"A THEORY OF FUTURES MARKET MANIPULATIONS"-- A DISCUSSION OF PETE KYLE'S PAPER

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"A THEORY OF FUTURES MARKET MANIPULATIONS"-- A DISCUSSION OF PETE KYLE'S PAPER

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Futures trading is among humanity's more impenetrable concepts. It involves selling what one does not own and, as a rule, buying what one does not want. It is deeply shrouded in terminology that conceals its meaning. It operates in an arena where opinion is everything, where supply and demand are hard to distinguish from supposition and doctrine and where inherent uncertainty has spawned an endless holy war between two religious sounding antagonists, the "fundamentalists" and the "chartists"... Into this world comes the general public, eager to enjoy its riches and often unprepared to become its poor.

I. INTRODUCTION

Pete Kyle has provided us with an extraordinary paper. It represents--to the best of my knowledge--the first logically coherent, formal model of a manipulation in the futures market. This model and the successors it will almost surely spawn may some day provide the information policy-makers need to judge in advance how proposed regulations would likely affect the conduct of participants in the futures markets.

Kyle's paper can be divided into two parts. Two-thirds of the paper--composed of eight sections--lucidly motivates the model, summarizes its story, and points out its policy implications. These sections are accessible to any interested reader and by themselves clarify its essential ideas. The lone remaining section--which comprises a full third of the paper--contains the formal model. Kyle entitles the section "A Simple Model"--presumably because it is a simplified representation of the complex behaviors engaged in by commodity traders. However, readers unaccustomed to models with asymmetric information, Nash conjectures, mixed strategies, and separation theorems may not find reading this section simple. One constructive function I can therefore serve as a discussant is to make this part of the paper more widely accessible. In the next section, I summarize the model so that it is accessible to any reader who understands why a profit-maximizing monopolist equates marginal revenue and marginal cost. After clarifying what *is* the equilibrium behavior of agents in the model, I turn briefly in Section III to a normative evaluation of the squeezer's behavior. In Section IV, I suggest several promising extensions of the model. In Section V, I present a more general characterization of manipulations and indicate other cases that warrant modeling in the future.

II. THE SIMPLE MODEL SIMPLIFIED

Kyle envisions a futures market frequented by three types of agents: hedgers, speculators, and an "informed trader." Hedgers make offers to sell contracts for future delivery. The informed trader--with private information about the aggregate size of such offers--decides whether to buy some of the contracts offered by the hedgers or to offer to sell additional contracts himself. Speculators observe the net amount of contracts offered. Using this information, they revise their expectation of the final futures price and bid against each other to purchase the net offering of the hedgers and informed trader. This bidding determines the initial futures price.

The contracts traded specify that the same amount (call it one unit) of either of two grades (call them plain and fancy) can be delivered. To take delivery a purchaser need merely hold his contract to maturity; if no delivery is desired the purchaser offsets his long position in the final (and only other) round of futures trading. Which grade is delivered is up to the short seller. Since the fancy grade under "normal" circumstances sells on the cash market for d more than the plain grade, shorts will ordinarily find the plain grade the cheaper to deliver. On some occasions, however, the delivery required exceeds the available supplies. Then the price of plain grade is bid up to the price of the fancy grade and the demand in excess of the available plain grade is satisfied with the fancy grade. It is assumed that the supply of the plain grade is inelastic at Z and this information is common knowledge. Moreover, there exists limitless amounts of the fancy grade.

Kyle refers to the informed trader as "the manipulator" but we will adhere to more neutral language. The informed trader places orders to buy or sell futures contracts. Based on his observation of the hedgers'

offers and his market experience he knows how much he will affect the initial futures price by a given offer to purchase or sell.

