A THEORY OF COURT SCHEDULING

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October 1971

P-4732

ABSTRACT

A relatively simple, analytical model of court scheduling (calendaring) is introduced and analyzed. The model is based on a representation of the process by which cases not disposed of at any appearance are scheduled for another appearance. This representation, although deterministic, possesses many of the important properties of court operations. Using this model, analytical expressions are derived for the number of cases scheduled to appear (the "calendar size") in each portion ("part") of the court each day, the steady-state calendar size in each part, the sizes of future calendars and case backlogs, and the average duration of and the average number of appearances made by each case. In addition, the effect of a dependence of certain processing parameters on the number of cases scheduled is investigated.

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Recently, a great deal of public attention has been focused on problems afflicting our systems of criminal justice in general and our criminal courts in particular. Of particular concern in this regard is the congestion and delay which threaten to strangle the judicial process. While there are a great many ways in which to attempt to solve this problem, one essential prerequisite is an understanding of the process by which cases are scheduled to appear in court. This process -- known as calendaring -- is the subject of this paper. In order to contribute to our understanding of court calendaring we present and analyze a relatively simple, deterministic model which possesses many of the important properties of actual court operations. While we shall discuss the model in the context of a criminal court, it may also be applied to the operations of civil courts and in other situations as well.

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*Any views expressed in this paper are those of the author. They should not be interpreted as reflecting the views of the New York City-Rand Institute.

I. A MODEL OF COURT CALENDARING

Each time a case appears in court there are two possible outcomes: either the case is disposed of (e.g., the case may be dismissed or the defendant may plead guilty and be sentenced), or the case is "adjourned" (sometimes called "continued") to a future date (i.e., placed on a future day's calendar). Most commonly, the future date is selected on the basis of the availability of all the necessary parties (e.g., the arresting police officer, witnesses, counsel, etc.). An analytical model which reflects this process is presented below.

An alternative view of court scheduling is that in which one specifies in advance the (maximum) number of cases to be scheduled each day (the maximum "calendar size"). Then, cases would be adjourned to the earliest date not yet "filled," consistent with the desires of the various parties. Many courts do, in fact, attempt to operate in such a manner. However, in large, congested courts, calendars have traditionally been subject to very imperfect controls. In the New York City Criminal Court, for example, the numbers of cases allowed on daily calendars were for a long time gradually increased as judges and clerks attempted to deal with rising case-loads by squeezing additional cases onto already crowded calendars. Further, in large, complex courts the calendar in a given portion of the court on a given day typically consists not only of cases previously adjourned in the portion of the court in question, but also of cases transferred to that portion of the court from other portions. Lacking access to up-to-the-minute information on the status of future calendars throughout the court, judges tend to select future dates primarily on the basis of the length of time until the next appearance.

In short, calendars are typically the product of adjournment policies, rather than vice versa. Eventually the availability of real-time computer systems with on-line access to judges and calendaring clerks may facilitate the establishment and maintenance of "optimum" calendar sizes. However, for the present, a model driven by adjournments seems more appropriate. Furthermore, even in cases where it is possible to impose strict limits on court calendars, one would need to determine whether the adjournment process -- unchecked -- would in fact tend to exceed those limits. Hence, our model of the adjournment process.

The structure of the model is shown diagrammatically in Figure 1. As indicated, we consider a court which is divided into a number of "parts" -- a court part being defined as a courtroom and all the staff necessary to conduct the business of the court. New cases entering the court pass through an arraignment stage first. During this first appearance, the defendant is informed of the charges against him. In some cases, these charges may be immediately dismissed and the defendant released. Or, if the defendant's lawyer is present and wishes to proceed, formal processing of the case may begin. If such processing is completed at the first appearance, the case is disposed of. If the processing of the case is not begun or is not completed, bail conditions are set and the case is adjourned for appearance on a future date in a hearing or trial part (HT part). Such parts may perform different, specialized functions, or each may perform the same functions. (While we shall assume the hearing and trial part(s) to be distinct from the arraignment part(s), the analysis below can be easily modified to allow other arrangements.) At each subsequent appearance, the case may be disposed of or it may be adjourned back to a hearing or trial part.

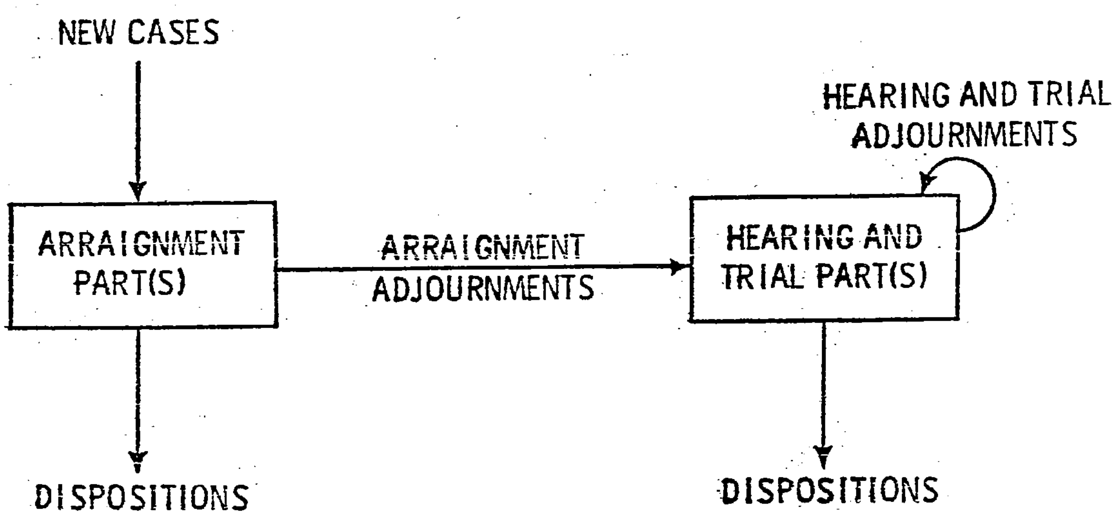

FIGURE 1. COURT MODEL

The remainder of this paper will deal only with the HT (hearing or trial) parts.

Major Assumptions

In order to facilitate the analysis of the model, the following simplifying assumptions are made:

1. Cases are indistinguishable from each other as far as dispositions and adjournments are concerned.

2. Numbers of cases are not restricted to integers. (This assumption should not significantly detract from the usefulness of the results, particularly when the volume of cases under consideration is large.)

3. Of those cases not disposed of at arraignment, a fixed fraction is adjourned to each HT part. While the fractions assigned to the various HT parts must add to unity, they need not be equal, and some may be zero.

4. Of those cases appearing on the calendar of any HT part on any day, a deterministic fraction is disposed of. The fraction need not be the same for any two parts. Initially we shall assume that each of these fractions is fixed; in portions of the analysis we shall allow each such fraction to depend on the number of cases on the day's calendar. All cases not disposed of are adjourned, and an additional appearance is scheduled.

5. Of those cases adjourned in any HT part, a fixed fraction is adjourned back to the same part and a fixed fraction is assigned to each other HT part. Again, these fractions must add to unity, need not be equal, and may include zeros.

6. The selection of a date on which an adjourned case is next scheduled

to appear is made without regard to the number of cases previously scheduled to appear. Rather, of those cases being adjourned at arraignment to an HT part or from one HT part to another (or back to itself), a fixed fraction is scheduled to appear on the following day, a fixed fraction on the second day hence, and so on. The set of fractions which apply to any combination of originating part and adjournment part must add to one but may include zeros and need not be equal to the set corresponding to any other combination of parts.

Notation

The notation to be used in the analysis which follows is summarized in Table I.

In order to assist the reader in distinguishing the equations which represent significant results from those which are included merely to indicate the manner in which the former are derived, the most significant equations are underscored.

