The Consolidated Support Model (CSM): A Three-Echelon, Multi-Item Model for Recoverable Items

John A. Muckstadt

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The model described in this report—the Consolidated Support Model (CSM)—was developed as part of Rand's contribution to the Air Staff-directed Maintenance Posture Improvement Program (MPIP) and Supply Support Improvement Program (SSIP). CSM can be used to examine the characteristics of an important class of logistics support structures, including: (1) the current structure with its three levels of maintenance (depot, intermediate, and base) and a two-echelon (depot, base) supply system, and (2) variations from the current structure in which resupply decisions occur from a central stockage point in a region, while repair is accomplished either at the central stockage decision point or at individual operating bases.

CSM provides a more refined methodology than the model used in earlier MPIP studies for assessing the supply impact of the centralized intermediate logistics concept. Improvement is achieved in two areas. First, CSM optimizes spares allocation in three-echelon supply systems. Second, it accounts for the hierarchical relationships—as evidenced in line replaceable units (LRUs) and shop replaceable units (SRUs)—that exist in many recoverable components of new weapon systems. CSM explicitly accounts for the interdependence of stocking inventories of LRUs and SRUs. Initial exploratory experiments indicate that for a particular subsystem of a weapon system, such as F-15 avionics, there are some configurations of regionalized resupply decision points and regionalized intermediate level maintenance activities where increased supply costs due to the extended pipelines are not so great as anticipated. Apparently, even though there is a greater investment in LRUs to support the longer pipeline time, there may be corresponding reductions in SRU investment because of the safety stock reductions that result from the regionalized (rather than base) control of SRUs.

A computer code of the algorithm described in this report has been written and is undergoing final testing and validation at Plans and Analysis, Air Force Acquisition Logistics Division, Air Force Logistics
Command (AFALD/AFLC). The earlier AFLC modeling efforts greatly facilitated the development of the CSM model.

This report is intended for Air Force personnel engaged in logistics planning and studies. It was prepared under the Project RAND study effort entitled "Development and Evaluation of Concepts and Methodologies for Improving the Air Force Spares Support Systems." It was partially supported by the Office of Naval Research, under Contract N00014-75-C-1172, Task NR-042-335 to Cornell University. The author is an Associate Professor of Operations Research and Industrial Engineering, Cornell University, and a consultant to The Rand Corporation.
The Consolidated Support Model (CSM) is a mathematical model of a general three-echelon supply system in which item demand is assumed to follow a Poisson distribution. It determines the optimal stock levels at each of the three echelons—base, regional supply point, and depot. The optimization takes the form of minimizing the total number of back-orders for a given level of funding on a particular type of recoverable item called Line Replaceable Units (LRUs) at operating locations where flying activities take place. At these locations, maintenance is normally limited to removing defective LRUs from aircraft and replacing them with serviceable ones. The defective LRUs are checked at the shop, and their malfunctioning subcomponents are replaced with serviceable ones. These subcomponents are called Shop Replaceable Units (SRUs). The defective SRUs are normally shipped to a depot for repair.

The time needed to restore the defective LRUs to a serviceable condition obviously depends on the availability of serviceable SRUs at that echelon. CSM recognizes this hierarchical relationship among the recoverable items and explicitly accounts for the relationship between the stocking of inventories of LRUs and SRUs at one echelon and the performance at that echelon, as well as at other echelons.

CSM derives a simple analytic solution for three-echelon stockage problems under the steady-state demand assumption. Under this assumption, the solution depends only on the mean resupply times rather than the resupply time distributions. Mathematical results are stated in terms of the Poisson assumption, but they can be readily extended to cover the compound Poisson case.

Because the development of CSM was initially motivated by the Centralized Intermediate Logistics Concept (CILC) [2], we discuss CSM within the CILC framework. It can, however, be applied to other logistics structures.

In the CILC, resources for performing intermediate level maintenance are no longer placed at operating bases; they are consolidated at the Centralized Intermediate Repair Facility (CIRF), which is usually some distance away. This type of three-echelon structure could be expected
to increase requirements of spares as compared to the current two-echelon system because of the extended pipeline time for the operating bases. But that conclusion does not necessarily hold when the situation is examined more closely using a model such as CSM, in which the hierarchical relationships of components are taken into account. Under the CILC, the average LRU resupply time would normally be longer than under the current two-echelon structure. Correspondingly, the average number of LRUs will be greater. On the other hand, since SRU demands will occur only at the CIRF and at the depot where repair can be performed, and SRUs will be stocked only at these two points, the total dollar value of SRU stock should be smaller under this three-echelon CILC-like structure. In other words, reducing the number of inventory points for SRUs can reduce the amount of safety stock needed for a given level of protection. Thus the outcome of comparing the total stockage requirements of the current two-echelon structure to the CILC type three-echelon structure is not obvious. CSM can be useful for systematically examining the relative merits of the two- and three-echelon supply structures under various operating conditions.

CSM can also assess implications on spares requirements due to delays caused by other factors, such as transportation delays that may take place between various locations in the system, and maintenance delays which may be induced by the degree of investment in maintenance resources at each location. The latter delay in resupply manifests itself in the proportion of reparables that can be repaired at a given location, and the average repair cycle time.

A computer program for implementing CSM has been developed. It generates a sequence of efficient points in a single computer run so that logistics planners can readily weigh investments in inventory, transportation, and maintenance resources against support effectiveness.

The use of CSM is illustrated with an analysis of a CILC-like three-echelon supply system using some F-15 data pertaining to avionics items. In the analysis, we assumed there were three operating bases (OBs): one OB had two squadrons; the remaining OBs had one squadron each. The analysis suggests that if a CIRF were collocated with the largest OB, and no more than two days needed to ship serviceables or
reparables between the CIRF and OBs, then no additional inventory investments would be required over the current structure. If this result can be validated for a wider variety of operating conditions, it will be worthwhile to reexamine the CILC.

Finally, two other approaches for computing requirements are compared with the hierarchical approach. One of these methods is used by the Air Force for computing initial provisioning stock levels; the second is an optimization model which ignores the hierarchical relationship. A comparison made using F-15 avionics system data indicates that these alternative models will perform substantially poorer than the model proposed in this report, that is, expected flying-related backorders will be much greater when using either of the alternative models.
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I. INTRODUCTION

During the past decade several significant factors have made reevaluation of the Air Force logistics system structure desirable:

1. Manpower costs have risen sharply as part of the efforts to achieve an all-volunteer military force.
2. New weapons are more technologically advanced than those procured during the 1950s and 1960s.
3. The quantity of each aircraft type procured in recent years has generally been smaller than the size of the fleet it has replaced. Many older aircraft have been retired from the active inventory, and flying activity has been curtailed for the remaining aircraft.
4. Advances in weapon technology continue to be accompanied by a need for more sophisticated repair equipment, more costly spares, and a requirement for highly skilled technicians.

In summary, although the Air Force currently has fewer aircraft in its inventory, the increased complexity of the newer weapons has resulted in an increasing proportion of the Air Force budget being spent on logistics.

In an attempt to reduce these support costs, Air Force planners are studying alternatives to the current logistics system structure, one of which is the regional centralization of intermediate level maintenance or the Centralized Intermediate Logistics Concept (CILC)—a method for increasing manpower utilization while reducing the requirement for manpower, expensive repair equipment, and the range of spare parts at many locations. Evaluating this or any other alternative to the current structure, however, will require extensive analysis of the implications of the proposed change on all aspects of the logistics system.

The main objective of this report is to develop a mathematical model that can be used to evaluate the implications of the CILC on
requirements for recoverable items, that is, items subject to repair when they fail. The model represents the logistics relationships among the various supply echelons, and also explicitly accounts for the hierarchical design of the items themselves.

