Air Reserve Personnel Study: Volume II. The Air Reserve Forces and the Economics of Secondary Labor Market Participation

Bernard Rostker and Robert Shishko

A Report prepared for
UNITED STATES AIR FORCE PROJECT RAND

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This report is the second in a series that presents work done under Rand's Air Reserve Personnel Study. The study was initiated at the request of the Deputy Chief of Staff/Personnel, Headquarters U.S. Air Force and is designed to explore the implications of a changing personnel environment on the airmen personnel structure of the Air Reserve Forces. This report examines the major economic features of the secondary labor market, a market the Air Force participates in when it recruits men for part-time employment in the Air National Guard and the Air Force Reserve. Although specific results cannot be directly applied to the Air Reserve Forces, this report provides a benchmark and independent check upon which to evaluate the theoretical and empirical analysis of reserve supply relationships. Volume III in this series will explicitly consider the supply of non-prior service airmen to the Air Reserve Forces.

SUMMARY

Using the Income Dynamics Panel data and the Tobit econometric technique, this report examines the determinants of the moonlighting supply curve. Conditions of primary employment are found to be major factors in determining the extent to which male heads of households participate in the secondary labor market. Of particular significance to the Air Force is the strong negative relationship between moonlighting and primary hours worked, which may indicate a cyclical aspect of moonlighting. In periods of rapid economic expansion, primary hours worked through voluntary and enforced overtime increase sharply, resulting in a decline in the moonlighting participation rate and the average number of hours spent in the secondary labor market. A corresponding proportional increase in the moonlighting wage rate would not be sufficient to restore the previous level of moonlighting.

The elasticity of supply with respect to moonlighting wages is about one (1.1 for hours and .9 for participation). If that holds true for the Air Reserve Forces, it will be very costly to replace draft-induced enlistees with volunteers obtained in the moonlighting labor market. However, since the absolute value of the moonlighting wage rate elasticity is greater than the absolute value of the primary wage rate elasticity, there will be some relief if the two increase at the same rate.

Reserve pay is relatively inflexible, being set by Congress for the entire nation, compared with civilian moonlighting wages. Therefore, the reserves may have a difficult time attracting people who have high wage moonlighting alternatives. The ordinary least squares equation of moonlighting wages indicates that high moonlighting wages are associated with being white, living in urban areas, being a high school graduate, and living in a western state. Among these groups the inflexibility of reserve pay would hinder recruiting. This study provides a benchmark and independent check upon which to evaluate the theoretical and empirical analysis of the relationships between reserve and moonlighting supply.
ACKNOWLEDGMENTS

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I. INTRODUCTION

A person holding two or more jobs is said to be moonlighting, or participating in the secondary labor market. Generally, he has a primary job from which he cannot completely satisfy his consumption desires. Moonlighting appears to be most prevalent when a person is limited to a standard work period and overtime is restricted.

The United States Air Force is a participant in the secondary labor market when it recruits men for service in the Air National Guard and the Air Force Reserve. ¹ Such military affiliation is moonlighting since reserve service is generally part-time and reservists usually have full-time positions. ² Reserve service is unique in several important ways: (1) New reservists must partake of at least five months of active duty training or have served in a regular component of the Armed Services; (2) new reservists without prior military service must be between the ages of 17 and 26; (3) reservists work only one weekend a month and 15 days each summer, compared with the more normal weekly pattern of the civilian labor market; (4) reservists commit themselves for specific periods of time and are subject to mobilization; and (5) patriotism presumably has some part in the decision to join a reserve (military) organization.

This study helps define general factors associated with moonlighting. Specifically, the study investigates the determinants of the aggregate supply function in terms of demographic and market factors and describes the relationship between primary and secondary employment.

Economic literature has treated moonlighting in two ways. First, there have been several attempts to extend traditional microeconomic theory to explain the individual moonlighter's supply curve. ³ Second,

¹At present there are programs to recruit women. However, these are relatively small and will not be examined.

²Although there is a small group of full-time employees of reserve units, the vast majority of reservists participate in their units on a part-time basis.

some researchers have presented demographic profiles of the typical moonlighter.\(^1\) To our knowledge, no one has combined these two approaches to estimate a moonlighting supply curve. In this report we attempt to estimate the moonlighting supply curve with a new data base, the Income Dynamics Panel (IDP), and the Tobit technique for estimating relationships for limited dependent variables. We first review the economics of moonlighting, the data base, and problems of statistically estimating the supply curve. Then we derive the estimates of the supply curve itself and discuss implications for the Air Reserve Forces.

II. THE ECONOMICS OF MOONLIGHTING

THE SUPPLY OF LABOR--AN INDIFFERENCE CURVE ANALYSIS OF THE PRIMARY LABOR MARKET

Traditionally, an individual receives purchasing power, or income, as a payment for work. Time spent to obtain this income can be viewed as a measure of leisure forgone—that is, the opportunity cost of working. Figure 1 shows a representative set of indifference curves indicating equal-utility combinations of income and leisure. The vertical axis indicates the level of income; the horizontal axis shows the hours of leisure or, given the complementary nature of work and leisure, the hours of work. The slope of an indifference curve at any point on the indifference map is the rate at which income and leisure are willingly substituted while a given level of satisfaction (utility) is maintained. That slope is generally referred to as the marginal rate of substitution of income for leisure.

Since income is the product of the hourly wage rate and hours worked, the slope of the line AB is equal to the prevailing wage rate. If an individual is assumed to maximize his satisfaction, he will choose a contract such that the prevailing wage rate is equal to his marginal rate of substitution of income for leisure. Such a contract is point P, the point of tangency between the indifference curve u'u' and the wage line AB. Given the wage rate, \( w_0 \), the individual will obtain the greatest satisfaction by working \( L^* \) hours and receiving \( Y^* \) income. If his wage rate increases he will reevaluate his decision and, depending on his individual preference patterns, may desire to work more or fewer hours. If he desires to work more hours he is said to have a forward-sloping supply curve. If a rise in the wage rate results in a decrease in hours worked, he has a backward-bending supply curve.