Table I

NOTATION

a_{ij}	= adjournment time for cases adjourned from HT part i to HT part j
α_i	= adjournment time for cases adjourned from the arraignment part(s) to HT part i
$b_i(t)$	= number of cases as of the end of day t on all future calendars in HT part i
$B(t)$	= m-element row-vector of which the ith element is $b_i(t)$
$b(t)$	= number of cases as of the end of day t on all future calendars in all HT parts
$c_i(t)$	= number of cases on the calendar of HT part i on day t
$C(t)$	= m-element row-vector of which the ith element is $c_i(t)$
$c_i^T(z)$	= geometric transform[(a)] of $c_i(t)$
$C^T(z)$	= m-element row-vector of which the ith element is $c_i^T(z)$
$c_i(\tau,t)$	= number of cases as of the end of day t scheduled to appear in HT part i on day $t + \tau$
$c_i^{TT}(y,z)$	= double geometric transform[(b)] of $c_i(\tau,t)$
$C_i^{T\cdot}(y,t)$	= partial geometric transform (on the first variable)[(c)] of $c_i(\tau,t)$

(a) The geometric transform (as a function of z) of a variable $x(t)$, where t is an integer variable, will be written $x^T(z)$ and is defined by the equation

$$x^T(z) = \sum_{t=0}^{\infty} z^t x(t).$$

(b) The double geometric transform of a function $x(\tau,t)$ of two integer variables τ and t will be written $x^{TT}(y,z)$ and is defined by the equation

$$x^{TT}(y,z) = \sum_{\tau=0}^{\infty} \sum_{t=0}^{\infty} y^\tau z^t x(\tau,t).$$

(c) The partial geometric transform (on the first variable) of a function $x(\tau,t)$ will be written $x^{T\cdot}(y,t)$ and is defined by the equation

$$x^{T\cdot}(y,t) = \sum_{\tau=0}^{\infty} y^\tau x(\tau,t).$$

Table I (continued)

$C^{TT}(y,z)$ = m-element row-vector of which the ith element is $c_i^{TT}(y,z)$

$C^{T\cdot}(y,t)$ = m-element row-vector of which the ith element is $c_i^{T\cdot}(y,t)$

f_i = fraction of cases disposed of in HT part i on any day

g_i = fraction of cases adjourned in HT part i on any day

I = identity matrix

$\bar{k}_i$ = average number of appearances made by any case in HT part i

$\bar{k}$ = average total number of appearances made by any case in all parts

m = number of HT parts

$M(z)$ = (mxm) matrix of which the (i,j)th element is $g_i P_{ij} Pa_{ij}^T(z)$

$n(t)$ = number of cases adjourned in the arraignment part(s) on day t

$n^T(z)$ = geometric transform of n(t)

$N(z)$ = m-element row-vector of which the ith element is $\rho_i\ P\alpha_i^T(z)$

P_{ij} = fraction of cases adjourned in HT part i that are assigned to HT part j

$Pa_{ij}(t)$ = fraction of cases for which a_{ij} equals t

$Pa_{ij}^T(z)$ = geometric transform of $Pa_{ij}(t)$

ρ_i = fraction of cases adjourned in the arraignment part(s) that are assigned to HT part i

$P\alpha_i(t)$ = fraction of cases for which α_i equals t

$P\alpha_i^T(t)$ = geometric transform of $P\alpha_i(t)$

s = index of time

t = index of time

τ = index of time

$\bar{T}$ = average total duration of any case

II. ANALYSIS OF CALENDAR SIZES

The General Case

Let us begin by writing an expression for the number of cases that will appear on the calendar (will be heard) on some day t ($t \geq 0$) in HT part j (j may be any integer from 1 to m). We shall refer to this number as the "size of the calendar" in HT part j on day t. This number will be composed of three groups of cases:

(a) Those which were already scheduled to appear on day t in part j prior to the start of the period under consideration. If we define the period of interest to begin on day zero ($t = 0$), the initial conditions which must be specified (i.e., the number of cases scheduled prior to day zero to appear in HT part j on day t) may be written in our notation as

$$c_j(t+1,-1).$$

(b) Those cases which were arraigned on or before day t, were adjourned, and were scheduled to have their first appearance in an HT part on day t in part j. The number of these cases may be written

$$\sum_{s=0}^{t} n(s)\rho_j P\alpha_j(t-s).$$

(c) Those cases which appeared in an HT part on or before day t, were adjourned, and were scheduled to have their next appearance in HT part j and on day t. This number may be written

$$\sum_{i=1}^{m} \sum_{s=0}^{t} c_i(s) g_i P_{ij} Pa_{ij}(t-s).$$

Putting these terms together, we have the equation

$$c_j(t) = c_j(t+1,-1) + \sum_{s=0}^{t} n(s)\rho_j P\alpha_j(t-s) + \sum_{i=1}^{m} \sum_{s=0}^{t} c_i(s) g_i P_{ij} Pa_{ij}(t-s).$$

Now, let us take the geometric transform of each side:

$$c_j^T(z) = \sum_{t=0}^{\infty} z^t c_j(t+1,-1) + \sum_{t=0}^{\infty} z^t \sum_{s=0}^{t} n(s)\rho_j P\alpha_j(t-s)$$

$$+ \sum_{t=0}^{\infty} z^t \sum_{i=1}^{m} \sum_{s=0}^{t} c_i(s) g_i P_{ij} Pa_{ij}(t-s).$$

This expression may be simplified to yield the result:

$$c_j^T(z) = \frac{1}{z} c_j^{T\cdot}(z,-1) + \rho_j P\alpha_j^T(z) n^T(z) + \sum_{i=1}^{m} g_i P_{ij} Pa_{ij}^T(z) c_i^T(z). \tag{1}$$

Equation (1) applies to each HT part j (j=1,2,..., m) and is now in a form which can readily be expressed in vector and matrix notation, thereby yielding the relationships for every HT part in a single equation. Using the symbols defined earlier, we may rewrite equation (1):

$$C^T(z) = \frac{1}{z} C^{T\cdot}(z,-1) + n^T(z)N(z) + C^T(z) \cdot M(z). \tag{2}$$

This equation may be manipulated to yield an expression for $C^T(z)$ solely in terms of known functions:

$$\underline{C^T(z) = [\frac{1}{z} C^{T\cdot}(z,-1) + n^T(z)N(z)] \cdot [I - M(z)]^{-1}.} \tag{3}$$

Recall that $C^{T\cdot}(z,-1)$ expresses the initial conditions, $n^T(z)$ the flow of new cases adjourned at arraignment, N(z) the fraction of new cases adjourned to each HT part and to each future date, and M(z) the fraction of cases adjourned from each HT part to itself and to each other part on each future date.

Thus, given numerical values for all of these variables and parameters, equation (3) could be solved to yield the transform of the daily calendar size in each HT part. These transforms could then be inverted to yield the actual number of cases to appear on the calendar of each HT part on each day beginning with day zero.

Let us now use equation (3) to investigate the behavior of the daily calendar size after the system has been operating for a very long time (i.e., the system is in "steady state"). To do this, let us assume that the total number of cases adjourned at arraignment each day approaches a constant; in other words, the limit

$$n = \lim_{t\to\infty} n(t)$$

exists. With this assumption, we may directly apply the final value theorem of geometric transforms, which may be stated as:

$$C(\infty) = \lim_{z\to 1} (1-z)C^T(z).$$

Applying this theorem to equation (3), we get

$$C(\infty) = \left(\lim_{z\to 1} (1-z)C^{T\cdot}(z,-1) + \lim_{z\to 1} (1-z)n^T(z)N(z)\right) \cdot \left(I - \lim_{z\to 1} M(z)\right)^{-1}$$

$$= [C(\infty,-1) + nN(1)] \cdot [I - M(1)]^{-1}.$$

If we assume that the initial conditions apply to only a finite period into the future (i.e., that adjournment periods are finite), then

$$C(\infty,-1) = 0,$$

and we obtain

$$C(\infty) = nN(1) \cdot [I - M(1)]^{-1}.$$

Or, writing out the vector N(1) and the matrix M(1), we have

$$C(\infty) = n\left(\rho_1, \rho_2, \ldots, \rho_m\right) \begin{pmatrix} 1-g_1P_{11} & -g_1P_{12} & \cdots & -g_1P_{1m} \\ -g_2P_{21} & 1-g_2P_{22} & \cdots & -g_2P_{2m} \\ \vdots & & & \vdots \\ -g_mP_{m1} & -g_mP_{m2} & \cdots & 1-g_mP_{mm} \end{pmatrix}^{-1}.$$

Thus, the "steady state" calendar size in each HT part is a constant determined by the total daily number of cases adjourned at arraignment, the fraction assigned to each HT part, and the fraction adjourned from each HT part to each other HT part.

<u>The Special Case: Identical HT Parts</u>

Let us now see what simplifications can be made in the expressions for $C^T(z)$ and $C(\infty)$ for the special case in which all the HT parts are identical. Specifically, "identical" will here mean that (for $m \geq 2$)

$$\rho_i = \frac{1}{m}, \quad \text{for } i=1,2,\ldots, m;$$

$$P\alpha_i^T(z) = P\alpha_1^T(z), \quad \text{for } i=1,2,\ldots, m;$$

$$C_i^{T\cdot}(z,-1) = C_1^{T\cdot}(z,-1), \quad \text{for } i=1,2,\ldots, m;$$

$g_i = g_1$, for $i=1,2,\ldots, m$;

$P_{ii} = P_{11}$, for $i=1,2,\ldots, m$;

$P_{ij} = \frac{1-P_{11}}{m-1}$, for $j\neq i$, $i=1,2,\ldots, m$;

$Pa^T_{ii}(z) = Pa^T_{11}(z)$, for $i=1,2,\ldots, m$;

$Pa^T_{ij}(z) = Pa^T_{12}(z)$, for $j\neq i$, $i-1,2,\ldots, m$.