By a hierarchically designed recoverable item we mean one that has components which are also recoverable items. When an aircraft fails, a recoverable item is often found to be faulty and is removed from the aircraft and replaced by a serviceable item of the same type. If this failed item has a hierarchical design, it is normally taken to a shop on the base where a faulty recoverable component may be identified. To return the failed assembly to a serviceable condition requires removing and replacing the defective recoverable component. Many avionics system components on newer aircraft have this type of hierarchical design. For example, the radar target digital processor on the F-15 has many recoverable components which are, for the most part, integrated circuit boards.

Recognizing and describing the hierarchical relationship among the recoverable items is a major element of the model. The relationship between the stocking of inventories of assemblies and recoverable components at one echelon and the performance at that and lower echelons is demonstrated through the equations for the average resupply time for each base. Once these equations—which are shown to be the backbone of the model—are developed, the mathematical statement of the three-echelon model is given for a single hierarchically designed item; this is followed by a description of an algorithm for computing stock levels for the assembly and its components. Finally, a method for computing stock levels for systems consisting of a large number of assemblies is presented and illustrated.

Next, the use of the CSM is illustrated with an analysis of a CILC-like three-echelon supply system using F-15 data pertaining to avionics items. The purpose of the analysis is to illustrate how the inventory investment required to achieve various levels of supply performance changes when adopting a three-echelon supply system rather than the current two-echelon structure. In this analysis, we assumed a specific four-squadron environment. The experiment shows that in
certain instances no additional investment would be required by a three-echelon system over the current two-echelon structure to achieve the same level of supply effectiveness.

Two other approaches for computing requirements are compared with the hierarchical approach. One of these methods is a simple deterministic model used by the Air Force for computing initial provisioning stock levels [1]. The other is an optimization model which ignores the hierarchical relationship. As we shall see, the performance of each of these alternate models is substantially poorer than the proposed model, that is, expected flying-related backorders are substantially greater in each case. Consequently, ignoring the hierarchical relationship and the interactions among the supply echelons in a model results in drastically degraded expected system performance. Stated another way, ignoring this relationship results in an over-investment in spares in an attempt to meet a system performance goal.
II. THE OPERATING ENVIRONMENT

The three-echelon system studied here consists of a group of bases and a depot. As in the current Air Force system, each base is capable of performing only certain types of maintenance. Some bases with a Centralized Intermediate Repair Facility (CIRF) have extensive repair capability, while the remaining operating bases (OBs) perform only flight line maintenance. By definition, an OB does not have a collocated CIRF. Lastly, the depots have the capability to perform all types of repair, including overhauls.

Certain recoverable components, called Line Replaceable Units (LRUs), are removed from aircraft when they fail. An OB would usually be restricted to removing these defective LRUs from the aircraft and replacing them with serviceable ones, but a CIRF would also be able to perform maintenance on LRUs, using diagnostic equipment located in off-aircraft shops to isolate their defective subcomponents. In some instances, these subcomponents—called Shop Replaceable Units or SRUs—may also be repaired at the CIRF, although they are normally repaired at the depot. For example, the SRUs in the avionics system on the F-15, F-111, and the planned B-1 will virtually always be repaired at a depot or at a contractor facility. In the system under study, we assume that LRU repair requiring removal and replacement of SRUs is performed only at the depot or a CIRF. Consequently, SRUs need to be stocked only at CIRFs and the depot, and LRUs will most likely be stocked at all three echelons. This is the major difference between the Air Force's current two-echelon supply system and the three-echelon system studied in this report. In the current system, all bases stock both LRUs and SRUs.

A demand for a serviceable LRU occurs whenever a failed LRU is removed from an aircraft. Once an LRU is removed, base supply must provide a serviceable replacement. If a serviceable spare LRU is immediately available from base supply, the aircraft is returned to an operational condition with minimal delay. On the other hand, if no serviceable stock is on hand, the LRU is placed in a backorder status
and repair of the aircraft is delayed. The failed LRU is then either repaired at the base or sent to some higher echelon to be repaired.

Correspondingly, resupply of base stock can occur in one of two ways. If a failed LRU is repaired at the base, resupply occurs from base maintenance; if the LRU is repaired at some other location, then resupply will also occur from some other location. In either case, the organization that resupplies the base supply activity does so by exchanging a serviceable part for a failed part on a one-for-one basis. The resupply time—that is, the time it takes to replace an LRU demanded from base supply with a serviceable one—depends on the source of resupply. For example, at a base having a CIRF, the average resupply time for base supply of an LRU requested by a flying unit equals the average base maintenance time if the LRU is repaired at the CIRF. This average base LRU repair time clearly depends on the availability of the SRUs needed to accomplish the repair. If adequate SRU stocks are on hand, repair will be completed with minimal delay. On the other hand, if repair of the LRU takes place at the depot, the average resupply time for base supply equals the average depot-to-base shipping time plus the expected waiting time before a serviceable LRU is available for shipment to the base. This expected waiting time depends on the depot LRU stock level.

Additionally, in this three-echelon system we will assume the structure of the supply and maintenance system for an LRU family—an LRU and its subordinate SRUs—can be represented by a tree as displayed in Fig. 1. Specifically, we assume that the set of bases can be partitioned into a collection of mutually exclusive and collectively exhaustive sets. Each set, which has exactly one CIRF and a collection of OBs logistically supported by the CIRF, is called a Consolidated Support Family. Each OB is assumed to receive all CIRF-level resupply from the CIRF in its Consolidated Support Family.

Bases at which a CIRF is located may or may not have a flying mission. If the base has a flying mission, the flying unit requests all spare LRUs from the base CIRF. The CIRF performs all resupply for the flying activity at that base and is also a resupply point for all the OBs in the same Consolidated Support Family. Some LRUs that fail
at the flying unit at a base having a CIRF are repaired at the base CIRF, but others may be sent to the depot for repair. We assume, for the sake of simplicity, that any failed LRU sent to a location for repair by a lower echelon base is not sent on and is repaired there.

Fig. 1 — The supply and maintenance system for an LRU family
III. THE MODEL

An extension of the two-echelon MOD-METRIC model [6], our model of the three-echelon system described in Sec. II is called the Consolidated Support Model (CSM). For simplicity, we temporarily consider only a single LRU family. Later we extend the results to the situation in which there are an arbitrary number of LRU families.

The objective of the CSM is to determine the depot, CIRF, and OB stock levels that minimize expected LRU backorders at the flying locations, subject to a constraint on total LRU and SRU investment. More precisely, a function measuring the expected backorder days is to be minimized. An LRU backorder exists whenever a demand for a serviceable LRU cannot be satisfied by base supply at the base at which the LRU failure occurred. LRU resupply delays at the depot or a CIRF are measured in the model only insofar as they influence base flying-related LRU backorders; SRU shortages are also measured indirectly. Observe that an LRU flying-related backorder indicates that an aircraft is missing an LRU and is consequently unavailable to carry out its flying mission. Since SRUs are only used to repair LRUs, an SRU backorder only delays repair of the LRU; it does not directly stop an aircraft from conducting its mission. Consequently, the impact of LRU and SRU backorders on aircraft availability is quite different. We describe the exact nature of the LRU/SRU interaction in detail later in this section.