MULTIPLE JOB HOLDING--THE SECONDARY LABOR MARKET

An individual's willingness to take a second job depends on whether he can work enough hours at his prevailing primary wage rate to satisfy

\[ w_0 = \arctan \frac{OA}{OB}. \]
Fig. 1 — Utility maximization in income and leisure
his consumption goals. Consider an individual whose primary job allows him to work only $L_0$ hours but would like to work $L^* - L_0$ additional hours. See Fig. 2. Ignoring any additional costs of securing the second job, or any additional job-related costs (such as transportation), an individual will accept a second job as long as it pays a wage above the marginal rate of substitution of income for leisure at point C—the intersection of the primary wage line and the allowable hours on the first job.\footnote{The minimum second job wage rate above which the individual will accept secondary employment is often called the second job reservation wage.} If the wage rate on the second job lies between this minimum, $w'_0$, and the primary wage rate, $w_0$, he will take a second job; and if he is not already on the backward bending portion of his supply curve the total hours worked will be somewhat fewer than the number he desired to work on his first job. If the wage rate on the second job exceeds that on his first job, depending on his preference pattern, he may decide to work even more hours than he originally desired. Moreover, if there are no limits on the amount of time he can spend on the second job, he may even substitute it for his primary employment. That sometimes occurs when people make their avocation their vocation. However, the general character of second jobs often limits the number of hours that can be worked (for example, seasonal work) or provides a complementary relationship with primary employment (for example, when a school teacher tutors students after class), or has an unacceptable uncertainty of income given the person's risk aversion. However, if an individual is completely free to determine the number of hours he wants to work, at a high enough secondary wage he may develop a backward bending supply curve, and an increase in his secondary wage might result in a decrease in the number of hours worked.

An employer who wishes to attract a person into the secondary labor market can offer a wage rate below the prevailing primary wage rate as long as it is above the minimum wage, $w'_0$. The minimum wage elicits only the first marginal unit of labor. Therefore, if an employer wishes a minimum commitment of $R - L_0$ hours he will have to pay $w''_0$, as illustrated in Fig. 3, since any wage rate below this is insufficient
Fig. 2 — Effect of restricting hours worked on primary job
Note:
\[ w_0 > w_c'' > w_0' \]

Fig. 3 — Effect of requiring specific commitment on second job
to obtain the minimum commitment. If the number of hours supplied to
the secondary labor market is still insufficient to meet the demand of
a secondary job, an employer may have to offer a moonlighting wage in
excess of the prevailing primary wage rate.

Changes in the primary wage alter the minimum wage necessary to
induce people to take a second job.\(^1\) In theory, an increase in the
primary wage rate can result in an increase or a decrease in the mini-
num acceptable second job wage rate. Moreover, an increase in the pri-
mary wage can increase or decrease the hours offered on the secondary
labor market. Wilensky has argued that, psychologically, moonlighters
feel that "consumption pressures and aspirations greatly exceed their
economic rewards"\(^2\) on their first job. Perlman points out that if an
individual tries to meet specific consumption goals he is likely to
have a backward bending supply curve. Conversely, if aspirations are
the lure the supply curve will be forward sloping, and a person will
have a greater desire to moonlight as his primary wage rate increases.\(^3\)

Just as a change in the primary wage rate affects both the reserva-
tion wage for moonlighting and the hours offered in the secondary labor
market, so will a change in the primary job hours affect these two vari-
ables. An increase in the first job hours, \(L_0\), can result in an in-
crease or a decrease in the moonlighting reservation wage, though a
large increase in \(L_0\) will probably result in an increase in that wage.

An increase in \(L_0\) can result in an increase or a decrease in the
moonlighting hours offered to the markets, depending only on whether
the moonlighting wage is greater or less than the primary job wage, and
whether leisure is a superior good\(^4\) or not. If the primary wage rate
exceeds the moonlighting wage rate and leisure is a superior good, moon-
lighting hours offered will unambiguously decrease. If the primary

\(^1\)A more complete mathematical treatment of this effect can be found
in Appendix A.

\(^2\)Harold L. Wilensky, "The Moonlighter: A Product of Relative
Deprivation," Industrial Relations, 3, No. 1, October 1963, 106.

\(^3\)Perlman, Labor Theory, p. 43.

\(^4\)A superior good is one that is consumed in greater quantities as
income rises.
wage rate equals the moonlighting wage rate, then the situation depicted in Fig. 1 prevails, and moonlighting hours offered will be decreased on a one-for-one basis as primary hours increase. Only if the primary wage rate is less than the moonlighting wage rate could an increase in primary hours result in an increase in moonlighting hours offered.

THE SECONDARY LABOR MARKET—A MATHEMATICAL TREATMENT

Although geometric techniques are useful in explaining economic relationships, a more precise formulation can be presented mathematically. Those uninterested in the mathematical presentation should see Section III. Consider a representative individual with a utility function embodying a general consumption good $x$, and leisure $\ell$. Suppose this person holds a primary job that contractually calls for $L_0$ hours of work at a fixed wage rate $w_0$. Assume further that he is underemployed in this situation (as in Fig. 2) and seeks $L_m$ hours of secondary employment. The magnitude of $L_m$ depends upon the moonlighting wage rate $w_m$, which the job holder is powerless to affect. The individual then

maximizes $U(x, \ell)$

subject to $A_0 + w_0 L_0 + w_m L_m - x \geq 0$ \hspace{1cm} (1)

and $N - L_m - L_0 - \ell \geq 0$ \hspace{1cm} (2)

$x, \ell, L_m \geq 0$ \hspace{1cm} (3)

where the price of $x$ is taken as the numeraire, $N$ is the fixed number of hours per unit of time available for work or leisure, and $A_0$ is non-labor income.