Using these relationships and equation (1), we may write:

$$c^T_j(z) = \frac{1}{z}\, c^{T\cdot}_1(z,-1) + \frac{1}{m}\, P\alpha^T_1(z)n^T(z) + g_1P_{11}Pa^T_{11}(z)c^T_j(z)$$

$$+ \sum_{\substack{i=1 \\ i\neq j}}^{m} g_1\, \frac{1-P_{11}}{m-1}\, Pa^T_{12}(z)c^T_i(z).$$

Observing (or proving) that our definition of "identical" HT parts requires that all the $c^T_j(z)$'s be equal, we may derive the transform of the daily calendar size in each HT part i on each future date:

$$\underline{c^T_i(z) = \frac{\frac{1}{m}\, n^T(z)P\alpha^T_1(z) + \frac{1}{z}\, c^{T\cdot}_1(z,-1)}{1-g_1P_{11}Pa^T_{11}(z)-g_1(1-P_{11})Pa^T_{12}(z)}}\quad , \text{ for } i = 1,2,\ldots, m. \qquad (4)$$

Further, in order to determine the steady-state calendar size in each HT part, we may apply the final value theorem as before, yielding:

$$\underline{c_i(\infty) = \frac{n/m}{1-g_1}}\quad \text{for } i = 1,2,\ldots, m. \qquad (5)$$

Recall that equations (4) and (5) apply to the special case of identical HT parts. As we shall see below, equation (5) could have been written directly on the basis of an intuitive argument. However, its derivation from the general formulas provides a check on their validity.

After a brief survey of the use of the model in the analysis of future calendar sizes (case backlogs) and case appearances and duration, we shall return to a study of the relationship between the steady-state calendar size and the disposition fraction.

III. ANALYSIS OF CASE BACKLOGS

So far we have considered the size of each HT part's calendar only on the day on which the calendared cases are actually heard. However, since each part adjourns cases into the future, establishing backlogs, it is of interest to examine the behavior of future calendars. Below, we develop expressions for the number of cases, $c_j(\tau,t)$, which have been scheduled by the end of day t to appear in HT part j on day $t+\tau$, and also for the total number of cases, $b_j(t)$, scheduled by the end of day t to appear in HT part j on all days in the future.

We begin by writing an incremental relationship for $c_j(\tau,t)$ to express the fact that the number of cases scheduled as of some day for a particular date in the future equals the number scheduled as of the previous day for that date plus the number of cases added during the most recent day:

$$c_j(\tau,t) = c_j(\tau+1,t-1) + n(t)\rho_j P\alpha_j(\tau) + \sum_{i=1}^{m} c_i(t)g_i P_{ij} Pa_{ij}(\tau).$$

This equation applies for any $t \geq 0$ and any $\tau \geq 0$; the initial conditions which must be specified are the values of $c_j(\tau,-1)$ for each part j and each day $(\tau-1)$.

We now take the double geometric transform of both sides of this equation:

$$\begin{aligned} c_j^{TT}(y,z) &= \sum_{t-0}^{\infty} \sum_{\tau=0}^{\infty} y^\tau z^t c_j(\tau,t) \\ &= \sum_{t=0}^{\infty} \sum_{\tau=0}^{\infty} y^\tau z^t c_j(\tau+1,t-1) + \sum_{t=0}^{\infty} \sum_{\tau=0}^{\infty} y^\tau z^t n(t)\rho_j P\alpha_j(\tau) \\ &\quad + \sum_{t=0}^{\infty} \sum_{\tau=0}^{\infty} y^\tau z^t \sum_{i=1}^{m} c_i(t) g_i P_{ij} Pa_{ij}(\tau). \end{aligned} \tag{6}$$

Noting that, by definition,

$$c_j(0,-1) = 0 \quad \text{and} \quad c_j(0,t) = c_j(t),$$

we may simplify equation (6) and express the result in matrix notation:

$$C^{TT}(y,z) = \frac{1}{y-z}\left[C^{T\cdot}(y,-1) - zC^{T}(z) + yn^{T}(z)N(y) + yC^{T}(z) \cdot M(y)\right]$$

To put this expression in a more useful form, we may use the expression for $C^T(z)$ derived earlier as equation (2) to yield:

$$\underline{C^{TT}(y,z) = \frac{1}{y-z}\left\{C^{T\cdot}(y,-1) - C^{T\cdot}(z,-1) + n^{T}(z)[yN(y) - zN(z)]\right.}$$

$$\underline{\left. + C^{T}(z) \cdot [yM(y) - zM(z)]\right\}.} \qquad (7)$$

Equation (7) gives us our desired result: an expression for (the double transform of) the future calendar sizes in each part on each day in the future, in terms of known quantities (the vector of actual calendar sizes, $C^T(z)$, may be found from equation (3)).

Now, let us use equation (7) to derive the expression for future calendars after the system has been operating for a very long time ("is in the steady state"): $C^{T\cdot}(y,\infty)$. To do this, we use the final value theorem used earlier:

$$C^{T\cdot}(y,\infty) = \lim_{z\to 1} (1-z)C^{TT}(y,z).$$

In component form, the result becomes:

$$c_j^{T\cdot}(y,\infty) = \rho_j n \frac{1-yP\alpha_j^T(y)}{1-y} + \sum_{i=1}^{m} g_i P_{ij} c_i(\infty) \frac{1-yPa_{ij}^T(y)}{1-y} . \qquad (8)$$

Finally, the transforms in equation (8) may be inverted (by recognition) to yield the same result in the time domain:

$$c_j(\tau,\infty) = \rho_j n\left(1 - \sum_{s=0}^{\tau-1} Pa_j(s)\right) + \sum_{i=1}^{m} g_i P_{ij} c_i(\infty)\left(1 - \sum_{s=0}^{\tau-1} Pa_{ij}(\tau)\right) .$$

As one might expect, the number of cases scheduled on future days is found to be monotonically decreasing with the length of time into the future, τ.

Now, let us derive corresponding expressions for the total backlog of cases scheduled to appear in each part. By definition,

$$b_j(t) = \sum_{\tau=1}^{\infty} c_j(\tau,t).$$

Taking the geometric transform of both sides and manipulating, we obtain:

$$b_j^T(z) = c_j^{TT}(1,z) - c_j^T(z);$$

or, in vector form,

$$B^T(z) = C^{TT}(1,z) - C^T(z).$$

Now, if we substitute the expression derived in equation (7) for $C^{TT}(1,z)$ and that in equation (2) for $C^T(z)$, we obtain:

$$B^T(z) = \frac{1}{1-z}\left\{C^{T\cdot}(1,-1) - \frac{1}{z}\, C^{T\cdot}(z,-1) + n^T(z)[N(1) - N(z)]\right.$$

$$\left. + C^T(z) \cdot [M(1) - M(z)]\right\} . \qquad (9)$$

Writing equation (9) in component form, we obtain the desired expression for the transform of the backlog in each part j:

$$b_j^T(z) = \frac{c_j^{T\cdot}(1,-1) - \frac{1}{z} c_j^{T\cdot}(z,-1)}{1-z} + \rho_j n^T(z) \frac{1-P\alpha_j^T(z)}{1-z}$$

$$+ \sum_{i=1}^{m} g_i P_{ij} c_i^T(z) \frac{1-Pa_{ij}^T(z)}{1-z} .$$

Finally, we may apply the final value theorem to derive the steady state backlog:

$$b_j(\infty) = \lim_{z\to 1} (1-z) b_j^T(z)$$

$$= \rho_j n \lim_{z\to 1} \frac{1-P\alpha_j^T(z)}{1-z} + \sum_{i=1}^{m} g_i P_{ij} c_i(\infty) \lim_{z\to 1} \frac{1-Pa_{ij}^T(z)}{1-z} . \tag{10}$$

Using L'Hopital's rule to evaluate the limits in equation (10), and defining the mean adjournment times $\overline{\alpha}_j$ and $\overline{a}_{ij}$ as

$$\overline{\alpha}_j = \sum_{t=0}^{\infty} t\, P\alpha_j(t),$$

and

$$\overline{a}_{ij} = \sum_{t=0}^{\infty} t\, Pa_{ij}(t),$$

we obtain the desired expression:

$$b_j(\infty) = \rho_j n\overline{\alpha}_j + \sum_{i-1}^{m} g_i P_{ij} c_i(\infty)\overline{a}_{ij}.$$

Lastly, if we define weighted average adjournment times $\overline{\alpha}$ and $\overline{a}_i$ as

$$\overline{\alpha} = \sum_{j=1}^{m} \rho_j \overline{\alpha}_j$$

and

$$\overline{a}_i = \sum_{j=1}^{m} P_{ij}\overline{a}_{ij},$$

we may write the total steady state backlog in all HT parts as

$$b(\infty) = n\overline{\alpha} + \sum_{i=1}^{m} g_i c_i(\infty)\overline{a}_i.$$

As in the case of actual calendar sizes, we could simplify these expressions for various special cases - for example, the case in which all the HT parts are identical.

