TWO NOTES ON INFERRING LONG RUN BEHAVIOR FROM SOCIAL EXPERIMENTS

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REMARKS ON INFERRING PERMANENT FROM TRANSITORY SUBSTITUTION EFFECTS

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In the Health Insurance Experiment, the change in net price of medical services is known, by the subject, to last only for a limited time period. Thus the measured substitution effect relates to a transitory price change, while the substitution effect it is desired to estimate relates to a permanent price change.

The same problem arises in negative income tax experiments and has been studied by Metcalf.[1] Metcalf assumed intertemporal additivity of the utility function. In reconsidering Metcalf's solution, I became aware of the importance of asking what data are available to the analyst. The following turns out to be the case: if data on demand for medical services are available for both the experiment period and the post-experiment period, then the permanent effect can be very easily inferred with <u>no</u> special assumptions (such as intertemporal additivity) on the utility function. Metcalf, on the contrary, does not assume a knowledge of substitution effects for the post-experimental period; however, he does require data on general consumption for the experimental period, data which are not easy to get.

All these observations are in the context of a two-period model. I conclude with some observations and questions about the multi-period formulation which is, I would argue, more realistic.

1. The General Two-period Model

Take the two periods to be the experimental and post-experimental periods. Let c_1, m_1 be demand for consumption other than medical services and demand for medical services in the experimental period, c_2 and m_2 the same for the post-experimental period. Take consumption as the numeraire good in both periods, R as the market rate of discount (in terms of consumption) for period 2 relative to period 1, and p_1 and

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[1] C.E. Metcalf, 1973, "Making inferences from controlled income maintenance experiments," American Economic Review 63: 478-483.

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p_2 as the prices of medical services in the two periods. The budget constraint, then, is:

$$c_1 + p_1m_1 + R(c_2 + p_2m_2) = W, \tag{1}$$

where W is the sum of present and discounted future income.

Make no special assumptions about the utility function. Let s_{ij} (i, j=1,2) be the compensated effect of a change in price p_j on the demand for medical services m_i. Let S be the compensated effect of a simultaneous and equal change in both p_1 and p_2 on m_1 (in a steady-state model this would also be the effect of such a change on m_2). What we want is to measure S; what our experiment certainly gives us is a measure of s_{11}.

By the ordinary laws of calculus,

$$S = s_{11} + s_{12}; \tag{2}$$

hence, our problem is to measure s_{12}.

We use Slutzky's relation. The discounted price of medical services in period 2 is Rp_2; hence, the relation is written:

$$s_{12}/R = s_{21}. \tag{3}$$

If we have post-experimental data on the demand for medical services, s_{21} can be measured; hence, from (2) and (3),

$$S = s_{11} + Rs_{21}, \tag{4}$$

is measurable when post-experimental data on medical services are available.

2. The Metcalf Model

I will rederive Metcalf's results in a quicker and more transparent way. He uses the assumption of intertemporal additivity to find an alternative expression for s_{21} to be substituted into (4).

The maximand is now assumed to be $U^1(c_1, m_1) + U^2(c_2, m_2)$. Let y_2 be expenditure in period 2,

$$y_2 = c_2 + p_2m_2. \tag{5}$$

Given y_2, it follows from intertemporal additivity that c_2 and m_2 maximize U^2 subject to (5). The maximizing choices are functions of

p_2 and y_2,

$$m_2 = m_2^*(p_2, y_2). \tag{6}$$

Clearly, a change in p_1 affects m_2 only through its effect on y_2, i.e., on savings behavior. Let

s'_{ij} = compensated effect on c_i of a change in p_j,

s''_{21} = compensated effect on y_2 of a change in p_1.

Then, by definition and (5),

$$s''_{21} = s'_{21} + p_2 s_{21}, \tag{7}$$

while from the previous remarks,

$$s_{21} = s''_{21}(\partial m_2^*/\partial y_2). \tag{8}$$

The substitution of (7) and (8) into (4) yields an answer, but one that involves post-experimental data from (7). Metcalf ingeniously suggests use of the identity that the price-weighted sum of the compensated effects of any price change must be zero. In this case the prices of c_1, m_1, c_2, and m_2 are 1, p_1, R, and Rp_2, respectively,

$$s'_{11} + p_1 s_{11} + R(s'_{21} + p_2 s_{21}) = 0. \tag{9}$$

Substitute from (7) into (9), solve for s''_{21} and substitute into (8), and then substitute into (4),

$$S = s_{11} - (s'_{11} + p_1 s_{11})(\partial m_2^*/\partial y_2). \tag{10}$$

This is essentially Metcalf's equation (17).