The informed trader finds that, when hedging is active, he can make a profit if he buys enough futures contracts and holds them to delivery--even though his long position drives futures prices up above the price expected for the plain grade. The informed trader makes profits because the futures price he pays for each contract is not driven as high as the expected price of the fancy grade and--for each contract he buys beyond Z--the shorts deliver an additional unit of the fancy grade in fulfillment of their contract. Hence he incurs losses on deliveries of Z units of the plain-grade in order to get deliveries of the fancy grade. In Kyle's model, the prices of the two grades differ by d with the price of the plain grade exogenous and random. Since the informed trader is risk neutral he would buy infinite amounts of futures contracts so as to secure title to limitless amounts of the fancy grade if he did not take account of the impact such a long position would have on the initial futures price and hence on his inframarginal costs.

When hedging is inactive, the informed trader finds that the futures price is high relative to what he expects the cash price to be on the plain grade. He finds that by selling futures contracts and then delivering the plain grade he maximizes profits. Again, he restrains his short sales because he takes account of the depressing effect they have on the initial futures price and hence on his inframarginal revenues.

The informed trader faces one schedule for the initial futures price if hedging is active and a different price schedule if it is inactive. Both schedules are increasing functions of his demand for futures contracts. The schedule when hedging is inactive can be derived from the schedule when hedging is active by shifting each point on the latter schedule to the

right by a constant amount. It is unnecessary that the informed trader ever notice this relationship between the two price schedules.

As it happens, the profit-maximizing long position which the informed trader takes when hedging is active and the profit-maximizing short position which he takes when hedging is inactive differ by this same constant. Consequently, the initial futures price which results from each position is identical.

We now formalize the foregoing description of the informed trader's problem. Kyle shows in Figure 2 how to derive a function whose lateral translations indicate the price in the futures market resulting from a given position when hedging is active on the one hand or inactive on the other. The function is not unique but as all the equilibrium price functions induce the same behavior we choose the simplest--a piecewise linear function. We take this price function as a given throughout this section but indicate how it can be derived in a footnote.[1/]

The Informed Trader's Profit Maximization Problem

Whether hedging is active or inactive the informed trader can earn R(x) if he takes delivery on x futures contracts and resells them on the cash market. The price on the cash market is uncertain but the plain grade has mean price EV and the fancy grade normally sells for d more. R(x) is therefore defined as follows:

$$R(x) = \begin{cases} xEV \text{ for } x < Z \\ xEV + (x - Z)d \text{ for } x \geq Z \end{cases}$$

That is, if the informed trader takes delivery of less than the deliverable supply of the plain grade, the mean per unit value is merely EV. But each

unit in excess of Z which he receives is worth EV + d.

Let H_1 (resp. H_0) denote the short position of the hedger when trading is active (resp. inactive) with $H_1 > H_0$.

It will be assumed throughout that the deliverable supply of the plain grade is small relative to the variation between active and inactive hedging. In particular,

(A1) $Z \leq (1 - \lambda)^2(H_1 - H_0)$, where λ is the probability that hedging is active.

When the exogenous parameters do not bear this relationship to each other Kyle shows that an equilibrium still exists but its characteristics differ.[2/]

o Behavior of the Informed Trader When Hedging is Active

When hedging is active, the informed trader faces the following piecewise continuous price function:

$$P_{H_1}(x) = \begin{cases} EV + d & \text{for } x > x_1^u \\ \frac{dx}{H_1 - H_0} + EV + (2\lambda - 1)d & \text{for } x_1^u \geq x \geq x_1^\ell \\ EV & \text{for } x < x_1^\ell \end{cases}$$

where:

$$x_1^u = 2(1 - \lambda)(H_1 - H_0)$$

and $$x_1^\ell = (1 - 2\lambda)(H_1 - H_0).$$

That is, if the informed trader goes very long he will drive the futures price up to the expected price of the fancy grade; if he goes very short

he will drive the futures price down to the expected price of the plain grade. And for intermediate positions, larger purchases raise the price at a constant rate.

