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A Report prepared for

UNITED STATES AIR FORCE PROJECT RAND

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PREFACE

This report was prepared as part of Rand's Manpower, Personnel, and Training Program, sponsored by the United States Air Force. It presents work done under the Supply and Retention of Air Force Officers project. The work was initiated at the request of the Deputy Chief of Staff/Personnel, Headquarters U.S. Air Force, and is designed to assist the Air Force in achieving stable accession and training rates, smooth career progression, and control of officer inventory by year group. This report is part of a planned series that examines the impact of changes in personnel policy on the supply and retention of officers and on the number and flow of officers by component, years of service, source of commission, and aeronautical rating.

The Officer Force Grade Limitations Model described here is one of a set of computer-based models designed to provide personnel planners with detailed officer inventories and flows reflecting the effects of policies and conditions that they may wish to investigate. The actual computer program is available upon request to The Rand Corporation.

The planned series of Rand reports will include the following tentative titles:

- *A System of U.S. Air Force Officer Personnel Planning Models: An Overview.*
- *The Officer Force Progression Model: A Steady-State Mathematical Model of the U.S. Air Force Officer Structure.*
- *The Constrained Officer Force Progression Model: A Steady-State Mathematical Model of the U.S. Air Force Officer Structure.*
- *The Officer Grade Limitations Model: A Steady-State Mathematical Model of the U.S. Air Force Officer Structure.*
- *The Retention of U.S. Air Force Officers.*

The ability to manage U.S. Air Force personnel in general, and officers in particular, is important because people represent the Air Force's most critical resource. Personnel costs have increased to the point where they constitute more than 50 percent of each annual Air Force or Defense budget.

The series of reports listed above will describe a system of officer personnel planning models designed to provide detailed, broadly based aggregated data for use by Air Force personnel planners in managing the Air Force officer force.

SUMMARY

The Officer Grade Limitations Model, described here, was developed at Rand as part of a series of Officer Force Structure Models; the other models are separately documented. The grade limitations model, like two of the others, is a steady-state model of the U.S. Air Force Officer Force Structure. By "steady-state" we mean "governed by the assumption that the force structure does not change from year to year, i.e., that accessions, loss rates, movements of officers, and inventories of officers remain constant from one year to the next, keeping the force in equilibrium."

In its computations, the model divides the force into states, each of which is defined by a unique set of the officers' attributes of grade, rating, source of commission, component, and year of service. The movements of officers from state to state: promotions, augmentations, rating transfers, etc.; are known as "flows." The inputs to the model, which include loss rates, descriptive parameters for the other types of flows, and grade authorizations, are all given in terms of the five attributes which define states.

The grade limitations model is unique in having the grade authorizations (number of officers in each grade) as inputs. Rather than taking accessions, promotion opportunities, etc., and computing from these inputs what the grade authorizations should be, this model starts with the grade authorizations and determines what the yearly accessions, promotion opportunities, etc., would have to be in order to meet the desired grade authorizations. This is an unusual approach to manpower modeling, which can be very useful in determining long-term policy when grade authorizations are imposed upon the Air Force planner by some outside authority (Congress, for example), and in solving other problems related to grade size.

In order to use grade authorizations as an input, the model must perform the calculations of the number of officers per state and the flows between them in a somewhat "backward" fashion (in fact the grade limitations model is sometimes referred to as the "backward model").

Instead of starting with yearly accessions and flowing them through the system first as lieutenants, then as captains, and on to the higher grades, the model starts with the authorized number of colonels and, after determining how many of them are lost to the service, replaces them with an equal number of promotions from the lieutenant colonel grade to maintain equilibrium. It then replaces these lieutenant colonels (and those who are lost to the service) with majors in such a way as to preserve the grade authorization for lieutenant colonels. In this manner, the model builds the officer force structure backwards, beginning with the highest grade of colonel* and going back to lieutenants. When the model has computed the number of officers who need to become lieutenants each year, the yearly accessions are known.