Two comments are in order: (1) As already suggested, determining s'_{11} requires having data on c_1. This is a different kind of observation than that of m_1. It is equivalent to knowing saving in the experiment period and can be determined only by a questionnaire response, which is apt to be inaccurate for many reasons. (2) Strictly speaking, determining $\partial m_2^*/\partial y_2$ requires post-experimental data. By an extension of the model it <u>might</u> be inferrable from the income effect on medical services during the experiment, though only if some identity of utility functions in the experiment and post-experiment situations is assumed.

3. A Word on Multi-period Models

Both the experiment period and the post-experiment periods are extended in time, and each can be represented as a sum of periods. This consideration may be especially important if it is recognized that the post-experiment period should be much longer than the experiment.

In the additive intertemporal case, the multi-period model is in fact equivalent to the two-period model, if suitable assumptions about the discount rates are made. There are two sets of discount rates, one for future utilities and one for future expenditures. As noted in a companion piece to this, if the two sets of discount rates are different, there will be steady accumulation or steady decumulation, either incompatible with a steady state equilibrium. Hence, we assume the same discount rate for both utilities and expenditures, so that the allocation problem is to maximize,

$$\sum_{t=1}^{\infty} R_t U(c_t, m_t), \tag{11}$$

subject to

$$\sum_{t=1}^{\infty} R_t (c_t + p_t m_t) = W, \tag{12}$$

where the life cycle is taken to be infinitely long. The experimental period will be the first T times, the post-experiment the rest; the periods are distinguished by the values of the price, p_t, of medical services, so it is assumed that

$$\begin{aligned} p_t &= p \text{ for } t \leqq T, \\ &= q \text{ for } t > T. \end{aligned} \tag{13}$$

The optimality conditions are

$$\partial U(c_t, m_t)/\partial m_t = \lambda p_t, \tag{14}$$

$$\partial U(c_t, m_t)/\partial c_t = \lambda. \tag{15}$$

Since λ is constant over time, these equations for determining c_t, m_t are the same for all $t \leqq T$, and also the same for all $t > T$.

Hence, c_t, m_t are constant over the experiment period, and also constant over the post-experiment. Letting c^1, m^1 be the values in the experiment period, c^2, m^2 over the post-experiment period, the allocation problem can be written, after dividing through in (11) and (12) by $\sum_{t=1}^{T} R_t$, as maximizing,

$$U(c^1, m^1) + R\, U(c^2, m^2),$$

subject to

$$c^1 + pm^1 + R(c^2 + qm^2) = W',$$

where

$$R = \left(\sum_{t=T}^{\infty} R_t\right) / \left(\sum_{t=1}^{T} R_t\right),$$

$$W' = W / \left(\sum_{t=1}^{T} R_t\right).$$

But this simple reduction to a two-period model is no longer valid when there is temporal dependence. In general, the allocations will not be constant within either the experiment or the post-experiment periods. While the Hicks composite commodity theorem allows a reduction to a two-period model, the process of constructing the aggregate commodities might be quite complex. This matter may need further investigation if in fact temporal dependence is found.

A TEST FOR INTERTEMPORAL DEPENDENCE IN THE UTILITY OF MEDICAL SERVICES

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Health insurance experiments, like negative income tax experiments, run for a period short compared to a human life. Thus, the parameters controlled, namely, income and the price of some commodity (leisure in the case of the negative income tax experiments, medical services in the case of the health insurance experiments) are varied from their prevailing levels only for a limited period. The subjects of the experiment are aware of this limitation; hence, in their rational planning, they should take it into account. It follows that the behavior (e.g., demand for medical services) during the experiment will not in general be the same as if the parameter changes were assumed to be permanent. But it is the latter effect that would be needed in judging the desirability of permanent policy changes, which is, after all, the supposed purpose of the experiments.