THE INDIVIDUAL SUPPLY CURVE OF MOONLIGHTING LABOR

Define the function

$$\psi (w_m; Y_o) = \text{maximum } U(x, \ell)$$
subject to \[ x + w_m \lambda = Y_0 \] \[ x, \lambda \geq 0, \] \[ Y_0 = w_0 L_0 + w_m (N - L_0) + A_0; \]
then the supply function for moonlighting labor is

\[ L_m = N - L_0 + \frac{\partial v}{\partial w_m} / \frac{\partial v}{\partial Y_0}. \]

The third term of the right-hand side of equation (6) is the negative of quantity of leisure demanded at price \( w_m \). Substituting this expression for \( \lambda \) in equation (2), which holds with equality at the optimum, the supply of moonlighting labor is obtained as the complement of the demand for leisure. The properties of this supply function can be uncovered by totally differentiating the first-order conditions.

\[ U_{\lambda} - U_{x_m} = 0 \]
\[ A_0 + w_0 L_0 + w_m L_m - x = 0 \]
\[ N - L_0 - L_m - \lambda = 0 \]

\[
\begin{bmatrix}
    w_m & 0 & -1 \\
    -1 & -1 & 0 \\
    0 & (w_m U_{\lambda} - U_{\lambda \lambda}) & (w_m U_{xx} - U_{x})
\end{bmatrix}
\begin{bmatrix}
dL_m \\
d\lambda \\
dx
\end{bmatrix}
= \begin{bmatrix}
dA_0 - w_0 dL_0 - L_0 dw_0 - L_m dw_m \\
dL_0 \\
- U_x dw_m
\end{bmatrix}
\]
or

\[
\Lambda = -dA_0 - w_0 dL_0 - L_0 dw_0 - L_m dw_m
\]

\[
\begin{bmatrix}
\frac{dL_m}{dx} \\
\frac{dL_0}{dx} \\
\frac{-U_x dw_m}{x_m}
\end{bmatrix}
\]

The second-order conditions at equilibrium guarantee that \( \Lambda > 0 \), since

\[
\Lambda = -w_m (\omega U_{xx} - U_{x}) + (\omega U_{xx} - U_{x}) = U^2 \nabla^2 H > 0,
\]

where \( \nabla^2 H \) is the bordered Hessian of the utility function \( U \). Let \( \Lambda_{ij} \) denote the cofactor of the i-jth element of \( \Lambda \), e.g., \( \Lambda_{11} = (w_{1 \times 1}^2 - U_{x1}) \). The slope of the moonlighting supply function with respect to \( w_m \) is given by

\[
\frac{\delta L_m}{\delta w_m} = \frac{U_x}{\Lambda} - L_m \left( \frac{h_{11}}{\Lambda} \right). \tag{11}
\]

This is very suggestive of the Hicks–Slutsky equation. The first term, \( \frac{U_x}{\Lambda} \), which is unambiguously positive, indicates that as the opportunity cost of not moonlighting increases, there is an increase in hours offered. The second term is the "income term." If leisure is a superior good, then \( \Lambda_{11}/\Lambda \) is positive; the sign of the right-hand side of equation (11) is then ambiguous. If leisure is an inferior good, then \( \Lambda_{11}/\Lambda \) is negative and the income and substitution efforts operate in the same direction.

The effect of an increase in the first-job wage rate \( w_0 \) can be seen from

\footnote{The large \( \Lambda \)'s denote matrices and the smaller \( \Lambda \)'s denote the corresponding determinants.}
\[ \frac{\partial L_m}{\partial w_0} = -L_0 \left( \frac{L_1}{\Lambda} \right) , \]  

(12)

which is negative if leisure is superior. The effect of a change in the number of first job hours is given by

\[ \frac{\partial L_m}{\partial L_0} = -1 + \left( \frac{L_1}{\Lambda} \right) (w_m - w_0) . \]  

(13)

From equation (13), it can be seen that if leisure is superior and the moonlighting job pays less than the first job, then an increase in the number of hours contractually required on the first job will mean a decrease in the number of moonlighting hours offered. However, if the moonlighting job pays more than the first job, then the result is ambiguous. If both jobs pay the same, there is a one-for-one tradeoff in the supply function.

An increase in non-wage income can be expected to affect the supply of moonlighting labor. The magnitude of this effect is simply given by the income term, since

\[ \frac{\partial L_m}{\partial A_0} = - \left( \frac{L_1}{\Lambda} \right) . \]  

(14)
III. ESTIMATING THE MOONLIGHTING CURVE

THE INCOME DYNAMICS PANEL

One of the major problems in estimating the moonlighting supply curve has been the difficulty in finding an appropriate data base. Most data sources, for example the decennial Census, do not disaggregate yearly earnings between primary and secondary jobs. In contrast, the Income Dynamics Panel (IDP) of the University of Michigan's Survey Research Center does provide detailed information on family compositions and earnings, as well as the wage rate and hours worked on the primary and secondary jobs of the family's head.¹

The Income Dynamics Panel contains a representative cross section of the United States as well as a supplemental sample of families known to have low incomes. Members of the Panel interviewed in the springs of 1968, 1969, and 1970 supplied information on their employment experience in 1967, 1968, and 1969. The interviews were designed to collect information that explained short-term changes in the economic status of individuals and families. Between 1968 and 1970 the representative cross section sample netted 2574 cases and the supplemental sample netted 1891 cases.