IV. ANALYSIS OF CASE APPEARANCES AND CASE DURATION

Up to this point, we have been concerned with the flow of cases from the point of view of the court. Let us now briefly examine the model from the point of view of a randomly selected case.

From this point of view, our model becomes probabilistic, rather than deterministic. Thus, for example, a case which is not disposed of at arraignment is assigned to HT part j with probability ρ_j; the probability that the time between arraignment and appearance in part j is t days is given by $P\alpha_j(t)$; and so on. Since we have assumed that the cases on a calendar are indistinguishable from each other and that the action taken on each is independent of each case's past history, our model falls into the class of probabilistic models known as discrete state, discrete time semi-Markov processes.

A Markov process of this type may be conveniently represented in terms of a "transition diagram" which depicts the various "states" in which each case may be, together with the probabilities with which the case may move from one state to another. Such a diagram for a court composed of two HT parts is shown in Figure 2. Here there are four states in which the case may be: "arraignment," either HT part, and "disposition;" each allowable transition is shown as a branch from one state to another, labeled with the appropriate probability. The only additional information needed to describe the process is the time distribution associated with

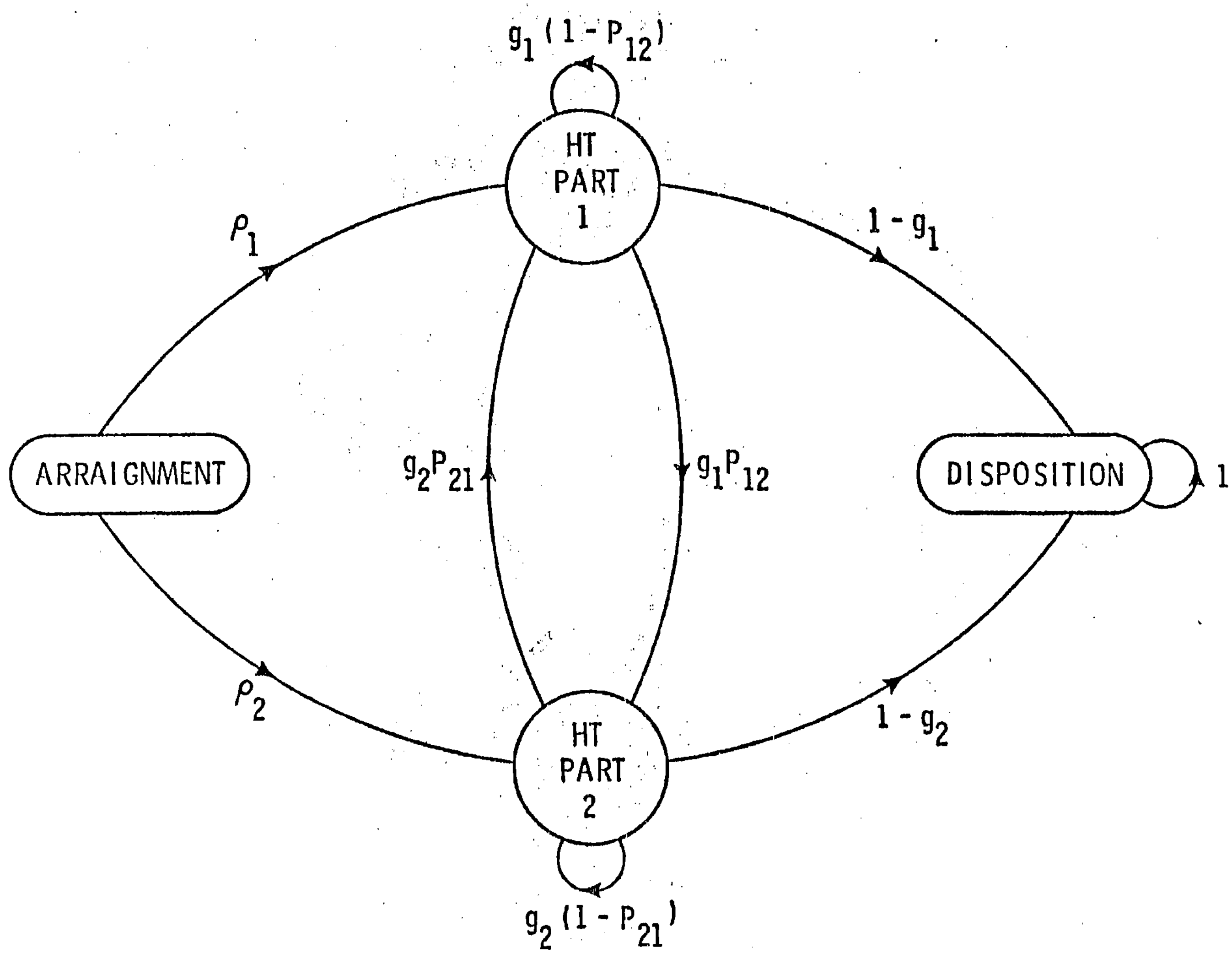

FIGURE 2. SAMPLE TRANSITION DIAGRAM

each "ρ_j" branch is the time distribution $P\alpha_j(t)$; with each "$g_i P_{ij}$" branch is the distribution $Pa_{ij}(t)$; transitions to the "disposition" state are instantaneous.

This information provides the complete description of the semi-Markov model, and well known results of the theory of Markov processes may be applied to derive expressions for such quantities as the average number of appearances per case until disposition, the average duration of the case, the probability distributions for these quantitites, and so on. For the general case, however, the form of the equations which result add little to one's understanding of the operations of court systems. Further, the general formulas can often be replaced with simpler procedures for special cases.

We shall, therefore, limit ourselves to a brief examination of the special case in which all the HT parts are identical. Here, if we do not need to distinguish between parts, we can simplify the transition diagram to that shown in Figure 3. Now, each new case makes one appearance at arraignment, followed by one or more appearances in the HT parts until it is disposed of. At each appearance in an HT part there is a probability g_1 that the case is adjourned for another appearance, and a probability $(1-g_1)$ that the case is disposed of. Thus, we may directly express the average total number of court appearances, $\overline{k}$, as

$$\overline{k} = 1 + \frac{1}{1-g_1} \quad .$$

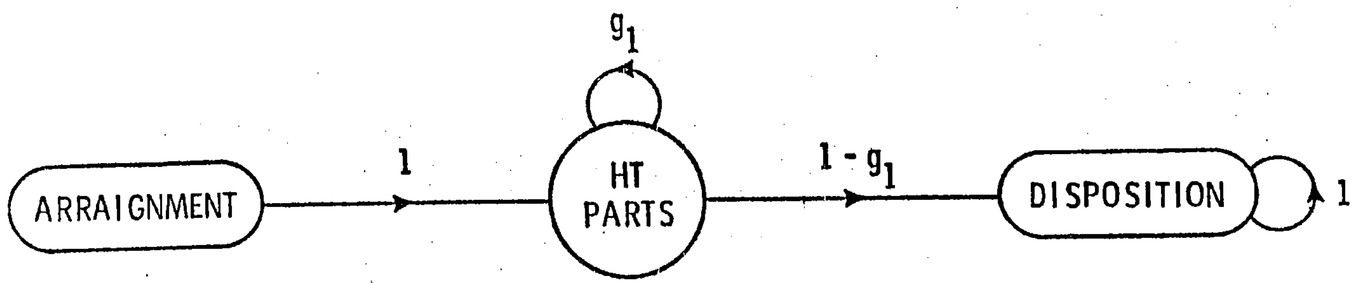

FIGURE 3. TRANSITION DIAGRAM FOR IDENTICAL HT PARTS

Further, the average number of appearances in each HT part, $\overline{k}_j$, must be

$$\overline{k}_j = \frac{1/m}{1-g_1} \ .$$

Finally, it is a simple matter to express the <u>average</u> total duration of a case, $\overline{T}$, in terms of the average number of appearances in each HT part. For even the <u>general</u> case, it must be true that (in days)

$$\overline{T} = 1 + \sum_{i=1}^{m} \rho_i \ \overline{\alpha}_i + \sum_{i=1}^{m} \overline{k}_i \sum_{j=1}^{m} g_i P_{ij} \overline{a}_{ij}$$

$$= 1 + \overline{\alpha} + \sum_{i=1}^{m} \overline{k}_i g_i \overline{a}_i .$$

For the <u>special</u> case of identical HT parts, we may use the expression for $\overline{k}_j$ given in equation (11) to yield

$$\overline{T} = 1 + \overline{\alpha} + \frac{g_1}{1-g_1} \overline{a}_1 .$$

VI. CALENDAR EQUILIBRIUM WHEN THE FRACTION OF CASES DISPOSED OF IS A FUNCTION OF THE CALENDAR SIZE

We now consider the case in which the fraction of cases disposed of in each HT part on any day may depend on the number of cases heard that day. This situation has particular practical value since the role of a judge in an HT part is, in practice, affected by the size of his calendar. For example, when the calendar is very light, the judge is able to devote a relatively large amount of time and attention to each case and encourage the completion of the steps necessary for disposition. On the other hand, when the calendar is particularly heavy, the time and attention which the judge can devote to each case is limited; and the fastest way to clear cases from a particular day's calendar is simply to adjourn them to future dates. Thus, as a judge's time is taken up with more and more administrative and clerical duties related to "moving" cases on the calendar, one should expect the fraction of cases disposed of to fall. There does, in fact, exist tentative evidence to support this view.