In the case of the negative income tax experiments, the adjustments have been studied in an ingenious paper by Metcalf [1], and the results can be transposed into the health insurance field. However, Metcalf postulated a utility function which is additively separable in time, so that goods and services at different points of time are neither complements nor substitutes (in the Edgeworth-Pareto sense). Intuitively, one can have reservations about this assumption, as applied to medical services. Some medical services have a durable goods aspect about them, e.g., dentistry, elective surgery, or psychotherapy; having the service at time t reduces the utility of medical services at time t+1.

Hence, it is desirable to have a test of the hypothesis that medical services in adjacent periods are substitutes. The data to be used in testing are to be drawn from the Health Insurance Experiments; hence, they have to satisfy some conditions: (1) They should be observed during the experiment; therefore, they should not include any observations made when the presently prevailing price is expected to rule indefinitely. (2) They should include only

observations on the demand for medical services and income (in addition to price data), not, therefore, data on consumption of other goods.[1]

For the utility function in each period, I will use the Stone-Geary form, a Cobb-Douglas function in which the origin has been displaced. It is well known that this function gives rise to linear demand functions (more precisely, to linear expenditure functions) without imposing the unsatisfactory condition of unitary income elasticities which would be implied by the straightforward Cobb-Douglas function. In each period, the goods are consumption of goods other than medical services and an index of medical service equivalents. The latter incorporates the possibility of substitution between adjacent periods. More precisely, the equivalent medical service is taken to be equal to current medical services plus some fraction of last period's medical services; in the case of additive separability over time, that fraction is zero. In symbols,

$$U_t = (c_t - \bar{c})^{\alpha} (M_t - \bar{M})^{\beta} \tag{1}$$

where,

$$M_t = m_t + \mu m_{t-1}, \tag{2}$$

and,

U_t = utility at time t,

c_t = consumption other than medical services at time t,

M_t = equivalent medical services,

m_t = current purchase of medical services,

μ = rate at which immediately past purchases of medical services substitute for current purchases.

Here, α, β, $\bar{c}$, and $\bar{M}$ are parameters of the utility function; concavity requires that $\alpha + \beta < 1$.

The maximand is a discounted sum of utilities over the entire lifetime of the individual, which will be taken as infinite. The experimental period is taken to be a short part of that lifetime,

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[1]Incidentally, this is a problem with Metcalf's results; they require data on the effects of parameter changes on consumption of other goods as well as the one (leisure or medical services) which is apt to be observed.

specifically, two units of time; in effect, the experiment is considered short compared to the post-experimental period for which the individual is planning.

There are, properly speaking, two discount rates, the discount of utilities and the discount of future expenditures, the latter entering into the budget constraint. They can, in principle, be different. If, however, there is a systematic difference between the two, then individuals will all be saving or all dissaving (in an infinite-horizon model), which cannot be true in equilibrium. (If individuals have different rates of subjective time discount, then indeed some could be saving and some dissaving, but we are, as is usual in consumer demand studies, assuming that all individuals are alike.) Hence, it will be assumed that the discount rates for utilities and for expenditures are equal. Let

R_t = rate at which both utilities and expenditures in time t are discounted.

Then the aim of the consumer is to

$$\text{Maximize} \quad \sum_{t=1}^{\infty} R_t U_t, \tag{3}$$

subject to a budget constraint,

$$\sum_{t=1}^{\infty} R_t (c_t + p_t m_t) = W, \tag{4}$$

where it is assumed that the price of consumption goods is 1 in every period (i.e., consumption goods other than medical services is taken as numeraire) and,

p_t = price of medical services in period t,

W = sum of discounted incomes.

The experiment determines the prices of medical services; for a given experimental plan and given post-experiment insurance policy,

$$\begin{aligned} p_t &= p \text{ for } t = 1, 2, \\ &= q \text{ for } t \geqq 3, \end{aligned} \tag{5}$$

where,

p = price of medical services (net of insurance) in experiment period,

q = price of medical services expected after experiment period.