Before developing the model, we comment on its potential uses, and their implications on its design. The CSM, like the METRIC [6] and MOD-METRIC [3] models, is a planning model designed to specify the proper quantities and allocation of stock for some future period of time. It can be effectively employed as an analysis tool for studying long-range logistics requirements, but it is not to be confused with a real-time decision model. Its system operation is conservative by design, that is, elaborate redistribution policies are not required in order to achieve predicted levels of performance. Rather it employs the simple \((S - 1,S)\) inventory policy described by Feeney and Sherbrooke [2], and no attempt is made to model the numerous opportunities
for improving system performance that exist in day-to-day operations. A long-range view is taken which by necessity ignores these intricate daily dynamics. Thus the performance—expected backorders—predicted by the model should be slightly greater than observed in the operating environment. Although this conservative view may be a shortcoming, we believe that forecasting specific future events and the decisions necessary to deal with them is impossible. Consequently the conservative view of the system's operation seems to be preferable when modeling this environment.

Before presenting the mathematical model for this decision problem, we first state the underlying assumptions and then develop the average LRU resupply time equations for all bases. As will be shown, these equations are the backbone of the model. They represent the manner in which LRU and SRU stock levels interact, and explicitly state how resupply capability for each echelon depends on the stock levels for all higher echelons. Having established these equations, we next determine the probability distributions describing the number of units in resupply for each location. Using these probabilities we can then calculate the expected number of LRU backorders outstanding at any time at each base; that is, we can state the model's objective function.

**BASIC ASSUMPTIONS**

The basic assumptions* underlying the CSM, in addition to those mentioned earlier, include:

1. LRU demand at each base is a stationary Poisson process.
2. There is no lateral resupply among bases.
3. All failed parts are repaired.
4. The probability of a failure of one LRU is independent of failures occurring for other LRUs.
5. Repair times are statistically independent.
6. There is no waiting or batching of items before starting the repair of any item.

---

*A complete discussion of these assumptions and their implications is given in Ref. 7.
7. The echelon at which repair is performed depends only on the complexity of the repair.

8. Each LRU failure repaired at a CIRF is caused by a failure of at most a single SRU.

AVERAGE LRU RESUPPLY TIME EQUATION

After defining some necessary notation, we first derive the average LRU resupply time equation for each CIRF and describe in detail the exact nature of the LRU/SRU interaction. Next we develop the average LRU resupply time equations for both the flying activity at CIRF bases and the O\(\overline{s}\) subordinate to the CIRF.

Let \(N(1)\) denote the set of locations having CIRFs, and let \(N(1, k)\) denote the set of locations resupplied by \(k \in N(1)\); let \(N(2) = \{n_1 + 1, \ldots, n\}\) be the set of O\(\overline{s}\). An index \(j\) will refer to an OB, an index \(k\) to a CIRF base, and \(0\) index will refer to the depot.

Let

\[
\lambda'_k \triangleq \text{expected daily LRU removals at the flying unit at CIRF base } k, k \in N(1);
\]

\[
\lambda'_j \triangleq \text{expected daily LRU removals at OB } j, j \in N(2);^{*}
\]

\[
w_{vt} \triangleq \text{probability that an LRU failure occurring at location } v \text{ is both repaired and resupplied by location } t;
\]

\[
\lambda_k \triangleq \text{expected daily LRU resupply requests levied on CIRF } k, k \in N(1).
\]

The expected number of requests for LRU resupply levied on CIRF \(k\) equals the expected number of daily removals of the LRU at CIRF base \(k\) plus the expected number of daily LRU resupply requests generated by lower echelon bases supported by CIRF \(k\). Thus,

\[\text{Note that } \lambda'_j \text{ and } \lambda'_k \text{ measure the expected daily removals requiring a resupply action to take place. Some LRU removals may not cause demands on supply to occur in the current system. This can happen when the LRU is taken to a shop and immediately determined to be serviceable. Consequently, the values of } \lambda'_j \text{ and } \lambda'_k \text{ may be larger in CILC than in the current Air Force two-echelon system.}\]
\[ \lambda_k = \lambda_k' + \sum_{j \in \mathbb{N}(1,k)} w_{jk} \lambda_j'. \]

Furthermore, let

\[ r_k' \triangleq \text{probability that an LRU failure at CIRF base } k \text{ is repaired at CIRF } k; \]
\[ r_k \triangleq \text{probability that an LRU arrival to CIRF } k \text{ is repaired there}; \]
\[ B_v \triangleq \text{the expected LRU repair cycle time at location } v, \text{ measured in days, including repair time delay for unavailable SRUs, } v = 1, \ldots, n; \]
\[ A_{vt} \triangleq \text{the expected LRU order-and-ship-time between } t \text{ and } v, \text{ measured in days, where } t = 0, \ldots, n, \text{ and } v = 1, \ldots, n; \]
\[ D \triangleq \text{the expected depot repair cycle time measured in days; } \]
\[ s_t \triangleq \text{the LRU stock level at location } t. \]

By assumption, all failed LRUs shipped from an OB to CIRF k are actually repaired at CIRF k; however, some LRUs that fail at CIRF base k's flying unit are sent to the depot for repair. Then the expected number of failed LRUs arriving at CIRF k each day that are repaired there equals the total number of CIRF k expected daily resupply requests minus the expected number of LRU failures per day occurring at CIRF base k's flying unit that require depot level repair. Thus the probability that an LRU arriving at CIRF k will actually be repaired there is

\[ r_k = \frac{\lambda_k - \lambda_k'(1 - r_k')}{\lambda_k'} = 1 - \frac{\lambda_k'}{\lambda_k} (1 - r_k'). \]

We are now ready to establish the equation for the average LRU resupply time for CIRF k, which we denote by \( T_k \). The expected LRU resupply time at CIRF k equals the probability \( r_k \) that the LRU will be repaired at CIRF k, times the average CIRF k LRU repair time \( B_k \), plus the probability the LRU will be repaired at the depot \( (1 - r_k) \), times the average depot-to-CIRF k resupply time.
The average depot-to-CIRF k resupply time equals the average LRU order-and-ship-time $A_{k0}$, plus the expected number of days before a serviceable LRU is available at the depot for shipment to the base. This depot delay time can be found using the following formula: expected delay days per demand equal the expected number of LRUs being delayed at any point in time— the expected number of depot backorders—divided by the expected daily depot demand rate. Let

$$\lambda = \sum_{t=1}^{n} \omega_{t0} \lambda'_{t},$$

the expected number of daily demands for LRU resupply placed on the depot. It follows from assumption 1 that $p(x|\lambda D)$—the probability that $x$ units are in resupply at the depot given that the expected demand over the depot resupply cycle is $\lambda D$—has a Poisson distribution with mean $\lambda D$. Thus the expected number of delay days experienced by each LRU resupplied by the depot can be expressed as

$$H(s_0) \overset{A}{=} \frac{\text{expected depot backorders given the depot LRU stock level } s_0}{\text{expected daily depot LRU demand}},$$

or

$$H(s_0) \overset{A}{=} \frac{1}{\lambda} \sum_{x > s_0} (x - s_0) p(x|\lambda D).$$

Combining these observations, we see that average LRU resupply time at CIRF k can be expressed as $T_k = r_k B_k + (1 - r_k)(A_{k0} + H(s_0))$. This equation indicates how the depot stock affects the average resupply time at CIRF k.

But $T_k$ also depends on the SRU stock levels. The average LRU repair cycle time $B_k$ at CIRF k is the sum of two terms. The first reflects the portion of the repair cycle time related to the operation of the maintenance and transportation systems. In particular, this term represents administrative delay time plus queuing time, plus fault isolation time, plus SRU remove-and-replace time. It also includes transportation
time if the LRU is sent to CIRF k for repair by an OB. Denote this portion of the average LRU repair cycle time by \( R_k \).

The second term reflects expected delay in completing LRU repair due to the shortage of serviceable SRUs. If a particular SRU is the cause of the LRU failure and no serviceable SRU of that type is on hand, then the LRU repair time is lengthened. Consequently \( B_{ik} \) depends on SRU stock levels at both CIRF k and the depot. Let \( C_k(s_{1k}, \ldots, s_{mk}; s_{01}, \ldots, s_{m0}) \) represent the average delay days per demand for CIRF k LRU repair given SRU i stock level \( s_{ik} \) at base k and SRU i depot stock level \( s_{i0} \), where \( m \) represents the number of different SRUs in the LRU. We will now develop an explicit expression for \( C_k \).