The outputs from the model show officer inventories, the flows into and out of the states, and various other properties of the force structure just computed. All of these outputs can be obtained in great detail, i.e., for any combination of the five officer attributes, or aggregated across certain attributes. In addition, the grade limitations model includes a statistical package referred to as the "Goodness Measure Package" which gives probabilities of taking certain paths through the officer structure (e.g., probability of promotion) and the mean extent of these paths. The purpose of these outputs is to aid the personnel planner in evaluating the desirability of such a force.

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*General officers may be included with colonels.

CONTENTS

I. INTRODUCTION

The management of U.S. Air Force personnel in general, and officers in particular, is an increasingly important problem. Over the years personnel costs have been increasing steadily until now they constitute more than 50 percent of the Air Force and Defense Department budgets. Moreover, since, in a military environment, there is virtually no lateral entry into the higher ranks, personnel management at low levels is a very important consideration for the Air Force. ". . . the services must acquire future senior officers at the initial entry level and then provide them with the military training, education and experience they cannot gain elsewhere. This requires an extremely long-range approach to officer personnel management since what is done today will have repercussions thirty years or more in the future."* The Air Force has long been plagued with personnel problems arising from officer management policies which addressed problems in the near term but neglected problems in the long term. Such policies have created an unstable force structure often necessitating drastic and undesirable policy actions such as reductions in force, and making it difficult to meet requirements for an effective and efficient Air Force officer force. "Humps and valleys" created during the Korean War are still with us, and they demonstrate the need for a smooth and even transition from one type of policy to another.

Projects such as Rand's Supply and Retention of Officers project, for which this report was written, are concerned with long-range effects of various policy actions and are designed to aid the Air Force in achieving and maintaining a stable and efficient officer force structure.

The Air Force has been an innovator in the development of long-term planning of officer force structure policy through the use of computer models and a system approach to personnel management. Initial

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*Quotation from an undated draft from the Office of the U.S. Secretary of Defense.

Air Force work in this area is recorded in the several volumes of *The USAF Personnel Plan.*[*] Specifically, volume two, "Officer Structure (TOPLINE) 1971," describes the philosophy and computer models that apply to the officer force structure. (TOPLINE is a short title for Total Officer Personnel objective structure for the line officer force.) The set of officer force personnel planning models, of which the Grade Limitations Model is a part, was developed as a continuation of such modeling efforts.

Basically, there are three types of personnel models that could be used by the Air Force, each with its own concept and individual uses:

1. Steady-state (or static). Steady-state personnel planning models are used to study long-range personnel objectives as well as to examine the effects of changes in various personnel policy parameters. These models assume that, for long-term planning purposes, ideal and steady-state conditions will apply. Steady-state conditions are hypothetical and imply that loss rates and other planning factors do not change from year to year. The resulting officer force structure is in equilibrium, implying that the size and shape of the officer force structure within such models is not dependent on time.

The TOPLINE static personnel planning model is used by Headquarters USAF to develop its long-range objective officer structure. DOPMS[†] is another, more recently developed, static model which allows the personnel planner more flexibility than TOPLINE. In addition to their usefulness as tools in designing long-term objectives, static models can also test the effects of changes in various policy variables and give policy planners a concept of the overall long-term effects of personnel policies. As a result, although the static nature of the models keeps them from

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[*] Department of the Air Force, *The USAF Personnel Plan, Vol. I, Personnel Management Objectives,* Washington, D.C., January 1970. (For Official Use Only.)

[†] DOPMS is a short title for Defense Officer Personnel Management System, which includes both a static and a dynamic model.

representing actual conditions, they can be very useful as a means of evaluating the long-term impacts of a personnel policy or proposed changes in such a policy.

2. Dynamic. Dynamic models, such as the TOPLINE and DOPMS dynamic personnel planning models (sisters of the TOPLINE and DOPMS static models), are used to study the short-term effects of a given personnel policy. These models apply a given policy to today's force to show the planner the direction in which such a policy would take the force were it applied right away. Actually, the policy is applied more than once, to each successive officer structure, so that the planner can see where it would take the force were it adopted now and used for a given number of years.