Apply the usual Lagrange multiplier analysis, with (1) and (2) substituted into (3). Differentiation with respect to m_t, c_t, respectively, yield,

$$\beta(c_t - \bar{c})^{\alpha}(M_t - \bar{M})^{\beta-1} + \mu\beta(c_{t+1} - \bar{c})^{\alpha}(M_{t+1} - \bar{M})^{\beta-1} = \lambda p_t, \quad (6)$$

$$\alpha(c_t - \bar{c})^{\alpha-1}(M_t - \bar{M})^{\beta} = \lambda, \quad (7)$$

where λ is a Lagrange multiplier (marginal utility of wealth).

Because of (7) for all t, we can legitimately divide the first term on the left-hand side of (6) by (7), the second term by (7) with t replaced by t + 1, and the right-hand side by λ.

$$\frac{\beta}{\alpha}\frac{c_t - \bar{c}}{M_t - \bar{M}} + \frac{\mu\beta}{\alpha}\frac{c_{t+1} - \bar{c}}{M_{t+1} - \bar{M}} = p_t. \quad (8)$$

Let,

$$x_t = \frac{\beta}{\alpha}\frac{c_t - \bar{c}}{M_t - \bar{M}}. \quad (9)$$

Substitute (9) into (8), and consider only values of $t \geqq 3$. Then, from (5),

$$x_{t+1} = (-1/\mu)x_t + (q/\mu). \quad (10)$$

The steady-state solution to this difference equation is,

$$x^* = q/(1+\mu). \quad (11)$$

We can certainly assume that $\mu < 1$. Hence, the difference equation (10) is unstable; if, therefore, $x_3 \neq x^*$, the path x_t would diverge, which is impossible for an optimal path. Therefore, $x_3 = x^*$, or, from (11) and (10),

$$x_t = q/(1+\mu) \text{ for } t \geqq 3. \quad (12)$$

From (8), (9), and (5),

$$x_t = p - \mu x_{t+1} \text{ for } t = 1, 2,$$

so that, from (12),

$$x_2 = p - [\mu/(1+\mu)]q, \tag{13}$$

$$x_1 = (1-\mu)p + [\mu^2/(1+\mu)]q. \tag{14}$$

Divide (13) by (14), substitute the definition (9) for x_2 and x_1, and solve for the ratio, $(c_2 - \bar{c})/(c_1 - \bar{c})$.

$$\frac{c_2 - \bar{c}}{c_1 - \bar{c}} = A^{-1} \frac{M_2 - \bar{M}}{M_1 - \bar{M}}, \tag{15}$$

where,

$$A = \frac{(1-\mu^2)p + \mu^2 q}{(1+\mu)p - \mu q}. \tag{16}$$

Now take equation (7) for t = 1 and for t = 2, and divide one by the other,

$$\left(\frac{c_2 - \bar{c}}{c_1 - \bar{c}}\right)^{\alpha-1} \left(\frac{M_2 - \bar{M}}{M_1 - \bar{M}}\right)^{\beta} = 1. \tag{17}$$

Substitute for $(c_2 - \bar{c})/(c_1 - \bar{c})$ from (15) into (17), and solve for $(M_2 - \bar{M})/(M_1 - \bar{M})$.

$$\frac{M_2 - \bar{M}}{M_1 - \bar{M}} = A^{\delta}, \tag{18}$$

where,

$$\delta = (1-\alpha)/(1-\alpha-\beta) > 1. \tag{19}$$

Finally, substitute for M_1 and M_2 their definitions in terms of current medical purchases, (2), and solve for m_2. This yields the basic relation,

$$m_2 = (A^{\delta} - \mu)m_1 + \mu A^{\delta} m_o + (1-A^{\delta})\bar{M}, \tag{20}$$

where A and δ are defined by (16) and (19), respectively.

This relation should hold for all individuals; it has the convenient property that income does not enter explicitly. If the experimental and post-experimental medical services prices are held constant, then we should observe a linear relation among m_2, m_1, and m_o. The relation cannot, however, be fitted by regression, because it is the result of joint decisions; indeed, one would expect collinearity between any two

of the variables.

