FIRE DEPARTMENT DEPLOYMENT ANALYSIS

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July 1979

P-6351

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ABSTRACT. In recent years fire departments in urban areas have experienced a sharp increase in demands for their services while their budgets have generally grown at a rate less than that of inflation. Many of these departments have turned to systems analysis for help, realizing that, if they do not use more effectively what resources they have, their level of service will diminish. The Rand Corporation has provided assistance to the New York City Fire Department and others over the past decade, concentrating on deployment policies, which tie available resources to their distribution and movement in the field. This lecture will first provide a brief overview of the deployment policies that have been analyzed with the help of mathematical models. This will be followed by a more complete discussion of one of the policy questions: how should available fire companies be temporarily relocated to provide coverage when many other companies are busy fighting large fires?

I. INTRODUCTION

Fire is the leading cause of catastrophic accidents (those in which five or more people die) in the United States, annually destroys over $4 billion in property, and costs the total economy an estimated $14 billion per year. Moreover, some of the underlying problems are becoming more severe. For example, technological change has created new fire risks, deterioration of inner-city neighborhoods has spawned rising numbers of fires, and the incidence of arson fires has risen dramatically.

At the same time, fire departments in many cities have experienced budget reductions or growth at a rate less than the rate of inflation. If they are unable to use what resources they have more effectively than in the past, their level of service must diminish.

Improved effectiveness, however, is not easy to achieve in fire departments. Fire service management and operations lean heavily on tradition and on rules of thumb, many of which have not changed very much since the turn of

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1980 Mathematics Subject Classification 90C50.

*Prepared for a lecture presented as part of the American Mathematical Society Short Course "Operations Research: Mathematics and Models," in Duluth, Minnesota, August 19-20, 1979.

the century. This is especially true of fire department deployment policies, which tie available resources to the actual distribution and use of firefighting services in the field. For example, the present number and arrangement of fire companies in most cities are based more on historical factors, such as where volunteer companies were first organized, than on a careful analysis of actual needs. In 1968, a seven-year, multimillion dollar effort was begun that was primarily devoted to developing and testing new models and methods for fire department deployment analysis. The work was performed at The New York City-Rand Institute. The results have been used in over fifty cities throughout the United States [2].*

The general subject of deployment analysis includes a variety of topics, some of which can be addressed using mathematical models and others not. Most of the issues that can be analyzed using mathematical models concern the manner in which fire companies are located and dispatched:

- How many fire companies should be on duty? This may be a planning decision related to the department's budget, or it may concern the appropriate variation in company levels by time of day or by season of the year.
- How many fire companies should be allocated to each region of the city? A simple nonlinear programming model called the Parametric Allocation Model was developed to address this question [4].
- Where should the city's fire companies be located? This question refers to choosing sites for fire stations. A descriptive deterministic model that calculates expected travel times to fires for different arrangements of fire companies was developed. The model is called the Firehouse Site Evaluation Model [9].
- How many fire companies should be sent to an incoming alarm? The answer may depend on the availability of companies at the time of the alarm and what is known about the nature of the incident at the time the dispatch is made. A semi-Markovian decision model was developed that considers explicitly information on the current alarm and expected future events [8]. The model assumes that alarms occur and are extinguished according to random processes. Its state space is the number of companies busy and the potential seriousness of the incoming alarm. The decision variable is the number of companies to dispatch, and the objective function measures travel time to serious fires.
- Which particular fire companies should be dispatched? Most fire departments dispatch the units that are closest to the location of an

*Numbers in square brackets identify references at the end of this paper.

incident. However, using insights from a queueing model, it was found that under certain conditions it pays *not* to dispatch the closest unit [1].

- Which fire companies should be temporarily relocated when a large fire depletes one part of the city of its fire protection? When one large fire or several small fires are being fought in a single region of a city, protection against future fires in the same region is considerably reduced. It is standard practice in most urban fire departments to protect the exposed region by temporarily relocating fire companies from outside the region into some of the vacant firehouses within the region [6].

All of the above issues are discussed in detail in [3]. The remainder of this talk will be devoted to a description of how we dealt with the last issue.

II. THE RELOCATION PROBLEM

Figure 1 presents an illustration of a situation that is, unfortunately, not unusual in New York City and other large cities. Two serious fires break out at about the same time in the South Bronx. Seven ladder companies are involved for several hours fighting the two blazes. This results in a large region being left without a ladder company close by to respond if another fire should break out in the area.

When this happens in a large city the fire department usually temporarily relocates (moves) some fire companies from their firehouses in parts of the city that are still adequately protected to some of the empty houses. In smaller cities it may be necessary to achieve the same effect by borrowing companies temporarily from neighboring communities via a mutual assistance agreement. The purpose of such temporary relocations is clear—to spread out the still available firefighting resources in order to reduce and balance the risks and consequences that would result if other fires occur.

Any relocation method must provide answers to the following three questions:

1. In what situations are relocations to be made?
2. Which of the empty firehouses should be filled?
3. Which of the available fire companies should be moved?
4. To which empty houses should each be moved?

When alarm rates are low, the fires requiring relocations of firefighting units are rare. They occur one at a time, and when they occur, no other fires are typically in progress. Thus, under low alarm rate conditions it is