POSSIBLE FORMS OF THE RELATIONSHIP BETWEEN CALENDAR SIZE AND DISPOSITION FRACTION

With regard to the form of the relationship between the total number of cases disposed of and the calendar size, there are four major possibilities, as shown in Figure 4. (We continue to assume that all relationships are deterministic. A more realistic view might be to consider the relationship between the calendar size and the <u>average</u> number or fraction disposed of, allowing some variability from day to day.)

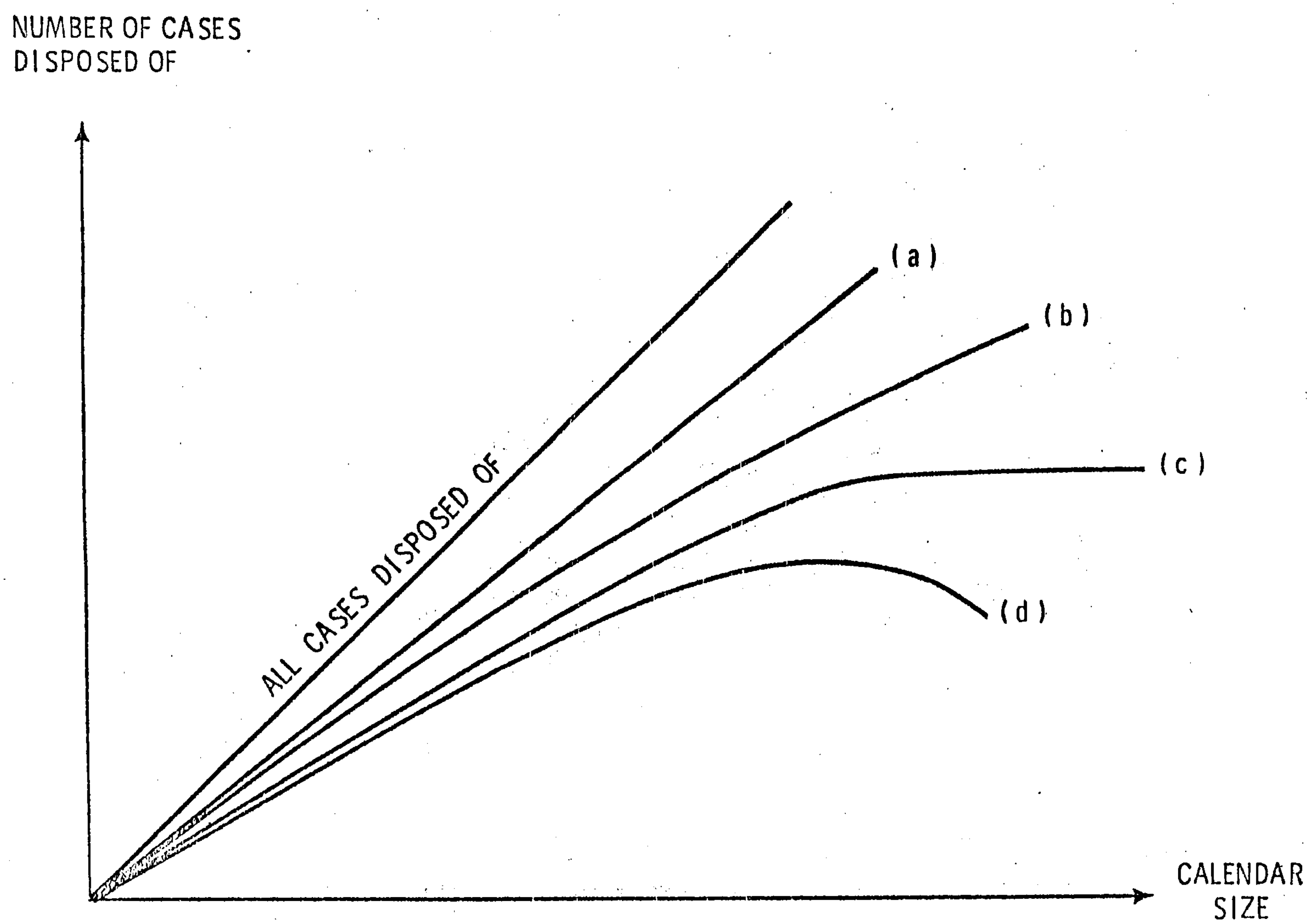

FIGURE 4. POSSIBLE RELATIONSHIPS BETWEEN CALENDAR SIZE AND NUMBER OF DISPOSITIONS PER DAY

Case (a) represents a situation in which a fixed fraction of cases is always disposed of, regardless of the calendar size. In cases (b), (c), and (d), the fraction of cases that are disposed of decreases as the calendar size increases. In case (b), it decreases in such a way that the number of cases disposed of continually increases; in case (c), the number reaches a maximum (asymptote) and remains there; in case (d), the number disposed of reaches a maximum and then falls.

Putting aside, for the moment, the form of the relationship, let us consider the "steady-state" number of cases on the calendar in a single HT part. We shall limit our discussion to the case of m identical parts and shall denote the steady-state value of the calendar size in each HT part by c^* and the steady-state value of the fraction disposed of (or adjourned) by f^* (or g^*).

Now, if a steady-state exists, the number of new cases added to the calendar in each part each day must equal the number disposed of. Thus, if c^* and f^* exist, they must obey the restriction:

$$n/m = c^*f^*,$$

or

$$f^* = \frac{n/m}{c^*} \quad .$$

Next, let us examine the steady-state condition in the case of each of the four forms suggested above for the relationship between dispositions and calendar size.

Case (a): Fixed Fraction Disposed of

In Figure 5 we have sketched a curve specifying the steady-state relationship in the c^*, f^* space, together with the relationship which gives the feasible value of the fraction disposed of as a function of the calendar size (in this case a horizontal line). We have assigned the notation f_a to the fixed fraction. Clearly, if there is a steady-state condition, the calendar size must be

$$c_a = \frac{n/m}{f_a} .$$

(For this case, we have shown earlier that the steady-state calendar size is given by equation (5), which is the same as the above.)

Note that if the calendar size is larger than c_a, the fraction disposed of will still be f_a, which is larger than the value needed to maintain the larger calendar size, and, as a result, the calendar size must fall. Similarly, if the calendar size is smaller than c_a, the fraction which can feasibly be disposed of will be "too small," and the calendar size will increase to c_a. Thus, the point (c_a, f_a) is referred to as the equilibrium point for this case.

Case (b); Decreasing Fraction Disposed of, Monotonically Increasing Number Disposed of

The feasible combinations of f^* and c^*, together with the steady-state relationship between f^* and c^*, are shown in Figure 6 for this case. In this case, the equilibrium calendar size is c_b, and the equilibrium fraction disposed of is f_b.