Recall we assume that if an LRU is repaired at a CIRF, at most one SRU needs to be replaced. Then the expected delay in LRU repair time at base k, given the failure of SRU i, is the expected number of SRUs of type i at CIRF k on which delay is being incurred at any point in time--the expected backorders for SRU i at CIRF k--divided by the expected SRU i daily removal rate at CIRF k. Denote this conditional delay by \( g_{ik} \), that is,

\[
g_{ik} \triangleq \frac{\text{expected backorders for SRU i at CIRF k at any point in time}}{\text{expected daily removal rate for SRU i at CIRF k}},
\]

or

\[
g_{ik} \triangleq \frac{1}{\lambda_{ik}} \sum_{x \geq s_{ik}} (x - s_{ik})p(x|\lambda_{ik}, T_{ik}) ,
\]

where

\[
\lambda_{ik} \triangleq \text{average number of daily removals of SRU i at base k};
\]

\[
r_{ik} \triangleq \text{probability that if SRU i fails, the SRU will be repaired at base k};
\]

\[
B_{ik} \triangleq \text{average SRU i repair time at base k};
\]

\[
A_{k0}^i \triangleq \text{average SRU i order and ship time from the depot to base k};
\]
The average resupply time equation $T_{ik}$ for SRU $i$ at CIRF $k$ equals the probability $r_{ik}$ that the SRU is repaired at CIRF $k$, times the average CIRF $k$ repair time $B_{ik}$ for SRU $i$, plus the probability the SRU will be repaired at the depot $(1 - r_{ik})$, times the sum of the depot-to-base order-and-ship-time $A_{k0}^i$, and the expected number of days before a serviceable SRU is available at the depot for shipment to the base. We denote this latter delay by $H_{ik}(s_{i0})$, where

$$H_{ik}(s_{i0}) = \frac{\text{expected number of unsatisfied depot demands for SRU } i \text{ at any point in time given the depot stock level for SRU } i \text{ is } s_{i0}}{\text{expected daily depot demand rate for SRU } i} ,$$

or

$$H_{ik}(s_{i0}) = \frac{1}{\theta_{i}} \sum_{x > s_{i0}} (x - s_{i0}) p(x|\theta_{i}, D_{i}) ,$$

and where

$$\theta_{i} = \sum_{k=1}^{n_{1}} \lambda_{ik}(1 - r_{ik}) ,$$

and

$$p(x|\theta_{i}, D_{i}) = \text{the probability of } x \text{ units in depot resupply at any point in time}$$

$$= e^{-\theta_{i} D_{i}} \frac{(\theta_{i} D_{i})^{x}}{x!} .$$

Thus

$$T_{ik} = r_{ik} B_{ik} + (1 - r_{ik})(A_{k0}^i + H_{ik}(s_{i0})) .$$
The probability that an LRU failure repaired at CIRF k is caused by SRU i is \( \lambda_{ik}/r_k \lambda_k \). Then the expected delay time in LRU repair at CIRF k due to the unavailability of SRU stock is found by multiplying the conditional delays \( g_{ik} \) by \( \lambda_{ik}/r_k \lambda_k \) and summing over SRU types. Thus

\[
G_k = \sum_{i=1}^{m} \frac{\lambda_{ik}}{r_k \lambda_k} g_{ik}.
\]

We have now seen that \( B_k \), the average CIRF k LRU repair time, can be represented as the sum of two terms, \( R_k \) and \( G_k \). We therefore have shown that the average LRU resupply time at CIRF k can be represented as

\[
T_k = r_k B_k + (1 - r_k)(A_{k0} + H(s_0)) = r_k (R_k + G_k) + (1 - r_k)(A_{k0} + H(s_0)).
\]

This equation indicates how the depot LRU stock level and the CIRF k and depot SRU stock levels affect the LRU resupply time at CIRF k.

We now develop the average resupply time equation for the flying activity located at base k, \( k \in N(1) \). Since the base flying unit at location k is physically collocated with the CIRF, no LRU stock will be allocated exclusively to it. Immediate resupply is assumed to be always available (zero lead time) for the flying activity from the CIRF, assuming serviceable stock is on hand. From a system's viewpoint there is no advantage to allocating exclusive stock to the flying activity since all LRUs assigned there, by assumption, would be unavailable for redistribution. This would degrade expected system performance. Since all resupply for the flying unit at base k comes from CIRF k, the average LRU resupply time for base k's flying unit, call it \( T_k' \), equals the expected number of delay days before a serviceable LRU becomes available at CIRF k. Therefore,
\[ T'_k = \sum_{x > s_k} \frac{(x - s_k)p(x | \lambda_k T_k)}{\lambda_k}, \quad k \in N(1). \]

The average resupply time equation for OB j, call it \( T_j \), can be found using the same method we used to determine \( T_k \). Let us temporarily assume that location j receives CIRF resupply from base k. Then \( T_j \) equals \( r_j \), the probability that the failed LRU is repaired at base j, times the average base LRU repair time \( B_j \), plus the probability \( w_{jk} \) that the LRU is repaired at CIRF k, times the sum of CIRF k to OB j order-and-ship time, \( A_{jk} \), and the expected delay in shipment due to the unavailability of a serviceable LRU at CIRF k (call this quantity \( H_k(s_k) \)), plus the probability that the LRU is shipped to the depot for repair, \( w_{j0} \), times the sum of the depot-to-base j order-and-ship time, \( A_{j0} \), and the expected delay before a serviceable LRU is available at the depot for shipment to the base, \( H(s_0) \). In general, let \( g(j) \in N(1) \) denote the CIRF base for which \( w_{jk} > 0 \). Then we may express the average resupply time for OB j as

\[ T_j = r_j B_j + w_{j0} g(j) (A_{j0} + H(s_0)) + \sum_{s_k} w_{jk} (A_{jk} + H_k(s_k)). \]

The average number of days an LRU resupply request levied on CIRF k is delayed before a serviceable LRU becomes available for shipment given the stock level \( s_k \) was denoted by \( H_k(s_k) \). This function is

\[ H_k(s_k) \triangleq \text{expected CIRF k LRU backorders at any point in time given the CIRF k stock level of } s_k \]

\[ \text{expected daily LRU demand at CSF k} \]

or

\[ H_k(s_k) \triangleq \frac{1}{\lambda_k} \sum_{x > s_k} (x - s_k)p(x | \lambda_k T_k), \]
where \( p(x|\lambda_k^T_k) \) is the probability of \( x \) LRU s in the CIRF \( k \) resupply system. In the expression, \( p(x|\lambda_k^T_k) \) is given by a Poisson distribution whose mean is \( \lambda_k^T_k \).

**Mathematical Statement of the Model**

The goal of the CSM is to find the LRU and SRU stock levels for each location that minimize the system's average number of flying-related LRU backorders outstanding at any point in time subject to a restriction on inventory investment. For each OB, that is, for each \( j \in N(2) \), we express the average number of outstanding flying-related LRU backorders at any time as

\[
\sum_{x > s_j} (x - s_j)p(x|\lambda_j^T_j),
\]

where \( s_j \) represents the LRU stock level at base \( j \). Recall that no stock is explicitly allocated to a flying unit at CIRF base \( k \). All stock at base \( k \) is under the administrative control of the CIRF. Thus the average LRU flying-related backorders at base \( k \) at any time are

\[
\sum_{x \geq 0} xp(x|\lambda_k^T_k) = \lambda_k^T_k.
\]

Therefore the objective function for the CSM is

\[
\sum_{j \in N(2)} \left\{ \sum_{x > s_j} (x - s_j)p(x|\lambda_j^T_j) \right\} + \sum_{k \in N(1)} \lambda_k^T_k.
\]

Note that the backorder expression for each base depends on its average resupply time.