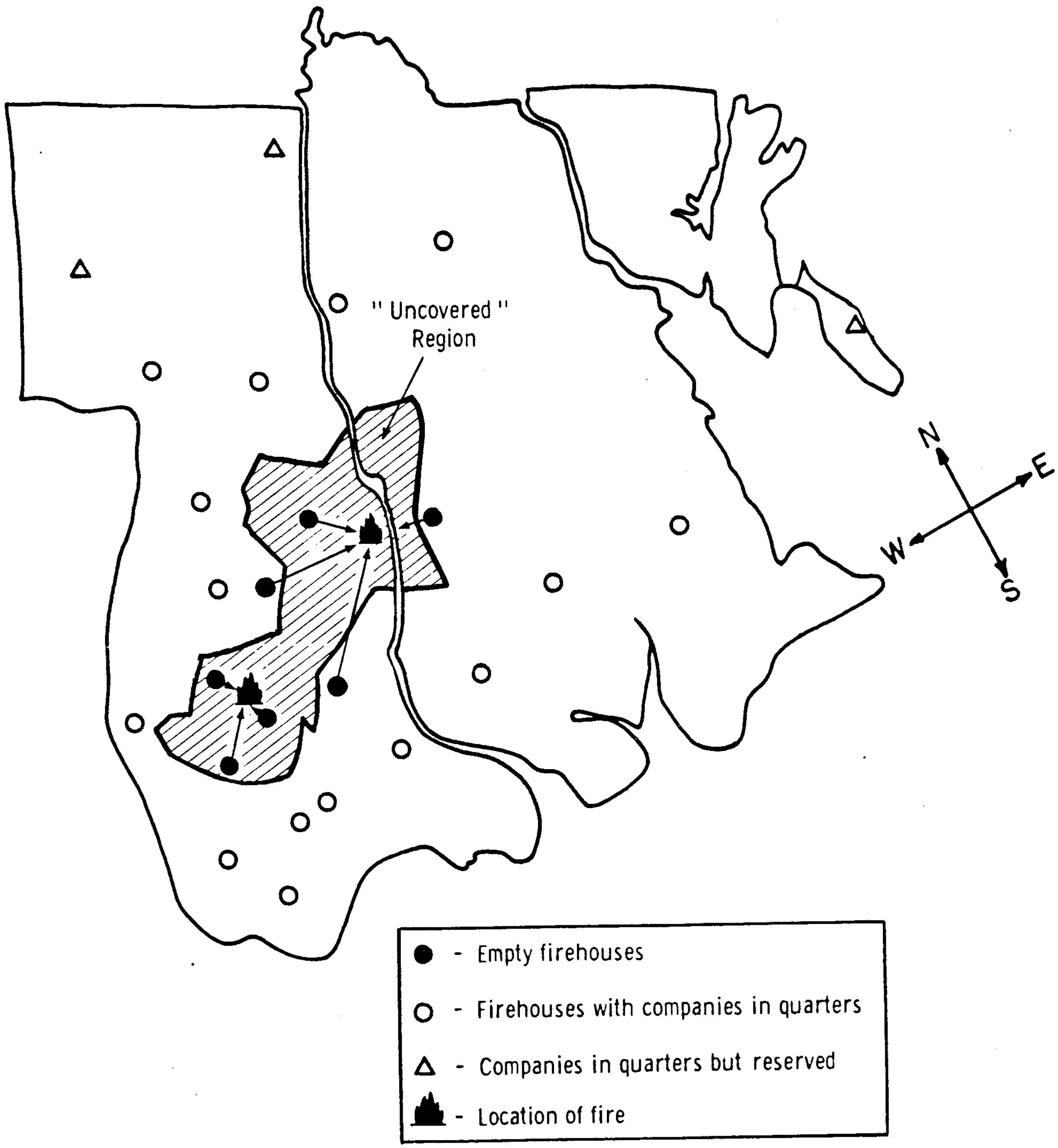

Fig. 1 - A sample relocation problem

possible to plan in advance for relocations with a reasonable expectation that the plan will be able to be carried out. Experienced fire officers imagine a hypothetical incident, say a three-alarm fire at a particular alarm box. Using their judgment, and assuming that all other fire companies not called to the third alarm will be available, they formulate a specific relocation plan. The plan consists of a list of temporary transfers of engine companies and ladder companies. Figure 2 is an example of an alarm assignment card used for dispatching and relocating fire companies in New York City. There is one of these cards for every alarm box in the city (over 14,000). The left side of the card shows the units (engine companies, ladder companies, and battalion chiefs) that are to be dispatched to alarms received from that box. The right hand side shows the companies that are to relocate when there is a serious fire in progress near that location. For example, the second line (corresponding to a "two-alarm" fire) shows that Engine 50 would be moved into the house of Engine 75, Ladder 49 would be moved into the house of Ladder 33, etc. When the alarm rate is low, alarm assignment cards work well. When the alarm rate is high, however, the plans often break down. The reason for the breakdown is simply that at high alarm rates several incidents (including small fires) may be in progress simultaneously, and the officers who created a relocation plan for one particular incident could not have anticipated this. To make a good and implementable relocation in this situation requires knowledge of the status of all the fire companies at the department's disposal and the nature of all incidents in progress *at the time action must be taken.* There are so many possible variations of the situation that can be encountered that there is no way to do this in advance.

Figure 3 shows two of the problems that can arise with pre-planned relocations.

1. A company that is supposed to relocate is already busy.
2. A company that is supposed to relocate is available, but moving it might create an even bigger gap in coverage.

We set out to design a new relocation procedure that could be used as part of an on-line real-time computer-assisted command and control system and that could be relied on to produce relocations that we, the fire department, and the public would agree were "good" (as opposed to "optimal") in all types of situations.

Our approach to developing a procedure was to view each of the questions raised above as a separate decision problem, with the solution to each problem being used as input to the next problem. We did this because the overall problem is a very large multiobjective problem that would be hard to implement

3311	CRESTON AVENUE and 192nd STREET									BRONX
ENGINE CO'S	Marine Co.	Res. Co.	LADDER CO'S	D.C	B. C.	Special Apparatus	Covering Chiefs	COMPANIES TO CHANGE LOCATION 11/67		
								ENGINE		LADDER
48 75 79			33 37	7	19		B. C. 15			
81 88 42			46				D. C. 6	50-75	38-79	49-33 32-37
43 46 62 95		3	38		18			41-46 67-95	90-62	
92 45 68 93			27					83-92 80-68	94-45 59-93	19-27
82 71 60 69			36					96-82 53-60	35-71 40-69	34-36