3. Transition. Transition models aid the planner in moving today's force toward a specified objective over a given number of years. Whereas dynamic models successively apply a given policy to today's force to see where that policy leads (it may not lead to a desirable force structure in an acceptable span of time), transition models take today's force, a long-term objective, and a target year; the model then determines what policies should be adopted each year to reach the objective by the target year. While a steady-state model will identify the policy that will maintain a desirable force structure once it has been reached, a transition model allows the planner to investigate alternative ways of moving toward that force objective beginning with today's officer inventory and policies.

Although transition models may well be the most useful of the three types, very little work has, as yet, gone into developing one. Building transition models is considerably more difficult than building either steady-state or dynamic models. Recently, however, with the knowledge gained by developing the steady-state models discussed in this report, the transition model has begun to look quite tractable. The next phase of the Supply and Retention of Officers Project will be to add a transition model to the set of officer force personnel planning models described below. A good understanding of steady-state

models is an important first step, because a transition model must be supplied with an objective for the officer force structure. Steady-state models are developed primarily to aid planners in designing and evaluating officer structure objectives. They have been quite successful and useful and are necessary tools for determining the inputs to a transition model.

OVERVIEW OF THE OFFICER FORCE PERSONNEL PLANNING MODELS

At the request of the Air Force's Deputy Chief of Staff, Personnel, Rand is developing a system, or family, of officer force personnel planning models to supplement and extend the TOPLINE and DOPMS static personnel planning models. These models provide increased capability to analyze the long-range effects on the officer force structure of changes in policy or in external or environmental influences. This system of officer force planning models is described in this and companion reports and includes the following computer models:

- Officer force progression model (short title: progression model)
- Constrained officer force progression model (short title: constraints model)
- Officer grade limitations model (short title: grade limitations model)
- Officer force behavioral model (short title: behavioral model)
- Officer force transition model (short title: transition model, under development)

This system of models is useful in providing increased planning capabilities by:

- Defining a more detailed force structure both in terms of inputs <u>and</u> outputs. This allows for a considerably increased amount of policy flexibility,
- Allowing for interaction of changes in policies, officer

behavior and the officer structure, and

- Providing for the impact of grade limitations.

The first four models in this system were designed to be used together as well as separately. Each one adds its own special dimension to officer force planning. The first two models, the progression and constraints models, are similar in outlook. Like TOPLINE and DOPMS, they take a cohort of entering officers and flow them through the system, promoting and attriting them and augmenting the regular force with reserves according to a given set of policies. Mathematically, they begin with the lowest grade of lieutenant and apply loss rates and policy factors such as promotion opportunity, in order to determine the structure of the next grade, captain. The process is repeated until all the grades have been structured, the structure of each grade depending upon that of the grades below it. These models are unconstrained by grade limitations, i.e., the number of officers allowed in each grade is free to vary. Their purpose is to present the user with the steady-state implications of the set of Air Force policies specified as inputs. Their outputs include grade limitations, officer inventories by attribute (grade, rating, etc.) and the number of promotions, losses, and other flows within the officer structure.

The difference between the progression and constraints models is that with the constraints model the user can also specify the total force size, the number of regular officers in the force, the number of pilots, the number of navigators, and the career reserve requirements. While the progression model's inputs include Academy, ROTC and OTS yearly accessions, the constraints model is given only Academy and ROTC accessions and may vary OTS accessions as needed to satisfy the added constraints.

The third model, the grade limitations model, looks at the officer force from a point of view unlike the other models. Rather than beginning with yearly accessions and moving them chronologically forward through the system according to a given set of policies, the grade limitations model begins with the grade limitations and determines the accessions and policies required to maintain the proper number of officers in each grade in a steady-state condition.

The purpose of the grade limitations model is to aid the Air Force in solving grade-size related problems. The Air Force is often faced with a set of officer grade limitations imposed upon it by Congress or some other authority, and the grade limitations model can help in determining the implications of these requirements. Outputs include yearly accessions, officer inventories, the flows within the structure, and the policy which generates the structure. By "policy" we mean the promotion opportunities, augmentation rates, etc., which determine the flows of officers through the force structure.

