MEASURING THE TRAVEL CHARACTERISTICS OF NEW YORK CITY'S FIRE COMPANIES

PETER KOLESAR, WARREN WALKER

R-1449-NYC
APRIL 1974

THE
NEW YORK CITY
RAND
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PREFACE

As part of its work to improve the deployment of New York City's fire-fighting resources, The New York City-Rand Institute designed an experiment for collecting and analyzing data to determine the relationship between travel times and travel distances for fire companies responding to alarms. The experiment was conducted by the Fire Department of the City of New York, but the techniques of data collection and data analysis are general and have already been applied by the Institute in other cities, and results will be documented in future reports. A user's manual for the computer program developed to analyze the experimental data is forthcoming.*

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*Hausner, J., "A Program to Analyze Data from Travel-Time Experiments," forthcoming.

SUMMARY

In order to obtain quantitative information about the relationship between travel times and travel distances in various regions of the City at different times of day, the Fire Department of New York conducted a stopwatch experiment during the summer of 1971. Data on over 2000 responses made by 15 units were collected and analyzed.

This Report describes the experiment, analyzes the results, and draws a number of conclusions about the travel characteristics of fire companies in New York City. Among the major findings were:

1. In most parts of the City, travel time increases with the square root of distance for short runs, and linearly for long runs.
2. Although average response velocities vary somewhat by time of day, the variations are smaller than expected and can be ignored for many planning purposes.
3. There are only small variations in the parameters of the function relating travel time to travel distance in different regions of the City. This implies that the average velocity for a given travel distance is almost constant throughout the City.

As a result, a single continuous function can adequately represent the relationship between travel time and travel distance at all times of day in all parts of the City. This function is a square-root relationship for response distances up to some point d, and linear for response distances greater than d; at the point d, the two functions intersect and have the same slope. When such a function was fitted to the New York City data, the best value of d was 0.88 miles, and the time/distance relationship was:

$$T(D) = \begin{cases} 2.88\sqrt{D}, & D \leq 0.88 \text{ miles} \\ 1.35 + 1.53D, & D > 0.88 \text{ miles} \end{cases}$$

where T is the travel time in minutes and D is the travel distance in miles.

ACKNOWLEDGMENTS

We are grateful to Deputy Assistant Chief Homer Bishop of the Fire Department of New York for explaining the mechanics of the experiment to the participating fire companies and for coordinating the data-gathering efforts. Jack Hausner, a member of the staff of The New York City-Rand Institute, wrote the computer programs to analyze the data.

We would particularly like to acknowledge the firemen and officers in the Fire Department of New York who participated so conscientiously in the data-gathering effort, and without whom we would have had nothing to analyze.

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I. INTRODUCTION AND DESCRIPTION OF THE EXPERIMENT

The time it takes for municipal emergency vehicles, such as fire engines, police cars, and ambulances, to respond to calls for service is an important and widely used indicator of the performance of emergency service agencies. Most of the models developed for analysis of the deployment of emergency vehicles implicitly or explicitly include travel time at one stage or another. Yet, little is known about actual travel times and how they vary with distance, by time of day, with weather, etc. In New York City, in particular, this information was needed as input to mathematical models which were being used to analyze fire-company deployment for the Fire Department of New York. No empirical data on travel times and travel speed were available, and we were unable to find reports of such data in the literature.

We encouraged the New York City Fire Department to correct this deficiency. During the summer of 1971, the Department carried out an experiment to measure the travel times, distances, and speeds of selected fire companies. Selected units measured their travel times with stopwatches and their response distances with vehicle odometers. The resulting data (over 2000 observations) were statistically analyzed at The New York City-Rand Institute to answer a number of questions:

1. *How does travel time depend on distance?* Most analysts make the assumption that travel time = travel distance ÷ average response velocity, i.e., that it takes twice as long to travel a mile as it takes to travel half a mile.[1] We found, on the contrary, that in most parts of New York City travel time increases with the square root of distance for short runs, and linearly only for long runs.

2. *How do response velocities vary by time of day?* Both we and the Department had presumed that velocities, and hence travel times, varied considerably by time of day as a result of differences in traffic conditions, street lighting, etc. We found, surprisingly, that although differences do exist, they are considerably less than expected and can be ignored for many planning purposes.

3. *How do travel velocities vary among regions of the City?* Only small variations among regions were detected in the travel time/distance relationship. This suggests that the average velocity for a given travel distance is almost constant throughout the City.

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[1] "Velocity" is used here as a synonym for "speed" and refers to miles per hour.

Details of the experimental results are presented in this Report. Before proceeding to them, we outline briefly how the experiment was carried out.

Mechanics of the Experiment

Fifteen units participated in the experiment: thirteen ladder companies and two Battalion Chief's cars. Each unit had an odometer that read in tenths of miles, so that reasonably accurate distance records could be produced. Only moderately busy companies were selected, since the process of data collection would have been unduly burdensome if the units were very busy, and it would have taken months to gather data on an adequate number of responses with units that were not busy enough. Each unit was provided with a stopwatch and copies of a form to keep a record of all responses made *from quarters*. Responses made when returning from an earlier run or from a position in the field were not included, because of the difficulty of recording accurately times, distances, and locations at time of dispatch.

Consideration was given to obtaining a good geographical spread of participating companies, but the need for odometers that recorded in tenths of miles was a limiting factor: only ladder companies and Battalion Chiefs participated, since no engines had such odometers. (Our failure to include engines in the experiment could introduce an element of bias in the results, since engines are generally smaller than ladders and are able to maneuver more easily in traffic and narrow streets, so that they may travel slightly faster. This should be remembered in applying the results of this experiment.)

In order to encourage cooperation in collecting data, the recording forms were kept simple and impersonal. Consequently, such information as date, identity of the officer recording the data, whether the company was the first due to respond, and special circumstances (such as weather or road conditions) were not recorded.

Of the thirteen ladder companies selected for the experiment, eight were located in Manhattan, two in Brooklyn, two in Queens, and one in the Bronx. One Battalion Chief each in Manhattan and the Bronx also participated. Company identities are given in Table 1, their locations are shown in Fig. 1, and summary statistics resulting from the experiment are given in Table 2.

Table 1

LADDER COMPANIES PARTICIPATING IN EXPERIMENT

Company	Division	Battalion	Location	1970 Operations	
				No. of Runs	No. of Workers
L1	1	1	100 Duane St., Manhattan	858	446
L2	3	8	165 East 51 St., Manhattan	1729	1219
L3	3	6	108 East 13 St., Manhattan	2133	1384
L4	3	9	788 8th Avenue, Manhattan	2126	1626
L11	1	4	222 East 2nd St., Manhattan	4878	4053
L12	3	7	146 West 19th St., Manhattan	----	----
L15	1	1	73 Water St., Manhattan	559	428
L25	4	10	205 West 77th St., Manhattan	2463	1295
L42	6	26	657 Prospect Avenue, Bronx	5950	4535
L110	10	31	365 Jay St., Brooklyn	3893	2050
L122	10	48	532 11th St., Brooklyn	2600	1622
L155	13	50	143-15 Rockaway Blvd., Queens	3109	2171
L162	16	54	218-44 97th Avenue, Queens	1793	829
B1	1	1	73 Water St., Manhattan	----	----
B45	14	45	10-40 47th Avenue, Long Island City, Queens	----	----

Figure 1

LOCATIONS OF LADDER COMPANIES PARTICIPATING IN EXPERIMENT

Table 2

SUMMARY OF RESPONSE CHARACTERISTICS

COMPANY	RESPONSE VELOCITY (M.P.H.)						RESPONSE DISTANCE (MILES)						RESPONSE TIME (MINUTES)						NUMBER OF RUNS		
	1st-Due Boxes		2nd-Due Boxes		All Boxes		1st-Due Boxes		2nd-Due Boxes		All Boxes		1st-Due Boxes		2nd-Due Boxes		All Boxes		1st-Due	2nd-Due	Total
	Avg.	St. Dev.	Avg.	St. Dev	Avg.	St. Dev.	Avg	St. Dev.	Avg.	St. Dev.	Avg.	St. Dev.	Avg.	St. Dev.	Avg.	St. Dev.	Avg.	St. Dev.			
MANHATTAN																					
LADDER 1	16.0	7.1	17.3	7.3	16.7	7.2	0.45	0.23	0.62	0.26	0.54	0.26	1.81	0.88	2.36	0.88	2.11	0.96	149	158	323
LADDER 2	18.3	6.5	24.6	6.4	19.9	6.9	0.56	0.26	0.91	0.26	0.68	0.30	1.78	0.57	2.35	0.81	2.05	0.72	12	8	23
LADDER 3	16.7	6.7	19.1	5.6	18.2	6.3	0.48	0.23	0.83	0.28	0.67	0.31	1.78	0.72	2.62	0.52	2.23	0.77	85	96	185
LADDER 4	18.1	7.1	18.7	7.6	18.1	7.2	0.46	0.20	0.71	0.28	0.54	0.26	1.67	0.79	2.48	0.97	1.93	0.97	84	32	119
LADDER 11	14.1	6.9	15.8	5.2	14.5	6.6	0.40	0.21	0.57	0.19	0.43	0.22	1.84	0.85	2.23	0.59	1.92	0.82	128	34	162
LADDER 12	15.2	5.1	15.5	5.6	15.4	5.3	0.45	0.21	0.74	0.27	0.59	0.29	1.80	0.63	2.96	0.79	2.36	0.92	100	87	197
LADDER 15	16.8	8.7	20.0	9.2	17.9	9.1	0.45	0.30	0.80	0.35	0.54	0.37	1.67	0.77	2.58	0.81	1.87	0.89	79	22	109
LADDER 25	19.1	6.7	----	----	19.6	6.5	0.51	0.31	----	----	0.77	0.50	1.66	0.98	----	----	2.41	1.59	66	----	124
ALL MANHATTAN LADDERS	16.3	7.0	17.6	6.8	17.0	7.0	0.45	0.24	0.71	0.28	0.58	0.32	1.76	0.81	2.54	0.81	2.13	0.95	703	----	1241
BROOKLYN																					
LADDER 110	17.5	5.8	17.0	5.2	17.3	5.5	0.67	0.27	0.95	0.39	0.79	0.35	2.36	0.73	3.32	0.69	2.78	0.85	59	46	105
LADDER 122	18.4	5.4	21.5	3.8	19.6	5.0	0.62	0.24	0.96	0.22	0.79	0.37	2.09	0.78	2.67	0.47	2.45	1.04	49	29	81
BRONX																					
LADDER 42	16.8	5.4	19.2	6.9	17.9	6.0	0.43	0.17	0.88	0.30	0.60	0.31	1.60	0.53	2.88	0.95	2.03	0.92	78	40	133
ALL LADDERS EXCEPT QUEENS	16.5	6.8	17.8	6.6	17.2	6.8	0.47	0.25	0.75	0.30	0.61	0.33	1.81	0.80	2.64	0.83	2.18	0.96	889	542	1560
QUEENS																					
LADDER 155	23.7	6.0	23.5	6.0	23.6	6.0	0.81	0.37	1.37	0.35	1.08	0.54	2.04	0.79	3.58	0.82	2.76	1.27	42	26	74
LADDER 162	25.7	6.0	29.5	5.4	27.4	6.0	1.94	1.13	2.07	0.73	2.00	0.96	4.41	2.17	4.22	1.30	4.33	1.80	73	61	138
BATTALION 1	21.4	7.4	----	----	----	----	0.77	0.42	----	----	----	----	2.24	1.14	----	----	----	----	138	----	----
BATTALION 45	25.6	7.0	----	----	----	----	1.45	0.70	----	----	----	----	3.44	1.85	----	----	----	----	96	----	----

The methods of data collection and data analysis associated with this experiment are general, and can be used by other fire departments to determine the response characteristics of their fire-fighting units. The computer program that was used to analyze the data is being documented, and a user's manual will be published soon.[2] The program has already been used to analyze data collected in similar experiments conducted by fire departments in Yonkers, New York, Wilmington, Delaware, and Denver, Colorado.

