ON GAMING AND GAME THEORY

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1. INTRODUCTION

Gaming and game theory are two extremely different yet highly intertwined disciplines. Many practitioners are well acquainted with one of these subjects but not the other. Their interrelationship is subtle and important, and a knowledge of both can easily aid work in either.

A third topic that is often mentioned in discussions of gaming is simulation. This article deals with gaming and game theory. However, in order to avoid the confusion that frequently exists, all three subjects are defined and the relationship among them is indicated in Figure 1 below.

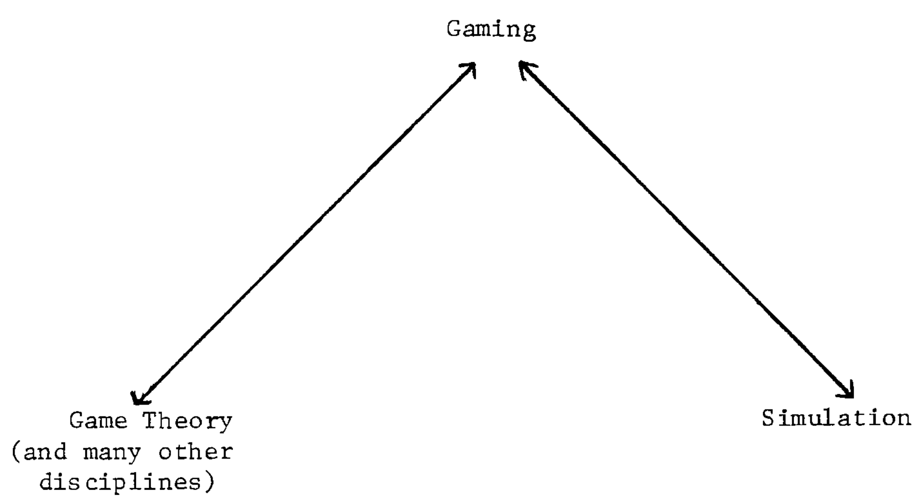

Fig. 1

Gaming: A gaming exercise employs human beings acting as themselves or playing simulated roles in an environment which is either actual or simulated. The players may be experimental subjects whose behavior is being studied or they may be participants in an exercise being run for teaching, training, or operational purposes.[1]

The discipline of gaming deals with the construction, organization, running, and analysis of games for these purposes. A developed methodology and theory of gaming hardly exists but is beginning to evolve.[2]

Game theory: Game theory is part of a large body of theory concerning decisionmaking. It provides a language for the description of conscious, goal-oriented decisionmaking processes involving more than one individual. It has provided a methodology to make relatively subtle concepts amenable to analysis. These are concepts such as the state of information, choice, move, strategy, outcome, and payoff.

It is a branch of mathematics which can be studied as such with no need to relate it to behavioral problems, to applications, or to games.

In general, game theoretic reasoning and analysis are of considerable use in constructing and analyzing games and gaming exercises constructed for any purpose. The solution concepts or theories offered in the theory of games may be regarded as normative or descriptive views of multiperson decisionmaking. When the solution concept is backed up with empirical evidence from gaming experiments or with evidence of behavior from other observations, it may be regarded as having some behavioral justification.

The key relationship between game theory and gaming hinges on the analytical conclusions of game theory that even with a peculiarly rationalistic set of assumptions concerning human behavior there is logically no simple extension of the concept of individual rational behavior to groups of individuals in general.

Simulation: Simulation involves the representation of a system or organization by another system or model which is deemed to have a relevant behavioral similarity to the original system. The simulator or model is usually far simpler than the system or organism it represents. It should be far more amenable to analysis and manipulation. Games utilize a simulated environment or simulated roles for the players, or both. In general, all games are simulations. However, it is not useful to use the reverse categorization. In other words, not all simulations should be regarded as games. In particular, there are many all-machine simulations of physical processes in which no freedom of human decisionmaking is postulated, nor is it relevant to the study of the process. Such simulations cannot be fruitfully construed as games.

Current usage is by no means clear concerning gaming and simulation. Some practitioners would call a simulation of a game theoretic air battle model performed by a computer a simulation; others might call it a gaming exercise, and still others might call it game theory. It is not particularly fruitful to engage in a taxonomic squabble unless some key aspects of understanding depend upon extreme care and clarity in terminology. Here this does not appear to be the case, hence, the general warning that one needs to check upon what an individual means when he uses the words gaming or simulation, and on some occasions even game theory should serve to allert the reader sufficiently.

2. RULES AND DESCRIPTIONS

Game theory provides a much needed methodology for constructing games, testing rules, and deciding upon the interpretation of the observed play. It deals with processes in which the individual decision unit has only partial control over the strategic factors affecting its environment. A formal description of a situation involving multiperson decisionmaking is of value not only to the game theorist but to anyone designing games for virtually any purpose.

There are three major formal descriptions of a game, all of which are of importance and of use to the game designer and user for different purposes. They are:

(1) The strategic or normal form,

(2) The extensive form,

and (3) The characteristic function form of a game.

Each of these descriptions is of use in a different way in describing a game, in designing and conducting experiments, or in teaching or using a game for operational purposes.

The representation of a game that is used most frequently in gaming experiments, especially in social psychology, is the matrix game. An example of a 2 x 2 matrix game is shown in Table 1.

The numbers in the cells represent the payoff, respectively, to the first and second players.

The 2 x 2 matrix game is used not only for experimentation but also in teaching, as it provides many examples of the different forms of environment for cooperation, coordination, and competition that can exist in an extremely simple situation. Rapaport and Guyer have calculated that there are strategically 78 different types of 2 x 2 matrix games.[3]

Though almost all of the experiments using matrix games have been done with the 2 x 2 versions with only two players, the formal definition holds for any number of players, each with any number of strategies.