When hedging is active, the cost of acquiring a position of x is simply $C_{H_1}(x)$:

$$C_{H_1}(x) = xP_{H_1}(x).$$

The informed agent's problem is therefore to

$$\underset{\{x\}}{\text{Maximize}}\ R(x) - C_{H_1}(x)$$

From the definitions of x_1^u, x_1^ℓ, and (A1) it is obvious that $x_1^u > Z > x_1^\ell$. Hence there are four regions to investigate ($x < x_1^\ell$, $x > x_1^u$, $x_1^u > x > Z$, and $Z > x > x_1^\ell$). The optimum occurs in the region where $x_1^u > x > Z$--at the point where $MR(x) = MC_{H_1}(x)$. Since $x > Z$ in this region, the informed trader's revenue function is $R(x) = xEV + (x - Z)d$. Since $x_1^u > x > x_1^\ell$ his cost function is $C_{H_1}(x) = \frac{dx^2}{H_1 - H_0} + (2\lambda - 1)xd + xEV$. By equating marginal revenue to marginal cost, it is straightforward to confirm that a unique local maximum (x_1) is achieved for this subinterval at $x_1 = (1 - \lambda)(H_1 - H_0)$ and results in a profit of $(1 - \lambda)^2(H_1 - H_0)d - Zd$. This profit is nonnegative by virtue of (A1). To verify that this is the global optimum we must determine the largest maximized profit achievable across the other subintervals and show that this value is strictly less than the profit at the asserted global optimum. Kyle omits these tedious but necessary details; to supplement his analysis, I relegate them to a footnote.[3/]

To summarize, when hedging is active the price function induces the informed trader to go long by x_1. In doing so he drives the initial futures price to $P_{H_1}(x_1) = EV + \lambda d$.

o Behavior of the Informed Trader When Hedging is Inactive

When hedging is inactive, the informed trader faces the following lateral translation of the previous price schedule--

$$P_{H_0}(x) = \begin{cases} EV + d & \text{for } x > x_0^u \\ \frac{dx}{H_1 - H_0} + EV + 2\lambda d & \text{for } x_0^\ell \le x \le x_0^u \\ EV & \text{for } x < x_0^\ell \end{cases}$$

where $x_0^u = x_1^u + (H_1 - H_0) = (1 - 2\lambda)(H_1 - H_0)$

$x_0^\ell = x_1^\ell + (H_1 - H_0) = -2\lambda(H_1 - H_0)$.

When hedging is inactive, the cost of acquiring a position of size x is simply $C_{H_0}(x)$:

$$C_{H_0}(x) = xP_{H_0}(x).$$

The informed agent's problem is therefore to

$$\underset{\{x\}}{\text{Maximize}}\ R(x) - C_{H_0}(x).$$

From the definitions of x_0^u, x_0^ℓ, and (A1) it is obvious that $Z > x_0^u > x_0^\ell$ and $x_0^\ell < 0$. Hence there are again four regions to investigate ($x < x_0^\ell$, $x > Z$, $Z > x > x_0^u$, and $x_0^u > x > x_0^\ell$).

The optimum occurs in the region where $x_0^\ell < x < x_0^u$ at the point where $MR(x) = MC_{H_0}(x)$. In this region the informed trader's revenue function is $R(x) = xEV$ and cost function is $C_{H_0}(x) = \frac{dx^2}{H_1 - H_0} + (EV + 2\lambda d)x$.

It can be verified by equating marginal revenue to marginal cost that a unique local maximum (x_0) is achieved for this subinterval at $x_0 = -\lambda(H_1 - H_0) < 0$ and results in a profit of $d\lambda^2(H_1 - H_0)$. To prove that this is the global optimum we must determine the largest maximized profit achievable across the

other subintervals and show that this value is strictly smaller than the profit at the asserted optimum. Again, such details are relegated to a footnote.[4/]

To summarize, when hedging is inactive the informed trader finds going short to be optimal. In doing so he drives down the price to $P_{H_0}(x_0) = EV + \lambda d$.