Fig. 2 - Alarm assignment card for Bronx box 3311

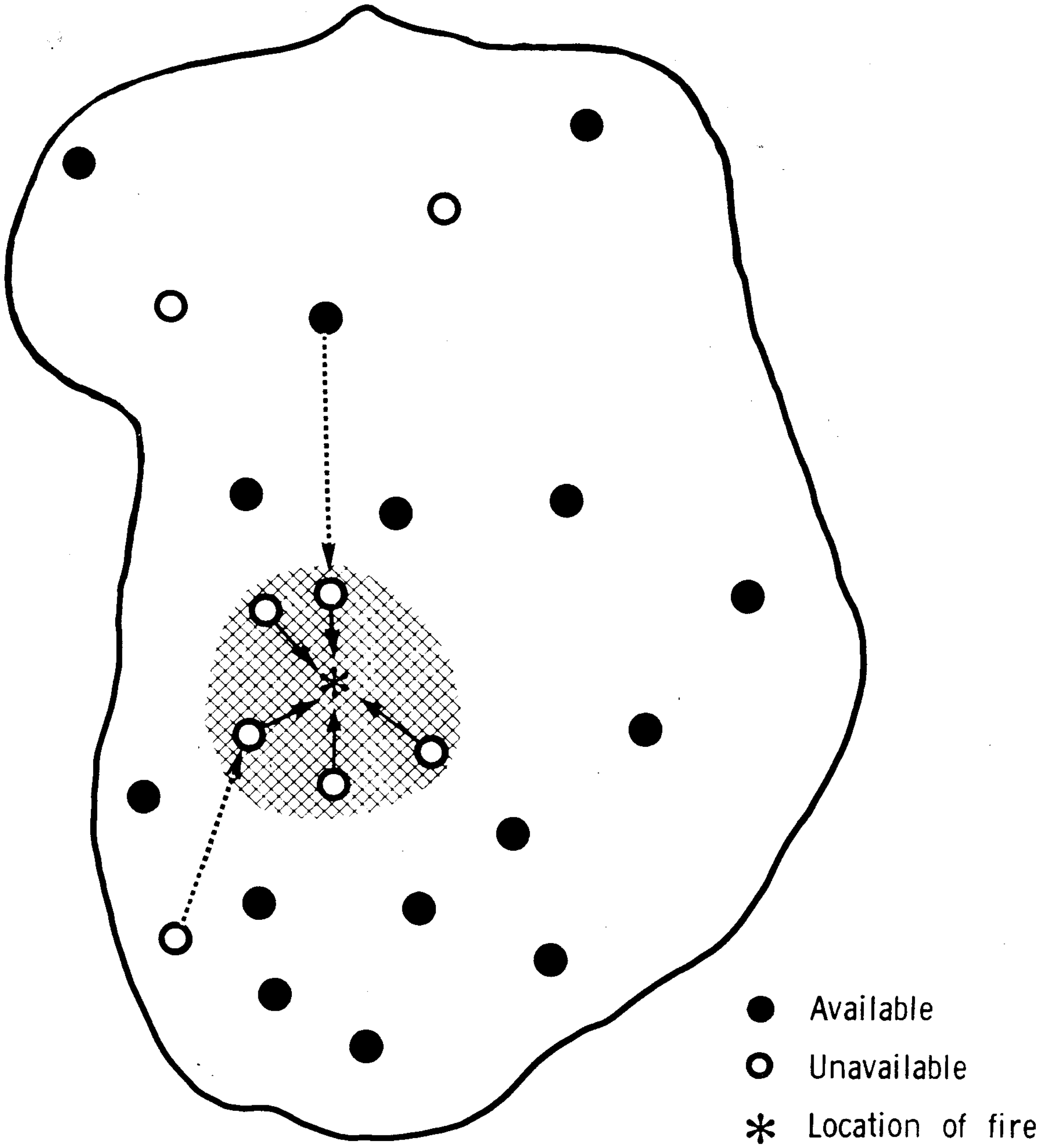

Fig. 3—Problems with the traditional relocation policy

within the computer time and space constraints of a real-time system (which would be performing other functions at the same time).

Most of the objectives to be satisfied by a "good" solution to the relocation problem were difficult to quantify. For example, in choosing a fire company to be moved, one would not want to move a company that was "too busy," that was protecting "too large" an area, or that would have to travel "too far" from its own house. In deciding which houses should be filled and which should remain empty one is faced with a conflict between efficiency and equity. The department would like to place its fire companies near where the fires are expected to occur, so they can reach them as fast as possible. However, they must provide an acceptable level of fire protection to all regions of the city --even those regions where the actual incidence of fires is low.

Separating the problem into four stages, each with its own objective function, made it easier for us to take all of the objectives into account.*

III. STAGE 1: WHEN TO RELOCATE

This question can be translated into the question, "when is an area underprotected?" One way of obtaining an operational answer is to set a minimum coverage standard (e.g., maximum travel time or travel distance) for every point in the city, based on the firefighting demands in the area. In practice, however, it would be difficult (and rather arbitrary) to specify minimum coverage standards. An attractive alternative is to let the way firefighting units are already allocated to areas implicitly define the minimum coverage standards for those areas. Usually fire companies are not uniformly distributed over a city but are concentrated in some areas and spread out in others. This distribution is the result of complex forces--some political, some operational, others historical. In working with its existing distribution of resources, a fire department has implicitly decided how it wishes to balance equity against efficiency, at least in the short run. In the long run, of course, the fire department may modify the distribution by building new firehouses. By assuming that the department is satisfied with the distribution of fire companies, we can define a minimum coverage standard for the relocation problem that will maintain approximately the same relative geographic distribution of fire companies as currently exists. This, in turn, will maintain approximately the same variation in travel times (or distances) as currently exists between the areas.

Therefore, the coverage criterion that we used required that for every alarm box in the city, at least one of the k closest engine houses and at least one of the p closest ladder houses contain an available company. k and p are parameters that are set by fire department policy. New York City has used

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*Complete details of the algorighm are contained in [6].

$k = p = 2$. Relocations are recommended whenever this coverage criterion is violated.

The application of this criterion is simplified by noticing that many alarm boxes will generally have the same k closest engines or p closest ladders. We call the aggregate of all alarm boxes having the same k closest engines an *engine response neighborhood* (written "engine RN" for brevity). A *ladder response neighborhood* ("ladder RN") is defined as the set of alarm boxes having the same p closest ladders. The set of engines and ladder response neighborhoods each form nonoverlapping partitions of the city. They are defined separately since the coverage standard is to be applied separately to engines and ladders in order to keep a balance of each unit type in each region. The definition of minimum coverage can now be restated as: *there must be no engine response neighborhood with all of its k engines unavailable and no ladder response neighborhood with all of its p ladders unavailable.*

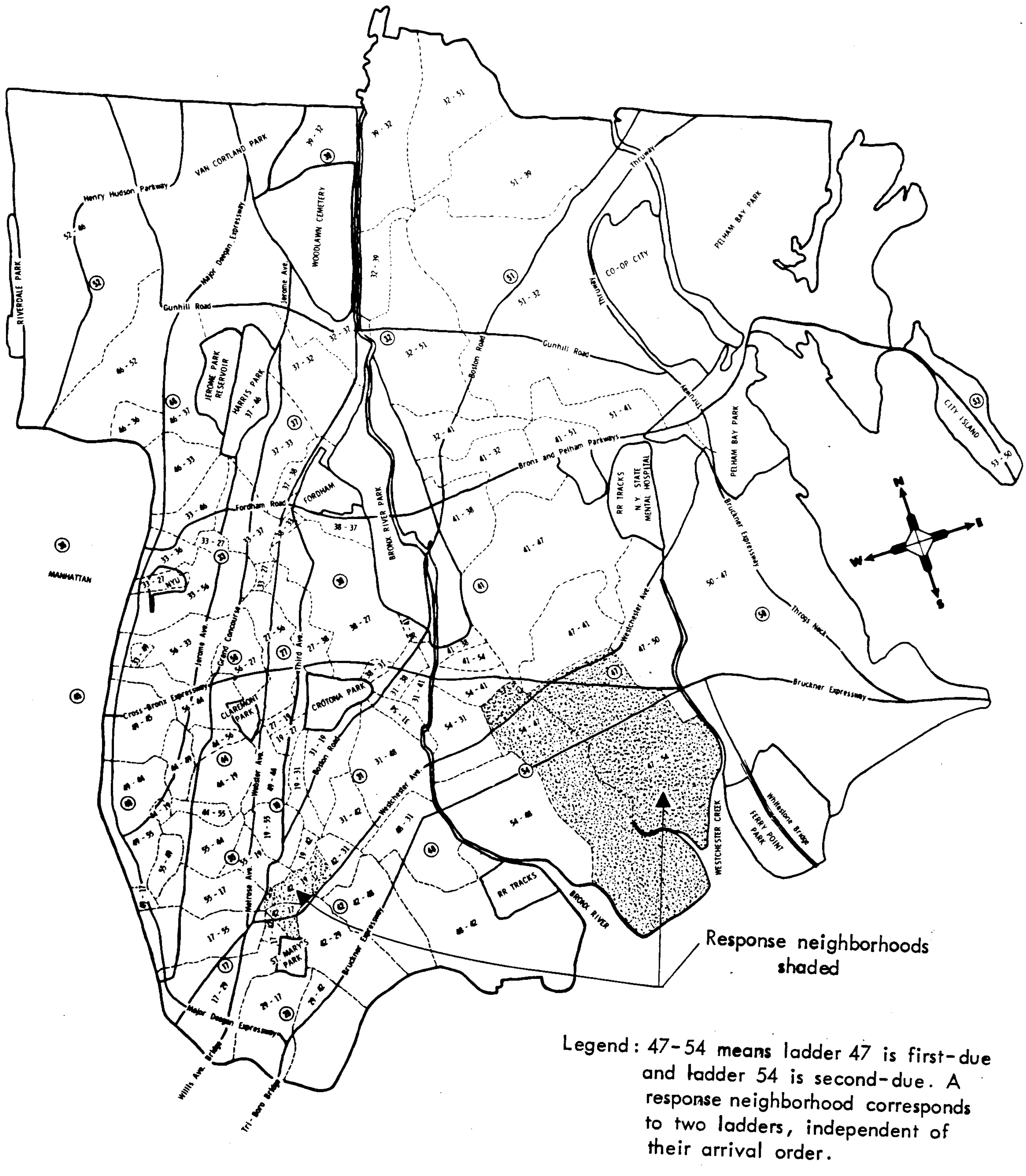
The use of response neighborhoods considerably reduces the calculations required to check on coverage. For example, in the Bronx there are over 2000 alarm boxes but with $p = 2$ fewer than 50 ladder RNs. Figure 4 shows the ladder RNs in the Bronx. Note that in regions where the ladder companies are close together the RNs are small, and where the companies are far apart they are larger.