Male heads of households were divided into two groups according to their participation in the secondary labor market. The first group consisted of 318 people who did some moonlighting in 1969. A second group of 1801 had not engaged in secondary employment. About 15 percent of those surveyed had second jobs, a somewhat greater percentage than may be expected in the general population. Table 1 presents the general profile of individuals in the two groups. This profile is consistent with previous characterizations of moonlighters. They tend to be younger and better educated than non-moonlighters. On the average they have larger families and spend a greater amount of a smaller income

Table 1
SELECTED CHARACTERISTICS OF HOUSEHOLDS
FROM THE INCOME DYNAMICS PANEL

<table>
<thead>
<tr>
<th>Selected Characteristics</th>
<th>Moonlighters</th>
<th>Non-Moonlighters</th>
</tr>
</thead>
<tbody>
<tr>
<td>Age</td>
<td>37.2</td>
<td>41.6</td>
</tr>
<tr>
<td>Percent non-white</td>
<td>25.6</td>
<td>33.5</td>
</tr>
<tr>
<td>Percent living in western United States</td>
<td>16.8</td>
<td>14.3</td>
</tr>
<tr>
<td>Percent living in urban areas</td>
<td>45.3</td>
<td>46.0</td>
</tr>
<tr>
<td>Percent high school graduate</td>
<td>64.2</td>
<td>50.2</td>
</tr>
<tr>
<td>Primary hourly wage rate</td>
<td>$3.60</td>
<td>$3.80</td>
</tr>
<tr>
<td>Weekly hours on primary job</td>
<td>40.9</td>
<td>43.6</td>
</tr>
<tr>
<td>Secondary hourly wage rate</td>
<td>$3.40</td>
<td>0.0</td>
</tr>
<tr>
<td>Weekly hours on secondary job</td>
<td>8.1</td>
<td>0.0</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Family</th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Family size</td>
<td>4.4</td>
<td>4.2</td>
</tr>
<tr>
<td>Non-moonlighting income</td>
<td>$9,852</td>
<td>$10,893</td>
</tr>
<tr>
<td>Asset income</td>
<td>$361</td>
<td>$516</td>
</tr>
<tr>
<td>Transfer payments</td>
<td>$236</td>
<td>$263</td>
</tr>
<tr>
<td>Labor income less moonlighting income</td>
<td>$9,254</td>
<td>$10,113</td>
</tr>
<tr>
<td>Annual cost of housing</td>
<td>$1,235</td>
<td>$1,146</td>
</tr>
</tbody>
</table>

Sample Size 318 1,801

on the major consumption item, housing. Conversely, non-moonlighters tend to have a higher hourly wage rate and work more hours per week on their primary job. In total, the picture one gets is of a young man with a small primary income facing major consumption pressures. One might expect that the average moonlighting supply curve is backward bending with respect to primary employment hours and wage rate.

**STATISTICAL MODEL**

The number of hours worked per week on the second job is the dependent variable in the statistical estimation of the moonlighting supply curve. This variable is characteristic of many economic variables obtained from surveys of households in that it has a limiting value and a large number of respondents grouped at that limit. In this case the
limiting value is zero moonlighting hours worked. In fact, 85 percent of the sample population were clustered at the limiting value. As Tobin has pointed out, "An explanatory variable in such a relationship may be expected to influence both the probability of limit responses and the size of nonlimit responses."\(^1\)

If the probability of participation in the secondary job market were the only variable of interest, the probit model would be sufficient.\(^2\) But using the probit model, Tobin points out, would be inefficient since information on the magnitude of the dependent variable—in this case, moonlighting hours—is being thrown away. Given a large concentration of observations at the zero moonlighting value, multiple regression analysis would not be appropriate either since the assumptions of that model would not be satisfied. In particular, a set of independent variables could take on values such that the expected value of the dependent variable, moonlighting hours, is below its limiting value, \(0\). Clearly this violates the assumption of a limited or bounded dependent variable.

The Tobit model combines the properties of the probit and multiple regression models and is therefore more appropriate for an estimation of the moonlighting supply curve. The Tobit model selects a set of coefficients \(\beta_i\) that maximizes the limited dependent variable likelihood function.\(^3\) The unconditional expected value of the dependent variable, \(E(y)\), can be computed directly from the coefficient vector for any vector \(X\) such that

\[\begin{align*}
\text{\textsuperscript{1} James Tobin, "Estimation of Relationships for Limited Dependent Variables," } \textit{Econometrica}, \text{ 26, No. 1, January 1958, 24.} \\
\text{\textsuperscript{2} In the probit model, the true proportion } p_i \text{ of heads of households with nonstochastic characteristics } x_{1i}, x_{2i}, x_{3i}, \ldots x_{ni} \text{ who moonlight would be estimated as } F(v_i) \text{ with } v_i = \beta_0 + \sum_{k=1}^{n} \beta_k x_{ki}, \text{ where } \beta_0' \\
\text{\textsuperscript{3} The first-order conditions for the maximization of the likelihood function are non-linear equations, but they can be solved by an iterative procedure.}
\end{align*}\]
\[ E(y) = P \cdot (X_B - L) + [(1 - P) \cdot (L)] + \sigma f \left( \frac{X_B - L}{\sigma} \right) , \quad (15) \]

where
\( L \) = limit value
\( P \) = the probability of observing \( y \geq L \) given \( X \)
\( \sigma \) = is the estimated standard error around the Tobit "index," \( X_E \).
\( f \) = ordinate of the unit normal density function.

One of the most important explanatory variables was the wage received or offered on the moonlighting job, and for non-moonlighters this information was not known. Therefore, we used the information on the 318 moonlighters to estimate a reduced form (OLS) equation with which we predicted the moonlighting wage for each member of the entire population. In effect, a two-stage procedure was performed. First, we projected moonlighting wages, based on a reduced form equation. Second, we fitted the data set of independent variables, including the reduced form prediction, to the Tobit model.