Behavior of Speculators

Speculators observe the excess of hedgers' offers over the informed trader's bids, and using this information try to infer whether hedging is active or inactive. Whenever hedging is active they expect the final futures price to be EV + d; whenever hedging is inactive they expect the final futures price to be EV. Hence they scrutinize the information at their disposal trying to determine which state has occurred so as to make a profit. When hedging is inactive, the excess of hedgers' offers over the bids of the informed trader is $H_0 - x_0 = \lambda H_1 + (1 - \lambda)H_0$. (This is larger than H_0--reflecting the fact that the informed trader in fact goes short himself.) When hedging is active, the excess of hedgers' offers over the bids of the informed trader is $H_1 - x_1 = \lambda H_1 + (1 - \lambda)H_0$. (This is smaller than H_1--reflecting the fact that the informed trader purchases some of what the hedgers offer). Since speculators see a net offer of $\lambda H_1 + (1 - \lambda)H_0$ in either situation, they cannot tell whether the hedging offers were active or inactive. Hence the speculators bid the initial futures price to a probability-weighted average (EV + λd) of the two possible final futures prices using unrevised priors about the probability of hedging being active. When hedging is active, they gain; when hedging is inactive, they lose. On average, they break even.

What happens on the final round of futures trading is critical to Kyle's story. He shows in Lemma 1 that the informed trader can do no better than to refrain from trading on the final period. If the informed trader always does refrain (indeed, as long as he adopts a pure strategy rather than randomizing among any of the equally good alternatives) then the speculators can infer whether hedging was initially active or inactive by observing the hedgers' demands for offsetting contracts.[5/] For example, if hedging was active, their demand for offsets will be large (H_1) and speculators will know that a squeeze is in progress. In this case, speculators close out their own limited long positions and hedgers who are unable to settle make deliveries to the informed trader.

At what prices do these exchanges between speculators and hedgers take place in the final period? A hedger trying to offset his short position would be willing to pay as much as EV + d--the price he would have to pay if instead he made delivery. A speculator trying to offset his long position would be willing to accept a smaller amount--as little as the market value of the mixture of grades he would expect to receive if instead he took delivery. If there were enough small speculators--each with an infinitesimal portion of the aggregate long speculative position ($H_1 - x_1$), each of them would be willing to settle for infinitesimally more than EV.[6/] Kyle argues that the futures price in the final round would settle at EV + d. Certainly, each speculator knows that hedgers will have to pay this amount if they cannot offset and that some hedgers <u>will</u> have to make deliveries. Nonetheless, one might be able to imagine final prices other than the one Kyle posits. Considering how much of the analysis hinges on what price occurs in the final round if a squeeze is in progress, I hope Kyle elaborates on the precise sequence of bargaining moves in subsequent work.

It is important to understand why the speculators--unlike the informed-trader--never take delivery. If a squeeze is on and a speculator takes delivery, he is virtually certain to get a mixture of the plain and fancy grades. As long as he can get a strictly better price selling his futures contract than he would expect to get by taking delivery of the mixture of grades and reselling them on the cash market, the speculator will avoid delivery. Since the futures price is driven to EV + d in a squeeze--no speculator would take delivery. As long as he expected any plain grade to be delivered, he would be better off selling his contract. The informed trader no doubt would also like to avoid delivery but realizes that if he unwinds his position the final futures price will collapse. To put this in more familiar terms, the informed trader holds up the price umbrella and the speculators get a free ride under it by unwinding their positions without having either to receive delivery of the plain grade or to cause a collapse of the final futures price.

Behavior of the Hedgers

If the informed trader makes positive expected profits and the speculators break even, someone must make losses. The hedgers are the losers. They cannot do as well as the speculators because of a first-mover disadvantage. It is true that the game is fair to the speculators. But even if the hedgers flipped a coin to determine whether to be active or inactive the informed trader--who acts knowing the outcome of the flip--is in a position to gain at their expense. Every time their flip directs them to take an inconsequential short position, the informed trader also goes short, and the final futures price falls to EV--leaving them with an inconsequential profit. Every time they take a significant short position, the informed trader squeezes, and the final futures price subsequently rises to

EV + d--leaving them with a significant loss.