The inventory investment constraint in the model states that the system investment in LRU s and SRU s cannot exceed some maximum value. If

\[ c = \text{the unit cost of an LRU}, \]
\[ c_i = \text{the unit cost of SRU } i, \]
\( s_{it} \) = stock level for SRU \( i \) at location \( t \), and
\( C \) = the available budget,

the mathematical representation of the investment constraint is

\[
\sum_{t=0}^{n} c_{t} + \sum_{i=1}^{m} c_{i} \sum_{t=0}^{n_{i}} s_{it} \leq C .
\]

Combining the above, we write the mathematical statement of the CSM as follows:

\[
\min \sum_{j \in N(2)} \left\{ \sum_{x > s_{j}} (x - s_{j})p(x | \lambda_{j}^{T}T_{j}) \right\} + \sum_{k \in N(1)} \lambda_{k}^{T}T_{k}
\]

\[
\text{subject to } \sum_{t=0}^{n} c_{t} + \sum_{i=1}^{m} c_{i} \sum_{t=0}^{n_{i}} s_{it} \leq C .
\]

where \( s_{t} \) and \( s_{it} \) are non-negative integers, and
\( t = 0, \ldots, n \) and \( i = 1, \ldots, m \).

We will call this problem \( P \).
IV. AN ALGORITHM FOR DETERMINING STOCK LEVELS

The objective function for the CSM represents the total system LRU flying-related backorders existing at any point in time. As stated in the previous section, the expected LRU backorder expression for each OB, that is, for each \( j \in \mathbb{N}(2) \), is

\[
\sum_{x>s_j} (x - s_j)p(x|\lambda_j^{T_j}),
\]

which depends on \( T_j \). But \( T_j \) is a function of both depot and CIRF LRU and SRU stock levels. Similarly, the expected LRU backorder expression for the flying unit for each \( k \in \mathbb{N}(1) \) depends on depot LRU and SRU stock levels as well as base SRU stock levels. Consequently, problem P is not a separable programming problem. Furthermore, the objective function need not be convex.

The strategy we employ to solve problem P circumvents these difficulties. Specifically, we will solve a finite sequence of subproblems, each corresponding to a fixed investment in LRUs. For a fixed total budget \( C \), it is possible to purchase either 0, 1, ..., or Q LRUs, where Q is the greatest integer less than or equal to \( C/c \). The proposed algorithm requires evaluating the solution—at least implicitly—to \( Q + 1 \) subproblems, one for each possible investment in LRUs. Each subproblem can be stated as follows:

\[
\min_{s_{it}} \left\{ \min_{s_{t}} \sum_{j \in \mathbb{N}(2)} \sum_{x>s_j} (x - s_j)p(x|\lambda_j^{T_j}) + \sum_{k \in \mathbb{N}(1)} \lambda_k^{T_k} \right\} ;
\]

\( s_{it} \) is fixed for all \( i \) and \( t \) (thereby establishing the SRU delay in \( T_k \)), and \( s_{t} \) and \( s_{it} \) are non-negative integers;

\[
\sum_{t=0}^{n} s_{t} = N ,
\]
where \( N \) represents the number of LRUs available for distribution, and \( N \) is the greatest integer less than or equal to

\[
C - \left\{ \sum_{i=1}^{m} c_i \sum_{t=0}^{n_i} s_{it} \right\}
\]

Consequently each subproblem can be partitioned into two parts, one corresponding to SRUs and the other to LRUs. The first part establishes the manner in which a limited budget \((C - cN)\) is allocated among the \( m \) SRUs. Once a specific allocation of SRUs to the depot and CIRF bases has been determined, the expected delay in LRU repair time due to SRUs is known. This in turn affects the resupply time and ultimately the expected LRU backorders at each base. The optimal allocation of the \( N \) LRUs among the bases—which corresponds to the second portion of the above problem—is obtained knowing the expected delay in LRU repair time at each CIRF.

Suppose \( U \triangleq C - cN \) dollars are available for investment in SRUs. How should it be allocated among the \( m \) SRUs? Clearly, we should make the investment so that the total expected LRU flying-related backorders are reduced by the greatest amount. If all \( n_i \) Consolidated Support Families are identical, it is not hard to show that this corresponds to an allocation in which the stock levels are selected so that total weighted expected LRU delay due to SRUs is minimized, where the weights reflect the expected number of daily LRU failures repaired at a CIRF. Although only an approximation in cases where the Consolidated Support Families are not identical, we will use this objective to determine the allocation of the available \( U \) dollars among the SRUs for each subproblem. A considerable amount of experimentation was accrued by the Air Force Logistics Command using this type of approximation in the MOD-METRIC model [4]. The approximation produced the optimal allocation in all cases. Thus, the SRU stock levels in each of the \( Q + 1 \) subproblems are obtained by solving the following problem, called problem Pl:
\[
\min \sum_{k \in \mathcal{N}(1)} r_k \lambda_k G_k = \min \sum_{k \in \mathcal{N}(1)} \sum_{l} \sum_{x > s_{ik}} (x - s_{ik})p(x \mid \lambda_{ik} T_{ik})
\]

subject to \[ \sum_{i=1}^{m} \left( c_i s_{i0} + \sum_{k=1}^{n_1} c_{ik} s_{ik} \right) \leq U, \quad (P1) \]

where \(s_{it}\) is a non-negative integer.

Observe that minimizing the total weighted expected delay due to SRUs is equivalent to minimizing total SRU backorders. The solution to this two-echelon SRU problem can be easily obtained using the method described in either Ref. 3 or Ref. 5.

When the SRU stock levels have been established, we must then determine the optimal method for allocating the \(N\) LRUs among the depot and bases. In particular, we must solve the following problem, called problem P2.

\[
\min \sum_{j \in \mathcal{N}(2)} \sum_{x > s_j} (x - s_j)p(x \mid \lambda_j T_j) + \sum_{k \in \mathcal{N}(1)} \lambda_k T_k
\]

subject to \[ \sum_{t=0}^{n_1} s_t = N, \quad (P2) \]

where \(s_t\) is a non-negative.

Due to the interaction of stock levels among echelons, problem P2 is neither convex nor separable. We therefore employ a simple partitioning procedure to obtain its solution. The algorithm for solving this three-echelon problem is based on the system's nested tree structure as displayed earlier in Fig. 1. The algorithm works up the tree by solving a sequence of independent two-echelon subproblems, one set of problems for each Consolidated Support Family; the solution to these problems are then combined in an appropriate way to solve problem P2. We now discuss the algorithm for solving problem P2 in detail.
Suppose the depot stock level is fixed at $s_0$, and assume that a total of $N_k$ LRU's are available for allocation to all bases in Consolidated Support Family $k$. Then the optimal allocation of the $N_k$ LRU's among the bases can be found by solving the following problem, called problem P3:

$$R_k(N_k; s_0) \triangleq \min \sum_{j \in N(1,k)} \sum_{x>s_j} (x - s_j)p(x|\lambda'_T j)$$

subject to  $s_0$ fixed, (P3)

$$\sum_{j \in N(1,k)} s_j + s_k = N_k, \text{ and}$$

$s_j$ a non-negative integer, $j \in N(1,k)$, $s_k \in R_k,$

where $R_k$ represents a set whose elements are the candidate values for $s_k$. This problem may not be convex and is not separable. To obtain its solution we solve the subproblems

$$h(s_k, s_0) \triangleq \min \sum_{j \in N(1,k)} \sum_{x>s_j} (x - s_j)p(x|\lambda'_T j)$$

subject to  $s_0$ and $s_k$ fixed, 

$$\sum_{j \in N(1,k)} s_j = N_k - s_k, \text{ and}$$

$s_j$ a non-negative integer, 

via marginal analysis (valid because of the convexity of the objective function). Then the solution to problem P3 is found by solving

$$\min_{s_k \in R_k} h(s_k, s_0) + \lambda'_T k$$
Since the optimal value of $N_k$ is unknown, problem P3 is solved for all values of $N_k \in \overline{R}_k$, where $\overline{R}_k$ represents the set of possible total family k stock levels.