Player 2's Choice

Player 1's Choice	1	2
1	5, 6	-11, 10
2	11, -9	-8, -8

Table 1

We may use a diagramatic representation of a matrix game rather than display the matrix. Figure 2 shows this for the game illustrated in Table 1 (this is the well-known Prisoner's Dilemma game). The four vertices correspond to the payoffs of the game arising from the four strategy pairs which can be chosen. Any point in or on the boundary of the area enclosed by the four lines shown in Figure 2 may represent the average payoff as a result of some extended series of play.

It is an open experimental question to determine if there is a significant difference in play between subjects given a game in matrix form or the same game with a diagramatic representation.

The fact that almost all human interaction involves a complex mixture of parallel and opposed interests is of fundamental importance to the behavioral sciences in general and to the interpretation of gaming exercises in particular. When interests are not directly opposed, individual rational or intelligent behavior may no longer be easily related with rational or intelligent social behavior. In the matrix shown in Table 1, in some sense it is intelligent to select the second strategy. If you are successful you clean out your competitor. If he is like-minded at least you get even with him. The Hobbsian man may be inclined to elect for his second strategy. However, it would be more socially intelligent for both to select their first strategies.

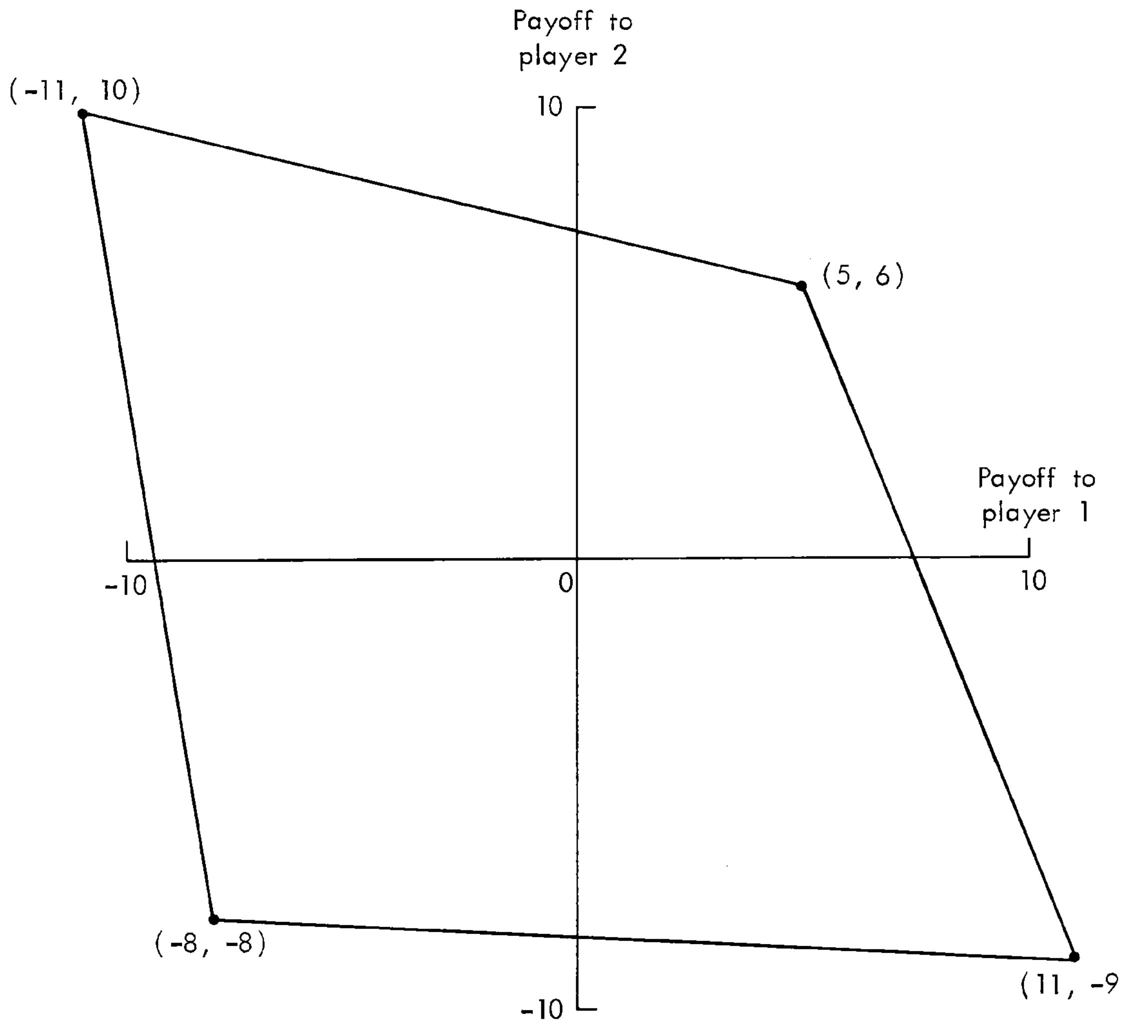
Payoff to
player 2
10
(-11, 10)
(5, 6)
Payoff to
player 1
-10
0
10
(-8, -8)
(11, -9)
-10

Fig. 2

An important class of games easily illustrated by the 2 x 2 matrix is the two-person zero-sum game where the payoffs in each cell are equal and opposite. Such a game is called a game of *pure opposition*. The maxmin strategy appears to be the rational strategy to employ if one assumes that the opponent is not going to commit blunders. In such a game, talk, language, and negotiations and other communication have no role to play. This is not true for the game illustrated in Table 1.

A fundamental difficulty in experimenting with non-zero sum matrix games is that societal rationality or intelligence may be established through cultural norms and other subtle aspects of the environment. A matrix game experiment is almost always neither rich enough nor played long enough to have the players establish norms in a laboratory setting.