III. DISCUSSION

One nice thing about a theoretical model like Kyle's is that it describes each agent's behavior precisely. No facts can be in dispute. Another nice thing is that the behaviors posited do not conflict with the self-interest of any agent. Hence the scenario described--unlike most "hypotheticals" about behavior during a manipulation--is not so implausible that it is foolish to consider. Given the clarity of Kyle's description, I would find it immensely interesting to know whether his informed-trader is doing something illegal and if so what. Is he doing something illegal when he shorts the market or only when he squeezes? Suppose there were two informed traders--each unaware of the other's existence. One understands only the mechanics of going short, the other only the mechanics of going long. When hedging is inactive, the first trader goes short and the second trader abstains. When hedging is active, the first trader abstains and the second trader goes long. Both profit when they trade. Are <u>both</u> doing something illegal or only one of them? I do not have the legal training or expertise to answer any of these questions but would like to hear from someone who does.

In the meantime, permit me to play devil's advocate for a moment by emphasizing what--in my opinion--Kyle's informed trader cannot legitimately be accused of. It is true that his informed trader acts in a way which leaves speculators unable to distinguish whether hedging is active or inactive. But as we have seen, he is led to act in this way by the invisible hand and certainly not by any intention to deceive. It is true that no other trader is assumed to have access to information

about hedging activity but there is no suggestion in the paper that the informed trader is in any way responsible for the failure of others to gain access to the same information. It is true that the informed trader realizes that his initial purchase or sale of futures contracts will affect their price--but so does any intelligent market trader; at any rate, this makes him a garden variety monopolist/monopsonist at worst--but "manipulator" seems a bit harsh to me. It is true that the informed trader holds contracts which result in delivery of Z units of a grade he does not want in order to acquire $x_1 - Z$ units of the fancy grade. But in doing so, his motives are essentially no different from those of a child who buys a cereal he does not like for the sake of the toy or coupon at the bottom of the box. Our informed trader deceives no one, excludes no one, and does nothing to impede delivery by the shorts. Indeed, he both wants and takes delivery by the shorts. The informed trader may be legally or morally culpable, but not for the foregoing reasons.

There are two aspects of his conduct which seem to me questionable. In part, these are hard to ferret out because Kyle ignores the first altogether and downplays the second. First, who must the informed trader be to acquire information about the offers of hedgers? A broker? A member of the exchange? William Casey? It is possible that the only people with access to such information must either have acquired it illegally or be breaking a law in trading on the basis of it. I can only raise these questions. I lack the requisite expertise to answer them and again defer to the experts present. The second aspect of the informed trader's conduct which I feel warrants closer scrutiny is what he does in the final round of trading. He does nothing. He holds x_1 contracts. If he offset $x_1 - Z - \varepsilon$ of them, the final futures price would remain EV + d and he

would earn from his partial offset exactly what he lost by foregoing delivery on those contracts. But if he sold any more, the final futures price would collapse to EV! This is not a situation which a garden-variety monopolist ever faces, let alone engineers.

IV. EXTENSIONS

Several extensions of this model are suggested by the previous discussion. The informed trader in Kyle's model does not consider legal consequences when determining his optimal strategy. If he does something which violates current laws, perhaps we should regard the model as describing how the informed trader would have behaved in the absence of such laws. It would then be interesting to ask how the imposition of the current laws alters equilibrium behavior and even whether the benefits of the current regulations are worth the various associated costs. This form of analysis could, of course, lead to use of the model to help design better laws. Here the law would not be judged by its implicit norms but by its positive consequences.

I would be very interested in an extension of Kyle's model in which there are n identical, informed traders who move simultaneously. It seems to me this seemingly innocuous extension might dramatically alter the nature of the equilibrium and would, in any case, clarify the fragility or stability of the manipulation scheme.