To solve problem P2 we use the solutions obtained for each Consolidated Support Family. More specifically, to solve P2 we solve problem P4:

$$B(N) \overset{\Delta}{=} \min_{k=1}^{n_1} B_k(N_k; s_0)$$

subject to $\sum_{k=1}^{n_1} N_k + s_0 = N$, \hspace{1cm} (P4)

$$N_k \in \overline{R}_k, \ s_0 \in R_0,$$

where $R_0$ represents the set of candidate depot LRU stock levels. A dynamic programming algorithm is used to compute the optimal solution.

The amount of effort required to solve problems P3 and P4 depends on the cardinality of the sets $R_0$, $R_k$, and $\overline{R}_k$. Fortunately, the number of stock levels that need to be explicitly considered for any location or Consolidated Support Family is generally not large. This is chiefly due to the nature of the functions $H(s_0)$ and $H_k(s_k)$, which rise very sharply for stock levels below the mean demand, and approach 0 rapidly for stock levels above the mean.* Experiments [4] on similar problems indicate that the cardinality of the $R_0$ and $R_k$ sets should rarely exceed 10.

To find $\overline{R}_k$, we may first compute the total expected daily removals for family k, call it $\overline{\lambda}_k$. An estimate of the average family k resupply time, $\overline{T}_k$, is found by weighting the expected resupply times for each location in the family by the proportion of family k daily demand occurring at that location, and then summing these quantities over locations. An estimate of the depot and base k optimal stock levels obtained, for example, using the method described in Ref. 4 is employed to estimate

*An illustration of this fact is given in Ref. 4, p. 479.
the value of \( T_j \) and \( T_k \) used in the averaging. Using these values we solve problem P5:

\[
\min \sum_{k=1}^{n_1} \sum_{x > s_k} (x - \bar{s}_k) p(x | \lambda_k T_k) \\
\text{subject to } \sum_{k=1}^{n_1} \bar{s}_k = N - \bar{s}_0, \quad \text{and (P5)}
\]

\( \bar{s}_k \) is a non-negative integer,

where \( \bar{s}_0 \) is the estimate of the optimal depot stock level. Marginal analysis is used to obtain the optimal solution since the objective function is convex. \( \bar{R}_k \) is constructed based on the estimate \( \bar{s}_k \). The minimum element of \( \bar{R}_k \) can be set at \( \max\{a \bar{s}_k, \bar{s}_k - b\} \) and the largest value at \( \min\{c \bar{s}_k, \bar{s}_k + d\} \). The values of \( a, b, c, \) and \( d \) can be selected as a function of the size of \( \bar{s}_k \). For larger values of \( \bar{s}_k \), the range should be larger. Limited computational experience on a similar problem using this technique has shown that a maximum cardinality of 15 for \( \bar{R}_k \) is adequate [6]. However, the best method for determining \( R_0, R_k, \) and \( \bar{R}_k \) remains an open question.

Combining the above observations, we can state a basic algorithm for determining item stock levels:

**Initialization Step:** Establish an upper and lower bound constraint on LRU investment. Let \( u \) and \( \ell \) represent these upper and lower limits, and \( z' = \infty \) and \( U = C - \ell \). Assume \( C \) is an integer multiple of \( c \).

**Step 1.** Solve problem P6:

\[
\min \sum_{k=1}^{m} r_k \lambda_k G_k \\
\text{subject to } \sum_{i=1}^{m} \left( c_i s_{i0} + \sum_{k=1}^{n_1} c_i s_{ik} \right) \leq U, \quad \text{(P6)}
\]

where \( s_{ik} \) is a non-negative integer.
Step 2. Solve problem P7:

$$\min z = \sum_{k=1}^{n_1} \lambda_k^* T_k + \sum_{j \in \mathbb{N}(2)} \left\{ \sum_{x > s_j} (x - s_j) p(x | \lambda_j^* T_j) \right\}$$

subject to $\sum_{t=0}^{n} c_{s_t} = C - U$, \hspace{1cm} (P7)

where $T_j$ and $T_k$ are calculated using the stock levels computed in Step 1, and $s_t$ is a non-negative integer.

Step 3. If $z \geq z'$, go to Step 4; otherwise, set $z' = z$ and retain the corresponding stock levels as the incumbent stock levels. Go to Step 4.

Step 4. Decrement $U$ by $c$. If $C - U > u$, stop; otherwise, return to Step 1.

The algorithm outlined above suggests a rather tedious method for establishing the optimal investment level in LRU's and SRU's. We will now see that the number of LRU budgets that need to be explicitly examined is generally quite small. First observe that the values of $u$ and $\ell$ should be selected considering the marginal impact of investment in SRU's on expected LRU flying-related backorders. The marginal impact is negligible when investment in SRU's is large; on the other hand, LRU resupply times are increased substantially, thereby increasing total LRU system backorders, when the investment in SRU's is relatively low. Roughly stated, we would like to allocate the available budget $C$ in such a way that the marginal reduction in LRU flying-related backorders per dollar invested in SRU's equals the marginal reduction per dollar invested in the LRU. The values of $u$ and $\ell$ should reflect this goal.

It is easy to obtain an estimate of the optimal total SRU investment. Suppose we estimate SRU and LRU depot stock levels, perhaps using the method described in Ref. 4. The total cost of this investment
can be determined and subtracted from the available budget C. We next assume that each of the $n_1$ Consolidated Support Families have the same total monthly flying program and the same number of OBs, and that all flying in each family takes place at the corresponding CIRF base. Then a crude estimate of the optimal investment in SRUs for each $k \in N(L)$ corresponds to the investment level for which the partial derivative of CIRF $k$'s average resupply time with respect to dollar investment in SRUs at the CIRF $k$ equals $1/c$. We can then easily estimate the optimal total system investment in SRUs by multiplying the Consolidated Support Family estimate by $n_1$ and adding to this value the estimated required depot SRU investment.

Once $u$ and $l$ have been established, the search for the optimal partitioning of the budget is simplified by exploiting the apparent strict quasi-convexity of the total expected LRU flying-related backorders as a function of investment in LRUs. Using the Fibonacci search algorithm, we see that it is necessary to examine only a very small number of LRU investment levels explicitly. For example, if $Q = 600$, only 13 problems need to be solved explicitly. Each problem requires solving two subproblems. The first subproblem has the form of problem P6 in Step 1 of the algorithm, and the second subproblem has the form of problem P7 in Step 2. The value of $U$, of course, corresponds to a specific total investment in LRUs.

Figure 2 displays the results of applying the proposed algorithm to one F-15 LRU/SRU family. The graph relates total expected LRU flying-related backorders to the proportion of the total system budget invested in LRUs. As indicated on the graph, approximately two-thirds of the total budget should be allocated to the LRU. Investing either a greater or lesser proportion of the total budget in the LRU increases total backorders. A substantial misallocation of the available budget can seriously degrade system performance. For example, investing one-half rather than two-thirds of the budget in the LRU causes expected LRU flying-related backorders to double.
Fig. 2 — LRU system backorders as function of the percentage of total budget allocated to LRU's.
V. MULTIPLE LRU PROBLEMS

We have developed a model of a three-echelon inventory system for one LRU and its subordinate SRUs. If this model could not be easily extended to multiple LRU problems, it would be of little practical use. We now demonstrate how it can be extended.