2.1. MODELING AND THE RULES OF THE GAME

The crucial consideration in the study of game theory or the application of gaming for any purpose is to make sure that the model being used is a good representation of the situation or topic to be taught or investigated. In the formal theory of games, it is assumed that all of the rules of the game are given and known to all players. This is an extremely strong assumption for most human affairs although not for formal games such as chess. Many important operational games take into explicit account the fact that the players do not know all of the rules. Frequently one of the major objectives of a gaming exercise is to explore the rules. For example, *Crisis Management Gaming* is usually devoted to this type of exploration.

2.2. THE EXTENSIVE FORM OF A GAME

The way in which we represent a game depends upon our interests and the type of analysis or experimentation to be performed. If the stress is on strategy and payoffs, the strategic form as illustrated by the matrix game will be used. If interest is on detail, information and fine structure, then the extensive form of a game will be employed. The characteristic function form serves to investigate coalitional behavior.

The extensive form of a game can be illustrated by means of a diagram known as the game tree. The representation of the game shown in Table 1 by means of a game tree is given in Figure 3.

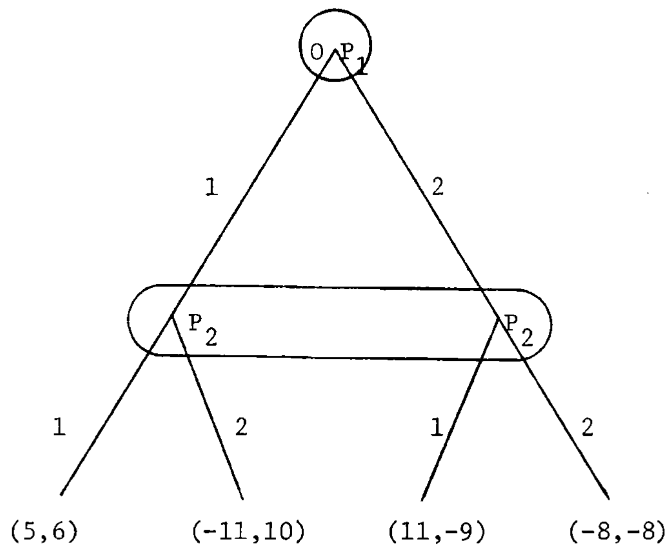

Fig. 3

In this representation of the game the first player is called upon to make his choice,which amounts to selecting one of the two branches emanating from the point 0. After he has made his choice, the second player has to choose a branch at one of the two locations marked by P_2. After they have both selected a branch, the payoff is given by the two numbers at the end of the branches. In order to indicate that the players move simultaneously, we enclose both of the nodes marked P_2 by a curve which portrays an information set. It implies that the second player, when called upon to move, cannot distinguish between the two nodes. In other words, he does not know what the first player has chosen. If he knew, we would then draw a separate information set around each of the nodes.

By employing this type of notation, the game theorist is in a position to talk about strategies. A strategy in the sense of game theory is a completely specified plan of action which covers all contingencies based upon the information state of the players throughout the game. Using this formal definition, the number of strategies for even a simple game such as tic-tac-toe is astronomical.

In most gaming exercises, be they aimed at urban development, diplomacy, or the study of military policy, the word "strategy" is often used. Frequently it is interchangeable with the word "plan." The difference between the gaming usage and the game theoretic usage comes in the treatment of the aggregation or the coding of information and in the delegation of responsibility to subordinates in an organization. The game theory definition of a strategy contains all of the minute details of tactics as well as the overall plan. The typical or military usage of the concept of a strategy or an overall plan sketches out the main aspects and leaves a certain amount of freedom of action to improvise to those entrusted with the task of carrying out the plan. It is extremely difficult to translate this far less precise but more operational concept of strategy into a formal mathematical model.

Although it is not easy to reconcile the game theoretic and the operational concept of strategy, the study of the difficulties in the reconciliation is both important and rewarding. How do organizations process and abstract information? How much leeway should be given to ambassadors or to field commanders?

In Figure 3 we have portrayed Player 1 as moving first, followed by Player 2 who nevertheless is not informed of his competitor's action. In the real world there are leaks of information. In bridge it is usually conceded that "a peek is worth two finesses." The formal apparatus of game theory can model information leaks. However, it requires the introduction of probabilities or a move by "Nature" which determines under what circumstances secrecy can be violated.

3. INDIVIDUAL PAYOFFS AND PREFERENCES

3.1. OUTCOMES AND PAYOFFS

A useful distinction can be made between the outcome of a game and the payoff or value attached by a player to a specific outcome; for example, when a chess game ends the three possible outcomes or final states of the board which can be classified as win, lose, or draw. These are physical states; nothing is stated about the value of these states to the players.

We could for example attach values of 1, -1, and 0 to the different outcomes. These numbers may stand for a point-scoring system in a tournament or represent sums of money. The relationship between payoffs and motivation is a key topic in every form of gaming.

In chess the meaning of the win is relatively well defined. In most human affairs this is not usually so. What constitutes a win in a war such as that in Vietnam is not easy to specify. In games which are not constant sum the goals of the individuals are rarely diametrically opposed.

In gaming and game theory, if we wish to interpret the meaning of an outcome to a game we must be in a position to describe individual preferences and know how to evaluate these preferences as well as to describe group norms and goals and their relationship to the goals of individuals. The first calls for a study of individual payoffs and the second for an investigation of the concept of solution. The latter is dealt with in parts 5 and 6.

No attempt is made here to offer an exhaustive discussion of preference conditions. The interested reader is referred to the considerable literature that already exists.[4] It is critical, however, that anyone seriously interested in the use of gaming be aware of some of the problems in the setting up of payoff functions or in the specification of the goals of a game.

Much of the work in economic theory and in decision theory has been based upon the concept that we may describe the ordering of preferences for an individual but may not be able to measure intensity.