We have delayed until now giving a precise definition of unit "availability" for purposes of minimum coverage. It makes no sense to relocate a unit into the house of a company responding to (but not yet working at) an alarm, returning from an alarm, or due back soon from a working fire. Therefore, we consider a company to be unavailable only if it is working at a fire expected to last for a "considerable" length of time (in practice, more than one hour).

IV. STAGE 2: WHICH HOUSES TO FILL

The primary objective of the relocation algorithm is to maintain minimum coverage as just defined. It makes sense to do this by moving as few companies as possible, since moving companies increases communication problems, places them in regions with which they may not be familiar, and takes them away from their home bases, food, and dry clothes. So we take as our criterion for the determination of empty houses to fill: have every response neighborhood covered, but move as few companies as possible. This translates into an integer program known as the set covering problem.

Suppose there are K uncovered RNs and L vacant houses whose busy companies cover or serve these RNs. For our decision variables let $x_j = 1$ if house j is to be filled and $x_j = 0$ otherwise. Then the problem is:



Response neighborhoods shaded

Legend: 47-54 means ladder 47 is first-due and ladder 54 is second-due. A response neighborhood corresponds to two ladders, independent of their arrival order.

Fig. 4 — Ladder response neighborhoods in the Bronx

$$\text{minimize} \quad \sum_{j=1}^{L} x_j$$

$$\text{subject to} \quad \sum_{j=1}^{L} a_{ij}\, x_j \geq 1 \qquad i = 1, 2, \ldots, K$$

$$x_j = 0, 1 \qquad j = 1, 2, \ldots, L$$

$$\text{where } a_{ij} = \begin{cases} 1 & \text{if the jth house's busy company covers or serves the ith RN,} \\ 0 & \text{otherwise} \end{cases}$$

The matrix of the a_{ij} is known as the *incidence matrix* for the covering problem. Each row of the matrix corresponds to an uncovered RN. There will be p elements equal to 1 in each row when we consider ladders, and k equal to 1 in each row when we consider engines. The columns correspond to the houses of the unavailable companies. There will be a 1 in each row-column position that marks the correspondence between an unavailable company and a RN it is supposed to cover when it is available. The output of stage 2 is a set of M ($M \leq L$) vacant houses to be filled, which is the input to stage 3.

This problem can, of course, be solved exactly. However, for reasons of speed and computer storage requirements, we developed simple heuristic procedures for solution of this problem, and those encountered in the next two stages. In testing the algorithms, the results obtained using exact procedures were compared to those obtained using the heuristics. Fortunately, in the tests the optimal solution was always obtained using the heuristic methods.

The basic heuristic rule for selection of a house to fill is to select first the house associated with the largest number of uncovered RNs. After application of this rule, the covering problem is reduced by the elimination of the house just selected to be filled and all RNs that will be covered as a consequence of filling it. In the same way, another house is selected to be covered. This procedure continues until all RNs are covered. The rule may be applied several times using alternate starting points.

V. STAGE 3: WHICH AVAILABLE COMPANIES TO MOVE

Once the houses to be filled have been selected on the basis of the minimum coverage criterion (which is basically an equity criterion), there may be many available companies that could be moved into those houses. Of course, no company should be moved if, by moving it, the minimum coverage criterion is violated. We therefore only consider moves that do not violate the standard. In addition, we wish to apply the following secondary "efficiency" criteria:

1. Do not move a company "too long" a distance.
2. Do not move a company that is "too busy."
3. Do not move a company that is protecting "too big" an area.

The way to tradeoff among these criteria is not immediately clear. A function that measures travel time was found to take all these secondary factors into account. We end up with a mathematical programming problem whose objective is to minimize the total expected travel time to alarms that occur in the regions affected by the moves. As in stage 3, the mathematical programming problem is solved heuristically.

The following simplified scenario provides the background for the development of the objective function. Referring to Fig. 5, suppose Ladder 31 has just responded to a serious fire. Its house is now empty and we wish to evaluate possible relocations into it. The houses of Ladder Companies 37 and 38 are currently covered, and either one may be moved into Ladder 31's house. We want to evaluate which move is superior and if indeed any move should be made.

First, consider the data required to make the evaluation. Let R_{31} denote the region in which Ladder 31 would be the closest company to all alarm boxes if it were available in its house. We must take into account which of Ladder 31's neighbors are currently available when we decide what constitutes R_{31}. Let A_{31} and λ denote respectively the physical area and the alarm rate of this region. (Note that this region changes as the pattern of available and busy companies changes.) Let R_{37} denote the region in which Ladder 37 is *currently* the closest available company, and A_{37} and λ_{37} be, respectively, the area and alarm rate in R_{37}. Once again, in deciding what constitutes R_{37} we must take into account which of Ladder 37's neighbors are available. Similar definitions apply for R_{38}, A_{38}, and λ_{38}. We assume that the alarm arrivals are a Poisson process and that all the above parameters have been estimated. In addition, r_{ij}, the time required to relocate a company from location i to location j, is assumed to be known.

The expected travel time of the first-arriving company to an alarm in the regions served by these companies depends on which companies are available to respond to the alarm. If the closest company is not in quarters, the second closest company must play the role of the closest company, and consequently the travel time will be longer. The computational requirements for the exact calculation of expected travel times in such a dynamically changing environment are formidable. We therefore used an approximate method based on two models*:

*The theoretical and empirical justification for the use of these models is provided in [5] and [7].