If, in fact, $\mu = 0$, then a strong conclusion follows. In that case, A = 1, from (16), so that, from (20),

$$m_1 = m_2 \text{ if } \mu = 0. \qquad (21)$$

This can be taken as a test of the hypothesis that $\mu = 0$. (21) should hold for all individuals; in a statistical model, one should probably require at least that the average values of m_1 and m_2 should be equal in each cell when the sample is stratified jointly on permanent income, experimental price, and post-experimental price.

One possible way of being a little more sophisticated (and of getting an estimate of μ if it is not zero) is sketched here. It is natural to use permanent income, W, as an instrumental variable. Let,

$$r = p/q \quad \text{ratio of experiment to post-experiment price;} \qquad (22)$$

note that A depends only on this ratio. For all individuals with a given value of r, find the covariances of m_t with W(t = 0, 1, 2). Let also A_r be the value of A for given r,

$$A_r = \frac{(1-\mu^2)r + \mu^2}{(1+\mu)r - \mu} . \qquad (23)$$

Then, from (20), we have, for each r,

$$s_2^r = (A_r^\delta - \mu)s_1^r + \mu A_r^\delta s_o^r , \qquad (24)$$

where s_t^r is the covariance between m_t and W within the subpopulation defined by holding r constant. Under the null hypothesis, $\mu = 0$, we have from (24) that,

$$s_2^r = s_1^r$$

for all r (since A = 1 when $\mu = 0$). This is a statistical analogue to the deterministic condition (21).

If μ is not 0, then the following is a possible way of estimating it. Solve (24) for A_r^δ; then, by taking logarithms, solve for δ,

$$\delta = \frac{\log(s_2^r + \mu s_1^r) - \log(s_1^r + \mu s_0^r)}{\log[(1-\mu^2)r + \mu^2]-\log[(1+\mu)r - \mu]} . \qquad (25)$$

This statement is, of course, true only up to a statistical error. This suggests the following procedure for estimating μ. Let d_r be the right-hand side of (25); it is a function of μ. Since it is supposed to be equal to a constant, independent of r, we can choose μ to minimize the variance of d_r, and use this choice as an estimate of μ. The variance should probably be weighted in some way, at least by the number of observations at each r level.

For small values of μ, it appears possible to simplify this minimization by taking linear approximations to the numerator and denominator of (25), as functions of μ. The numerator is approximately,

$$\log \frac{s_2^r}{s_1^r} + \mu\left(\frac{s_1^r}{s_2^r} - \frac{s_0^r}{s_1^r}\right) ,$$

while the denominator is approximately $(1-r)\mu$. Substituting these approximations,

$$d_r \approx \nu a_r + b_r , \tag{26}$$

where

$$\nu = 1/\mu , \tag{27}$$

$$a_r = \frac{1}{1-r} \log \frac{s_2^r}{s_1^r} ,$$

$$b_r = \frac{1}{1-r}\left(\frac{s_1^r}{s_2^r} - \frac{s_0^r}{s_1^r}\right) .$$

Take the sample variance of both sides of (26) (over r),

$$s_d^2 \approx \nu^2 s_a^2 + 2\nu s_{ab} + s_b^2 ,$$

where s_{ab} is the covariance between a and b, and s_x^2 is the variance of x, where x may be d, a, or b. We choose ν to minimize s_d^2:

$$\nu = -s_{ab}/s_a^2 ,$$

or, from (27), the estimate of μ is given by,

$$\mu = -s_a^2/s_{ab} .$$

Final Note: The chief concern I have about this model is that of a constant tradeoff between past and current medical expenditures. For very small r (low experiment price relative to post-experiment price), it leads to the conclusion that it is cheaper to get a sufficient amount of medical services in the last experiment period and have none at all in the first post-experiment period. A more realistic model would call for diminishing returns in equivalent current medical services for increases in past medical services, i.e., replacing (2) perhaps by,

$$M_t = m_t + f(m_{t-1}),$$

where f is concave, with f' approaching zero. But this model is harder to analyze, since the optimal path does not settle down to a steady state immediately upon arriving at the post-experiment period (see (12) above).

REFERENCE

Metcalf, C.E., 1973, "Making inferences from controlled income maintenance experiments," American Economic Review 63:478-483.