In particular this means that a statement such as "a is preferred to b" is meaningful but a statement that "a is liked twice as much as b" has no meaning. Consideration of risk, bargaining, and welfare has led many individuals to the conclusion that at least as an approximation for some situations, preference scales can be obtained and even a comparison of preferences is of importance. Statements such as "this is only a minor inconvenience to you but of major value to me" are both relevant to and useful for resolving problems of bargaining, welfare, and planning.

What do the numbers in the payoff matrices mean in terms of motivation and how are they to be measured? How are the numbers to be compared? For example, consider the two matrix games shown in Table 2. The matrix in Table 2 (b) is obtained by multiplying the numbers awarded to the second player in Table 2 (a) by 100. If we ran an experiment

	1	2
1	3, 3	-1, 1
2	4, -10	0, 0

(a)

	1	2
1	3, 300	-1, 100
2	4,-1000	0, 0

(b)

Table 2

where payment to the experimental subjects was an hourly wage, do we expect to find any difference in the behavior between those who play the game shown in 2 (a) and those who play the game in 2 (b)? Some game theoretic solutions based upon comparability of payoffs might suggest yes, others would strongly suggest no.

In most non-military gaming for operational and other purposes, the description of the payoffs is extremely complex. It might be argued that the prime purpose of gaming situations such as community development is to determine the payoff functions. By the act of playing, the players may be able to clarify what the goals are.

The needs for payoff measurement vary for different games. In many tactical and military games such as games of pursuit, the games can be modeled as two-person zero-sum games with payoffs relatively well defined from military tactical considerations. In non-zero sum games, even those describable by 2 x 2 matrices, the meaning of the numbers is extremely difficult to come by when the games are presented as representations of reality. It is hard to attach very much meaning to a statement such as "the loss of Berlin is -10 brownie points to the Western allies."

Many experimental games have simple monetary payoffs. Usually the stakes are quite low and are designed so that the experimental subject earns around what he would have earned by being paid on an hourly basis. What happens to the quality of play when serious money is riding on the outcomes? What are the motivations of colonels playing in war games? How do the numbers in the payoff tables and the scoring systems influence operational games, teaching games, or experimental games" We know very little about the answers to these questions. However, at least the development both in game theory and gaming make many of these questions well defined, and it is conceivable that with careful experimentation they can be answered.

4. PAYOFFS AND COALITIONS

An important aspect of gaming at the level of bureaucracy, international bargaining, or diplomacy is the role of side payments or transfers among the participants. The side payments may be a formal part of the game to be studied or they may occur as an informal aspect of a gaming exercise. For example, when a game is being played for points with no particular tangible prize offered to the participants beyond "a greater understanding of their problem," there may be a set of unstated prizes such as promotion, picking up of information, meeting the right people, and so forth. The distribution of these prizes may be determined by an active market outside of the immediate game. However, the trading that takes place in this market can easily influence the play of the game.

A side payment is a transfer of wealth or other forms of value among the players beyond or after the actual play of the game. Side payments enlarge the scope for cooperation and at the same time they enable individuals to threaten and blackmail each other more effectively. The example provided in Table 3 and Figure 4 illustrates this. In Table 3,

	1	2
1	1, 17	0, 5
2	5, 0	0, 0

Table 3

we observe that by using their first strategies, together the players can achieve an outcome of 18. If they are not permitted to make side payments there is a great temptation for the first player to use his second strategy thereby obtaining 5 instead of 1. By merely playing the game the players can achieve any outcome shown in Figure 4 by the area ABCD. If, however, side payments are permitted, then any point along the line EF can be achieved. In particular, Player 1 may demand a side payment from Player 2 in return for his cooperation.

When dealing with simple examples in game theory there is a tendency to attach picturesque language to the point being illustrated. We might wish to call the behavior of Player 1 blackmail or threatening behavior. However, from a different point of view we might say that both play equal roles in the obtaining of a joint payoff of 18, hence both should have a more or less equal share and this share involves a side payment.