Editing the Data

The raw data were edited to eliminate obviously erroneous records. We used a number of consistency checks in this process. For example, we eliminated records for which the average velocities attained were higher than 60 mph. In addition, observations for runs to the same alarm box were grouped and, if distances varied by more than 1/4 mile, an independent check of the possibility of such readings was made. Less than 5 percent of the original data were eliminated by this process.

[2]Hausner, J., "A Program to Analyze Data from Travel-Time Experiments," forthcoming.

II. THE RELATIONSHIP BETWEEN TRAVEL TIME AND RESPONSE DISTANCE

Other things being equal, the farther a fire engine travels, the longer it takes to make the trip. Thus, any mathematical relationship that reflects this fact is a candidate for use to predict travel times from distances. The time/distance relationship normally employed assumes that a unit makes an entire trip at a constant velocity and, therefore, that travel time increases proportionally with the distance traveled. In this study, we attempt to determine whether this relationship is valid, or if some other, more complicated model should be used.

We hypothesized the following: suppose that, for short runs, a unit never reaches a cruising velocity, but rather increases its speed for the first half of the trip, as it accelerates, gets onto main thoroughfares, etc., and then decelerates for the last half of the trip, as it approaches its destination, gets off main thoroughfares, etc. Suppose further that, for longer runs, there is a similar initial "acceleration" phase, but that the unit then runs at cruising speed for some distance before decelerating as it nears its destination. These hypotheses can be expressed mathematically as follows:

Let:

a = acceleration
D = length of the run
D_c = distance required to achieve cruising velocity
v_c = cruising velocity
T = travel time.

Then, using basic mathematical relations, and assuming constant acceleration and deceleration, a, during the initial and final phases of travel, and a constant cruising velocity, v_c, during the middle phase, we derive, for travel time as a function of distance,

$$T(D) = \begin{cases} 2\sqrt{\frac{D}{a}}, & \text{if } D \leq 2D_c, \\ \frac{v_c}{a} + \frac{D}{v_c}, & \text{if } D \geq 2D_c. \end{cases} \qquad (1)$$

From this, we obtain for $\bar{v}(D)$ the average velocity as a function of distance (average velocity being defined as $\bar{v}(D) = D/T(D)$),

$$\bar{v}(D) = \begin{cases} \frac{1}{2}\sqrt{aD}, & \text{if } D \le D_c , \\ \\ \dfrac{aD}{v_c + \dfrac{aD}{v_c}} , & \text{if } D \ge D_c . \end{cases} \qquad (2)$$

The simple linear relationship traditionally assumed is $T(D) = D/\bar{v}(D)$. A generalization of this relationship is $T(D) = a + bD$.

With these hypotheses in mind, we proceeded to examine the experimental data. Least squares regression fits were made of the relationships:

$$T(D) = c\sqrt{D}, \qquad (3)$$

$$T(D) = a + bD . \qquad (4)$$

Regressions were done separately for each participating company. In addition, separate regressions were done (1) for runs (responses) to alarms to which the company was the closest ladder, (2) for runs to alarms to which it was the second closest ladder, and (3) for all runs, including runs made to more distant alarms. The purpose of these separate analyses was to determine how the travel-time patterns varied among companies, and how they differed, if at all, for short runs and for longer runs. The results are summarized in Tables 3 and 4 and in Figures 2-17, below. The four major conclusions supported by these results are:

1. *In regions of the City where average response distances are short (about 1/2 mile or less), $T(D) = c\sqrt{D}$ provides the better fit to the data.* Generally, response distances are short in most regions, except in eastern Queens, Staten Island, and some parts of Brooklyn and the north Bronx. For the longer runs that are typical in the latter regions, the relationship between distance and time looks linear, but the square-root fit is still very good, since the slope of the square-root function changes slowly at such distances.

2. *In regions of the City in which average response distances are longer, $T(D) = a + bD$ is the better time/distance model.* These are the regions where the average distance to first-due alarm boxes is more than about 1/2 mile.

3. Although the parameter values for different companies within each of the two types of region exhibit statistically significant variations, these differences are not very large and, for many purposes, *a single function can be used for all companies within each type of region.*

4. *A single continuous function that is piecewise square-root and linear (as in Equation (1) above) produces good estimates of average travel times for all regions of the City.*

We now proceed to describe the results that support these four conclusions. Table 3 contains--for each participating ladder company, and for some groupings of ladder companies--the following information for first-due, second-due, and all runs resulting from regressions of $T(D) = c\sqrt{D}$ and $T(D) = a + bD$: $\hat{c}$, the estimated value of the coefficient, c, of the square-root function; $\hat{\sigma}_{\hat{c}}$, its standard error; and r, the ratio of the sum of squared errors of the original data from the model $T(D) = C\sqrt{D}$ to the sum of squared errors from the model $T(D) = a + bD$. The value of r is a rough measure for comparison of the two models. If $r < 1$, the square-root model is "better."[3]

We make the following observations from Table 3:

1. Using r as a rough measure of which model is better, we see that the square-root model fits first-due runs better than it fits second-due runs. This was to be expected.
2. Again using r as a measure of which model is better, we find that the square-root model is preferred for L1, L2, L3, L11, L12, L15, and L110, and the linear model is preferred for L25, L122, L155, and L162. For both L4 and L42, r = 1.05, indicating that the linear model is slightly better. But, when we examine plots of the regression fits, there is very little difference between the models. Therefore, we prefer the square-root fit, because it has one less parameter (we use the parsimony of the model as a principle of selection) and because the linear model

[3]There are some technical difficulties involved in interpreting r, because we are comparing a one-parameter non-linear model to a two-parameter linear model.

Table 3

SUMMARY OF SQUARE-ROOT MODEL REGRESSION RESULTS

Company	1st-Due Runs			2nd-Due Runs			All Runs		
	$\hat{c}$	$\hat{\sigma}_{\hat{c}}$	r	$\hat{c}$	$\hat{\sigma}_{\hat{c}}$	r	$\hat{c}$	$\hat{\sigma}_{\hat{c}}$	r
Manhattan									
L1	2.85	0.08	0.96	3.01	0.08	1.00	2.97	0.06	0.95
L2	2.45	0.13	0.75	2.52	0.22	1.16	2.57	0.12	0.95
L3	2.65	0.15	0.95	2.88	0.06	1.16	2.80	0.05	0.94
L4	2.55	0.10	1.02	3.00	0.18	1.00	2.77	0.09	1.05
L11	3.08	0.10	0.98	2.98	0.12	0.96	3.00	0.08	0.98
L12	2.76	0.07	0.92	3.46	0.09	1.06	3.19	0.06	0.92
L15	2.55	0.11	0.76	2.88	0.20	1.18	2.64	0.09	0.96
L25	2.56	0.11	1.33	----	----	----	3.07	0.11	1.24
All Manhattan Ladders	2.73	0.04	0.95	3.05	0.04	1.02	2.92	0.03	0.95
Brooklyn									
L110	2.91	0.10	0.98	3.43	0.09	1.13	3.19	0.07	0.95
L122	2.74	0.10	1.10	2.76	0.07	0.91	2.90	0.08	1.49
Bronx									
L42	2.47	0.07	1.06	3.10	0.15	1.03	2.76	0.07	1.05
Queens									
L155	2.38	0.08	1.16	3.05	0.14	1.11	2.82	0.09	1.25
L162	3.44	0.09	1.67	3.02	0.07	1.18	3.24	0.06	1.50
All Ladders	2.88	0.03	1.19	3.06	0.03	1.01	2.99	0.02	1.03
Battalion 1	----	----	----	----	----	----	2.72	0.07	----
Battalion 45	----	----	----	----	----	----	3.03	0.12	----
L25 (0800 - 1800)	2.59	0.16	N	----	----	----	3.25	0.21	----
L25 (1800 - 0800)	2.54	0.14	N	----	----	----	2.94	0.10	----

is inconsistent with the physical acceleration/deceleration process; the linear model requires an infinite acceleration. The results are consistent across ladders, with two exceptions:

a. Ladder 25 in Manhattan, with a short average length of run, appears to follow the linear model (see Fig. 9). (This company was not part of the original experiment. Data were gathered by Ladder 25 during the fall of 1972 in order to test whether the traffic on West 77th Street, on which it is located, was slowing its responses.)

b. Ladder 110 in Brooklyn, with a longer average run, appears to follow the square-root model (see Fig. 11).

3. Among the nine ladder companies for which the square-root function provides a better fit, the range of values of the parameter $\hat{c}$ generally varies by less than 10 percent from the overall average value of 2.93. There is no consistent geographical pattern to the variations; we conclude, therefore, that a single parameter value estimated from the data for all nine companies can be used in each region where the square-root model applies. Estimation of such a parameter value is discussed later in this section.

Table 4 gives a summary of the regression results for the linear model $T(D) = a + bD$. It contains, for each company for which the linear model is preferred, the following information for first-due, second-due, and all runs: $\hat{a}$, the estimated value of the constant term a; $\hat{b}$, the estimated value of the slope b; and $\hat{\sigma}_a$ and $\hat{\sigma}_b$, the standard errors of the above estimates.

The results summarized in Table 4 are more variable than those given in Table 3; the parameter values are estimated less precisely and they exhibit greater variation among companies. In addition, the parameter values for first-due runs and second-due runs for the same company are different. Nevertheless, the results of the linear regression done with all four companies combined--the last line of Table 4--fit the data for each individual company surprisingly well, and also fit the combined data well. The resulting regression line, $T(D) = .94 + 1.75D$, explains 93 percent of the variation of average travel times for the four companies.

Table 4

SUMMARY OF LINEAR MODEL REGRESSION RESULTS

Company	1st-Due Runs				2nd-Due Runs				All Runs			
	$\hat{a}$	$\sigma_{\hat{a}}$	$\hat{b}$	$\sigma_{\hat{b}}$	$\hat{a}$	$\sigma_{\hat{a}}$	$\hat{b}$	$\sigma_{\hat{b}}$	$\hat{a}$	$\sigma_{\hat{a}}$	$\hat{b}$	$\sigma_{\hat{b}}$
L162	.97	.20	1.78	.09	1.14	.28	1.49	.13	.97	.20	1.78	.09
L155	.57	.10	1.81	.18	2.88	.66	.51	.47	.57	.16	1.82	.18
L25	.32	.12	2.62	.21	--	--	--	--	.41	.16	2.60	.17
L122	.67	.21	2.30	.33	1.45	.32	1.51	.33	.67	.22	2.30	.17
All 4 combined	.75	.07	1.87	.05	1.47	.15	1.36	.09	.94	.06	1.75	.04

Figures 2 to 14 display the experimental data for the average travel time associated with each response distance for each of the selected ladder companies. In addition, each graph has a plot of the preferred fitted time/distance function for each company. These graphs support and illustrate the observations made above.