THE SUPPLY CURVE

The OLS results for the reduced form of the moonlighting wages are:

\[
w_m = 47.07 + .403 \, w_0 - 90.06 \, \text{Race} + 75.51 \, \text{Urban} + 47.33 \, \text{Hisch} \\
\quad \text{(.062)}^{\dagger} \quad (24.47) \quad (21.60) \quad (23.42) \\
+ 113.64 \, \text{Reg} + 2.26 \, \text{Age}, \\
\quad (27.62) \quad (.94)
\]

where

\[ w_m \] = moonlighting wage (\$/hr)
\[ w_0 \] = primary wage (\$/hr)
\[ \text{Race} \] = \begin{cases} 0 & \text{white} \\ 1 & \text{non-white} \end{cases}
\[ \text{Reg} \] = \begin{cases} 0 & \text{non-west} \\ 1 & \text{west} \end{cases}

\[ \dagger \] Numbers in parentheses indicate the standard error of the coefficient.
Urban = \{ \begin{array}{l}
0 \text{ non-urban} \\
1 \text{ urban}
\end{array} \quad R^2 = 0.34
\}

Hisch = \{ \begin{array}{l}
0 \text{ non-graduate} \\
1 \text{ high school graduate}
\end{array} \quad F = 25.82
\}

D.F. = 312

Age = age (years)

The above equation was used to predict the moonlighting wage rate, \( w_m \), for the entire sample population.\(^1\) Table 2 presents the results of fitting the IDP data with the computed moonlighting wage rate to the Tobit model.

The model can be evaluated at the means of the independent variables, in which case it is possible to calculate the unconditional expected value of moonlighting hours worked per week,\(^2\) the predicted probability of moonlighting, the elasticity of the expected value, and the elasticity of the predicted probability.\(^3\) Furthermore, the unconditional

---

1. This procedure may introduce a selectivity bias, which arises if moonlighting wage offers are stochastic and there is a systematic propensity for actually observed moonlighting wage rates to come from the upper (or lower) tail of the distribution. Imputing a moonlighting wage rate from observed wage rates usually overestimates the actual offers faced by those who do not moonlight. The effect of this overimputation is to bias toward zero the effect of the moonlighting wage rate on moonlighting hours in the Tobit equation.

2. "Unconditional" on holding a second job refers to the entire sample, including both moonlighters and non-moonlighters.

3. The elasticity of the expected value with respect to the variable \( X_1 \) can be obtained by differentiating equation (15) and converting to elasticity form. The elasticity of the predicted probability with respect to \( X_1 \) can be obtained as follows:

Let \[ P = \int_{-\infty}^{\frac{X_1B}{\sigma}} f(\xi) \, d\xi = F\left(\frac{X_1B}{\sigma}\right) \]
expected value divided by the predicted probability should approximate
the average hours worked per week by those who moonlight. In this case,
the unconditional expected value is 1.14 hours per week, the predicted
probability of moonlighting is .1351, and the average moonlighter can
be expected to work 8.4 hours per week on his second job.

The results presented in Table 2 also indicate that the supply curve
is forward sloping with respect to the moonlighting wage rate and back-
ward sloping with respect to primary earnings. In the first instance,
a 10 percent increase in moonlighting wages will result in an 11 per-
cent increase in the unconditional expected moonlighting hours worked
and a 9 percent increase in the predicted probability of moonlighting.\footnote{Based upon calculations of elasticities evaluated at the means of
the independent variables.}
This indicates that the average moonlighter would then work about 8.6
hours per week on his second job.

In the second instance, a given increase in first job earnings will
have a negative effect, as seen by the signs on the primary wage, the
primary hours, and the family labor income variables. However, the
magnitude of the change depends upon whether the increase was affected
by a change in the primary wage rate or by a change in hours worked on
the primary job.\footnote{Note that \( Y_0 = w_0 L_0 \) and \( dY_0/Y_0 = dL_0/L_0 + dw_0/w_0 \).} The negative elasticity with respect to primary hours
appears to be greater than the negative elasticity with respect to pri-
mary wages because a change in the latter affects only earnings but a
change in the former reduces the time available to moonlight.

People who have relatively fixed consumption goals may be expected
to have a backward sloping supply curve with respect to primary earn-
ings. Such people reduce their commitment to the secondary labor mar-
ket as they meet their goals. This conclusion is supported not only

\[
\frac{\partial P}{\partial X_i} = f\left(\frac{X_i}{\sigma}\right) \frac{\beta_i}{\sigma}
\]

and

\[
\frac{X_i}{P} \frac{\partial P}{\partial X_i} = \frac{\beta_i X_i}{\sigma} f\left(\frac{X_i}{\sigma}\right) \frac{f\left(\frac{X_i}{\sigma}\right)}{P} = \frac{\beta_i X_i}{\sigma} f\left(\frac{X_i}{\sigma}\right) \frac{f\left(\frac{X_i}{\sigma}\right)}{F\left(\frac{X_i}{\sigma}\right)}.
\]
Table 2
TOBIT MODEL
(Independent variable, weekly hours on second job)