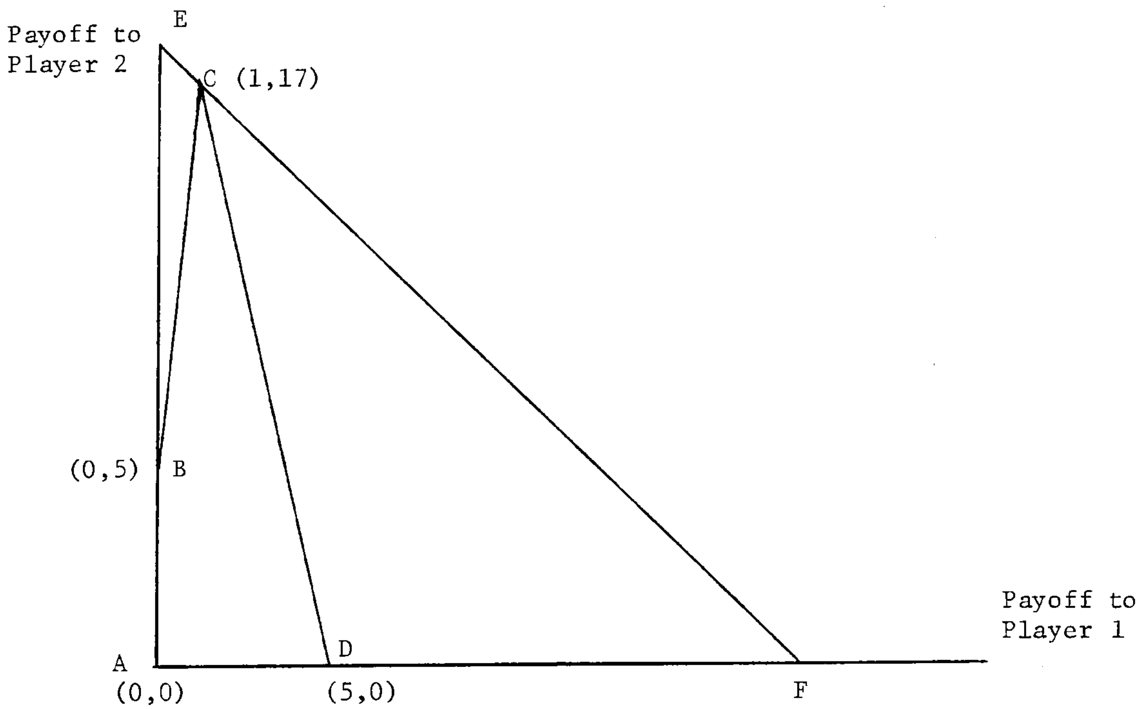

Fig. 4

In this case we have assumed that not only can the individuals compare their payoffs but that a one-to-one transfer of points or money that has a constant value to each regardless of the amount transferred, is possible. This was implicitly shown by drawing the line ECF as a straight line. It might be that as the amount paid from one player to the other increases, the successive units have more value to the first and less to the recipient. In this case the line would not be straight but would be a curve.

There are three important distinctions to be made concerning side payments. They are:

(1) The possibility for the transfer of goods or services outside of the actual playing of the game.
(2) The possibility of making comparisons among the payoffs of the individual.
(3) The existence of some item or good for which there is a linear relationship between the individual's evaluation of the worth of the good and the quantity of the good.

4.1. SIDE PAYMENTS AND "THE NAME OF THE GAME"

In game theory the use of side payments serves as a modeling device in order to abstract certain parts of the process of bargaining, action, and distribution. Side payments are used by the game theorist to separate the study of the actual playing of the game from the study of how the proceeds are to be distributed. Many game theoretic solutions are based upon the idea that the players will play in a way that will first maximize the take for all and will then concentrate on bargaining concerning shares. In such a situation, the actual play of the game is relatively uninteresting. It is the settling up of the shares that commands the interest.

When running gaming exercises, the distinction between behavior in the game and related behavior outside of the game is often overlooked. The world is rich in the manifestations of *quid pro quo*. How this is manifested in a game must be checked by its directors.

In gaming activities such as urban or other social development programs it is desirable to try to uncover whether there are implicit or explicit welfare comparison assumptions in the structure of the side payments. There are two types of comparisons which appear in a game. The first is *extrinsic* to the game, i.e., the comparison is imposed, from the outside, by the game designer or by agreement. For example, it may be decided that a heavy tank may be traded for 17 light tanks whether or not the trading ratio has any meaning to reality. On the other hand, sometimes there may emerge from the actual structure of the game an intrinsic ratio for comparing payoffs. In such an instance the discovery of the ratio itself may be an important result of the game. For example, it may turn out that there is built into a game an exchange ratio between saving an extra life on the highway and saving an extra life by hospital construction. This ratio might have been an *intrinsic* property of the game structure, but is not perceived by the designers or the players until after the play.

4.2. DAMAGE EXCHANGE RATES AND MAXMIN THE DIFFERENCE

Tied in with the question of comparability, yet stressing an extremely competitive point of view, is the specification of goals based upon differences in scores rather than the absolute value of individual payoffs. When dealing with two-person zero-sum games an attempt to maximize your score is equivalent to trying to maximize the difference between your score and that of your opponent. This is not the case with non-constant sum games. Further, if in an experimental (or other) game the stakes are small or have little intrinsic interest to the players, it is quite likely that the players will convert a non-constant sum game into a game of pure opposition. This can be done by adopting as a goal the maximization of the difference in score rather than the maximization of one's own score. The example in Table 4 illustrates the point.

5, 5	3, 4
4, 3	0, 0

(a)

0	-1
1	0

(b)

Table 4

Table 4 (a) shows the actual payoff matrix for the game (we might consider that the players are being paid in points or in pennies). Table 4 (b) shows the different matrix where the goals of the players are "to beat each other." It is a safe rule to follow that if you have not checked the results of a low stake experimental game for the possibility that the players are trying to maximize the difference in their score, you may easily be misinterpreting your results.

The idea of the damage exchange rate is appealing in tactical analysis and is sound when studying two-person zero-sum games. It is misleading and frequently false when the games are not zero sum.

When there are more than two players it is important to check for the possibility that the players may be trying to "beat the average," or in some instances they may convert the game into a game of status where the object is to be first at any cost.[5]

It is almost always necessary when gaming to take a great deal on faith concerning the relationship between outcomes and payoffs. It is of paramount importance that the gamer understand the nature of his assumptions. One way in which he can improve his chances for understanding is to apply several alternative models to explain the payoff structure of his game as it might be perceived by the players.

5. SOLUTIONS

The selection of the best description of a game and the specification of payoffs are the necessary preliminaries for the utilization of a game for game theory or gaming purposes.

For some purposes the "solution" to an investigation of a social, political, or military situation is its description as a game. The amount of analysis and understanding of the real world required to produce a good model is considerable. This may be as far as you wish to go.

Sometimes we may wish to go further. We may want a prediction, or to explore the sensitivity of the system, or to test out plans.

The term "solution" in game theory is used to describe some form of analysis that will be performed on a game given in one of the three forms noted in Section 2.*

There are four broad categories of solution of interest to both the gamer and game theorist. They are:

(1) Non-cooperative solutions,

(2) Cooperative solutions,

(3) Mechanistic solutions,

and (4) Dynamic and behavioral solutions.

Samples of the first two types of solution concepts are given below. The third type of solution refers to various theories in which the human is replaced by a mechanism with no strategic freedom of his own. The price system in an economy provides an example.[7]

Dynamic and behavioral solutions are at the same time the most important to gamers and the most difficult to work with. Which comes first, the game theory or the gaming? Often it is the gaming if we wish to devise and explore the validity of solution concepts. The game theory is useful in setting up the game for study, but the running of the game or gaming experiments provide our means to explore behavior.