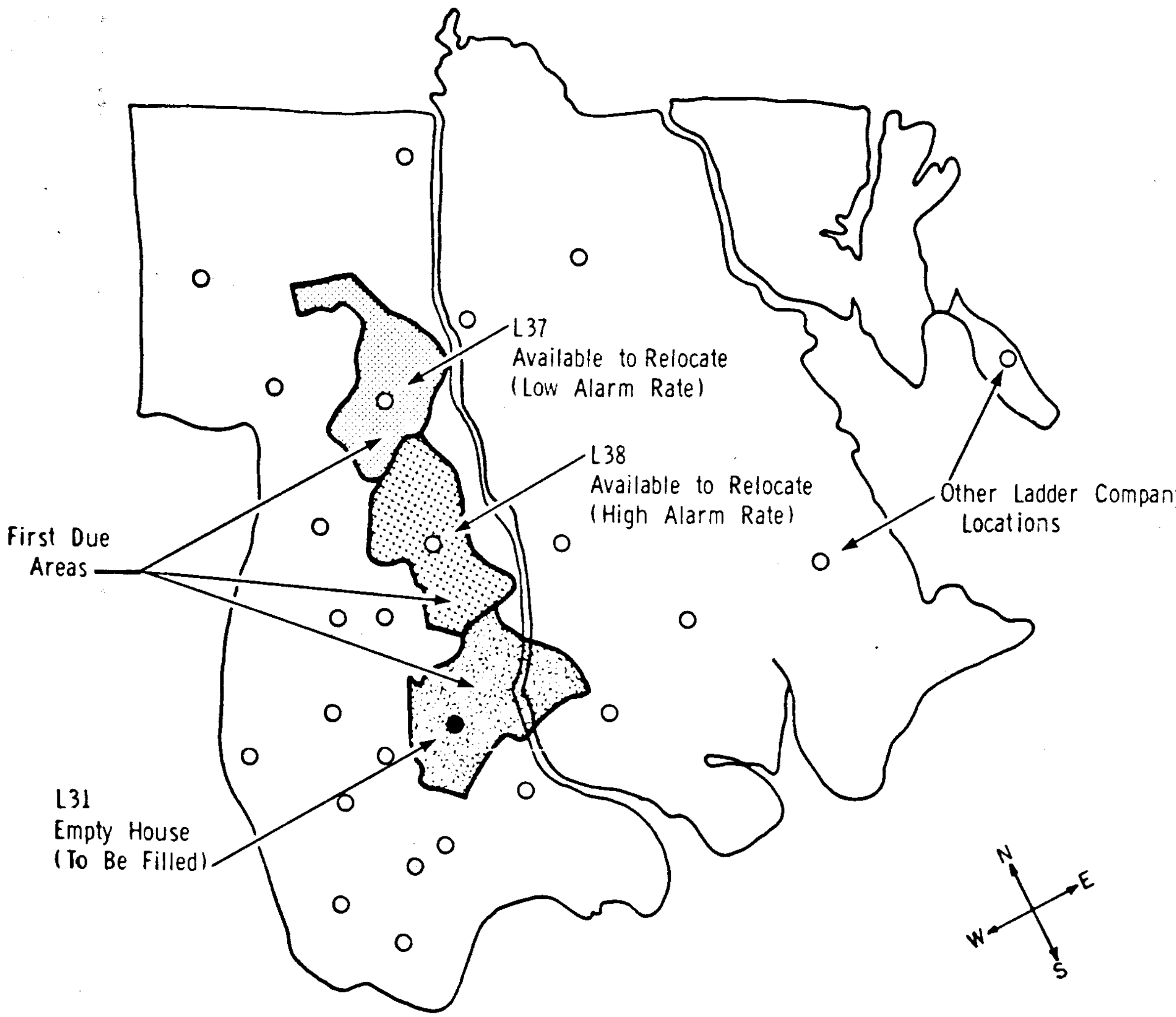


Fig. 5 - A simple relocation problem: Which of the two available companies should fill the empty house?

1. average travel distances in a region are directly proportional to the square-root of the area of the region and inversely proportional to the number of companies located in the region; and
2. average travel times in a region increase linearly with the average travel distance.

The travel-distance model estimates the regional average travel distance for the first-arriving unit as $c_1\sqrt{A/N}$ when the region has area A and has N units available. To estimate the average second-arriving travel distance c_1 is replaced by c_2. In our case N = 1, so the expected travel distance of the closest responding unit in R_i when Company i is available is estimated by $c_1\sqrt{A_i}$. If it is unavailable, but its neighbors are available (as is more or less the case if minimum coverage is being guaranteed), then $c_2\sqrt{A_i}$ is the expected response distance of the closest responding unit. Denoting the average response velocity in R_i by v_i, we have $\tau_1(i) = c_1\sqrt{A_i}/v_i$ as the expected travel time to alarms in R_i if Company i is available, and $\tau_2(i) = c_2\sqrt{A_i}/v_i$ as the expected travel time to alarms in R_i if Company i is busy.

If the alarms in R_i are arriving according to a Poisson process with an average of λ_i alarms per hour, then $\lambda_i T$ alarms would occur on the average in the region during a period of length T hours. Ignoring some of the complicated dynamic behavior that could occur in R_i during the time a large fire is in progress, we have $\tau_1\lambda_i T$ or $\tau_2\lambda_i T$ as the expected total first-arriving travel time to alarms occuring in R_i during the interval T--the duration of the fire that is causing the relocation problem.

We now return to the simple scenario of Fig. 5 where Ladder Companies 37 and 38 are candidates to relocate into Ladder 31's house, and calculate the cost of relocating Ladder 37 in terms of expected total travel time. The relevant information for this calculation is given in the table below.

i	$r_{i,31}$ (hours)	λ_i (alarms/hr.)	A_i (sq. miles)	$\tau_1(i)$ (mins.)	$\tau_2(i)$ (mins.)
37	0.2	0.2	0.9	1.7	2.8
38	0.1	1.2	1.3	2.1	3.5
31	--	1.7	0.9	1.8	2.9

Assume that the fire at which Ladder 31 is working will last one hour (i.e., T = 1 hour). In order to compare the cost of relocating Ladder Company 37 to the cost of relocating Ladder Company 38 we will consider alarms that occur in the interval $T' = [0, T + \max(r_{37,31}, r_{38,31})] = [0, 1.2]$. That is, T' has to be long enough to encompass all of the effects of any relocations in the system.

The cost of moving Ladder 37 to Ladder 31 ($c_{37,31}(T')$) is based on the assumption that Ladder 37 spends a time $r_{37,31}$ traveling to Ladder 31's house, stays at that house until Ladder 31 returns from the fire at time T, and then returns home. So R_{31} is covered by a second-closest company during the interval $[0,r_{37,31}]$ and by a closest company during the interval $[r_{37,31},T']$; R_{37} is covered by a second-closest company during the interval $[0,T']$; and R_{38} is covered by a closest company during the interval $[0,T']$. We are using an important property of the Poisson process, namely, that when two or more independent processes are observed simultaneously, the "joint" process (the results of both taken together) is also a Poisson process and has as its rate the sum of the rates of the individual processes. The components of the cost are:

For Ladder 37's area:	$\tau_2(37)\lambda_{37}T' = (2.8)(0.2)(1.2)$	$= 0.67$
For Ladder 31's area:	$\tau_2(31)\lambda_{31}r_{37,31} + \tau_1(31)\lambda_{31}(T' - r_{37,31})$	
	$= (2.9)(1.7)(0.2) + (1.8)(1.7)(1.0)$	$= 4.05$
For Ladder 38's area:	$\tau_1(38)\lambda_{38}T' = (2.1)(1.2)(1.2)$	$= \underline{3.02}$
Total cost (minutes) $= c_{37,31}(T')$		$= 7.74$

If Ladder 38 relocated into Ladder 31's house, the expected total first-arriving ladder travel time over the interval $[0,T']$ would be calculated in a similar fashion. The result is that $c_{38,31}(T') = 9.14$ minutes, which is significantly higher than $c_{37,31}(T')$. The cost of making no relocation can also be calculated. In this case the cost would be 9.35 minutes. So, in this case, the best policy would be to relocate Ladder 37 into the house of Ladder 31.