In practice, problem P is solved for a finite number of budgets $C_1^i$, $C_2^i$, ..., $C_q_i^i$ for each LRU family $i$. The number of budgets explicitly examined, $q_i$, depends, in practice, on the expected LRU failure rate. Using the data obtained when solving these $q_i$ problems, it is possible to plot performance versus investment. Figure 3 illustrates this trade-off data for an F-15 LRU family. A piece-wise linear function can then be constructed to approximate the entire performance versus investment trade-off curve for this LRU family as shown in Fig. 4. If this curve is not convex, then replace it by its greatest convex minorant.

![Graph](image-url)  

**Fig. 3**—LRU system backorders as a function of various levels of total investment
After the convex performance/investment trade-off curves are developed for each LRU family, they are combined to produce a curve relating total flying-related backorders for all LRUs as a function of investment in all LRUs and SRUs. This curve is constructed by applying a simple marginal analysis algorithm. The first point on the system performance curve corresponds to the total expected flying-related LRU backorders when investing the minimal amount, $C_1^i$, in LRU family $i$.

Let $B_i(C_j^i)$ represent the total expected flying-related backorders for LRU $i$ given the investment in LRU family $i$ is $C_j^i$. Then the first point on the system performance curve is $\sum_i B_i(C_1^i)$ corresponding to an investment of $\sum_i C_1^i$. Next compute

$$\Delta_1^i \triangleq \frac{B_i(C_1^i) - B_i(C_2^i)}{C_2^i - C_1^i}$$
for each LRU family. Then $\Delta_1^i$ measures the marginal reduction in system LRU flying-related backorders per dollar invested in LRU family $i$. Suppose $\Delta_1^k = \max \Delta_1^i$. Then the second point on the curve is
\[ \sum_i B_i(c_1^i) - \Delta_1^k \]
corresponding to an investment of $\sum_i c_1^i + c_2^i - c_3^i$. Now compute
\[ \Delta_2^k = \frac{B_k(c_2^k) - B_k(c_3^k)}{c_3^k - c_2^k}, \]
and find the minimum of $\Delta_1^1, \Delta_1^2, \ldots, \Delta_1^k, \ldots, \Delta_1^I$, where $I$ represents the number of LRU families in the problem. The third data point is determined by computing the new total backorders and new total investment. Continue in this manner until all the available individual LRU family data have been used.

We illustrate the algorithm with a two-LRU family example. Table 1 shows the results of solving problem P several times for each of the two families. In particular, the data in the table represent the investment versus backorder data for each individual family. Table 2 contains the values of the marginal reduction in backorders per dollar invested for each incremental investment level for each LRU. These values are

<table>
<thead>
<tr>
<th>LRU Family 1</th>
<th>LRU Family 2</th>
</tr>
</thead>
<tbody>
<tr>
<td>Budget ($)</td>
<td>Budget ($)</td>
</tr>
<tr>
<td>LRU Backorders</td>
<td>LRU Backorders</td>
</tr>
<tr>
<td>231,804</td>
<td>1,036,100</td>
</tr>
<tr>
<td>.1747</td>
<td>.8580</td>
</tr>
<tr>
<td>251,204</td>
<td>1,168,100</td>
</tr>
<tr>
<td>.1108</td>
<td>.6018</td>
</tr>
<tr>
<td>270,604</td>
<td>1,300,100</td>
</tr>
<tr>
<td>.0736</td>
<td>.3642</td>
</tr>
<tr>
<td>290,004</td>
<td>1,432,100</td>
</tr>
<tr>
<td>.0448</td>
<td>.2415</td>
</tr>
<tr>
<td>309,404</td>
<td>1,564,100</td>
</tr>
<tr>
<td>.0303</td>
<td>.1465</td>
</tr>
<tr>
<td>328,804</td>
<td>1,682,400</td>
</tr>
<tr>
<td>.0178</td>
<td>.0878</td>
</tr>
<tr>
<td>350,530</td>
<td>1,814,400</td>
</tr>
<tr>
<td>.0114</td>
<td>.0531</td>
</tr>
<tr>
<td>367,604</td>
<td>--</td>
</tr>
<tr>
<td>.0069</td>
<td>--</td>
</tr>
</tbody>
</table>
Table 2

REDUCTION IN BACKORDERS PER DOLLAR INVESTED FOR EACH INCREMENT IN INVESTMENT FOR EACH LRU

<table>
<thead>
<tr>
<th>LRU Family 1</th>
<th>LRU Family 2</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\Delta_1^1 = 3.2938 \times 10^{-6}$</td>
<td>$\Delta_1^2 = 1.9409 \times 10^{-6}$</td>
</tr>
<tr>
<td>$\Delta_2^1 = 1.9175 \times 10^{-6}$</td>
<td>$\Delta_2^2 = 1.8000 \times 10^{-6}$</td>
</tr>
<tr>
<td>$\Delta_3^1 = 1.4845 \times 10^{-6}$</td>
<td>$\Delta_3^2 = 9.2955 \times 10^{-7}$</td>
</tr>
<tr>
<td>$\Delta_4^1 = 7.4742 \times 10^{-7}$</td>
<td>$\Delta_4^2 = 7.1970 \times 10^{-7}$</td>
</tr>
<tr>
<td>$\Delta_5^1 = 6.4432 \times 10^{-7}$</td>
<td>$\Delta_5^2 = 4.9619 \times 10^{-7}$</td>
</tr>
<tr>
<td>$\Delta_6^1 = 2.9457 \times 10^{-7}$</td>
<td>$\Delta_6^2 = 2.6288 \times 10^{-7}$</td>
</tr>
<tr>
<td>$\Delta_7^1 = 2.6355 \times 10^{-7}$</td>
<td>$\Delta_7^2 = --$</td>
</tr>
</tbody>
</table>

denoted $\Delta_j^i$, where

$$\Delta_j^i = \frac{B_i(C_j^i) - B_i(C_{j+1}^i)}{C_{j+1}^i - C_j^i}.$$

The results of applying the algorithm to this two-LRU example are given in Table 3. These data show how system performance—total LRU backorders—depend on system investment.

These data can then be used to determine what the individual LRU family investment levels should be so that a target system budget or performance goal is achieved. For example, suppose planners decide that approximately $1.9$ million is available for investment in these two LRUs. The closest tabulated value corresponds to a total investment of $1,892,904$. This point in turn corresponds to an investment of $328,804$ in the first LRU family and an investment of $1,564,100$ in the second LRU family.
Table 3
BACKORDER AND INVESTMENT DATA
FOR THE COMBINED SYSTEM
(Two LRU families)

<table>
<thead>
<tr>
<th>Investment ($)</th>
<th>Backorders</th>
</tr>
</thead>
<tbody>
<tr>
<td>1,267,904</td>
<td>1.0327</td>
</tr>
<tr>
<td>1,287,304</td>
<td>.9688</td>
</tr>
<tr>
<td>1,419,304</td>
<td>.7126</td>
</tr>
<tr>
<td>1,438,704</td>
<td>.6754</td>
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<tr>
<td>1,570,704</td>
<td>.4378</td>
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<tr>
<td>1,590,104</td>
<td>.4090</td>
</tr>
<tr>
<td>1,722,104</td>
<td>.2863</td>
</tr>
<tr>
<td>1,741,504</td>
<td>.2718</td>
</tr>
<tr>
<td>1,873,504</td>
<td>.1768</td>
</tr>
<tr>
<td>1,892,904</td>
<td>.1643</td>
</tr>
<tr>
<td>2,011,204</td>
<td>.1056</td>
</tr>
<tr>
<td>2,032,930</td>
<td>.0992</td>
</tr>
<tr>
<td>2,050,004</td>
<td>.0947</td>
</tr>
<tr>
<td>2,182,004</td>
<td>.0600</td>
</tr>
</tbody>
</table>