For simplicity, suppose these are two informed traders. Suppose that whenever hedging is active they always end the final round of trading with identical long positions. Denote each position as $\hat{x}$. If a profitable squeeze is in effect $2\hat{x} > Z$. It is straightforward to see that this cannot be a Nash equilibrium--for any $\hat{x}$. For, each informed trader has an incentive to sell additional contracts, thus unilaterally altering his final

position. Each trader would conjecture that as long as the other fellow maintained his position, he could reduce his own by $2\hat{x} - Z - \varepsilon$ without causing the price to collapse. For each contract he sold he would get EV + d--more than he would get from reselling the hodge-podge he would receive if he took delivery. This is not to say that an equilibrium fails to exist in the final round; but it must either involve asymmetric pure strategies or symmetric mixed strategies. From a policy point of view, what happens in the final round with n informed traders is important because it indicates how fragile is this particular manipulation scheme.

There is another interesting aspect of this extension. By assumption, each of the two informed traders initially observes whether hedging is active or inactive and each moves simultaneously. But neither can directly observe what position the other has taken. Each can, however, make an inference by observing the responses of the speculators to their simultaneous moves. This inference aspect of the problem is absent in the case of a single informed trader.

There is a final group of extensions which warrants mention. Kyle occasionally refers to what the speculators observe as "open interest." But open interest reflects completed transactions--not outstanding offers to sell. (To understand why, for example, one cannot interpret what the informed trader sees as completed sales by hedgers it is sufficient to note that--at the time of his observation--neither the informed trader nor the speculators have yet moved and consequently there is no one who could be on the other side of the hedgers' transactions). Presumably, what Kyle has in mind is a more complex multiperiod story in which what agents observe is in fact open interest. I would be interested to learn whether squeezes can occur in such a model. In this regard it is important to

remember that speculators in reality have access to information on both the volume and the concentration of open interest. It would be interesting to see the characteristics of the equilibrium under the assumption that both types of information are available. Perhaps, the concentration data would eliminate squeezes; alternatively in the new equilibrium perhaps the invisible hand would induce the informed trader to use multiple accounts.

V. TOWARDS AN EXPANDED THEORY OF MANIPULATIONS

In entitling his essay, "A Theory of Futures Market Manipulations," Kyle implies that he has not said the last word on the subject. Indeed, despite the importance of this pioneering contribution, I believe some real-world manipulations may fall outside the scope of his model. In this final section, therefore, I want to broaden the discussion so as to include other manipulations which may merit consideration in the future.

In any long squeeze, traders who hold futures contracts succeed in raising the costs which the shorts expect to pay to deliver on their contracts. Given their revised expectations, the shorts then find themselves in a situation where they are willing to pay dearly to offset their positions. Kyle's model fits this characterization. His short hedgers initially expect delivery will cost EV + λd but learn unambiguously whenever the squeeze is on that the true delivery cost will be EV + d. The long trader accomplishes the squeeze in Kyle's model by taking delivery on the cheapest-to-deliver grade and consequently bidding up its spot price.

But the foregoing characterization also fits other situations besides the one envisioned by Kyle. Delivery costs can be bid up without affecting the spot price of the commodity--by increasing packing, shipping, or storage costs. Indeed, such manipulations of the delivery mechanism are a common complaint in squeeze cases. And--if the spot price of the commodity is increased--this can be accomplished without taking delivery. In the May 1976 fiasco in the Maine potato market, for example, the increase in the spot price which competitive holders of the inventory found acceptable during the brief delivery period seemed to be in part related to a timely rumor--reported in local newspapers but apparently without factual basis--that foreign buyers desperate for potatoes were heading toward the port in Searsport, Maine.[7]

Finally, it may be unnecessary for the actual delivery costs of the shorts to increase. A squeeze can occur if the shorts revise upward their expectations of the costs of future delivery. If, for example, the shorts had limited information about future delivery costs but observed some signal which made an upward revision in their cost expectations rational, then they might be induced to offset their contracts even if delivery costs did not happen, in fact, to be higher. One such signal might be relatively high settlement demands by a long known to have superior information. In such situations, there might still be considerable ambiguity about the magnitude of delivery costs at the close of futures trading. This differs from the situation depicted by Kyle where all such ambiguity is resolved prior to entry into the delivery period. When such ambiguity persists, we might expect to see wild gyrations in the final futures

price, acrimony and charges of bluffing in the final round of trading, and occasional episodes where the shorts go to delivery (and possibly default) in what turns out to be a mistaken belief that delivery costs are in fact low. It is my impression that at least the Maine potato futures market has displayed each of these characteristics.