<table>
<thead>
<tr>
<th>Independent Variable</th>
<th>Asymptotically Normal z-Score</th>
<th>Elasticity of Expected Value(^a)</th>
<th>Elasticity of Predicted Probability(^a)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Age</td>
<td>-.2639</td>
<td>-5.237</td>
<td>-1.280</td>
</tr>
<tr>
<td>Family size</td>
<td>.6556</td>
<td>2.619</td>
<td>.328</td>
</tr>
<tr>
<td>Cost of housing ($/yr)</td>
<td>.0022</td>
<td>2.460</td>
<td>.305</td>
</tr>
<tr>
<td>Primary wage rate (c/hr)</td>
<td>-.01924</td>
<td>-3.129</td>
<td>-.860</td>
</tr>
<tr>
<td>Secondary wage rate (c/hr)</td>
<td>.02723</td>
<td>3.499</td>
<td>1.105</td>
</tr>
<tr>
<td>Weekly hours on primary job</td>
<td>-.2635</td>
<td>-4.084</td>
<td>-1.348</td>
</tr>
<tr>
<td>Family income ($/yr)</td>
<td>Asset income</td>
<td>.00027</td>
<td>.837</td>
</tr>
<tr>
<td>Transfer payments</td>
<td>Labor income less moonlighting income</td>
<td>-.00077</td>
<td>-1.044</td>
</tr>
<tr>
<td>Constant</td>
<td>-1.4114</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Predicted probability evaluated at mean X(I) .1351
Observed probability .1501
Mean dependent variable 1.21
Expected value, E(y) evaluated at mean X(I) 1.14
Estimated standard error around Tobit "index" σ 16.70
Equation \(\chi^2\) 74.03
Degrees of freedom 9.

\(^a\)Evaluated at mean values of the independent variables, X(I).
by the significant relationships reported above, but also by the fact that housing (a major consumption item) and family size (a proxy for consumption) are significant and positively related to moonlighting hours. Furthermore, as is consistent with the life cycle consumption hypothesis, age shows a significantly negative relationship to moonlighting hours and can be considered an inverse proxy for unmet family needs. The estimates indicate that transfer payments and asset income by themselves are not a significant determinant of moonlighting hours either statistically or numerically.
IV. MOONLIGHTING AND THE AIR RESERVE FORCE

Using the Income Dynamics Panel data and the Tobit econometric technique, we have attempted to examine the determinants of the moonlighting supply curve. Our first major finding is that conditions of primary employment are major factors in determining the extent to which male heads of households participate in the secondary labor market. Of particular significance to the Air Force is the strong negative relationship between moonlighting and primary hours worked, which may indicate a cyclical aspect of moonlighting. In periods of rapid economic expansion, primary hours worked through voluntary and enforced overtime increase sharply, resulting in a decline in the moonlighting participation rate and the average number of hours spent in the secondary labor market. A corresponding proportional increase in the moonlighting wage rate would not be sufficient to restore the previous level of moonlighting.

A second major finding is that the elasticity of supply with respect to moonlighting wages is about one (1.1 for hours and .9 for participation). If that holds true for the Air Reserve Forces, it will be very costly to replace draft-induced enlistees with volunteers obtained in the moonlighting labor market. However, since the absolute value of the elasticity of the moonlighting wage rate is greater than the absolute value of the elasticity of the primary wage rate, there will be some relief if the two increase at the same rate.

Reserve pay is relatively inflexible, being set by Congress for the entire nation, compared with civilian moonlighting wages. Therefore, the reserves may have a difficult time attracting people who have high wage moonlighting alternatives. The ordinary least squares equation of moonlighting wages indicates that high moonlighting wages are associated with being white, living in urban areas, being a high school graduate, and living in a western state. Among these groups the inflexibility of reserve pay would hinder recruiting. Although reserve service differs from civilian moonlighting in several major ways, these findings provide a benchmark and independent check upon which to evaluate the theoretical
and empirical analysis of the relationship between reserve and moon-lighting supply. Further research to be reported in this series will examine the supply curve of non-prior service airmen. The results presented here will be correlated with results of the non-prior service reserve supply study to determine if in fact the same general pattern of response holds for the Air Reserve Forces as appears in the general labor market.
Appendix A

A MATHEMATICAL ANALYSIS OF THE RESERVATION WAGE
FOR SECOND JOB HOLDERS

In this appendix we attempt to answer the question of how an individual's reservation wage for a second job changes as his first job wage rate changes, all other things held constant.\(^1\) As we did earlier, we are discussing a representative individual with a utility function involving goods and leisure. We also confine the analysis to a person who is underemployed in his primary job.\(^2\)

On the indifference map in Fig. A-1, assume that an individual's first job contract calls for \(I_0\) hours of work per period, leaving him \(l_0\) hours of leisure, but that his true supply curve of labor SS' over the relevant range of wage offers lies entirely to the left of \(\xi_0\). The person with a wage \(w_0\) will accept a second job when an offer exceeding the marginal rate of substitution (MRS) at point A is made. The reservation wage is the MRS at point A. At the higher wage \(w_1\), but with the same number of hours of work required as on the first job, the reservation wage for this individual is the MRS at point B.

To answer the question posed in this appendix, we must ascertain how the MRS changes along the vertical \(\xi = \xi_0\). If the MRS increases along the vertical, the reservation wage for moonlighting labor increases as the first job wage increases; the reverse holds if the MRS falls along the vertical. In determining whether the MRS increases or decreases along the vertical, the conditions at equilibrium will clearly not be of any value.

THE MODEL—ONE GOOD

Suppose there is but one good \(x\) to be consumed (in addition to leisure); let the utility function of a representative individual be

\[\text{reservation wage here refers to the smallest wage rate above which an individual will accept some second job employment. This is determined naturally by his utility function.}\]

\[\text{For reference, see Perlman, Labor Theory, pp. 34-48.}\]
Fig. A-1—Representative indifference map
\( U = U(x, -L_0) \). Let \( \mu \) be the marginal utility of leisure, \( U_x \) the marginal utility of \( x \), \( U_{xx} \) the second partial with respect to \( x \), and so on. These are assumed to be nonzero. We would like to know what determines the sign of \( \partial MRS / \partial x \bigg|_{\hat{z} = \hat{z}' = 0} \).