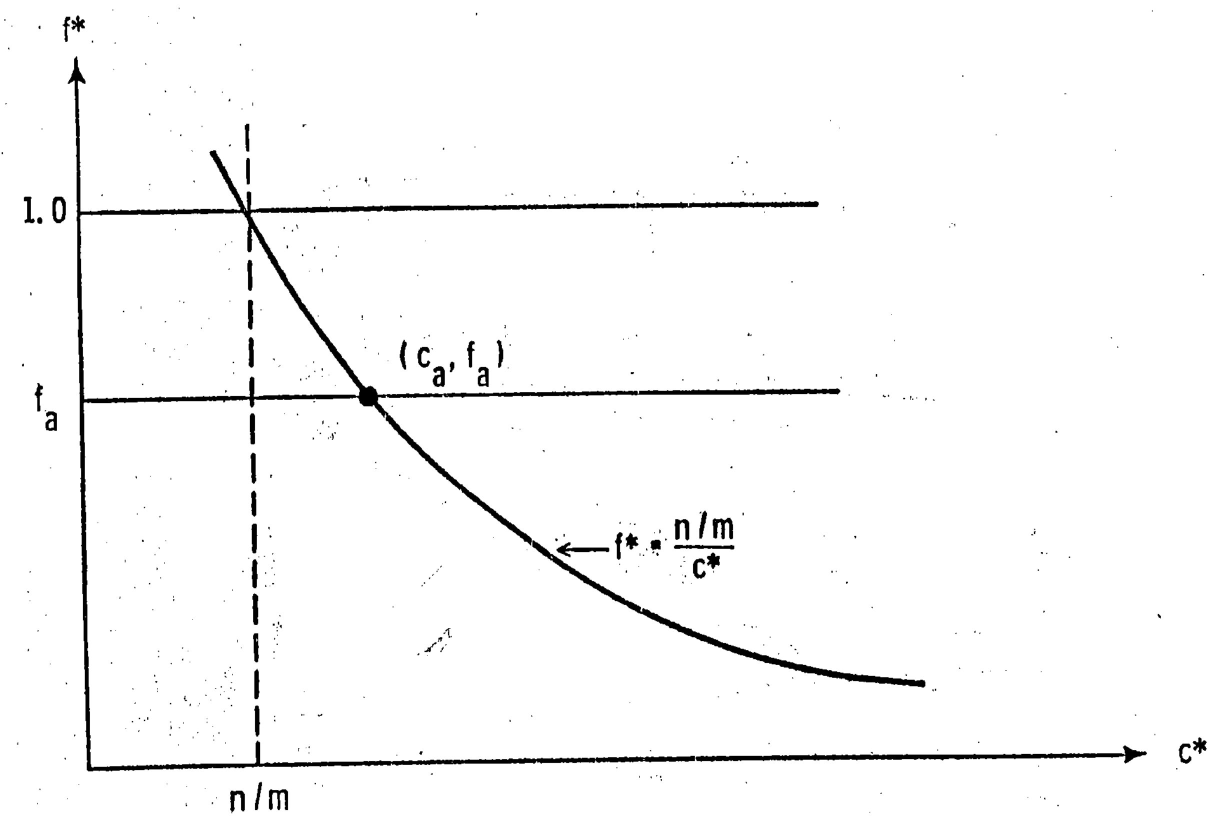

FIGURE 5. CALENDAR EQUILIBRIUM, CASE (A)

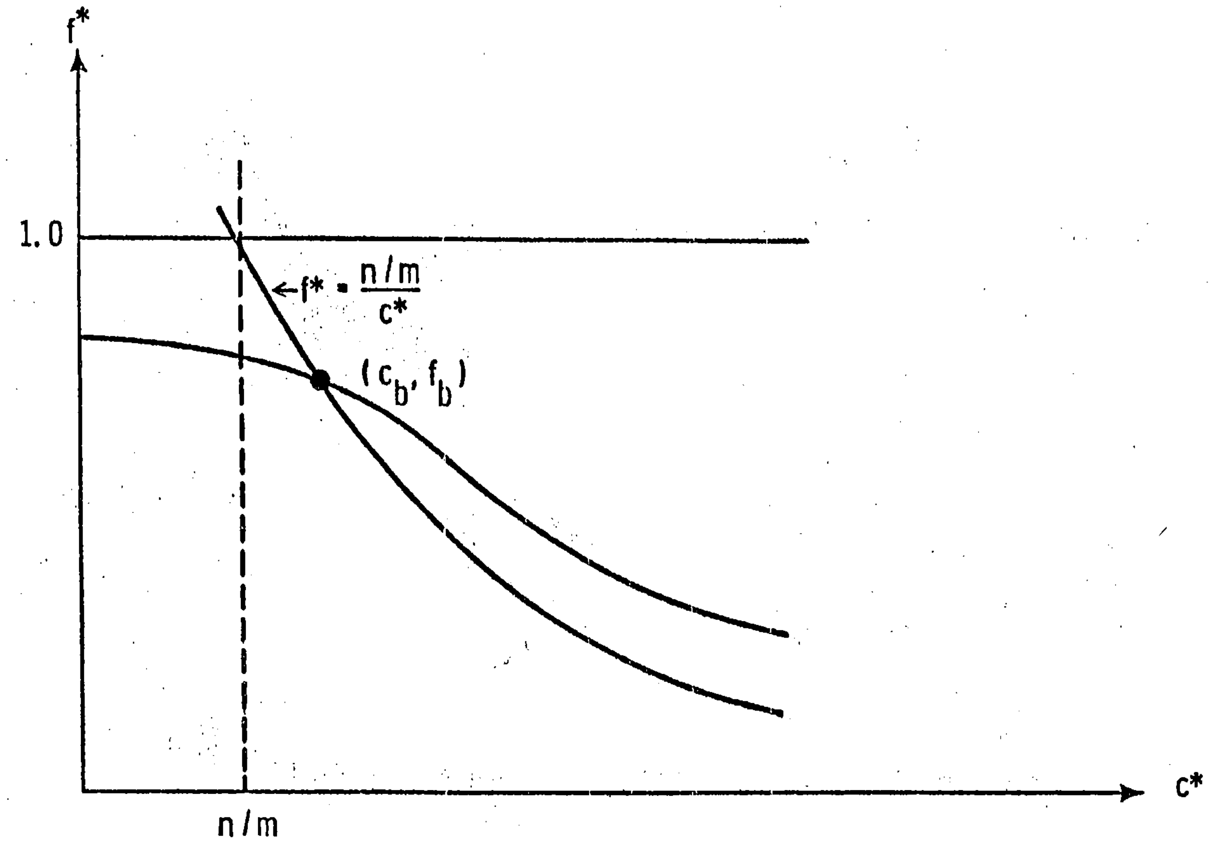

FIGURE 6. CALENDAR EQUILIBRIUM, CASE (B)

Case (c): Decreasing Fraction Disposed of; Asymptotic Number Disposed of

For this case, the feasible and steady-state relationships between the calendar size and fraction disposed of may intersect in two ways, as shown in Figure 7. Here, since the situation shown in Figure 7-1 has the same characteristics as that in Figure 6, let us confine our attention to the case shown in Figure 7-2. Here, the maximum number of cases which each part can dispose of each day exactly equals the number it must dispose of: n/m . In this case, any point (c^*, f^*) such that

$$f^* = \frac{n/m}{c^*}$$

and for which $c^* \geq \dot{c}_c$ (see the figure) is an equilibrium point.

At first glance, it may appear that it would be pure coincidence for such a case to actually occur. On the contrary, however, this case may be the rule, rather than the exception, in large urban courts. One way in which it might arise is the following: Suppose at one point in time the situation is that depicted in Figure 7-1 (or that to be discussed under case (d), below); and further suppose that the input of new cases (n) proceeds to rise faster than the number of available judges (m). Eventually, the curve representing the steady-state relationship between the calendar size and the disposition fraction will move far enough to the right that it no longer intersects the curve representing the feasible combinations of the calendar size and the fraction of cases which each judge can (tends to) dispose of. As long as this situation persists, calendar sizes will continue to rise and disposition fractions will correspondingly fall. As this condition (and the resulting rise in case backlogs) is detected, reforms of various types (including longer