Since the previous analysis indicates, broadly speaking, that a square-root function fits the data well for short responses and a linear function fits better for long responses, we attempted to fit a piecewise square root-linear function with a continuous first derivative to all of the data. Such a function is consistent with our original hypothesis, about the time/distance relationship. It is of the form

$$T(D) = \begin{cases} c\sqrt{D}, & D \leq d \\ a + bD, & D > d. \end{cases} \qquad (5)$$

To see if such a function was reasonable, given the experimental data, we compared a fit of the square-root model to the data for first-due runs of ladder companies for which the square root is preferred to the fit of the linear model to the data for the second-due runs of the other ladder companies. The two functions are: $T(D) = 2.74\sqrt{D}$ and $T(D) = 1.47 + 1.36D$. These two functions are nearly tangent at $D = 1$ mile. (For the

Figure 2

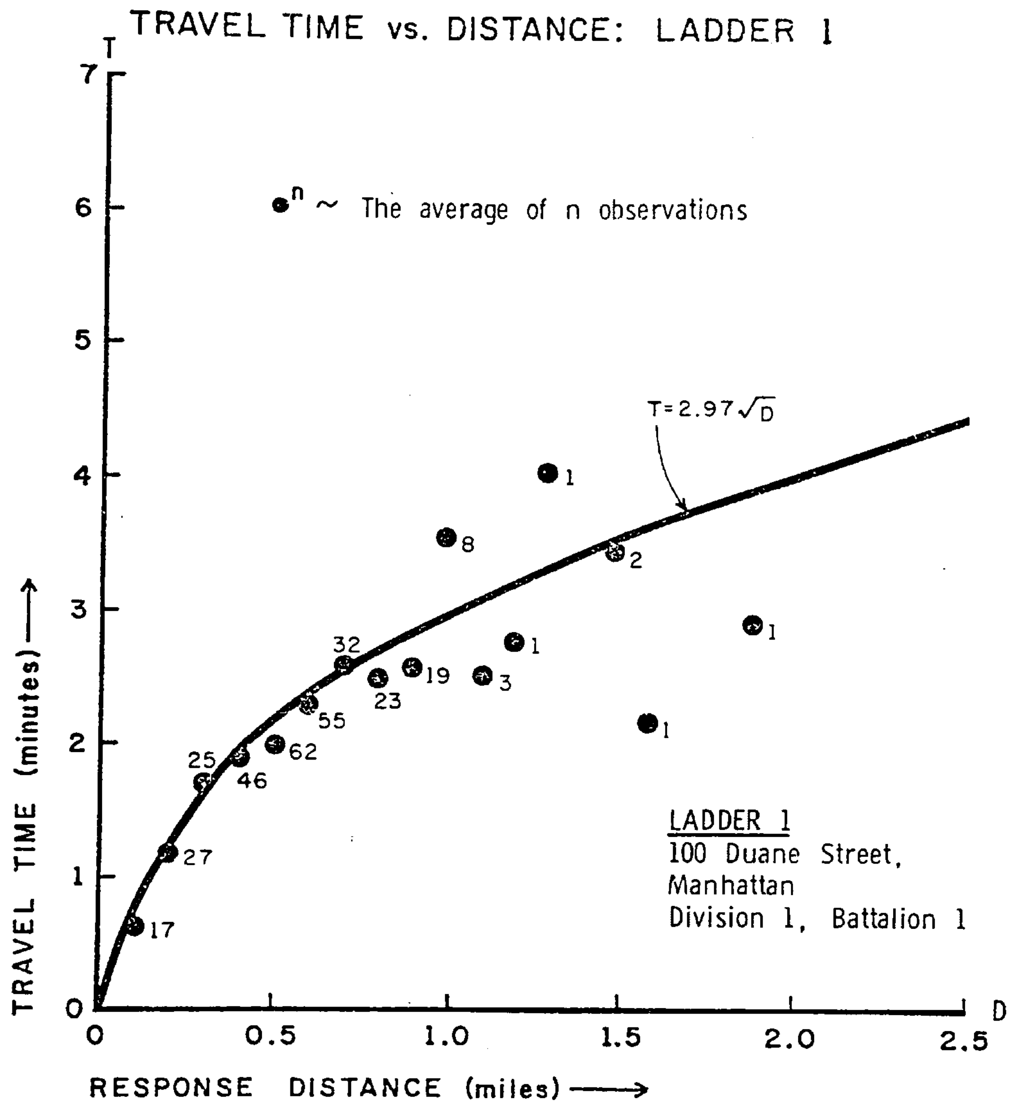

TRAVEL TIME vs. DISTANCE: LADDER 1
T
7
6
5
4
3
2
1
0
TRAVEL TIME (minutes) ⟶
The average of n observations
T=2.97√D
17
27
25
46
62
55
32
23
19
3
1
8
1
2
1
1
LADDER 1
100 Duane Street,
Manhattan
Division 1, Battalion 1
0
0.5
1.0
1.5
2.0
2.5
D
RESPONSE DISTANCE (miles) ⟶

Figure 3

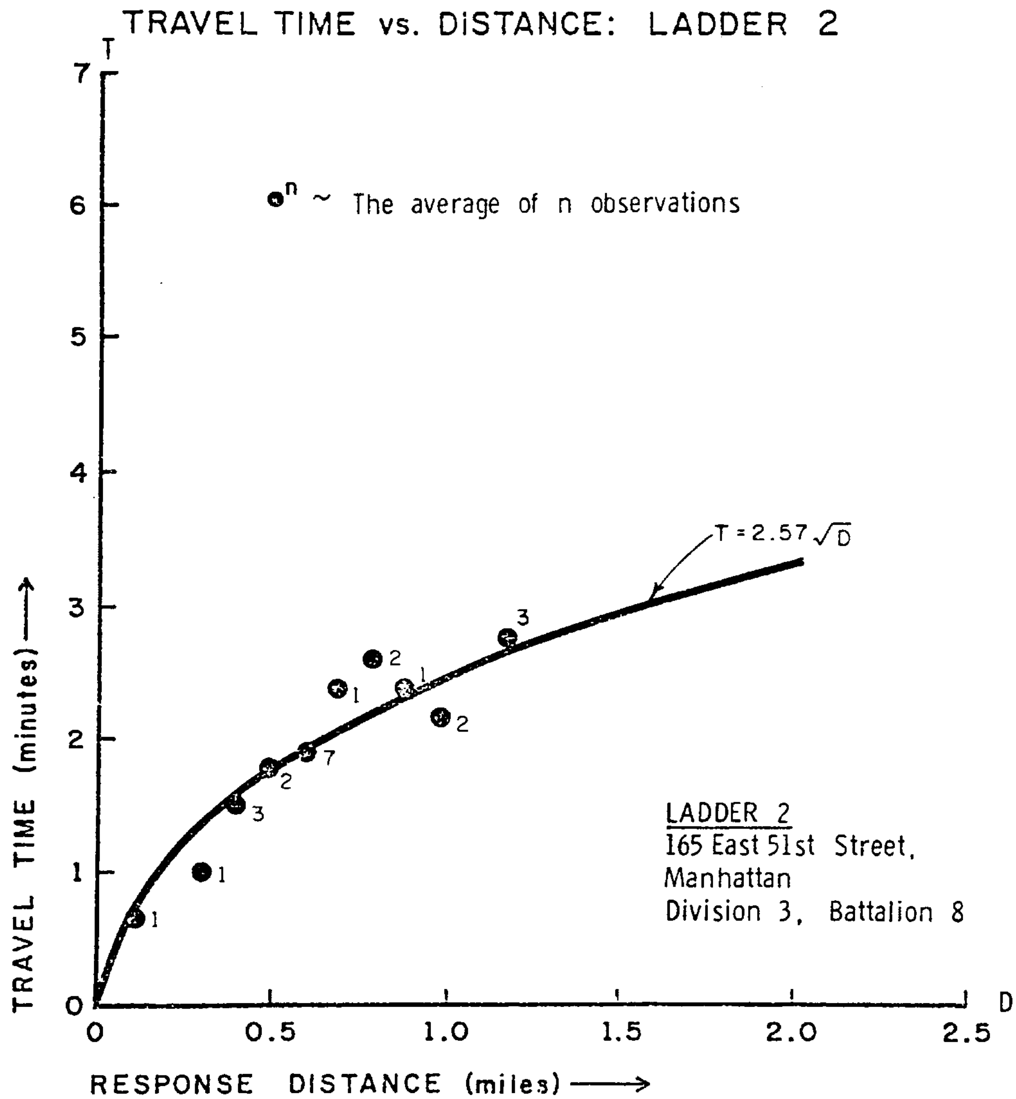

TRAVEL TIME vs. DISTANCE: LADDER 2
T
7
6
5
4
3
2
1
0
TRAVEL TIME (minutes) ⟶
The average of n observations
T = 2.57 √D
LADDER 2
165 East 51st Street,
Manhattan
Division 3, Battalion 8
D
0
0.5
1.0
1.5
2.0
2.5
RESPONSE DISTANCE (miles) ⟶

Figure 4

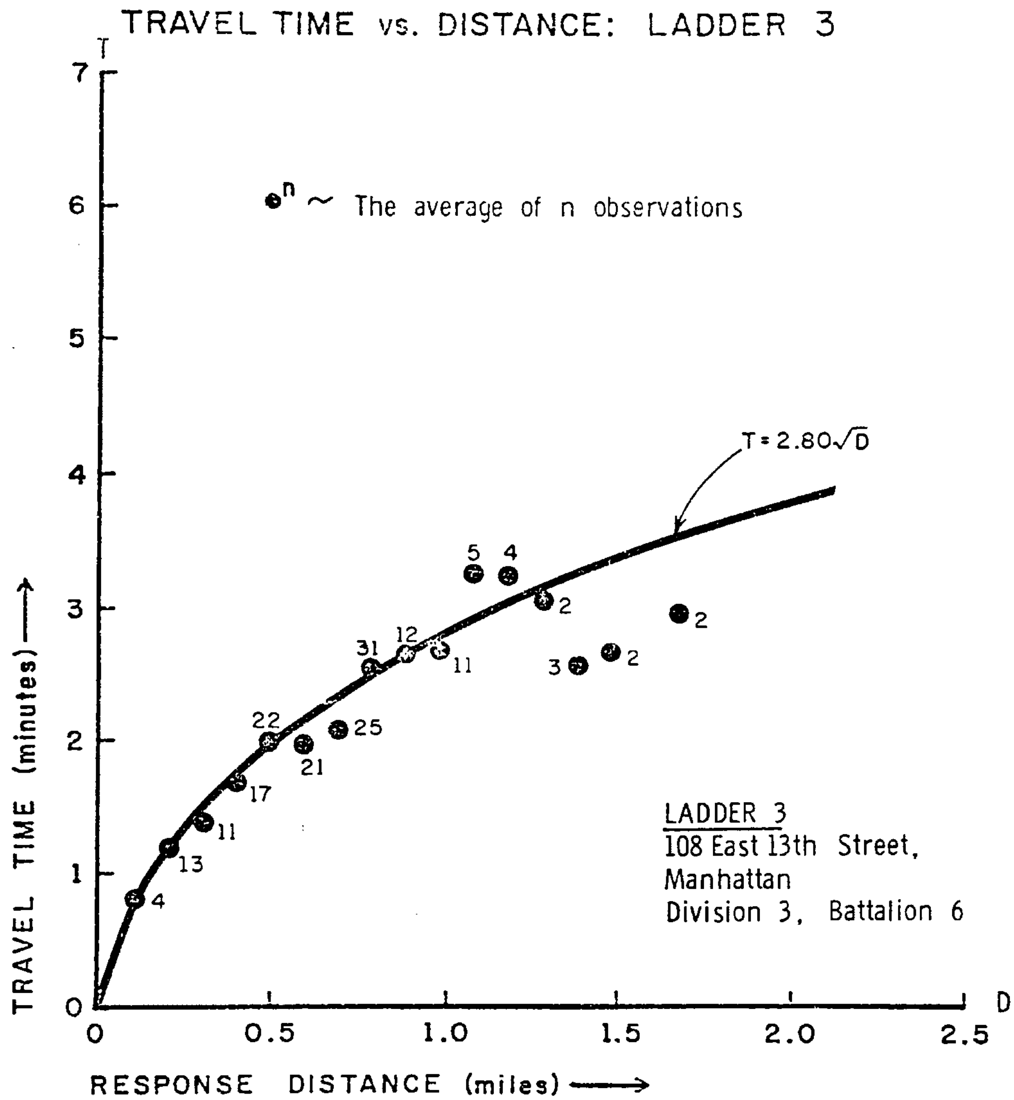

TRAVEL TIME vs. DISTANCE: LADDER 3
T
7
6
5
4
3
2
1
0
n ~ The average of n observations
T = 2.80√D
5
4
2
2
31
12
11
3
2
22
25
21
17
11
13
4
LADDER 3
108 East 13th Street,
Manhattan
Division 3, Battalion 6
D
0
0.5
1.0
1.5
2.0
2.5
TRAVEL TIME (minutes) ⟶
RESPONSE DISTANCE (miles) ⟶