First,

\[
\left(1\right) \quad \frac{\partial MRS}{\partial x} \bigg|_{\hat{z} = \hat{z}' = 0} = \frac{1}{U_x} \left( \frac{\partial \mu}{\partial \hat{z}} \bigg|_{\hat{z} = \hat{z}' = 0} - \mu \left( \frac{U_{xx}}{U_x^2} \right) \right)
\]

The sign of the right-hand side is ambiguous since we do not know the sign of \( \partial \mu / \partial x \). However, if we make the usual assumption of strict quasiconcavity of the utility function, we have\(^1\)

\[
\left(2\right) \quad \begin{vmatrix} 0 & U_x & \mu \\ U_x & U_{xx} & \mu_x \\ \mu & \mu_x & \mu_{\hat{z}} \end{vmatrix} > 0
\]

or

\[
\left(3\right) \quad 2U_{xx} \mu + A > 0
\]

where \( A = -\mu^2 U_{xx} - \mu \mu_{\hat{z}} U_x \).

Combining equations (1) and (3), we have

\(^{1}\)For quasiconcavity, we would only have

\[
\begin{vmatrix} 0 & U_x & \mu \\ U_x & U_{xx} & \mu_x \\ \mu & \mu_x & \mu_{\hat{z}} \end{vmatrix} \geq 0
\]
\[
\frac{\partial \text{MRS}}{\partial x} \bigg|_{\ell = \ell_0} > -\frac{A}{2\mu U_x^2} - \mu \left( \frac{U_{xx}}{U_x^2} \right)
\]

\[
> \frac{1}{2} \left[ \frac{\mu}{\mu} - \frac{U_{xx}}{U_x^2} \right]_{\ell = \ell_0}
\]

With strict quasiconcavity, a sufficient condition for \(\frac{\partial \text{MRS}}{\partial x} \bigg|_{\ell = \ell_0} \) to be positive is that the expression in brackets is greater than or equal to 0. When we rearrange terms, we can write that sufficient condition as:

\[
\Delta_{\ell = \ell_0} = \left| \begin{array}{cc} U_{\ell} & U_{xx} \\ \mu & U_x^2 \end{array} \right| = \left| \begin{array}{cc} U_{\ell} & U_{xx} \\ U_{\ell} & U_x^2 \end{array} \right| \geq 0
\]

A necessary condition for \(\frac{\partial \text{MRS}}{\partial x} \bigg|_{\ell = \ell_0} \leq 0 \) follows as the contrapositive of the above theorem, namely for \(\frac{\partial \text{MRS}}{\partial x} \leq 0 \) and with strict quasiconcavity we must have \(\Delta < 0\). With quasiconcavity, \(\Delta \geq 0\) implies \(\frac{\partial \text{MRS}}{\partial x} \geq 0\); consequently, \(\frac{\partial \text{MRS}}{\partial x} < 0\) implies \(\Delta < 0\). If the utility function is quasiconcave but not strictly, then \(\Delta \geq 0\) implies \(\frac{\partial \text{MRS}}{\partial x} < 0\).

---

1 An alternative view of the sufficiency condition can be taken: Let \(A^*_0(x)\) and \(w^*_0(x)\) be the non-labor income and primary wage required to make \(\ell = \ell_0\) precisely optimal, i.e., \(U(\ell, \ell_0)/U(\ell_0, \ell) = w^*(x)\). Then condition (5) implies

\[
\frac{\partial \ell}{\partial A^*_0} \geq 0
\]

At \((x, \ell_0)\), leisure is a superior good. However, the fact that leisure is a superior good over some region does not imply that the MRS along \(\ell = \ell_0\) is always increasing in that region. Note that the sign of \(\Delta\) is invariant under any order-preserving transformation of the utility function.
THE MODEL—n GOODS

Let the utility function of the representative individual described in Fig. 1 be \( U = U(x_1, x_2, \ldots, x_n, -L_0) \). Let \( \mu \) have the same meaning as before, the marginal utility of leisure. Let \( U_1 \) and \( U_{ii} \) be the first and second partials of the utility function with respect to the \( i \)th good, \( i = 1, \ldots, n \). Let \( y = w_0 L_0 \) be the income of the individual and \( \lambda \) the marginal utility of income.

Even though the person is in a nonequilibrium situation with respect to work and income, we assume he allocates his income over the available goods \( x_1, \ldots, x_n \) at fixed prices \( p_1, \ldots, p_n \) so as to maximize his utility function. We then have as first order conditions of this maximization:

\[
\begin{align*}
U_1 - \lambda p_1 &= 0 \\
U_2 - \lambda p_2 &= 0 \\
&\vdots \\
U_n - \lambda p_n &= 0 \\
y - \sum_{i=1}^{n} p_i x_i &= 0
\end{align*}
\]

(6)

The variable we are interested in is \( \Delta \text{MRS}/\delta y \big|_{\lambda \rightarrow 0} \) where \( y \), it should be recalled, is \( w_0 L_0 \) and represents first job purchasing power.