The tabulated values can be used in a second way as well. Suppose the planners decide they want no more than .1 expected flying-related backorders attributed to these two LRUs at any point in time. Then a budget of $2,032,930 must be made available for these LRU families with $350,530 and $1,682,400 budgeted for the first and second families, respectively.
VI. AN EXAMPLE OF ANALYSIS USING CSM

To illustrate the use of CSM in assessing the resource implications of certain changes in the structure of the current logistics system, we postulated an operating environment involving three bases. Two squadrons of F-15 are stationed in the first base; one squadron of F-15 is stationed in each of the other two bases. Flying activities per aircraft are assumed to be the same at each of the three bases. Intermediate maintenance is performed at each base. The average repair cycle time is assumed to be 4 days, order and shipping time from depot to each of the bases is 12 days, and the depot repair cycle time is 52 days.

We selected 18 LRUs related to the F-15 avionics system as our data base. The number of SRUs associated with these 18 LRUs is 224 items. For these LRUs, 80 to 95 percent of the malfunctioning items can be repaired at intermediate (or base) level. These avionics type items were selected for analysis because it would make sense to consolidate their intermediate maintenance at some central location.

We made five sets of computer runs, and for each run we generated a tradeoff curve between inventory investment and performance expressed in terms of the expected number of backorders on LRUs at the three bases:

Case 1 is the base case in which the structure of the logistics system remains the same as the current one. In other words, intermediate maintenance is performed at each of the operating locations, and other system parameters are the same as those described above.

In Case 2 it is assumed that intermediate maintenance will be performed at the largest base, namely base 1. It is assumed to take an average of four days to ship defective LRUs to base 1 from bases 2 and 3, and to ship serviceables back from base 1 to bases 2 and 3. Order and shipping time from base 1 to depot, and depot repair times remain the same as in the base case. This situation resembles the CILC test that PACAF is undertaking in the Far East at its Kadens, Osan, and Kunsan air bases.
In Case 3 all parameters remain unchanged from Case 2 except that it is assumed that the NRTS (Not Reparable This Station) rate for every LRU has been reduced by 50 percent. The reduction of 50 percent in NRTS rate is purely hypothetical and is not based on any engineering study. Under the CILC structure, however, the intermediate maintenance capability at a CIRF will be improved, thus reducing the proportion of items that have to be returned to the depot for repair.

In Case 4 we based our calculation on the same NRTS rates as in the base case and also using the same system parameters, except the order and shipping time from bases 2 and 3 to base 1, and the transportation time for shipping reparable items to base 1 from bases 2 and 3, have been reduced from 4 to 2 days. This was done to check the effect of the responsiveness of the transportation system on the CILC type structure.

Finally, in Case 5 we assume that order and shipping time can be set at two days, and it takes only two days to ship reparables to the CIRF. Furthermore, we assume that the NRTS rate has been reduced as in Case 3.

The results are shown in Fig. 5. Each curve summarizes the analysis corresponding to each of the five cases described above. Each curve portrays the impact on the performance of the support system as a function of investment in inventory of spares and conditioned on system parameters as described above. The performance is stated in terms of the number of backorders on LRUs throughout the system. For example, if we take the base case trade-off curve, we see that for an investment of $45 million, there will be 9 backorders on the average. How does one relate these 9 LRU backorders to some more operationally related term? We can interpret them to mean that at most there will be 9 NORS (Not Operationally Ready Supply) aircraft. Since we are talking about a wing of aircraft in the system, 9 NORS aircraft would mean the NORS rate of approximately 13 percent. This is a high-side estimate, however, because if one considers the possibility of cannibalization, it is obvious that 9 LRU backorders do not have to be distributed as one backorder per aircraft; they can be consolidated on fewer aircraft.
Fig. 5 – Analysis of structures: maintenance centralization vs decentralization
A comparison of Case 2 with the base case shows that introducing a CIRF-type structural change would imply that additional spares requirements of nearly $5 million would be needed to maintain the same level of performance. However, Case 3 results show that if the maintenance capability at the intermediate level could be enhanced to the extent that a greater proportion of defective LRUs could be fixed at base level instead of having to be shipped all the way to a depot, additional spares requirements would be minimal. Even without the assumed improvement in the maintenance productivity, if it takes only two days to ship serviceables and reparable from operating bases to the base where a CIRF is located, then the CIRF structure does not require any additional spares requirements as depicted in the comparison between Case 1 and Case 4. It was mentioned earlier that in the CIRF structure, an additional requirement for LRUs may be offset by a reduction in SRU stockage. When Case 4 was compared to Case 1 at a performance level of 25 LRU back-orders, it was found that the composition of stockage had changed as follows: For Case 1, inventory investment for SRU was $13.8 million and for LRU, $27.5 million. For Case 4, they were $11.3 million and $28.4 million, respectively. Thus under the hypothesized operating conditions, a saving in SRU stockage more than offset a need for more LRUs.

Finally, Case 5 suggests that if the CIRF structure can improve maintenance productivity as well as rely on a highly responsive transportation system, economic gains in the area of spares requirements are possible.
VII. SUMMARY AND CONCLUDING COMMENTS

A three-echelon, two-indentured inventory model was developed that can be used to establish LRU and SRU stock levels for an arbitrary number of LRU families. The model's development was based on demonstrating how SRU and LRU stock levels influence the average resupply time equations and ultimately the expected flying-related LRU backorders outstanding at any time at any location. Furthermore, an algorithm was presented for computing item stock levels for each location.

The model was compared with two other approaches: (1) the Air Force's initial provisioning technique [1] under (2) a METRIC-like optimization model.

The Air Force initial provisioning technique is not an optimization model. Requirements are established by determining the quantity of each item needed to fill the resupply system pipeline. Neither cost nor the LRU/SRU interactions are considered in this approach. Normally, too large a fraction of total investment is allocated to LRUs when using this approach. The second approach is an optimization model, whose objective is to minimize total LRU and SRU backorders subject to a constraint on total inventory investment. However, the LRU/SRU relationship is not considered. The model usually allocates too large a proportion of a given budget to SRUs because they are generally less expensive than LRUs, and both LRU and SRU backorders are considered to be equally undesirable in the model.

To illustrate these observations, the two alternate approaches were compared with the proposed model. For a set of F-15 fire control system data, flying-related LRU backorders more than doubled when using the two alternate approaches. The same total target budget was, of course, used in all cases. As this test indicates, ignoring the hierarchical relationship between the LRUs and its SRUs can degrade expected system performance substantially for a given level of investment. Stated in another way, ignoring this relationship causes an over-investment in spares to achieve a specific system performance goal.
Obstensibly, the model's main use would be to determine inventory levels for each location. The model, however, was developed primarily as a tool for investigating the impact on both supply performance and investment of changing the Air Force's two-echelon supply system to a three-echelon system. Specifically, Air Force planners are interested in examining how such a change affects the requirement for logistics resources, and inventory investment in particular. Thus, in addition to being simply a mechanism for computing stock levels, the model can be effectively employed to answer many questions related to the design of a logistics system. For example, issues that can be addressed concern the number and citing of CIRFs, the impact of changing pipeline times on inventory investment, and the way that repair capability--measured in the model in terms of the probability that an item is repaired at a particular location--alters the investment in inventory.
REFERENCES