In general, if we let $\alpha_k = \lambda_k\sqrt{A_k}/v_k$, we can show that the cost (in expected total travel time) of relocating available company i into the empty house of company j is given by

$$c_{ij}(T) = (c_2 - c_1)\ \{\alpha_i(T + r_{ij}) + \alpha_j r_{ij}\},$$

and the cost of making no relocation is just $(c_2 - c_1)\alpha_j T$.

Notice that each of the three secondary criteria—relocation travel distance (r_{ij}), the "busyness" of a company (λ_i), and the size of the region protected by a company (A_i)—are all explicitly included in the cost function. In addition, another element appears that perhaps had not been anticipated: the duration of the fire causing the relocation problem. According to the cost function, it is possible that a different relocation would be suggested for a short incident than for a long incident. In fact, using this function it is possible to determine what the predicted length of the incident must be before it becomes advantageous to relocate.

Figure 6 illustrates the typical relocation costs for the situation in which Ladder Company 31 is working at a fire and Ladder Companies 37 and 38 are available to relocate. The average total first-arriving ladder travel time is shown for fires that occur during the duration of the incident leading to the relocation for four alternatives:

1. No relocation (Ladder 31's house remains uncovered);
2. Move Ladder 38, which is closer to Ladder 31, but is a busy company;
3. Move Ladder 37, which is farther away from Ladder 31, but is less busy;
4. Relocate Ladder 37 into Ladder 38's house, and relocate Ladder 38 into Ladder 31—called a "successive moveup."

For any given value of T, the best relocation is the one for which the function is the smallest. Examination of the graph indicates that it certainly does not pay to make a relocation for an incident of predicted duration less than 15 minutes. If the fire will last longer than that, the best plan is to move Ladder 37 into Ladder 31's house. If the incident lasts more than a half hour, there is a clear advantage to this relocation. Note that if Ladder 37 could not be moved it would not be worthwhile to make any relocation for a fire lasting a half hour or less. Figure 6 also shows that the successive moveup of L37 to L38 to L31 is slightly worse than just relocating Ladder 37 to Ladder 31's house.

Of course, these observations depend on the characteristics of the particular problem we have been examining. Nevertheless, examination of cost functions for many situations suggests some generalizations. First, "successive moveups" have about the same travel-time cost as simple relocations. They are sometimes a little better, more often a little worse. But they also move twice as many companies, thereby increasing the inconvenience to the men—as well as increasing communication and control problems. For these reasons, and after consultation with fire department senior officers, successive moveups were eliminated from further consideration in New York City. Second, in most situations in the city there seemed to be a clear travel-time advantage to relocating only if the relocation were to last more than one hour. (In the example, the advantage showed at about a half hour, but this is due to the fact that Ladder 31 was at that time the busiest ladder company in the city.) In general, an incident that will last an hour or more is identifiable by a chief when he first arrives at the fire. So, rather than requiring the exact duration of all incidents (the value of T in the cost function), it was suggested that relocations be made only for fires expected to last more than one hour.

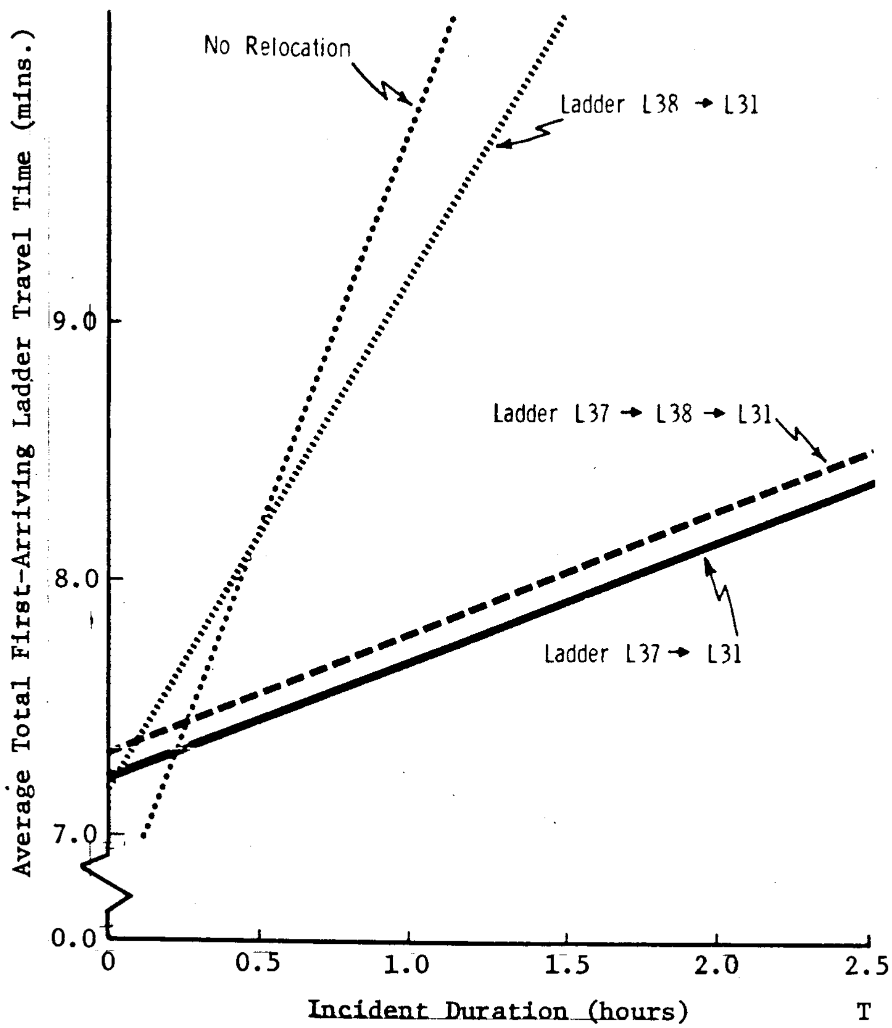


Fig. 6—A comparison of relocations into the house of Ladder 31

The actual duration of such "serious fires" does not change the *relative* travel-time cost rankings very much. That is to say, if moving Ladder 37 to the Ladder 31 house looks better than moving Ladder 38 there for a one-hour fire, it also looks better for a two-hour fire. This is important since, if it were not true, the identities of the relocating companies would depend on accurate predictions of fire duration (which are not generally easy to make). In the method we developed, travel-time cost calculations are based on the average duration of a serious fire--about one hour.