Figure 5

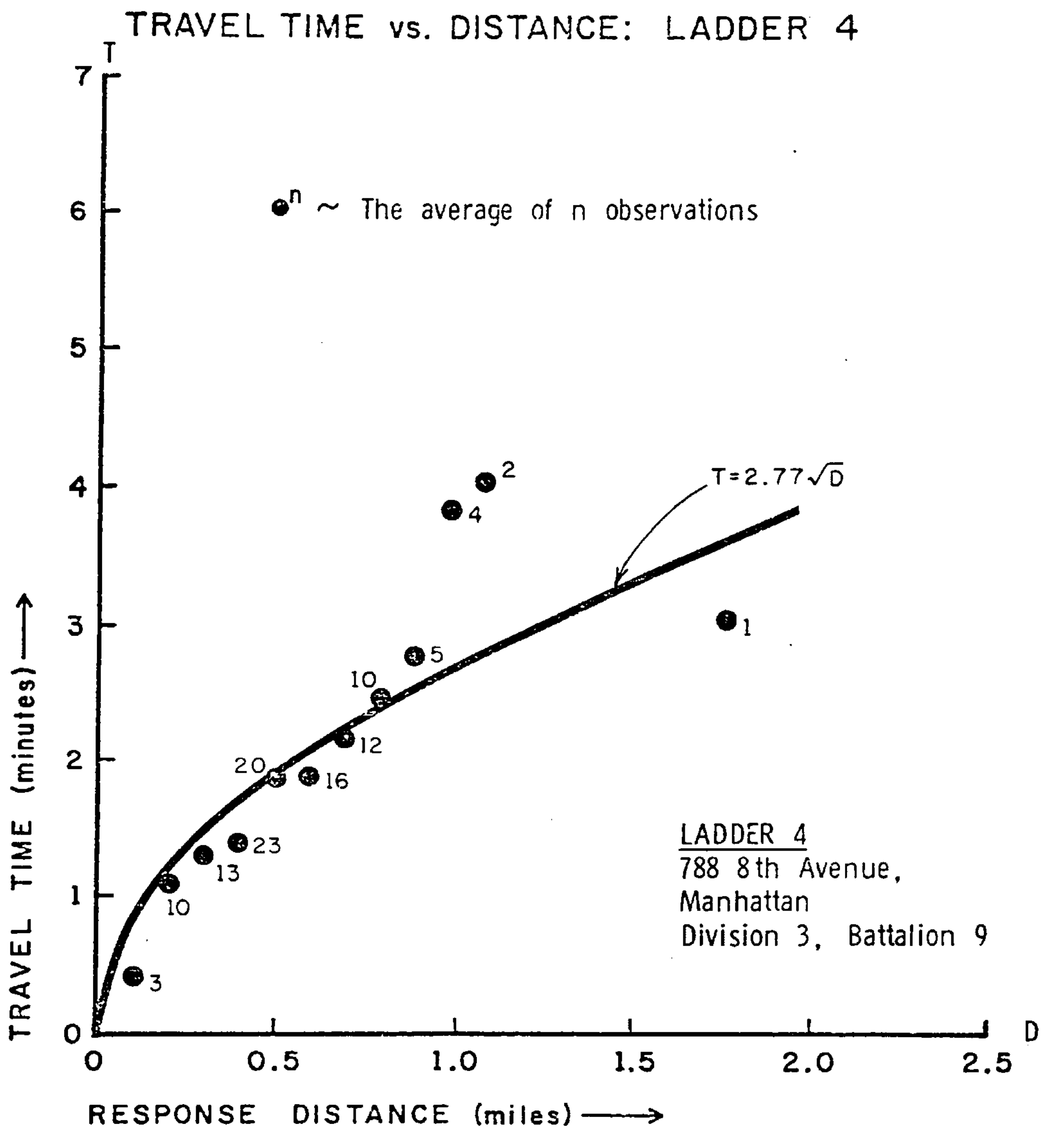

TRAVEL TIME vs. DISTANCE: LADDER 4
T
7
6
5
4
3
2
1
0
~ The average of n observations
2
4
T=2.77√D
1
5
10
12
20
16
23
13
10
3
LADDER 4
788 8th Avenue,
Manhattan
Division 3, Battalion 9
D
0
0.5
1.0
1.5
2.0
2.5
TRAVEL TIME (minutes) ⟶
RESPONSE DISTANCE (miles) ⟶

Figure 6

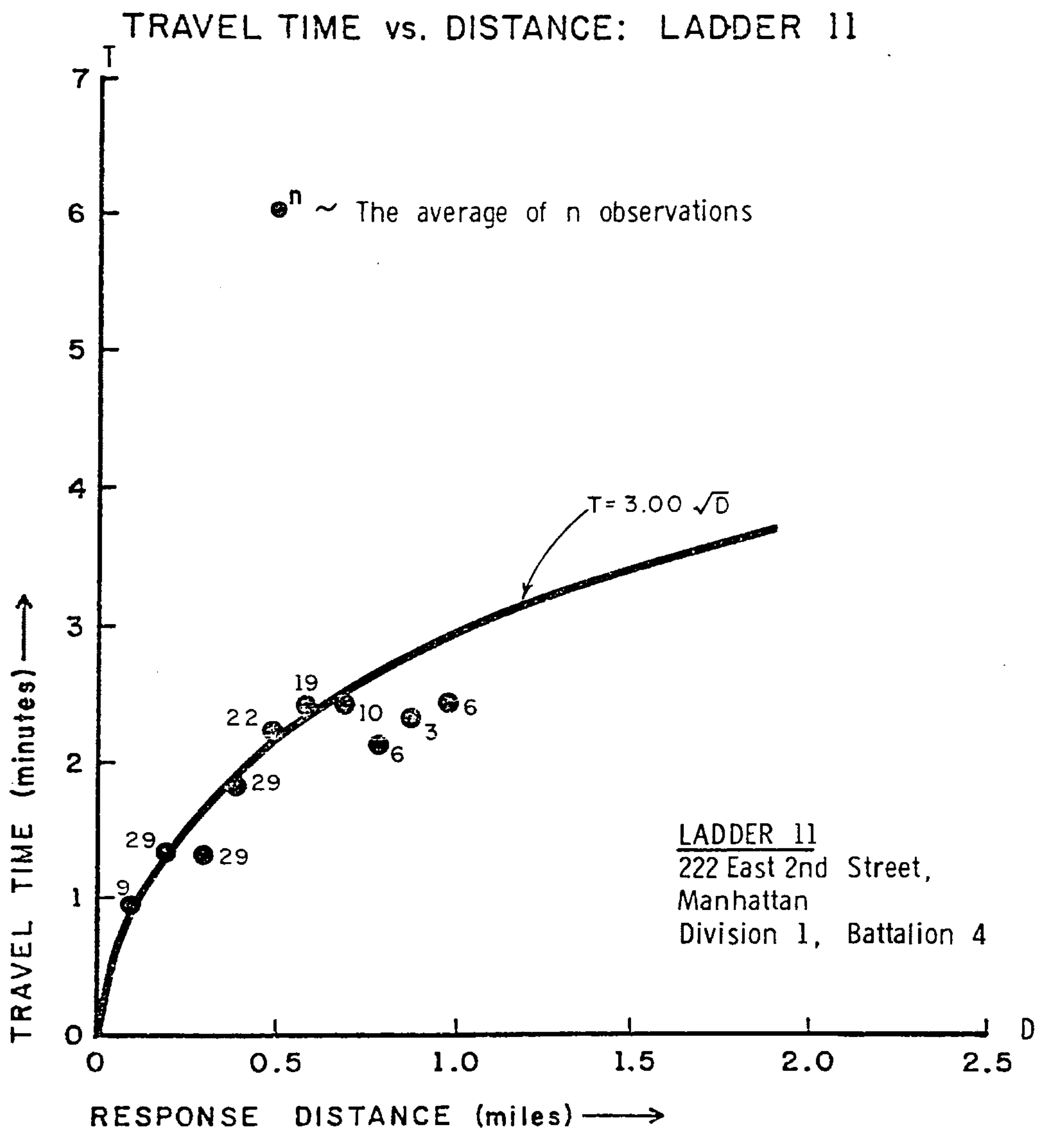

TRAVEL TIME vs. DISTANCE: LADDER 11
T
7
6
5
4
3
2
1
0
The average of n observations
T= 3.00 √D
19
22
10
3
6
6
29
29
29
9
LADDER 11
222 East 2nd Street,
Manhattan
Division 1, Battalion 4
TRAVEL TIME (minutes)
D
0
0.5
1.0
1.5
2.0
2.5
RESPONSE DISTANCE (miles)

Figure 7

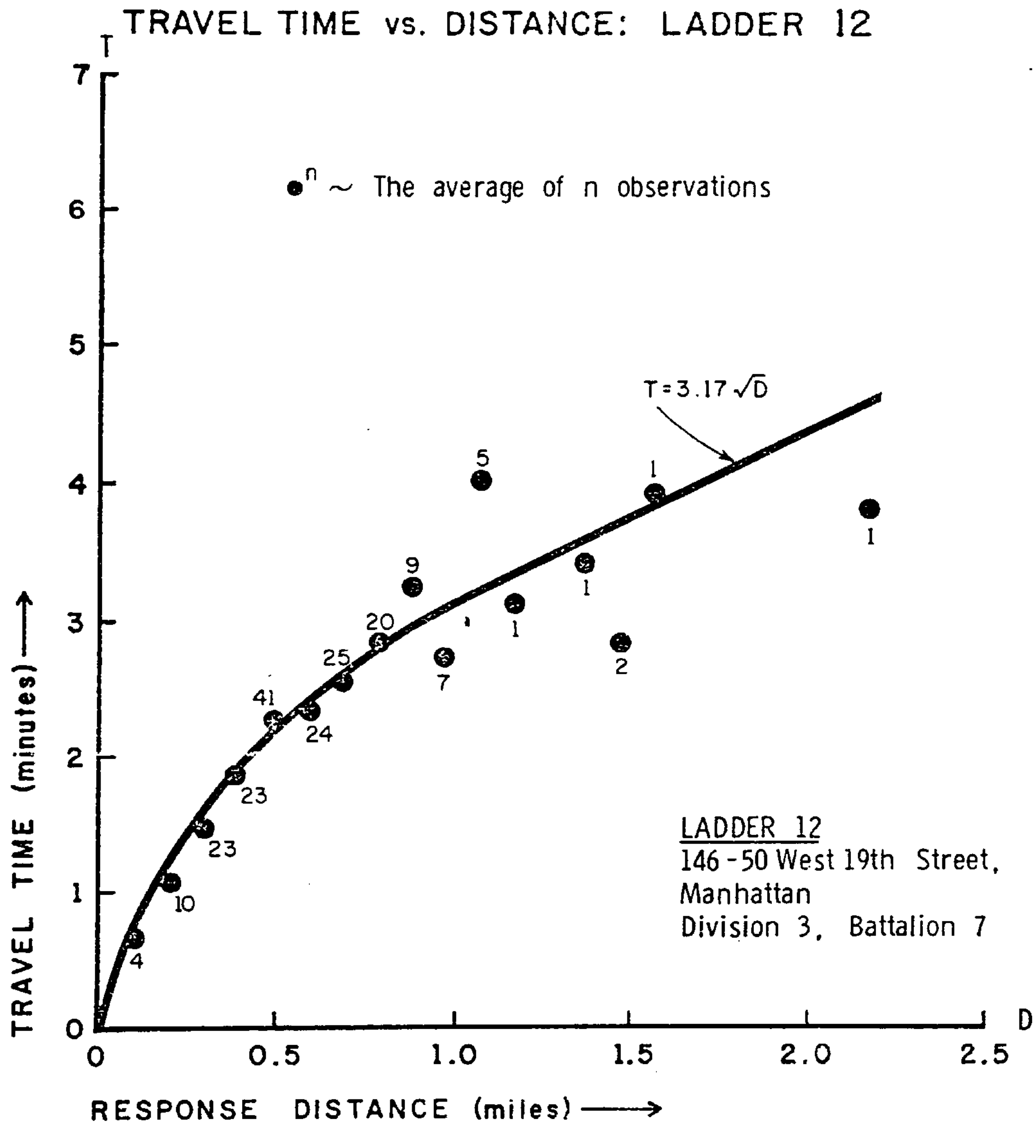

TRAVEL TIME vs. DISTANCE: LADDER 12
T
•ⁿ ~ The average of n observations
T=3.17√D
LADDER 12
146-50 West 19th Street, Manhattan
Division 3, Battalion 7
TRAVEL TIME (minutes)
RESPONSE DISTANCE (miles)
D
0
0.5
1.0
1.5
2.0
2.5
1
2
3
4
5
6
7