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*There are other forms that can be used to represent games; they are discussed elsewhere.[6]

5.1. TWO-PERSON ZERO-SUM GAMES

It is still common that individuals who have heard of game theory are under the impression that the bulk of the theory is concerned with the extremely pessimistic behavior characterized by the "maxmin" strategy. This is true only in the zero-sum two-person game. The goals of the players are diametrically opposed; they are, in the purest sense of the word, opponents.

When a zero-sum game is played experimentally, frequently the players fail to choose optimal strategies. There are two major reasons for this. First, it may well be that the points in the payoff matrix are not true payoffs, merely outcomes. If the outcomes were transformed into payoffs the game might no longer be zero sum. Second, if the players are not necessarily quick-thinking dispassionate, rational men, they may commit mistakes, they may fail to perceive opportunities, or one may be unnerved by a "ploy" of the other. Game theory usually makes the assumption of "external symmetry." In other words, it abstracts out of many real human failings. In experimental gaming these aspects of human behavior come back in and make the analysis far more difficult.

A useful game both to experiment with and to earn a modest income at your local bar is given in Table 5.

	1	2
1	2	-3
2	-1	2

Table 5

This game serves to make out the normative case for the maxmin solution. We observe that it does not have a solution in pure strategies. Furthermore, we observe that it might be possible to convince an opponent that this is a fair game. However, it is considerably stacked in favor of Player 1. If he uses a mixed strategy, choosing his first strategy with a probability of 3/8 and his second strategy with a probability of 5/8 he is in a position to guarantee an expected win of 1/8. This is shown in the calculation below:

Against strategy 1 of Player 2 $\quad 3/8(2) + 5/8(-1) = 1/8$

Against strategy 2 of Player 2 $\quad 3/8(-3) + 5/8(2) = 1/8$

The game is stacked in favor of Player 1 better than roulette for the house!

An important type of continuous zero-sum game which has a pure strategy equilibrium point is the duel. An excellent summary of much of the work on dueling is given by Dresher.[8]

Game theory, gaming, and simulation join together in the analysis of duels, games of allocation, and weapons evaluation. Frequently a game theoretic model may serve to provide the basis on which a simulation is built. The man-machine interactive simulation may be used to check out the way humans behave in contrast with the way the ideal game players behave.

5.2. CHESS, GAMING, AND ARTIFICIAL INTELLIGENCE

It can be shown that chess could be expressed as a matrix game with a saddle point. From a game theoretic point of view, chess is uninteresting as no one need bother to play as soon as sides have been chosen and optimal strategies declared. There is, however, still plenty of gaming evidence that chess is an interesting game. Many attempts have been made to construct computer programs to play chess.[9] These programs have been part of the attempt in the study of "artificial intelligence" to portray behavior that at least by human standards could be described as good chess playing.

5.3. A NON-COOPERATIVE SOLUTION FOR NON-CONSTANT SUM GAMES

There is no operational meaning to the difference in cooperation or non-cooperation in the context of a two-person zero-sum game. This changes considerably when non-constant sum games are investigated. There no longer is a clear conflict of interest between the players. At even the most abstract level no single normative prescription for individual rational behavior in non-constant sum games is known.

Both the gamer and game theorist have devoted much time to the questions of how individuals should or do behave in two-person non-constant sum games under varying information, communication, and repeated play conditions.

The favorite game for experimentation has been the 2 x 2 matrix game, generally run with the players not in direct view or verbal contact with each other, but playing the same matrix game several times.

Consider the two games presented in Table 1 and in Table 4 (a). If we applied a prescription of individual rationality, it is easy to note that in Table 1, each player is better off choosing his second strategy regardless of the actions of his competitor. In Table 4 (a) he is better off choosing his first strategy regardless of the action of his competitor. In the first case, this spells joint disaster. They obtain -8 each. In the second case they obtain the joint maximum of 5 each. In one-shot games there is a reasonable amount of evidence that players in the two instances noted actually do play this way. If, however, we iterate the game several times, the pattern of behavior in the Prisoner's Dilemma (Table 1) appears not only to depend upon game theoretic features such as the communication conditions, but also upon a host of sociopsychological phenomena.

The most popular and powerful solution concept suggested for non-constant sum games with little stress on cooperation or communication is the Nash Non-cooperative Equilibrium Point.[10] It is described below for the two-person case but obviously generalizes immediately for any number of individuals. Let s_i be a strategy for player i. In order to denote a specific strategy as contrasted with a general strategy, we use the notation $\bar{s}_i$. A point $(\bar{s}_1, \bar{s}_2)$ is an equilibrium point if it satisfies the following conditions:

$$\max_{s_1} P_1(s_1, \bar{s}_2) \text{ implies } s_1 = \bar{s}_1$$

$$\max_{s_2} P_2(\bar{s}_1, s_2) \text{ implies } s_2 = \bar{s}_2.$$

Formulated in this manner the solution is static and gives us no insight into how the dynamics of play might bring it about.

We may require extra conditions for an equilibrium point, and experimental evidence[11] shows that in games where some or all of these conditions are present, the prediction power of the non-cooperative equilibrium is considerably increased. The extra conditions are:

(1) The equilibrium strategy for an individual should dominate all other strategies. (Row and column domination.)

(2) The equilibrium should be socially rational or Pareto optimal.

(3) The equilibrium should be unique.

(4) The equilibrium should not employ mixed strategies.

0, 0	1, 4
4, 1	0, 0

Table 6

In Table 6 there are two equilibrium points, both of which are socially rational. However, the strategies of the players are not row-or column-dominant as they are in Tables 1 and 4 (a). The non-cooperative equilibrium that has all of the properties listed above is the one in Table 4 (a).

5.4 SCENARIOS, DEGREES OF FREEDOM, AND EXPERIMENTATION WITH MATRIX GAMES

Much experimental work has been done with 2 x 2 matrix games. Owing to their special and extremely simple structure, several factors must be taken into account when using them.