The mathematical formulation of the stage three problem (which available companies to move) is similar in structure to a "transportation" problem with additional constraints to assure that the coverage criteria are not violated.

We let $j = 1, 2, \ldots, M$ refer to the empty houses to be filled, $j = M + 1, \ldots, M + N$ refer to the available companies, and $k = 1, 2, \ldots, L$ refer to the RNs associated with the available companies. The objective function to be minimized is the total expected travel-time during the relocation incident. If we fill empty house j with available company $i_j (j = 1, 2, \ldots, M)$, the total cost of this set of moves is

$$\sum_{j=1}^{M} c_{i_j j} = (c_2 - c_1) \sum_{j=1}^{M} \{\alpha_{i_j} (T + r_{i_j j}) + \alpha_j r_{i_j j}\}$$

The decisions are framed in terms of which available company is assigned to which empty house. The decision variables will be x_{ij}, where $x_{ij} = 1$ if available company i is assigned to relocate into empty house j and $x_{ij} = 0$ otherwise. A "dummy" empty house ($j = 0$) is used so that if available company i is to remain in its own house $x_{i0} = 1$. The integer linear program to be solved is:

$$\text{min.} \quad \sum_{j=1}^{M} \sum_{i=M+1}^{M+N} c_{ij} x_{ij}$$

$$\text{s.t.} \quad \sum_{i=M+1}^{M+N} x_{ij} = 1 \qquad j = 1, 2, \ldots, M$$

$$\sum_{j=0}^{M} x_{ij} = 1 \qquad i = M+1, M+2, \ldots, M+N$$

$$\sum_{i=M+1}^{M+N} a_{ik} x_{i0} + \sum_{i=M+1}^{M+N} \sum_{j=1}^{M} a_{jk} x_{ij} \geq 1 \qquad k = 1, 2, \ldots, L$$

$$x_{ij} = 0, 1 \qquad \text{for all } i, j.$$

The objective function and the first two sets of constraints have the structure of a transportation problem. The first set of constraints requires that all M of the empty houses are filled. The second set of constraints guarantees that all available companies are assigned somewhere. As in stage 2, the coefficients a_{ij} form the incidence matrix between firehouses and response neighborhoods. $a_{ik} = 1$ if available company i serves response neighborhood k and is zero otherwise. The last set of constraints requires that none of the RNs that were being covered by available companies be uncovered.

Rather than solve such a large integer programming problem exactly, we use a heuristic algorithm. The heuristic rule that we use for determining the available company to move into a given house is to try to fill each empty house with the available company having the lowest relocation cost associated with the move. The relocation costs are the c_{ij} described above. To help select the relocatees (companies to be relocated) a ranked list of candidate relocatees can be created for each house to be filled. The companies on the list are the available ones, ordered by their c_{ij} values.

Each move must be checked against the coverage criterion to assure that no RNs become uncovered. A company on any relocatee list may be relocated without violating minimum coverage. But, if the selections are made independently for each vacant house to be filled, the resulting set of moves might have the same company moving into more than one house, or might leave one or more RNs uncovered by moving neighboring companies.

A feasible relocation is generated by successive applications of the heuristic and the feasibility test. The procedure begins with one house to be filled and progresses in sequence through the others one by one. If the lowest cost move for each house produces a feasible relocation, that relocation is optimal and no further computations are necessary. Otherwise, since the algorithm is fast, several feasible relocations are produced by changing the order in which houses being filled are considered, and by changing the heuristic for the first house being considered to "choose the available company with the qth lowest relocation cost." The least-cost relocation generated after all trials is used as the stage 3 solution.

VI. STAGE 4: SPECIFIC RELOCATION ASSIGNMENTS

The output of stage 3 is a specific set of assignments or "moves" of available companies to the empty houses being filled. However, the assignments sometimes make the relocating companies travel further than another possible assignment of the same set of companies to the same empty houses. In some rare instances the assignment can even make relocating companies travel on paths that cross. This is because travel distance is only one of the components of

c_{ij}. While the relocating travel distance matters, it is actually distance times alarm rate times the square-root of area that is being considered, and so the algorithm can sacrifice relocation travel distance for gains in expected travel times to alarms.

Yet for several reasons fire departments are concerned with the distance that relocating companies must move. One reason is that shorter relocation distances mean less of a burden on relocating companies and larger availability times. Another is that keeping the relocation distance down tends to keep companies in areas where they are familiar with street patterns as well as with particular firefighting problems. To solve this problem we view stage 3 as a device for selecting the companies to relocate and ignore the specific moves it suggests. We then determine the specific assignments that minimize total travel distance. Of course this would not make sense if the overall relocation cost were much higher. But in most cases the resulting "reassignment" increases the relocation cost very little, and can significantly reduce the total distance traveled.

We now let the index j = 1, 2, ..., M refer to the M *empty houses selected by stage 2* and the index i = 1, 2, ..., M refer to the M *available companies selected by stage 3*. Again, r_{ij} denotes the time required for a unit to relocate from "full" house i to "empty" house j, and the decision variable $x_{ij} = 1$ if available company i is assigned to empty house j and is zero otherwise.

Mathematically, stage 4 involves solving a traditional assignment problem:

Find $\{x_{ij}\}$ to

$$\text{min.} \quad \sum_{j=1}^{M} \sum_{i=1}^{M} r_{ij}\, x_{ij}$$

$$\text{s.t.} \quad \sum_{j=1}^{M} x_{ij} = 1 \qquad i = 1, 2, \ldots, M$$

$$\sum_{i=1}^{M} x_{ij} = 1 \qquad j = 1, 2, \ldots, M$$

$$x_{ij} = 0, 1.$$

For small problems, say up to M = 5, solutions can be obtained quickly by complete enumeration of all M! permutations (5! = 120). In fact it would be unusual to have to make even five relocations at one time. Usually a fire progresses through a number of stages (first alarm, second alarm, third alarm, etc.)

At each stage the number of relocations needed would be small. For example, the hypothetical case of two serious fires that break out simultaneously in the South Bronx, which was presented in Section II, led to the need for only four relocations. Figure 7 shows both the least cost set of relocations resulting from stage 3 of the algorithm (the dotted lines), and the least travel distance solution produced by stage 4 (the solid lines). The solution produced by stage 4 results in a reduction of 22 percent in travel distance with only a 9 percent increase in the cost function.

VI. TESTING AND IMPLEMENTATION

Before the New York City Fire Department would consider using the relocation algorithm in its Management Information and Control System, it insisted on extensive testing. We subjected the algorithm to a number of tests, first with problems that were designed to present difficult or interesting situations, second in a simulation model in which over 3600 alarms were generated at random according to historical patterns, and third, to provide a strenuous realistic test, in specifying relocations for one of the worst evenings ever experienced in the Bronx. In the latter, the sequence of incidents was reconstructed and a simulation was run to determine what would have occurred if the relocation algorithm had been operating during one of the most trying periods in recent departmental history. Finally, the algorithm was run in parallel with the existing manual system in one of the fire department's communications offices.