Figure 8

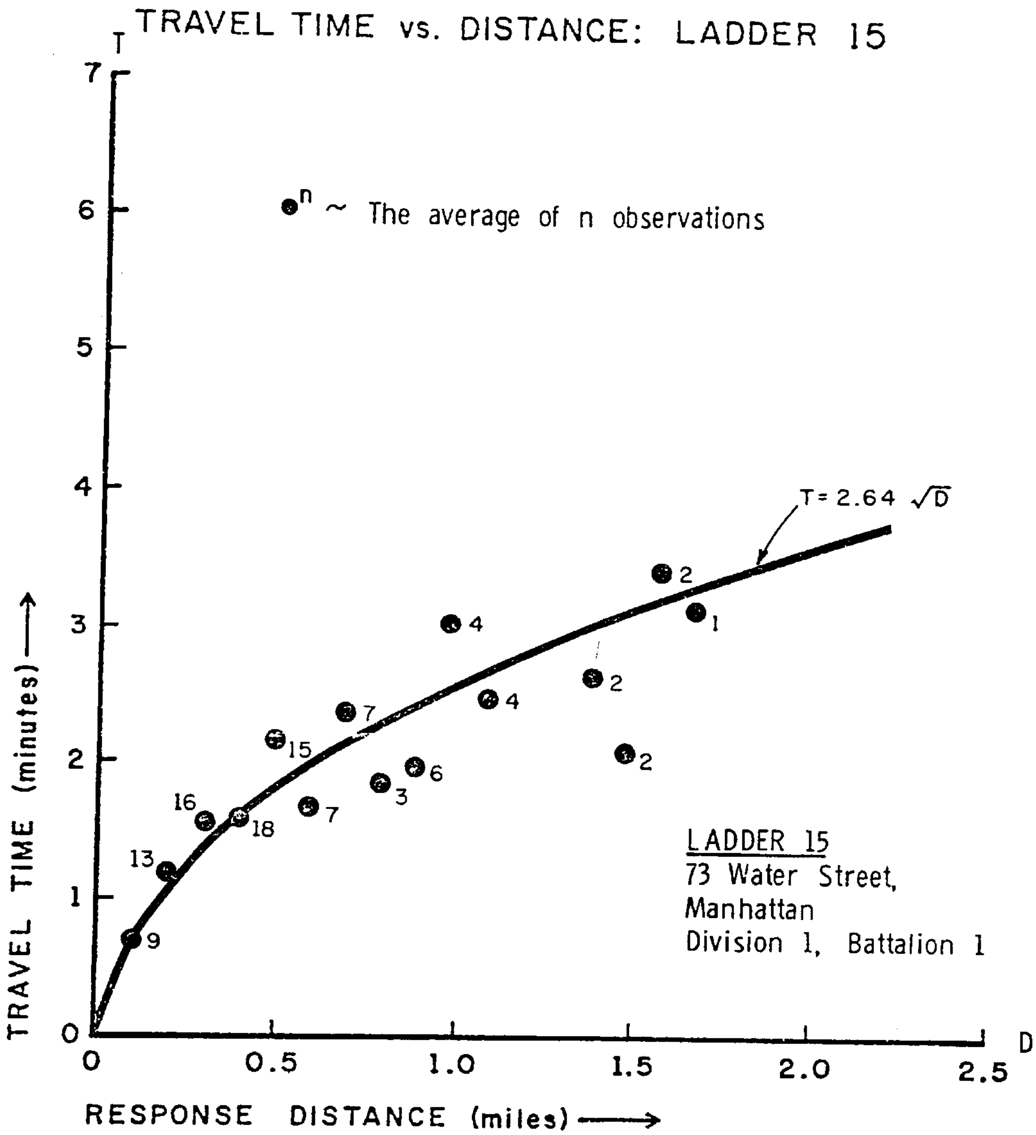

TRAVEL TIME vs. DISTANCE: LADDER 15
T
7
6
5
4
3
2
1
0
• n ~ The average of n observations
T = 2.64 √D
9
13
16
18
15
7
7
3
6
4
4
2
2
2
1
LADDER 15
73 Water Street,
Manhattan
Division 1, Battalion 1
TRAVEL TIME (minutes) ⟶
0
0.5
1.0
1.5
2.0
2.5
D
RESPONSE DISTANCE (miles) ⟶

Figure 9

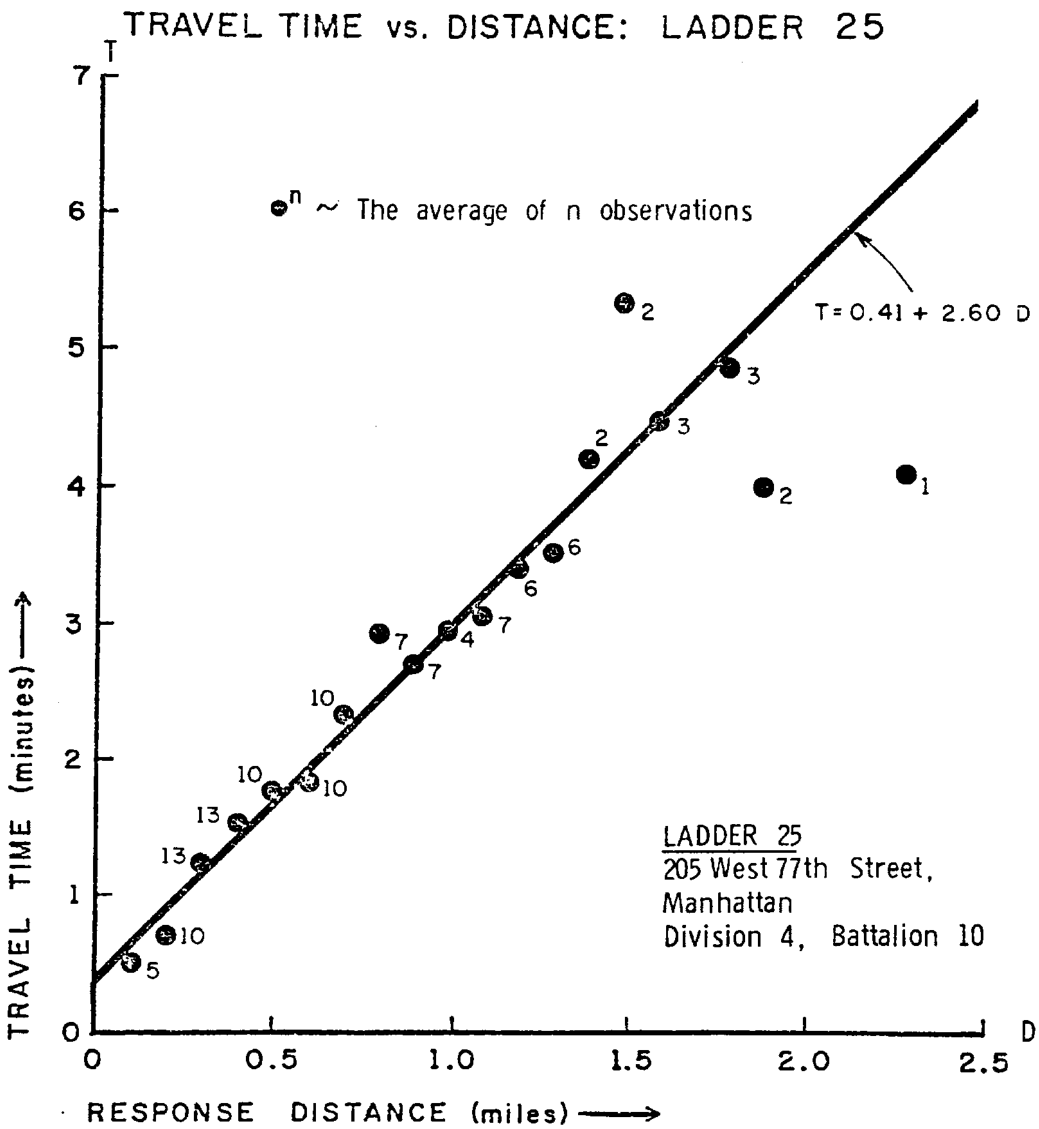

TRAVEL TIME vs. DISTANCE: LADDER 25
T
D
TRAVEL TIME (minutes)
RESPONSE DISTANCE (miles)
0
1
2
3
4
5
6
7
0.5
1.0
1.5
2.0
2.5
The average of n observations
T=0.41 + 2.60 D
LADDER 25
205 West 77th Street,
Manhattan
Division 4, Battalion 10