Is the writing of a scenario pertaining to a simple matrix game a determining factor in the way it is played? We might, for example, attempt to describe the game in terms of a military situation, a business situation, a social situation, or an abstract problem. It is extremely difficult to describe the influence of different descriptions on the play of the game.

There are other noncooperative theories which we do not describe here. For example, both players might try to minimize the score of the other regardless of costs, or both players might be so conservative that they both might choose to play maxmin even in a nonconstant sum game. There are also many cooperative solutions some of which are noted in Section 6. However, a 2 x 2 matrix contains only four cells. This means that in some instances even with repeated play there is not enough resolution power in an experiment to distinguish one theory from another. When one is also trying to control for the effect of symmetry, comparability of payoffs, salience, decision time, and a host of other factors, one begins to appreciate that although the relationship between game theory and gaming is of considerable importance, an enormous amount of care must be taken before we are in a position to interpret the results of a simple experimental game in game theoretic terms.

6. COOPERATIVE SOLUTIONS

In our discussion of noncooperative solutions, much use was made of the strategic form of a game. Many of the experiments in gaming and social psychology use the matrix representation of the game. This representation is also usually used in the study of search games, allocation games, and duels.

If the subject to be studied involves negotiations, bargaining, or fair division procedures, it is important to look at the game in characteristic function form. The characteristic function assigns a value to every coalition that can be formed in a game. A simple three-person example serves to illustrate this.

Table 7

$v(\overline{1}) = 2 \qquad v(\overline{2}) = 2 \qquad v(\overline{3}) = 4$

$v(\overline{12}) = 8 \qquad v(\overline{13}) = 8 \qquad v(\overline{23}) = 4$

$v(\overline{123}) = 11$

The symbol v(13) stands for the amount that a coalition consisting of players 1 and 3 can obtain together. This is denoted by a single number, in this instance 8. However, there is an implication in using a single number that the players can transfer and compare wealth. If this is not the case, there is a more complicated representation of the characteristic function which can be used.[12] If we consider the two-person game illustrated in Table 3, the following characteristic function represents the coalitional information.

Table 8

$v(\overline{1}) = 0 \qquad v(\overline{2}) = 0$

$v(\overline{12}) = 18$

Here we observe that a single player can guarantee nothing for himself. The two together, however, can obtain 18. If side payments are permitted, then the characteristic function representation in Table 8 reflects the basic symmetry of the game shown in Table 3.

The number of entries in a characteristic function will increase rapidly with the number of players. There will be $2^n -1$ coalition for an n-person game. The examples in Tables 7 and 8 show the two-and three-person games.

The characteristic function may be regarded as an important presolution if the stress is on examining bargaining and negotiation. Even a reasonable formulation of this function tells us a great deal about the situation. For example, suppose we were to study a five-person game involving China, the Soviet Union, the United States, Vietnam, and the rest of the world. Even an elementary attempt to calculate the coalition values may raise basic questions concerning the interest of the Soviet Union in helping to terminate the Vietnamese war. Usually, however, more analysis is required than is obtained by merely formulating the characteristic function. Two examples of solutions which tell us more are given. They are the *core* of a game and the *value* of a game.

6.1 THE CORE

Individual rationality calls for the individual to refuse to take less than he can get by himself. Societal rationality suggests that the group as a whole should always be efficient regardless of how they divide total product among the participants. Can we extend this type of concept of rationality to include groups of a size larger than one but smaller than the whole society? The game theoretic formulation of this problem shows clearly the conditions under which it is possible and when it is not possible to do so. Table 9 shows two characteristic functions for a three-person symmetric game. The second line has two values attached to the amount that a two person coalition can get. In each game, the 1- and 3-person coalitions get fixed amounts. The first value completes the characteristic function for the first game and the second completes the characteristic function for the second game.

Table 9

$$v(\overline{1}) = v(\overline{2}) = v(\overline{3}) = 0$$

$$v(\overline{12}) = v(\overline{13}) = v(\overline{23}) = 2 \text{ or } 5$$

$$v(\overline{123}) = 6$$

Suppose that a point denoted by (a_1, a_2, a_3) were in the core. If group rationality, individual rationality, and societal rationality are all to be satisfied, then the following inequalities must hold true.

$$a_1 \geq 0 \qquad\qquad a_1 \geq 0$$

$$a_2 \geq 0 \qquad\qquad a_2 \geq 0$$

$$a_3 \geq 0 \qquad\qquad a_3 \geq 0$$

$$a_1 + a_2 \geq 2 \qquad\qquad a_1 + a_2 \geq 5$$

$$a_1 + a_3 \geq 2 \qquad\qquad a_1 + a_3 \geq 5$$

$$a_2 + a_3 \geq 2 \qquad\qquad a_2 + a_3 \geq 5$$

$$a_1 + a_2 + a_3 = 6 \qquad\qquad a_1 + a_2 + a_3 = 6$$

The inequalities and equation on the left are the conditions for the first game and those on the right are the conditions for the second game. If we sum the three inequalities in each instance representing the conditions on the two person coalitions, we obtain the following two inequalities:

$$2(a_1+a_2+a_3) \geq 6 \qquad\qquad 2(a_1+a_2+a_3) \geq 15$$

In the first instance it is easy to see that there are many triplets of numbers which satisfy this condition and the condition that the three players acting together can obtain 6. An example of one such division is (2,3,1). On the other hand, when we examine the inequality we have obtained for the second game, it is impossible to simultaneously satisfy all of the demands of the two-person coalitions. This would call for a division of proceeds which must add up to at least 7-1/2, whereas there are no more than 6 available for the whole game.

It is surprising to note that very little controlled experimentation has been done to verify whether or not players in simple many-person games choose outcomes within the core. Most of the simple three-person experiments have been with games without a core. Perhaps it is felt that the condition is "so obvious" if the core exists that it is not worth checking. I believe that some carefully set up experiments with three-, four-, and five-person games with cores would be worthwhile exploring.