(7) \[
\Delta \text{MRS} \bigg|_{\lambda \rightarrow 0} = \frac{\partial \left( \frac{\mu}{\lambda} \right)}{\partial y} \bigg|_{\lambda \rightarrow 0} = \frac{1}{\lambda} \mu y - \mu \frac{\lambda}{\lambda y^2}.
\]

To eliminate \( \lambda y \) from equation (7), totally differentiate equation set (6), recalling that prices are fixed.
\[
\begin{bmatrix}
U_{11} & U_{12} & \cdots & U_{1n} & -p_1 \\
U_{21} & U_{22} & \cdots & U_{2n} & -p_2 \\
\vdots & \vdots & \ddots & \vdots & \vdots \\
U_{n1} & \cdots & U_{nn} & -p_n \\
-p_1 & \cdots & -p_n & 0
\end{bmatrix}
\begin{bmatrix}
dx_1 \\
dx_2 \\
\vdots \\
dx_n \\
d\lambda
\end{bmatrix}
= 
\begin{bmatrix}
0 \\
\vdots \\
\vdots \\
0 \\
d-y
\end{bmatrix}.
\]

Then

\[
\frac{\partial \lambda}{\partial y} = \lambda^2 \left(-\frac{\mathcal{H}}{\mathcal{H}^*}\right),
\]

where \( \mathcal{H} \) is the Hessian determinant

\[
\begin{bmatrix}
U_{11} & U_{12} & \cdots & U_{1n} \\
U_{21} & \cdots & \cdots \\
\vdots & \vdots & \ddots & \vdots \\
U_{n1} & \cdots & U_{nn}
\end{bmatrix}.
\]
and $H^*$ is the bordered Hessian determinant\(^1\)

\[
\begin{vmatrix}
U_{11} & U_{12} & \cdots & U_{1n} & U_1 \\
U_{21} & \ddots & \cdots & \ddots & \vdots \\
\vdots & \ddots & \ddots & \ddots & \vdots \\
\vdots & \ddots & \ddots & \ddots & \ddots \\
U_{n1} & \cdots & U_{nn} & U_n & U_1 \\
\end{vmatrix}
\]

Substituting equation (9) into equation (7), we obtain

\[
(10) \quad \frac{\partial MRS}{\partial y} \bigg|_{\lambda=\lambda^*} = \frac{1}{\lambda} \mu y - \mu \left( \frac{-H}{H^*} \right).
\]

We make the usual assumption that the utility function is strictly quasiconcave, so that $H^*$ (but not $H$) is necessarily negative definite.

The first term on the right-hand side in equation (10) can be re-written by multiplying both numerator and denominator by the price of any one of the goods—say, for convenience, good $x_1$.

\(^1\)The determinant of the linear system in equation (8), $\Gamma$, where

\[
\Gamma = 
\begin{vmatrix}
U_{11} & U_{12} & \cdots & U_{1n} & -P_1 \\
U_{21} & \ddots & \cdots & \vdots & \vdots \\
\vdots & \ddots & \ddots & \ddots & \vdots \\
\vdots & \ddots & \ddots & \ddots & \ddots \\
U_{n1} & \cdots & U_{nn} & -P_n & U_1 \\
-P_1 & \cdots & -P_n & 0 & U_1 \\
\end{vmatrix}
\]

is related to $H^*$ by $\lambda^2 \Gamma = H^*$, which explains the presence of $\lambda^2$ in equation (9).
\( \frac{\partial \text{MRS}}{\partial y} \bigg|_{\hat{x}=\hat{\xi}_0} = \frac{p_{1}y}{p_{1} \lambda} - \mu \left( \frac{-H}{H^*} \right) \)

\[ = \frac{\mu x_{1}}{U_{1}} - \mu \left( \frac{-H}{H^*} \right). \]

By quasiconcavity of the utility function, the third principal sub-determinate must be positive

\[ \begin{vmatrix} 0 & \mu & U_{1} \\ \mu & \mu_{2} & \mu_{x_{1}} \\ U_{1} & \mu_{x_{1}} & U_{11} \end{vmatrix} > 0 \]

or

\[ \begin{equation} 2\mu \mu_{x_{1}} U_{1} - \mu \mu_{2} U_{11} - \mu_{x_{1}} U_{11} > 0 \end{equation} \]

Following the argument in the one-good world, a sufficient condition for \( \frac{\partial \text{MRS}/\partial y}{\hat{x}=\hat{\xi}_0} > 0 \) is \( \Delta^* > 0 \), where \( \Delta^* \) is a bordered \( \Delta \) given by:

\[ \Delta^* = \begin{vmatrix} -\frac{H}{H^*} & \Delta \\ -\frac{1}{\mu} & \frac{2}{-U_{1}} \end{vmatrix} \]

A necessary condition for \( \frac{\partial \text{MRS}/\partial y}{\hat{x}=\hat{\xi}_0} \leq 0 \) is that \( \Delta^* < 0 \).

\(^{1}\)Note again that the sign of \( \Delta^* \) is invariant under any order-preserving transformation of the utility function, though the proof is more tedious than for \( \Delta \).
A COMMENT ON HOMOTHETIC UTILITY FUNCTIONS

A utility function is called homothetic if it can be written as $h[g(x)]$ where $x = (x_1, \ldots, x_n)$; $h$ is continuous, positive-valued, and nondecreasing; and $g$ is homogeneous. ¹ With a homothetic utility function that is strictly quasiconcave, the MRS along any vertical always increases. This can be seen in Fig. A-2.

Since homothetic utility functions have the property that the MRS along any ray remains the same, the MRS at point $A$ equals the MRS at point $B$. By strict quasiconcavity (that is, diminishing MRS along any indifference curve), the MRS at point $C$ must be higher than at point $A$.

Unfortunately it is not true that $\Delta$ (or $\Delta^*$) is always positive for homothetic utility functions. For example, the Cobb-Douglas utility function

(15) \[ U = x_1^\alpha x_2^\beta \quad 0 < \alpha, \beta < 1 \]

has $\Delta \geq 0$ for $\beta \geq \alpha$ and $\Delta < 0$ for $\beta < \alpha$. Even though we know the MRS increases along a vertical, it does not follow from our theorem.

¹A function $g$ is homogeneous if and only if $g(\omega x) = \varphi(\omega) g(x)$, $\omega > 0$. For more details, see Amoz Kats, "Comments on the Definition of Homogeneous and Homothetic Functions," *Journal of Economic Theory*, Vol. 2, No. 3, 1970, pp. 310-313.
Fig. A-2—Homothetic indifference map