The problem whose heuristic solution is shown in Fig. 7 was also solved using an exact integer programming computer code. The result obtained was identical to the minimum cost solution found by the heuristic algorithm. However, the heuristic required only one-quarter of the CPU time and only one-half the amount of computer core storage.

When the algorithm was tested using a simulation model that recreated the actual situation in the Bronx on July 4, 1969, it was found that, if it had been operating at the time, it would have avoided almost all of the problems that actually occurred that evening. For example, in the actual situation at one point in the evening only 79 percent of the alarm boxes in the Bronx had at least one of their two closest ladder companies available in quarters. Using the algorithm over 90 percent of the boxes did. Of course, the algorithm never leaves a response neighborhood without "minimum coverage." However, on that night, a total of 16 RNs were actually left uncovered for periods ranging from 30 minutes to 1.6 hours. In addition, the algorithm generated its relocations gradually and continually over time, while relocations made by the dispatchers were generally made in spurts. For example, at one point both methods had produced 23 relocations but the algorithm had called for relocations to be made at

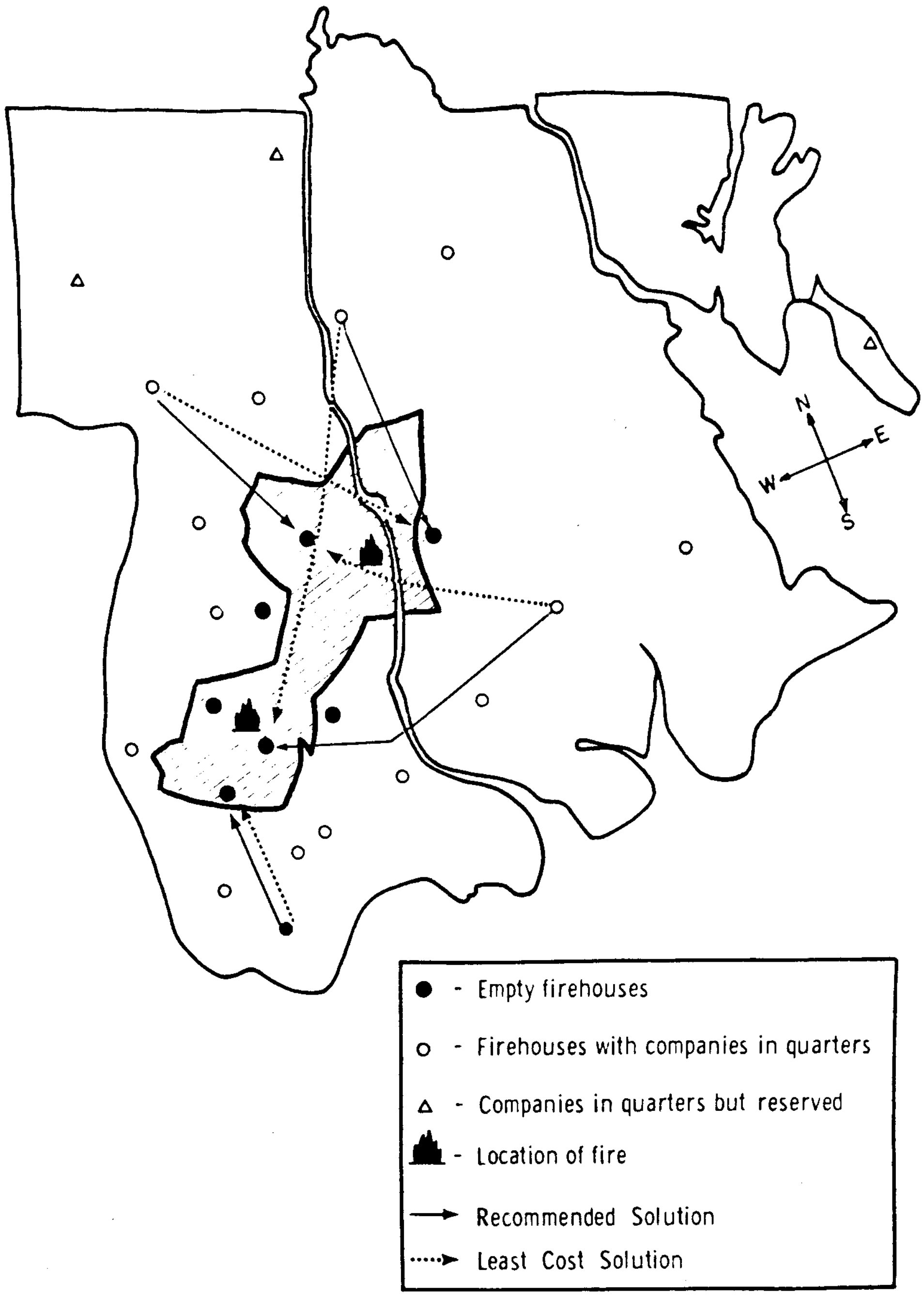


Fig. 7 - A comparison of the solutions from stages 3 and 4

16 separate times, while the dispatchers had made their relocations at only five different times. (Ten of their relocations were made at one time.)

The relocation algorithm was implemented as part of the New York City Fire Department's real-time computer-based Management Information and Control System in the middle of 1977. In addition to recommending relocations, the MICS recommends dispatches, maintains the status of fire companies and alarms in progress, and updates statistical records. The system, which was implemented in the Brooklyn communications office, is to be expanded citywide by 1981.

REFERENCES

1. Carter, Grace, Jan Chaiken, and Edward Ignall, "Response Areas for Two Emergency Units," *Operations Research*, Vol. 20, No. 3, 571-594 (1972).

2. Chaiken, Jan, "Transfer of Emergency Service Deployment Models to Operating Agencies," *Management Science*, Vol. 24, No. 7, 719-731 (1978).

3. The Rand Fire Project (Warren Walker, Jan Chaiken, and Edward Ignall, editors), *Fire Department Deployment Analysis*, Elsevier North-Holland, New York (1979).

4. Rider, Kenneth, "A Parametric Allocation Model for the Allocation of Fire Companies in New York City," *Management Science*, Vol. 23, No. 2, 146-158 (1976).

5. Kolesar, Peter, and Edward Blum, "Square Root Laws for Fire Engine Response Distances," *Management Science*, Vol. 19, No. 12, 1368-1378 (1973).

6. Kolesar, Peter, and Warren Walker, "An Algorithm for the Dynamic Relocation of Fire Companies," *Operations Research*, Vol. 22, No. 2, 249-274 (1974).

7. Kolesar, Peter, Warren Walker, and Jack Hausner, "Determining the Relation Between Fire Engine Travel Times and Travel Distances in New York City," *Operations Research*, Vol. 23, No. 4, 614-627 (1975).

8. Swersey, Arthur, "A Markovian Decision Model for Deciding How Many Units to Dispatch," in *Models for Reducing Fire Engine Response Times*, Ph.D. Thesis, Columbia University, New York (1972).

9. Walker, Warren, *Firehouse Site Evaluation Model: Executive Summary*, Report R-1618/1-HUD, The Rand Corporation, Santa Monica (1975).

RAND/P-6351

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