Figure 10

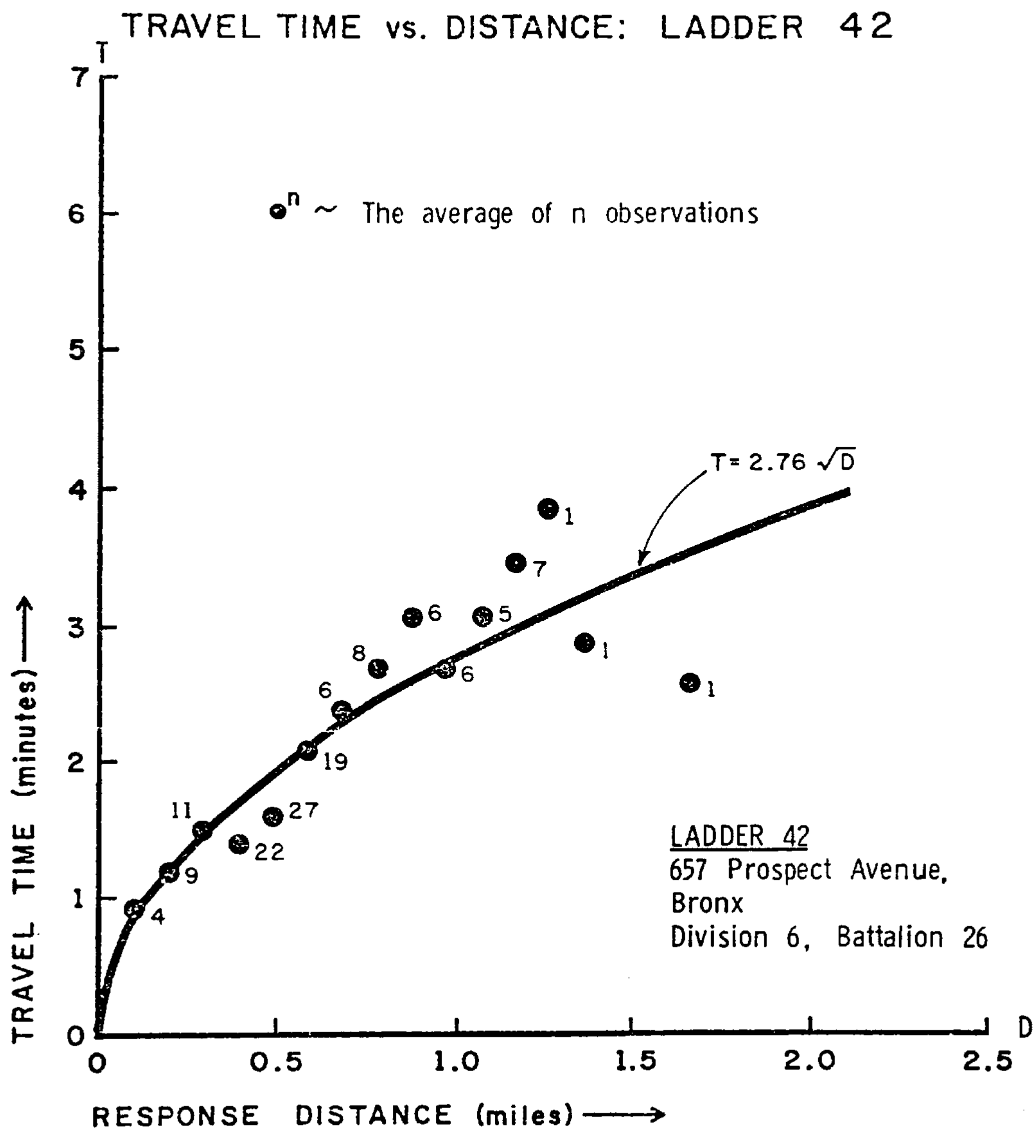

TRAVEL TIME vs. DISTANCE: LADDER 42
The average of n observations
T= 2.76 √D
LADDER 42
657 Prospect Avenue,
Bronx
Division 6, Battalion 26
TRAVEL TIME (minutes)
RESPONSE DISTANCE (miles)
T
D
0
1
2
3
4
5
6
7
0.5
1.0
1.5
2.0
2.5
4
9
11
22
27
19
6
8
6
6
5
7
1
1
1

Figure 11

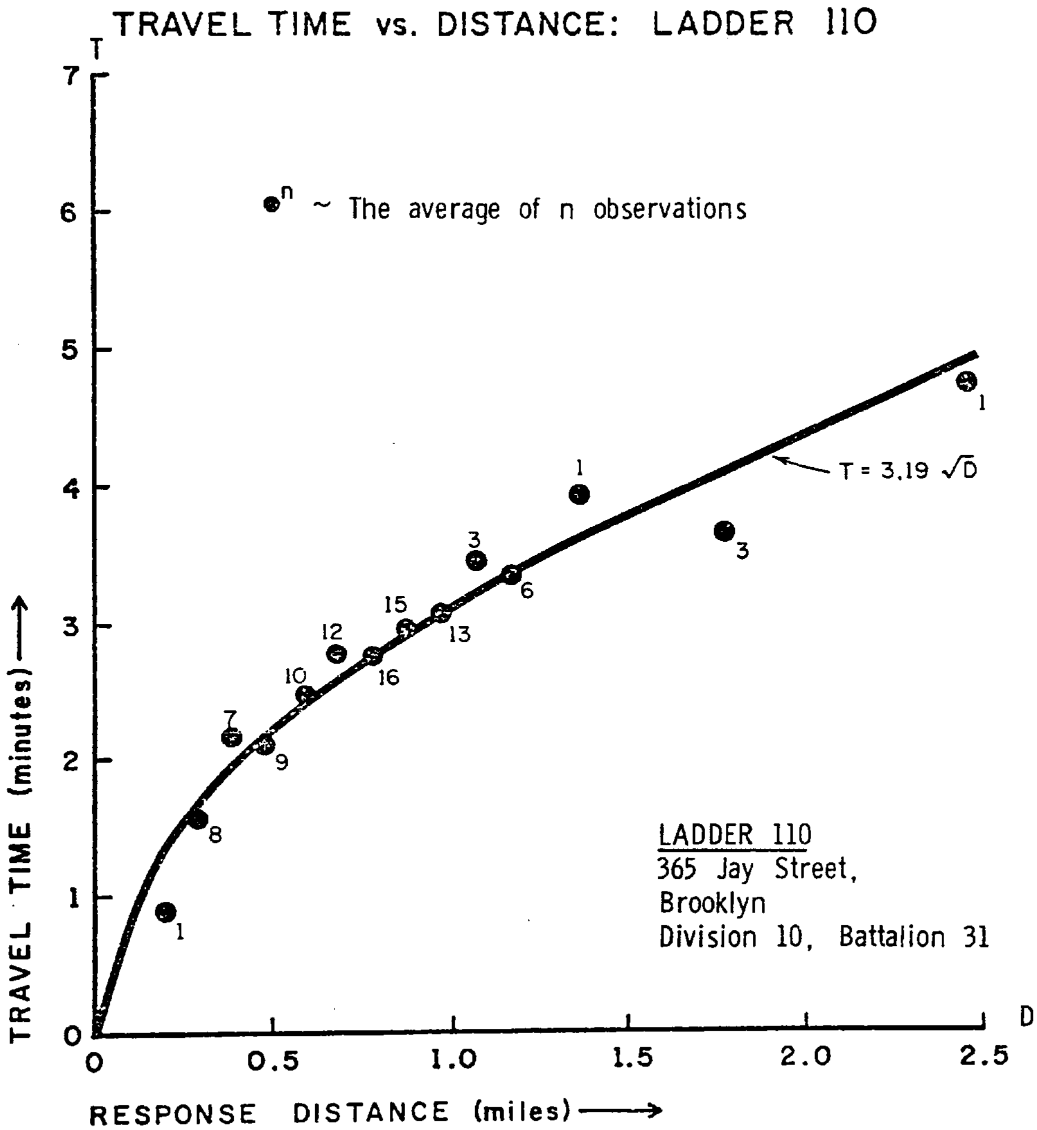

TRAVEL TIME vs. DISTANCE: LADDER 110
T
7
6
5
4
3
2
1
0
TRAVEL TIME (minutes)
The average of n observations
T = 3.19 √D
1
8
7
9
10
12
16
15
13
3
6
1
3
1
LADDER 110
365 Jay Street,
Brooklyn
Division 10, Battalion 31
D
0
0.5
1.0
1.5
2.0
2.5
RESPONSE DISTANCE (miles)

Figure 12

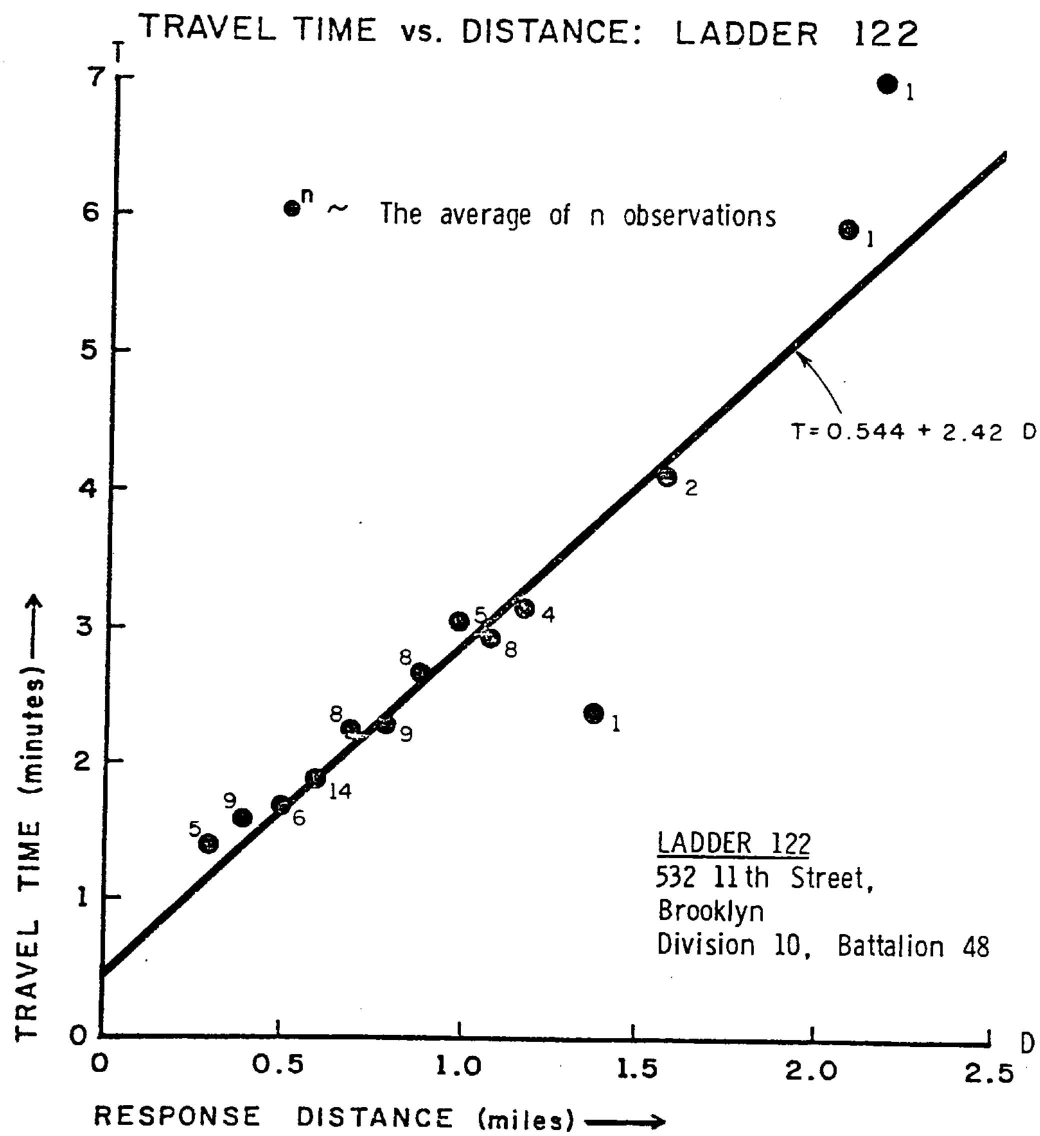

TRAVEL TIME vs. DISTANCE: LADDER 122
The average of n observations
T=0.544 + 2.42 D
LADDER 122
532 11th Street,
Brooklyn
Division 10, Battalion 48
TRAVEL TIME (minutes)
RESPONSE DISTANCE (miles)
T
D
0
1
2
3
4
5
6
7
0.5
1.0
1.5
2.0
2.5