At the end of Kyle's paper, he discusses the effects of alternative policy proposals such as cash settlements instead of required delivery, modification of delivery differentials, implementation of position limits, and so forth. I have argued that squeezes other than the type modeled by Kyle merit serious attention in the future. Policies which Kyle finds to be ineffective against his particular form of squeeze may turn out to be effective against other forms of squeeze. For example, in Kyle's model, there is no cost to making delivery except the cost of purchasing the deliverable at the spot price; there is no way for a squeezer to manipulate the delivery mechanism by increasing packing, shipping, or storage costs. Kyle finds that--in his model--implementing a policy of cash settlements at spot prices does not eliminate squeezes. While it is valuable to discover that there exists at least one form of squeeze which cannot be affected by a policy of cash settlements, one cannot logically conclude from his analysis that such a policy would be similarly ineffective against <u>every</u> form of squeeze. Until the other forms of squeeze are subjected to a parallel analysis and the relative importance of each assessed empirically, policy recommendations seem premature.

FOOTNOTES

1/ Of the three types of equilibria which Kyle shows can arise for different values of the exogenous parameters, each can be supported by a continuous price function (and its lateral translation) which has a constant value of EV to the left of some point, a constant value of EV + d to the right of some point, and a linear segment in between the two points. Such a function can always be parameterized by the two points where the flat segments terminate. For example, consider the following price function ($P_{H_1}(x)$) and its lateral translation ($P_{H_0}(x)$), both of which are parameterized by the numbers x_1^u and x_1^ℓ.

$$P_{H_1}(x) = \begin{cases} EV + d & \text{for } x > x_1^u \\ \dfrac{xd}{x_1^u - x_1^\ell} + EV - \dfrac{d x_1^\ell}{x_1^u - x_1^\ell} & \text{for } x_1^\ell \leq x \leq x_1^u \\ EV & \text{for } x < x_1^\ell \end{cases}$$

$$P_{H_0}(x) = \begin{cases} EV + d & \text{for } x > x_0^u \\ \dfrac{xd}{x_1^u - x_1^\ell} + EV - \dfrac{d[x_1^\ell - (H_1 - H_0)]}{x_1^u - x_1^\ell} & \text{for } x_0^\ell \leq x \leq x_0^u \\ EV & \text{for } x < x_0^\ell \end{cases}$$

where $x_0^{\ell} = x_1^{\ell} + H_1 - H_0$

$x_0^{u} = x_1^{u} + H_1 - H_0$.

In Section II, we assume these functions are given by specifying $x_1^{u} = 2(1 - \lambda)(H_1 - H_0)$ and $x_1^{\ell} = (1 - 2\lambda)(H_1 - H_0)$. We could alternatively have solved for these two numbers, thereby determining the equilibrium price functions. For any pair of numbers (x_1^{u}, x_1^{ℓ}) the optimal positions for the informed trader (x_0, x_1) and the resulting initial futures price in each state can be determined as in the text. x_1^{u} and x_1^{ℓ} can then be determined so as to satisfy the twin conditions that the futures price equal $EV + \lambda d$ in each state. For the two other types of equilibria, corresponding conditions permit determination of x_1^{u} and x_1^{ℓ}.

2/ As we will see, when (A1) holds (1) the informed trader goes long by more than Z whenever hedging is active and goes short whenever it is inactive, (2) each position depends only on $H_1 - H_0$ and not on Z, and (3) the price in each state is $EV + \lambda d$. As Z is increased within the region (holding $H_1 - H_0$ fixed) profits when hedging is inactive do not change. Profits when hedging is active decline, however, because more of the unchanged optimal position x_1 results in delivery of the plain grade and less in delivery of the fancy grade. In particular, if π_1 denotes profits hedging is active:

$$\pi_1 = \{x_1 EV + (x_1 - Z)d\} - x_1(EV + \lambda d)$$

$$= [x_1(1 - \lambda) - Z]d.$$

That is, profits equal revenues (enclosed in curly brackets) less costs. Note that profits in this state decline linearly as Z increases. Since $x_1 = (1 - \lambda)(H_1 - H_0)$, profits from the long squeeze fall to zero when (A1) holds with equality.