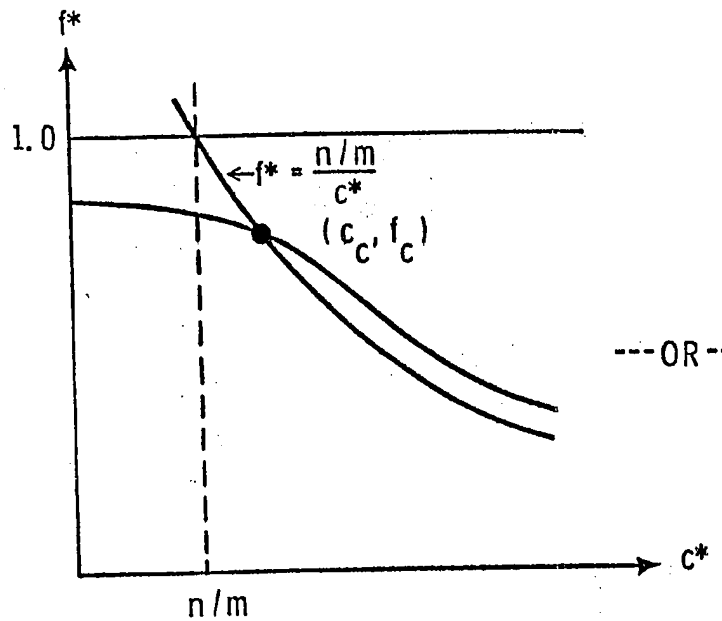

Fig. 7 - 1

---OR---

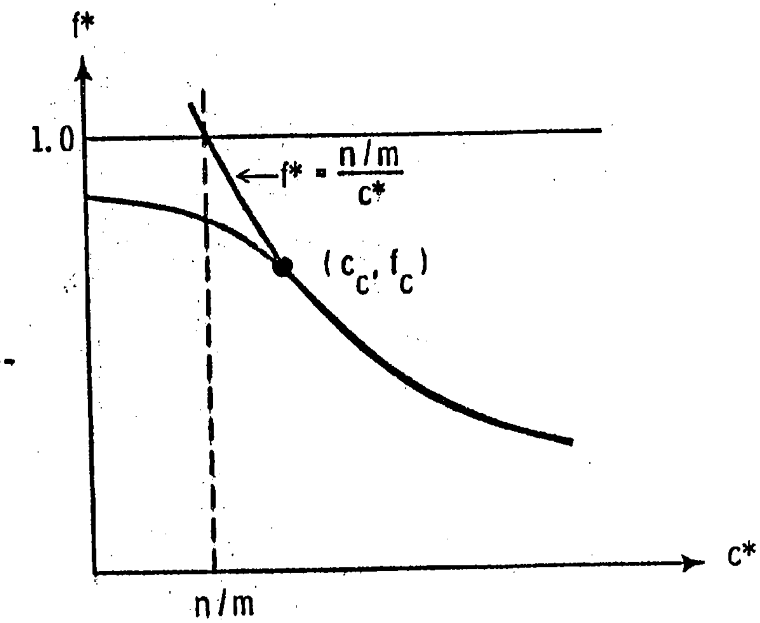

Fig. 7 - 2

FIGURE 7. CALENDAR EQUILIBRIUM, CASE (C)

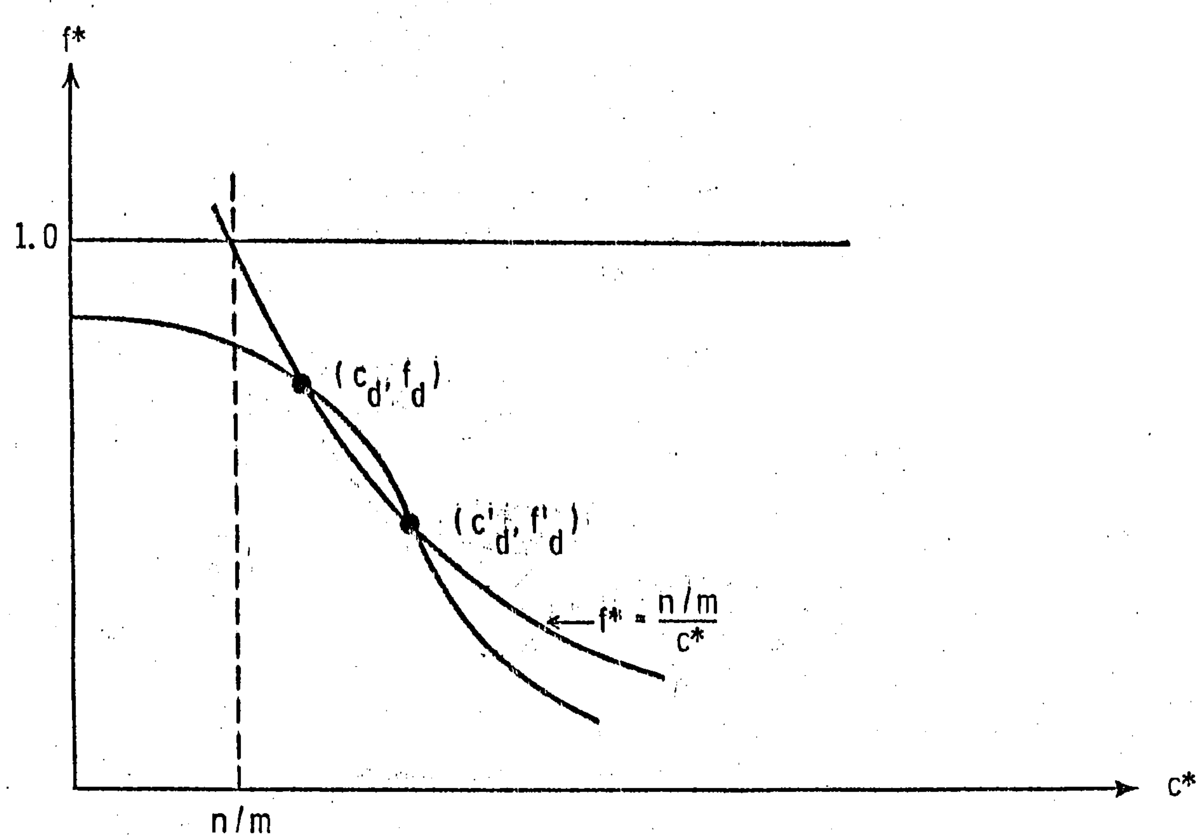

FIGURE 8. CALENDAR EQUILIBRIUM, CASE (D)

hours for the judges) may be implemented in order to raise the feasible relationship just enough so that n/m cases can be disposed of each day.

Case (d): Decreasing Fraction Disposed of;
Increasing, then Decreasing, Number Disposed of

The curve of feasible combinations of calendar size and disposition fraction is sketched, together with the steady-state relationship between these variables for this case, in Figure 8. Here, as shown, there will, in general, be two equilibrium points: (c_d,f_d) and (c_d',f_d'). If the two curves are just tangent, these two points coincide.

It should be noted, however, that point (c_d',f_d') is what is known as an unstable equilibrium point: if a system operating with this combination of calendar size and disposition fraction is disturbed in either direction, the system will tend to move away from this point and not return. Specifically, if the calendar size is artificially raised, for example, the system can never dispose of sufficient cases to keep pace with new cases, and the calendar size rises (and the disposition fraction decreases) without limit; if the calendar size falls below c_d', the system will move to (stable) equilibrium point (c_d,f_d). Thus, in theory, it would seem to be very unlikely for a court system to operate at the unstable point. However, in practice, the continual interaction between the pressures of rising caseloads and opposing reforms may act to maintain operation at such an unstable point.

IMPLICATIONS OF ALTERNATIVE EQUILIBRIA

In cases (c) and (d), above, we showed that there may exist alternative equilibria at which a court system may operate. Let us now examine the relative merits of alternative points.

Earlier we showed that, for identical HT parts in the steady-state, the average number of court appearances per case, $\bar{k}$, may be expressed as

$$\bar{k} = 1 + \frac{1}{f^*} \quad .$$

Clearly, then, operation at that equilibrium point which has the largest disposition fraction (f^*) will minimize the number of court appearances required in each case. Since unnecessary court appearances place needless burdens on all parties concerned, minimization of this quantity, other things being equal, is a desirable end.

Associated with a maximum disposition fraction is a minimum daily calendar size in each part. This property has the additional benefits of minimizing the number of persons in the individual court rooms, as well as the building as a whole, all of whose movements must be coordinated; and, at the same time, allowing the judges to devote more sustained attention to each case.

Thus, it would appear that, if two or more equilibria are feasible, the most desirable is that which provides the smallest calendars and largest disposition fractions.

MOVING FROM ONE EQUILIBRIUM POINT TO ANOTHER

Let us now consider a court system of the type described in case (c) or case (d), for which the calendar equilibria are shown in Figure 9.