6.2 THE VALUE

The value solution to a game is closely related to the concepts of equity and fair division. It provides a one-point solution or prediction of the outcome. At its very simplest, the value calls for a 50-50 split when it is applied to the problem of dividing goods between two individuals.

Shapley, using a set of reasonable assumptions about fair division, was able to derive a formula suggesting the amount to be awarded to each individual in any n-person game. The details of the derivation do not concern us here. The idea behind it, however, can be expressed quite simply. The value is calculated by considering all of the different ways in which a player might enter a coalition. Each player is assigned the increment of wealth that his presence brings to the coalition. We add up all of these increments for each player and average them over all of the coalitions. In other words, the value is a measure of the average incremental worth of each individual.

A simple example helps to convey the flavor of the value calculations. Consider a committee of four in which the first individual has two votes and the others, one each. The value will assign an imputation of (1/2, 1/6, 1/6, 1/6) to the players. This reflects the relative worth of contribution of an individual to the success of the committee. The two-vote member is critical one-half of the time. This can be shown by looking at all of the 24 ways in which a vote can take place.

Some years ago Kalisch, Milnor, Nash, and Nering ran some experiments with n-person games in characteristic function form.[13] They discovered that there appeared to be a reasonably good fit between the observed data and the value. They also noted that the outcome appeared to depend considerably on personalities in individual games. Although these experiments are highly suggestive and merit replication and extension, little further experimentation of this type has been performed.

6.3 THE BARGAINING SET

Another solution in which the concepts of game theory and the interests of experimental gaming are closely linked is the solution known as the bargaining set.

This particular solution was suggested to its authors when running experimental bargaining games in characteristic function form.[14] The experimental evidence obtained suggests that the outcome of the bargains in the games run lies within the bargaining set.

A point is a bargaining point if a certain form of stability or bargaining stalemate exists. The details are given elsewhere;[15] however, it is worth noting that this solution is highly suggestive of the adversary process and the interaction of pressure groups in a larger society.

6.4 BEHAVIORAL SOLUTIONS

In many ways the type of individual portrayed as a "game theory man" is a very simple soul who can be described as "son of utilitarian man." He has no personality; he really does not learn anything or change his opinion in the course of play. He invariably knows all of the rules of the game; he usually is able to compute and calculate accurately at great speed. He is assumed to always know what he wants and to know what the others want.

All of these sound like extremely limiting conditions. Depending upon the experiment or the game, sometimes they are reasonable, sometimes they are not. Nevertheless, almost always the game formulation helps to explain a great amount of the phenomenon being observed. It is important to remember that "one man's variance is another man's experiment."

The game theorist, when running his experiments, is not in competition with the social psychologists, psychologists, or psychiatrists. Each professional in his own way acknowledges the extreme complexity of the manifestations of human behavior. Each may contribute to explaining some part of what is seen in the laboratory.

The discipline of game theory provides a far broader basis for the gamer and the social psychologist than has yet been appreciated.

Leaving aside the many important problems that are central to social psychology, there still remains an enormous amount of useful work that can be done to investigate the degree in which formal game theoretic solutions may explain social behavior.

7. GAMING AND GAME THEORY: CONCLUSIONS

The basic understanding of the concepts of game theory enable someone who wishes to utilize gaming to provide guidelines for the construction of games. It helps him to avoid many pitfalls in modeling and it provides him with a set of basic measures which serve as an aid in constructing relevant questions for the analysis of games.

In particular, the selection of the strategic, extensive or characteristic function form of a game already serves as a means for deciding upon the level of aggregation needed to study the situation at hand.

The formal game theoretic description of the rules of the game, the information states, moves, and strategies provide the game designer and control team with guidance in the specification of the gaming exercise.

The definition of outcomes and payoff forces the gamer to verify that his gaming model is an adequate representation of the situation he wishes to study. In particular, the game theorist is extremely careful in invoking the concepts of "external symmetry"; in other words, except when the differences are explicitly stated, the players are assumed to be equal in all other respects. One important reason for running a game is to observe the difference in skills among players. Thus the game theory assumption of symmetry provides the benchmark or the "null hypothesis." In a completely symmetric game, most game theory solutions will include the symmetric outcome as an important possibility. A comparison of the actual outcome with the symmetric outcome may provide a measure for factors such as skill, social structure, bargaining power, etc.

Four words key to the description of the contribution of game theory to gaming are:

Explicitness,

Aggregation,

Symmetry

and Sensitivity.

The power of explicitness comes in the formulation of good models. Aggregation is of importance in the selection of the appropriate abstraction.

Symmetry plays an extremely important role in the construction of formal models and in their analysis. Two types of symmetry must be noted. External symmetry calls for the explicit acknowledgment that those features of the players or the outside environment which are not specified are assumed to be the same for all. Internal symmetry is used as an alternative to aggregation. For example, when one wants to analyze a game with many players it is often easy to do so if we assume that many of them are alike. This is not the same as aggregating a group of players into a single player; nevertheless, it makes the study of their behavior considerably easier.

Last is the problem of sensitivity. It is important for the gamer to know that some minor change in the description of his game will not lead to enormous differences in manifest behavior. Sensitivity analysis is keyed to virtually any use of a gaming exercise. For this purpose, formal game theoretic analysis can be extremely useful.

The relationship between game theory and gaming goes in both directions. Game theory provides an extremely useful background for the structuring the building and analysis of games. Yet at the same time gaming provides important evidence for the construction of new solution concepts for games and for the isolation of sociological, psychological and other variables which are not taken into account in the game theoretic model.

Game theory is by no means the only discipline of use to a gamer. Nevertheless, though an operational gamer, a social psychologist or other behavioral scientist may wish to stress different and more complicated models of man than those used by the game theorist, they may find that a basic understanding of game theory serves as a critical fund of knowledge for the design and control of gaming for virtually any purpose.

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