Figure 13

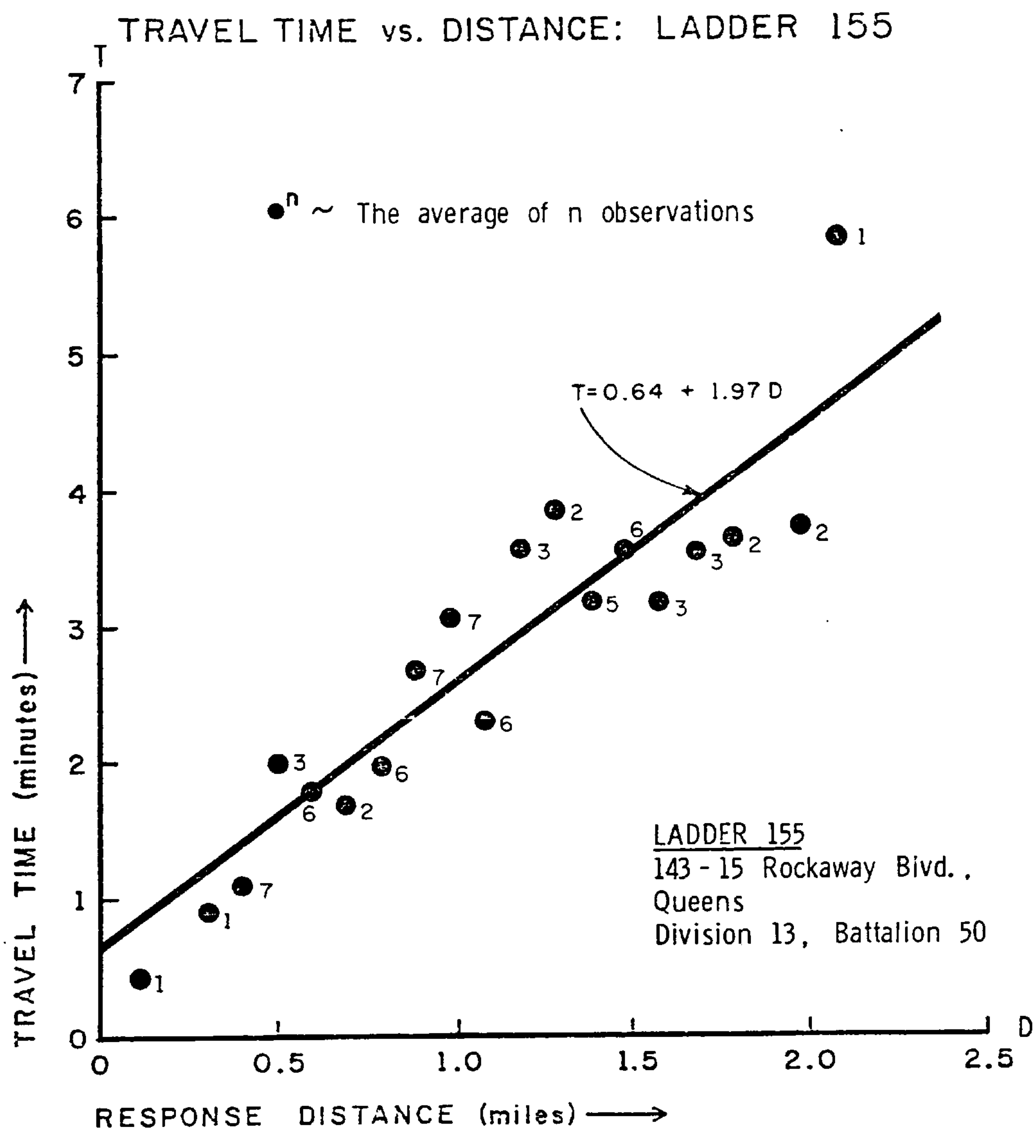
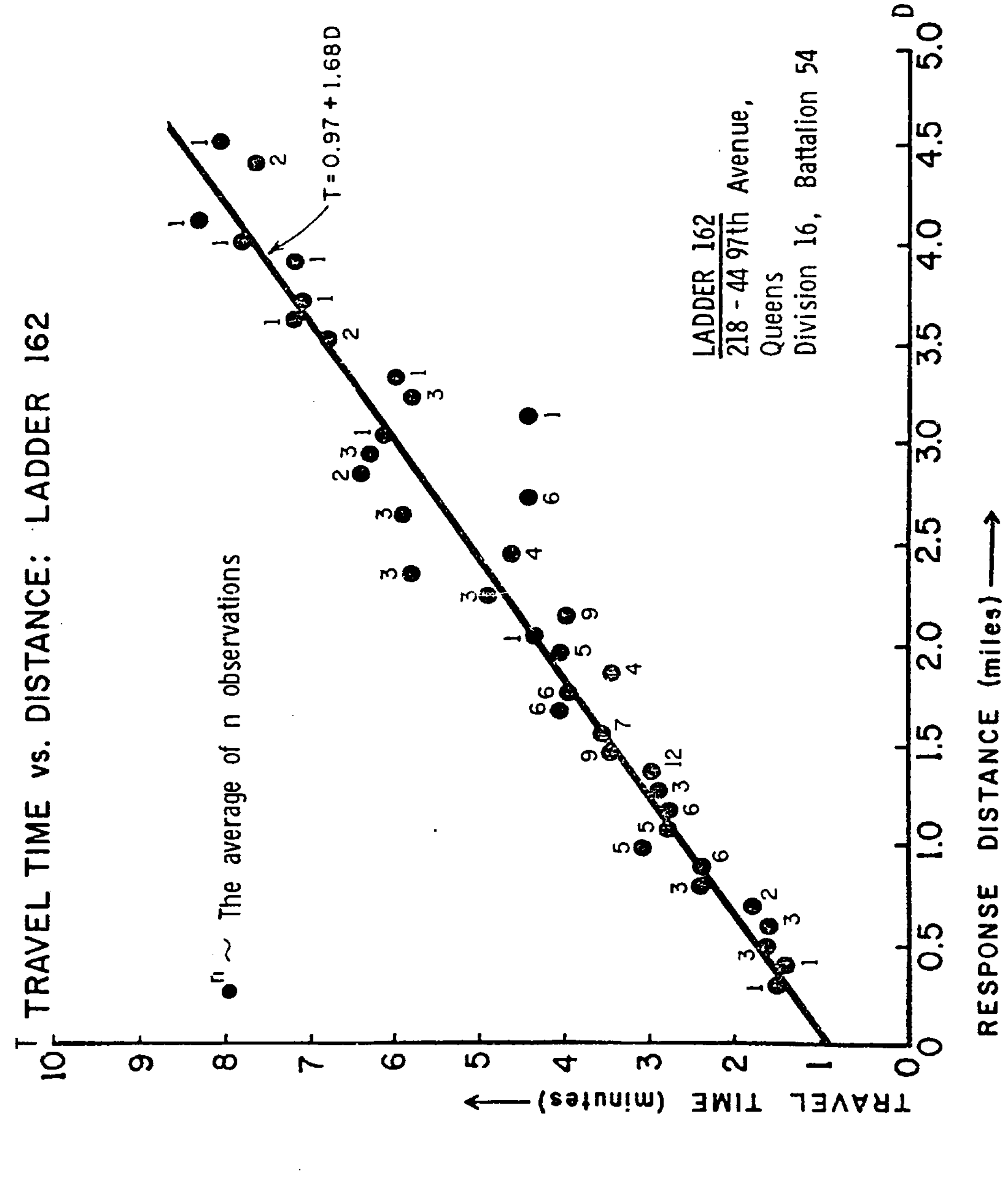

TRAVEL TIME vs. DISTANCE: LADDER 155
T
7
6
5
4
3
2
1
0
TRAVEL TIME (minutes) ⟶
The average of n observations
T=0.64 + 1.97 D
LADDER 155
143 - 15 Rockaway Bivd.,
Queens
Division 13, Battalion 50
0
0.5
1.0
1.5
2.0
2.5
D
RESPONSE DISTANCE (miles) ⟶

Figure 14


TRAVEL TIME vs. DISTANCE: LADDER 162
• n ~ The average of n observations
T = 0.97 + 1.68D
LADDER 162
218 - 44 97th Avenue,
Queens
Division 16, Battalion 54
TRAVEL TIME (minutes)
RESPONSE DISTANCE (miles)
T
D

square-root model T(1) = 2.74; for the linear model T(1) = 2.83. The slope of the linear function is T'(1) = 1.36; that of the square-root function is T'(1) = 1.37. The slopes of both functions are identical at .9 miles.)

We used the data for all runs to fit a square-root function for short runs and a linear function for long runs simultaneously, while forcing the functions to intersect and have the same slope at a single point. The fit of this piecewise function was made using least squares. Mathematically, the problem is:

Find a, b, c, d to

$$\text{minimize} \quad \sum_{i=1}^{N_d} (T_i - c\sqrt{D_i})^2 + \sum_{i=N_d+1}^{N} (T_i - a - bD_i)^2,$$

subject to:

$$c\sqrt{d} = a + bd,$$

and

$$\frac{c}{2\sqrt{d}} = b.$$

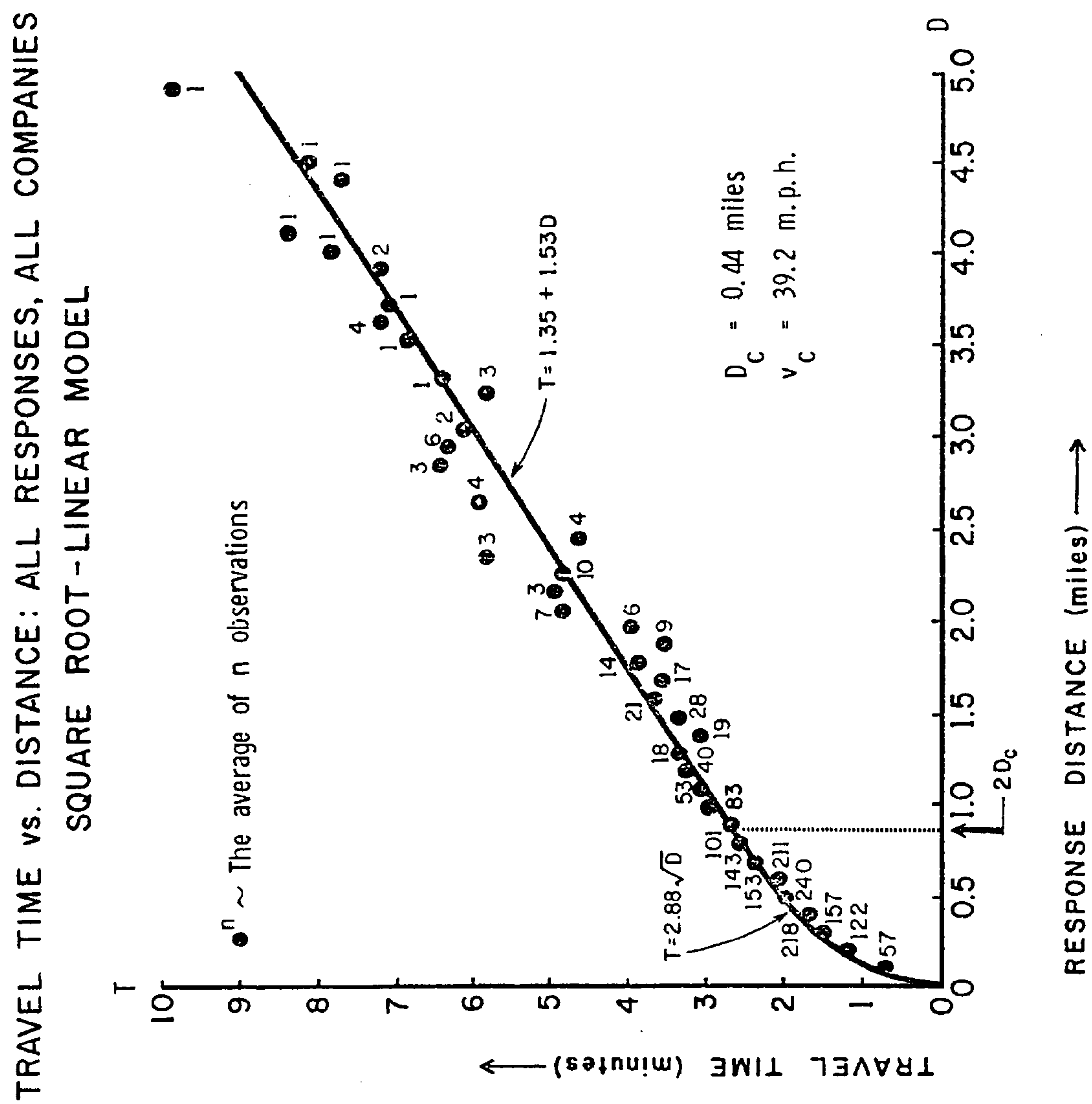
Here, (T_i, D_i) i=1,..., N are the observed travel time/travel distance pairs ordered by increasing distance; c is the parameter of the square-root portion of the function; a and b are the parameters of the linear portion of the function, and d is the distance at which the two segments of the function are tangent. For any given d, N_d is the number of observed distances that are less than or equal to d. The constraints specify the tangency conditions.