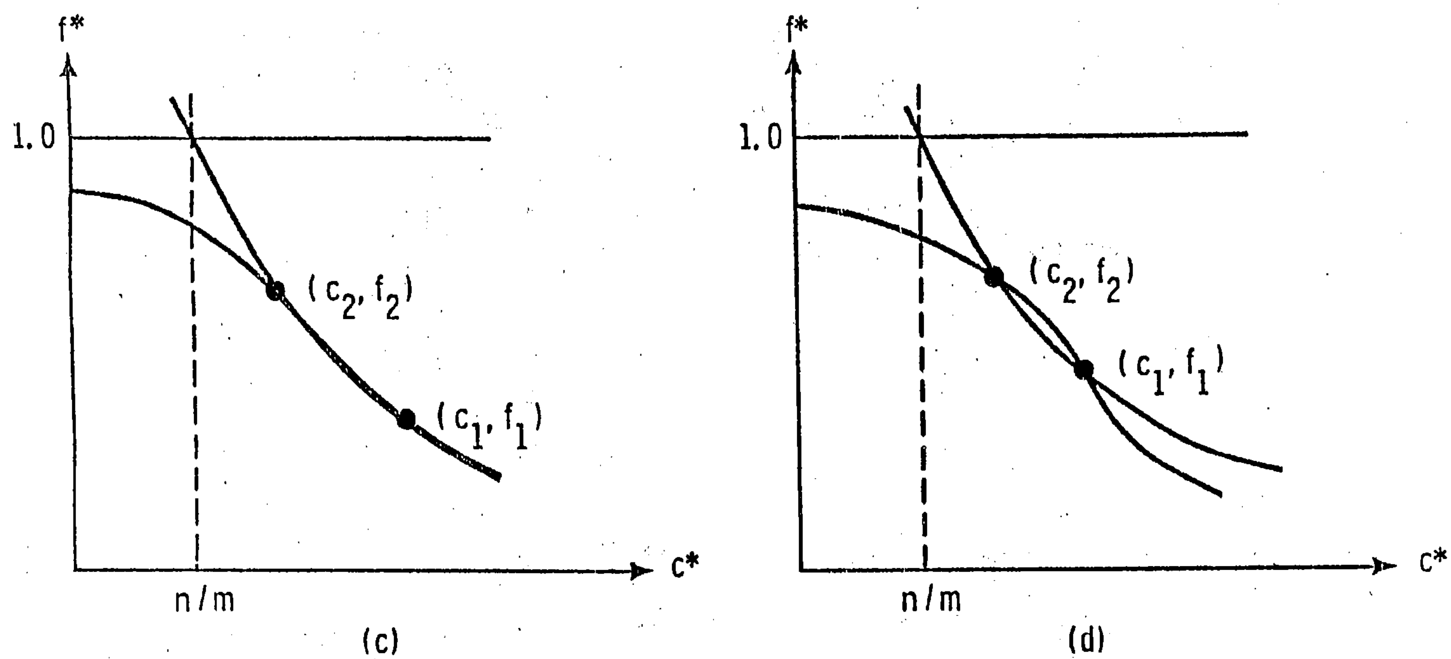

FIGURE 9. MULTIPLE CALENDAR EQUILIBRIA

Further, suppose that the system is presently operating at point (c_1,f_1) and that we would like to shift to point (c_2,f_2).

First, if the length of time between court appearances is flexible, one means of moving from point (c_1,f_1) to point (c_2,f_2) is simply to limit each part's calendar to c_2 cases. However, given a particular backlog, a reduction of the calendar size must be accompanied by an increase in the average adjournment period. Such an increase in the time between appearances is, in general, undesirable. One means of reducing the calendar size without affecting the adjournment period is through a temporary increase in the number of judges (in HT parts). This method is analyzed below.

REDUCING CALENDARS BY TEMPORARILY INCREASING THE NUMBER OF HT PARTS

To review, we are concerned with a system of m identical HT parts which, in the steady state, have come to equilibrium at the point (c_1,f_1).

At such a time, the (transform of the) future schedule of cases for any part has been given for the general case by equation (8);

$$c_j^{T\cdot}(y,\infty) = \rho_j n \frac{1-yP\alpha_j^T(y)}{1-y} + \sum_{i=1}^{m} g_i P_{ij} c_i(\infty) \frac{1-yPa_{ij}^T(y)}{1-y} . \tag{8}$$

For identical HT parts, this becomes:

$$c_j^{T\cdot}(y,\infty) = \frac{n}{m}\,\frac{1-P\alpha_1^T(y)}{1-y} + g_1 P_{11}\,\frac{n/m}{1-g_1}\,\frac{1-yPa_{11}^T(y)}{1-y}$$

$$+ \; g_1(1-P_{11})\,\frac{n/m}{1-g_1}\,\frac{1-yPa_{12}^T(y)}{1-y}\;. \qquad (12)$$

If the system were allowed to continue to operate without adjustment, the daily calendar size would continue to be c_1 cases, of which g_1c_1 would be adjourned each day. However, we wish to reduce the calendar size to $c_2 < c_1$. Suppose we do this as follows: each day we simply take those cases on the calendar that are in excess of c_2 and assign them to the new HT parts. (We could also simply increase the number of existing parts by any number of new parts and divide the cases scheduled each day equally.) Let us now determine the number of such cases which will be encountered.

If the system had been operating at (c_2,g_2), the steady-state future calendars in each part would have been

$$c_2^{T\cdot}(y,\infty) = \frac{n}{m}\,\frac{1-P\alpha_1^T(y)}{1-y} + g_2 P_{11}\,\frac{n/m}{1-g_2}\,\frac{1-yPa_{11}^T(y)}{1-y}$$

$$+ \; g_2(1-P_{11})\,\frac{n/m}{1-g_2}\,\frac{1-yPa_{12}^T(y)}{1-y} \qquad (13)$$

Therefore, the number of cases to be assigned to the new parts, $\nu^T(y)$, is simply

$$\nu^T(y) = m\left\{c_1^{T\cdot}(y,\infty) - c_2^{T\cdot}(y,\infty)\right\}.$$

Substituting from equations (12) and (13), and simplifying, we obtain:

$$\nu^T(y) = n\left\{\frac{g_1}{1-g_1} - \frac{g_2}{1-g_2}\right\}\left\{P_{11}\frac{1-yPa_{11}^T(y)}{1-y} + (1-P_{11})\frac{1-yPa_{12}^T(y)}{1-y}\right\}.$$

This transformed expression may be inverted back to the time domain by recognition, yielding

$$\nu(t) = n\left\{\frac{g_1}{1-g_1} - \frac{g_2}{1-g_2}\right\}\left\{P_{11}\sum_{s=t}^{\infty} Pa_{11}(s) + (1-P_{11})\sum_{s=t}^{\infty} Pa_{12}(s)\right\}.$$

Since this function is monotonically decreasing over time the largest number of new cases to be assigned to the new parts in a day is $\nu(0)$:

$$\nu(0) = n\left\{\frac{g_1}{1-g_1} - \frac{g_2}{1-g_2}\right\}.$$

The length of the period over which new cases are assigned to the new parts is simply the maximum adjournment period.

Now, we could, if we wished, analyze the behavior of the calendars in the new parts for any number μ thereof. However, the analysis would be identical to that already undertaken for the regular parts; the only adjustments needed would be to replace $n(t)$ with $\nu(t)$, to replace m with μ, and to use zeros for the initial conditions.

Another alternative would be to adjust the number of new parts in such a way that the daily calendar size in each is c_2, the same as in the regular parts. In such a case, the new parts would not have to be isolated from the regular parts -- every part could adjourn cases to every other part. The number of new parts needed each day could be analyzed using the equations developed earlier on a day by day basis.

VI. SUMMARY

This paper presents a simple model of court calendars and adjournments. The model is based on an adjournment process which schedules case appearances in accordance with deterministic distributions of inter-appearance times and deterministic distributions which specify the way in which cases are transferred from one part of the court to another. While the sizes of the resulting calendars are allowed to vary, the deterministic nature of the model eventually produces a "steady-state condition" in which calendar sizes and other quantities approach constants.

The analysis of the model is divided into four parts:

In Section II the size of the daily calendar in each part of the court is analyzed. The results include expressions which make it possible to calculate the size of the daily calendar in each part on each day, and the steady-state size of the calendar in each part. These expressions are developed first for the general case and are then simplified for the special case in which all parts of the court (excluding the arraignment, or intake, part(s)) are identical. One of the most useful expressions is that for the steady-state calendar size in the case of identical parts. Representing the calendar size by $c_i(\infty)$, the daily number of new cases by n, the number of parts of the court (or judges) by m, and the fraction of cases not disposed of in each part each day by g, it is shown that

$$c_i(\infty) = \frac{n/m}{1-g} .$$

Section III presents an analysis of the size of future calendars and case backlogs. Again, specific expressions are developed for both of these quantities over time and in the steady state. For example, the steady-state backlog in each part of the court, $b_i(\infty)$, for the special case in which all parts are identical, is found to be

$$b_i(\infty) = \frac{n}{m}\,\bar{\alpha} + g\,c_i(\infty)\bar{a}_i;$$

here, $\bar{\alpha}$ is the average time between the arraignment (or intake) appearance and a subsequent appearance, and $\bar{a}_i$ is the average time between an appearance in part i and a subsequent appearance in another part.

In Section IV the model is viewed from the point of view of the individual case, rather than that of the court, with the goal of developing expressions for such quantities as the average number of appearances made by a new case in each part, the average duration of a new case, and so on. While procedures are cited which would yield the desired information for the general case, expressions for these quantities are developed only for the special case of identical parts. Thus, it is shown that the average number of appearances made by a new case in each (identical) part, $\bar{k}_i$, is given by

$$\bar{k}_i = \frac{1/m}{1-g},$$

and that the average total duration in days, $\bar{T}$, is given by

$$\bar{T} = 1 + \bar{\alpha} + \frac{g}{1-g}\,\bar{a}_i.$$

Finally, Section V presents an examination of the interaction between calendar size and disposition fraction when the latter is allowed to depend on, as well as determine, the former. This analysis is presented in terms of the establishment of an equilibrium at which the values of the calendar size and the disposition fraction (again for the case of identical parts of the court) are such that the combination is not only feasible but enables each part to dispose of as many cases as it receives. Here it is shown that there may, in fact, exist alternative equilibria at which a court may operate. After observing that several criteria favor that alternative with the smallest calendar size, several means are suggested for moving from an equilibrium point with a larger calendar size to one with a smaller calendar size. One of these means - a temporary increase in the number of judges - is analyzed.