For larger values of Z, this type of equilibrium cannot exist since the informed trader would prefer not to trade when hedging is active. Kyle shows that another type of equilibrium does exist, however, in which--when hedging is active--the informed trader sometimes goes long and sometimes abstains from trade. Since squeezes are less frequent, speculators bid the initial futures price up by less and the informed trader makes zero profits--instead of losses--when he goes long. As Z increases, the frequency of squeezes must decline so that the initial futures price is lowered and the long squeeze continues to result in zero profits; otherwise it would result in losses. Eventually, the frequency of squeezes reaches zero. Beyond this, we are in Kyle's third region where squeezes never occur.

3/ If $x < x_1^{\ell}$, $\quad R(x) - C_{H_1}(x) = xEV - xEV = 0.$

If $x > x_1^{u}$, $\quad R(x) - C_{H_1}(x) = xEV + (x - Z)d - xEV - xd = -Zd < 0.$

If $Z > x > x_1^{\ell}$, $\quad R(x) - C_{H_1}(x) = xEV - \frac{dx^2}{H_1 - H_0} - xEV - (2\lambda - 1)xd$

$$= \frac{dx^2}{H_1 - H_0} - (2\lambda - 1)xd.$$

This function reaches a relative maximum at $\frac{1}{2}x_1^{\ell}$.
If $x_1^{\ell} > 0$, the relative maximum is not achieved within the interval under consideration and a constrained maximum of zero is achieved at the lower boundary with $x = x_1^{\ell}$.

If $x_1^{\ell} < 0$, the relative maximum is achieved within the interval at a profit of $\frac{d}{4}(1 - 2\lambda)^2(H_1 - H_0) > 0$. Since this is strictly smaller than the profits at x_1, the global maximum occurs at x_1 as asserted in the text.

4/ If $x < x_0^{\ell}$, $\quad R(x) - C_{H_0}(x) = xEV - xEV = 0$

If $x > Z$, $\quad R(x) - C_{H_0}(x) = xEV + xd - Zd - xEV - dx = -Zd < 0$

If $x_0^u \leq x \leq Z$, $\quad R(x) - C_{H_0}(x) = xEV - x(EV + d) = -dx \leq -dx_0^u$

Maximized profit in these regions equals Max $(0, -dx_0^u)$. Since this is strictly smaller than the profits at x_0, the global maximum occurs at x_0 as asserted in the text.

5/ If the informed trader does randomize, it is unclear to me whether the speculators could tell whether a squeeze was in progress; if not, perhaps there are other equilibria which have been excluded by the assumption that the informed trader adopts a pure strategy (always abstaining) in the final round.

6/ Suppose there were n identical speculators and that all went to delivery. The informed trader is long x_1 contracts. Since the hedgers are short H_1 contracts, speculators collectively are long $H_1 - x_1$ contracts. Hence, each speculator holds $\frac{H_1 - x_1}{n}$ of the contracts and expects to receive $\left(\frac{\frac{H_1 - x_1}{n}}{H_1}\right)$ of the total deliveries of each grade. Since Z plain grade and $H_1 - Z$ fancy grade will be delivered, the speculator expects to receive a mixture worth $\left(\frac{H_1 - x_1}{nH_1}\right)(H_1 - Z)\cdot d + EV$ on each contract. Hence the expected value per contract to a speculator who takes delivery of the mixture approaches EV if the number of other speculators is large enough.

7/ I base these tentative conclusions on interviews I conducted (in person and by telephone) in June 1976 with journalists in Maine, CFTC investigators in Washington, and officials at the port of Searsport.

"A THEORY OF FUTURES MARKET MANIPULATIONS" A DISCUSSION OF PETE KYLE'S PAPER