We developed an iterative method for solving this constrained minimization problem for estimation of the non-linear parameters (see the Appendix for details). The results of this procedure produced the estimates

$$T = 2.88\sqrt{D} \qquad \text{if } D \leq 0.88 \text{ miles},$$
$$T = 1.35 + 1.53D \qquad \text{if } D > 0.88 \text{ miles}.$$

Referring to the notation used above, the acceleration cutoff distance, D_c, is 0.44 miles; the cruising velocity, v_c, is 39.2 miles/hr., and the acceleration, a, is 29.0 miles per hour/min. This function, fitted to the original data, is shown in Fig. 15. Although we do not document it in

Figure 15


TRAVEL TIME vs. DISTANCE: ALL RESPONSES, ALL COMPANIES
SQUARE ROOT-LINEAR MODEL
n ~ The average of n observations
T=1.35 + 1.53D
T=2.88√D
D_c = 0.44 miles
v_c = 39.2 m.p.h.
2D_c
TRAVEL TIME (minutes)
RESPONSE DISTANCE (miles)
T
D

this Report, we should remark that the "goodness of fit" is largely insensitive to the choice of D_c in the range from 0.3 to 0.6 miles. Further, for different values of D_c in this range, the values of the parameters are relatively stable. The reason for this is the near linearity of the square-root function in this range. Table 5 presents a summary of the data used for these fits. Included in the average travel time for each distance are data collected on all runs by all the participating ladder companies. For each travel distance, the table gives the number of observations and the resulting average and standard deviation of travel time (see also Fig. 15).

Table 5

SUMMARY OF TRAVEL CHARACTERISTICS:
ALL RUNS FOR ALL PARTICIPATING LADDER COMPANIES

Travel Distance (miles)	Number of Observations	Average Travel Time (minutes)	Standard Deviation of Travel Time (minutes)
0.10	57.0	0.75	0.42
0.20	122.0	1.22	0.62
0.30	157.0	1.52	0.60
0.40	218.0	1.73	0.68
0.50	240.0	2.08	0.72
0.60	211.0	2.19	0.63
0.70	153.0	2.47	0.65
0.80	143.0	2.65	0.68
0.90	101.0	2.79	0.72
1.00	83.0	3.06	0.86
1.10	53.0	3.11	0.88
1.20	40.0	3.31	0.79
1.30	18.0	3.40	0.75
1.40	19.0	3.19	0.68
1.50	28.0	3.48	0.96
1.60	21.0	3.75	0.77
1.70	17.0	3.65	0.63
1.80	14.0	4.00	0.96
1.90	9.0	3.62	0.61
2.00	6.0	4.03	0.69
2.10	7.0	4.94	0.91
2.20	3.0	5.09	1.81
2.30	10.0	4.99	1.11
2.40	3.0	5.97	1.28
2.50	4.0	4.76	0.64
2.70	4.0	6.04	1.46
2.80	3.0	4.58	0.58
2.90	6.0	6.52	0.35
3.00	2.0	6.46	0.65
3.10	3.0	6.29	0.73
3.20	1.0	4.52	0.00
3.30	1.0	5.92	0.00
3.40	4.0	6.57	0.83
3.60	1.0	7.00	0.00
3.70	2.0	7.37	1.24
3.80	1.0	7.25	0.00
4.00	1.0	7.32	0.00
4.10	1.0	8.00	0.00
4.20	1.0	8.50	0.00
4.50	1.0	7.83	0.00
4.60	2.0	8.21	2.89
4.90	1.0	10.00	0.00

III. VARIATION IN RESPONSE VELOCITIES BY TIME OF DAY

Since the New York City Fire Department has been considering varying the number of companies on duty and the number of engines dispatched to an alarm at different hours of the day, an understanding of how response velocities vary by hour of day was of particular interest.

1. Do fire engines travel faster or slower in daylight than in the dark?
2. How much slower do fire engines travel during rush hours?

The results of our analysis are simple and surprising. First, there is no practical difference between travel velocities under conditions of daylight and darkness. Second, while velocities are lower during rush hours, they are not as much lower as we or the Department expected. The reduction in average velocity (of about 20 percent) is greatest during the 8 a.m. - 9 a.m. period.[4] The data in Tables 6 and 7 support these conclusions.

Table 6 shows the average and standard deviation of velocity for runs grouped by two-hour intervals and by division of the day into the following four periods: 5 a.m. to 8 p.m., excluding the "rush hours" (these are taken to be daylight hours); 8 p.m. to 5 a.m. (these are taken to be hours of darkness); and two rush-hour periods, 8 a.m. to 9 a.m. and 4:30 p.m. to 5:30 p.m. The data are presented separately for (1) the nine ladder companies for which the square-root time/distance model is preferred, (2) the four ladder companies for which the linear time/distance model is preferred, and (3) for all companies. The combined results are typical of those obtained for individual companies, and they indicate that, although there are time-of-day effects, they are not strong.

We note in passing that the number of observations in each two-hour period listed in Table 6 illustrates the dramatic difference in the demand on the Fire Department by hour of day. The peak of 239 calls in the period 2000 to 2200 is more than 5 times the 45 calls received during the period 0600 to 0800.

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[4] One qualification needs to be made about these observations. Since dates were not recorded, we could not separate weekends and weekdays. A reasonable assumption is that the rush-hour effect would be stronger on weekdays. Since most of our observations came from weekdays, we were unable to sort out the weekend effect.

Table 6

SUMMARY OF RESPONSE VELOCITIES BY TIME OF DAY

Hours	The 9 "Square-Root" Ladder Cos.			The 4 "Linear" Ladder Cos.			All 13 Ladder Cos.		
	$\bar{v}$	σ_v	n*	$\bar{v}$	σ_v	n	$\bar{v}$	σ_v	n
0000-0200	17.8	7.2	100	23.2	6.0	36	19.2	7.3	136
0200-0400	15.9	5.2	82	24.2	8.3	21	17.7	6.8	104
0400-0600	17.0	7.4	52	23.2	7.3	10	18.0	7.7	62
0600-0800	17.2	7.1	34	19.5	7.0	11	17.7	7.1	45
0800-1000	15.1	5.8	51	21.7	8.7	10	16.2	6.7	61
1000-1200	17.0	7.5	88	22.0	6.7	29	18.3	7.6	117
1200-1400	17.2	6.3	129	24.8	5.5	36	18.9	6.9	165
1400-1600	17.2	7.3	158	22.4	5.9	47	18.4	7.3	205
1600-1800	15.8	7.1	161	22.0	7.5	39	17.0	7.6	200
1800-2000	17.7	6.8	167	22.9	7.7	49	18.9	7.3	216
2000-2200	16.6	6.9	173	23.5	6.7	66	18.5	7.5	239
2200-2400	17.3	6.6	161	22.8	7.3	61	18.8	7.2	272
0500-2000 (rush hours excluded)	16.9	7.0	704	22.9	6.7	200	18.2	7.3	904
2000-0500 (dark)	17.0	6.7	550	23.2	6.9	192	18.6	7.3	742
0800-0900 (morning rush hour)	14.5	5.4	25	12.3	2.1	2	14.3	5.2	27
1630-1730 (evening rush hour)	17.1	7.0	77	22.0	8.4	22	18.2	7.6	99
All hours	16.9	6.8	1356	22.9	6.9	416	18.3	7.3	1772

*$\bar{v}$ = average velocity

σ_v = standard deviation of velocity

n = number of observations

Table 7 shows the results of additional analysis done to check for possible compounding of velocity and distance effects with time-of-day effects. Separate regressions were made of the square-root time/distance model for hours of darkness, for daylight hours excluding rush hours, and for rush hours. The table gives the estimated values of $\hat{c}$, the square-root model parameter, as well as of $\bar{v}$, the average velocity, $\bar{D}$, the average response distance, and $\bar{T}$, the average travel time. The results indicate clearly that there is not a strong dependence of response velocity on time of day.

Table 7

RECRESSIONS FOR TIME OF DAY--9 LADDER COMPANIES IN GROUP S

Hours	1st-Due Runs				2nd-Due Runs				All Runs			
	$\hat{c}$	$\bar{v}$	$\bar{D}$	$\bar{T}$ *	$\hat{c}$	$\bar{v}$	$\bar{D}$	$\bar{T}$	$\hat{c}$	$\bar{v}$	$\bar{D}$	$\bar{T}$
0500-2000 no rush hrs. (daylight)	2.73	16.1	.45	1.77	3.11	17.8	.75	2.67	2.95	16.9	.58	2.16
2000-0500 (dark)	2.77	16.4	.48	1.85	3.08	17.6	.73	2.59	2.91	17.0	.58	2.14
rush hours	2.61	16.0	.44	1.70	3.17	16.7	.73	2.70	2.90	16.6	.57	2.12
all hours combined	2.74	16.2	.46	1.80	3.10	17.7	.74	2.64	2.93	16.9	.58	2.15

* $\hat{c}$ = fitted parameter of the square-root model $T = c\sqrt{D}$, can be interpreted as the average travel time for a one-mile run in minutes

$\bar{v}$ = average response velocity (mph)

$\bar{D}$ = average response distance in miles

$\bar{T}$ = average travel time in minutes

APPENDIX

FITTING THE PIECEWISE TRAVEL-TIME FUNCTION

The problem of fitting a continuous piecewise square root-linear travel-time curve to the experimental data can be expressed mathematically as follows:

Given N sets of observations (T_i, D_i, M_i), $i=1, 2, \ldots, N$ where T_i denotes the average travel time (in minutes) of the M_i responses having a response distance of D_i (in miles), find values of the parameters a, b, c, and d to

$$\text{minimize} \quad \sum_{i=1}^{N} M_i(T_i - f(D_i))^2 \tag{1}$$

subject to:

$$f(D) = \begin{cases} c\sqrt{D}, & \text{if } D \le d, \\ a + bD, & \text{if } D > d, \end{cases}$$

and

$$a + bd = c\sqrt{d}$$
$$b = c/(2\sqrt{d}).$$

The first two constraints specify the form of the piecewise function to be fitted, and the second two specify that the two pieces of the curve are to be tangent at the break point d (they must meet and have the same slope at d). After eliminating a and c by solving for them in terms of b and d, the problem can be written as:

Find b and d to (2)

$$\text{minimize } q(b,d) = \sum_{i=1}^{N_d} M_i(T_i - 2b\sqrt{dD_i})^2 + \sum_{i=N_d+1}^{N} M_i(T_i - bd - bD_i)^2$$

where, assuming that the sets of observations are ordered by increasing value of D_i, N_d is the largest value of i such that $D_i \le d$. If we fix a value of d (and, hence, of N_d), we can determine $b^*(d)$, the optimal value of b for that value of d, by differentiating (1) with respect to b and equating the derivative to zero. The result is

$$b^*(d) = \frac{2\sqrt{d}\sum_{i=1}^{N_d} M_i T_i \sqrt{D}_i + \sum_{i=N_d+1}^{N} M_i T_i (d + D_i)}{4d \sum_{i=1}^{N_d} M_i D_i + \sum_{i=N_d+1}^{N} M_i (d + D_i)^2}$$

Then, by varying d, an optimal pair of values b^* and d^* can be determined. We performed this search for d^*, using an interactive computer program which, for a given d, computed $b^*(d)$ and $q(b^*(d))$. Using this program, we mapped out $q(b^*(d))$ as a function of d in the range of interest.