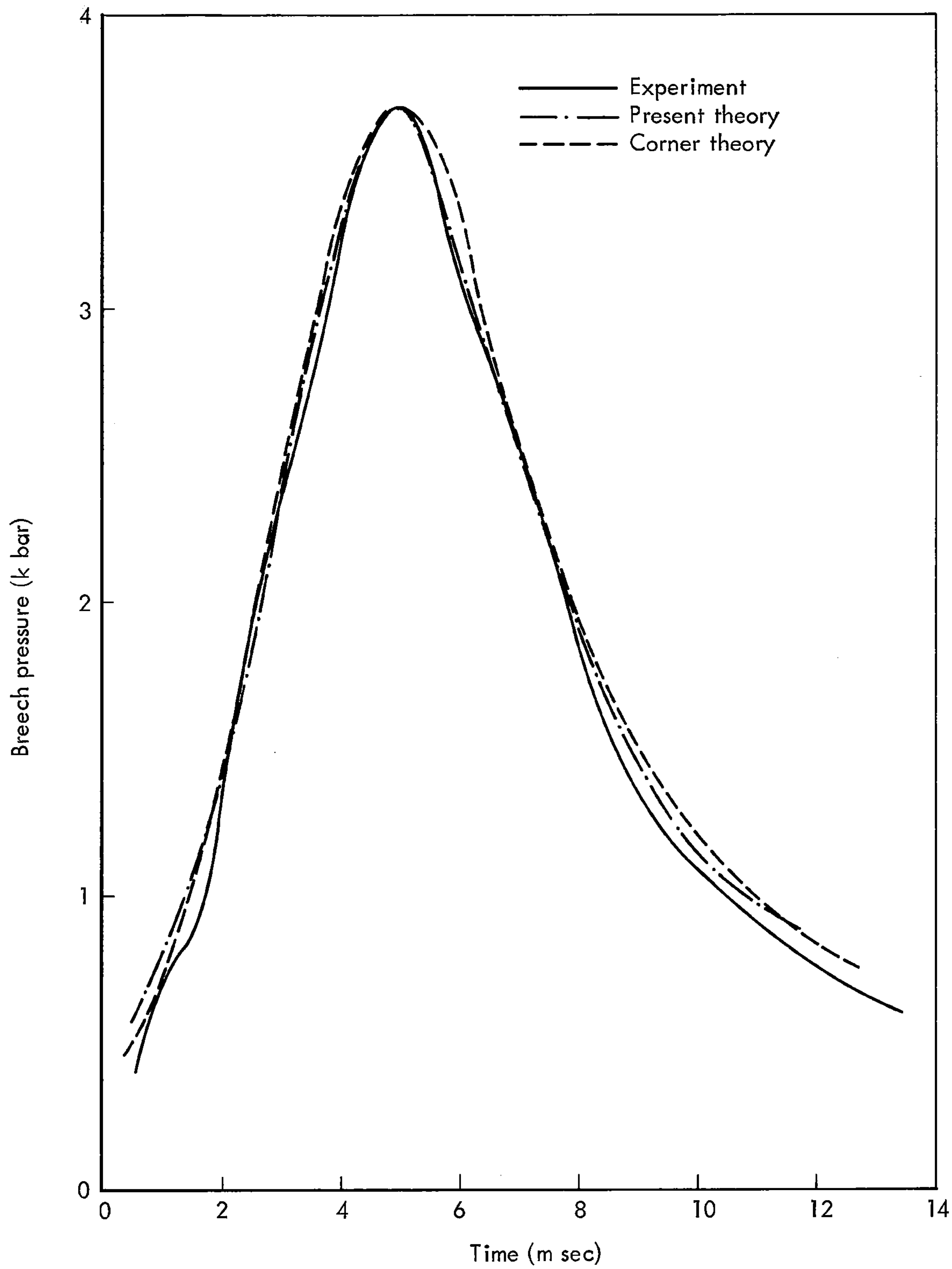

Fig. 1—Pressure-time histories

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