To compute a structure and policy in keeping with the grade limitations, the model must begin with the highest grade of colonels* and work its way back to the lowest grade of lieutenant. The reason for this "backward" approach is that all movements of officers take place to fill up vacancies in higher grades. This implies that flows of officers out of a grade determine flows into it and that the flows out must be known before the flows in can be calculated. All vacancies in the highest grade result from losses to the force and not from promotions. Since losses can be computed from the input grade limitation and loss rates, colonel vacancies are known before any other calculations take place. This determines the flows into the grade of colonel which is the same as the flows out of lieutenant colonel. In this manner, the model works its way to lieutenants with the final calculation being the yearly accessions into lieutenant needed to maintain the computed force structure.

Steady-state deterministic simulation models, such as the three just described, are driven by loss rates, which are the fractions of officers from each state who leave the Air Force each year. If the rates are unrealistic or inaccurate, the outputs may be inaccurate. Obtaining accurate loss rates has always seriously plagued analysts who desired to use personnel planning models. The fourth model of the officer force personnel planning system, the behavioral model, predicts loss rates as a function of changes in the promotion and augmentation policies and in such external factors as officer's pay

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*General officers may be included in the colonel inventory.

and expected civilian pay. "A major assumption employed in the model is that officers base their expectations about the promotion system on the current experiences of officers more senior in years of service. Given this assumption, the logical process is to estimate the probabilities of promotion, augmentation, and . . . [losses] . . . by year of service."* These probabilities are used to derive the loss rates.

The behavioral model adds considerable power to the planning system and increases the usefulness of the other three models by providing reasonable loss rates based on realistic conditions. Since loss rates are influenced by the officer force structure and the officer force structure is, in turn, affected by the loss rates, the models can be used to iterate until convergence is reached. This is done by running the behavioral model and using the predicted loss rates as an input to one of the steady-state models. The resulting officer force is then made an input to the behavioral model and a new set of loss rates obtained. This continues until a mutually consistent set of personnel policies and loss rates is achieved.

These four models make excellent tools for studying the long-term officer force management problem. Such study is especially important since, as a result of the long-lasting effects of policy decisions, the long-run situation, including objectives and policy interactions, must be thoroughly understood before the solution of short-term personnel problems is attempted. Once the long-term has been studied and an officer force objective decided upon, the personnel planner could use a fifth officer force planning mode, the transition model, to help him in making the short-term policy decisions which both meet today's requirements and move the force toward the desired objective. The transition model, like the other models, will be designed with ease of mutual interaction in mind. The most obvious connection between the transition model and the other models is that the force objective of the transition model will come from one of the three steady-state models. In addition, as the transition model moves

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*G. A. Gotz, *An Analysis of the [illegible] Air Force Officers: An Empirical Model*, unpublished report, 1974.

today's force into the future it will interact with the behavioral model on a yearly basis to adjust loss rates in accordance with the changing officer force structure and policies.

Perhaps the most powerful feature of the officer force personnel planning models is their ability to be used in conjunction with each other. Together, these models provide the personnel planner with tools to determine the interaction of personnel policies, officer behavior, and the officer force structure.

The rest of this report will be devoted to a detailed discussion of the grade limitations model. Section II presents the terminology, assumptions, and concepts used by the model. Section III summarizes the mathematical basis of the grade limitations model. It includes a simple numerical example as well as a discussion of infeasible solutions and how to handle them. In Section IV an added feature of the grade limitations model, the "goodness measure" package, is discussed. The goodness measure package comprises a set of routines which compute various numerical parameters of the computed officer force (for example, the probability of promotion to grade G once grade G-1 has been reached). The purpose of these statistics is to aid the personnel planner in making decisions about the relative desirability of the officer structure just computed. Section V is a very brief concluding section, followed by a set of appendixes. The appendixes include discussions of the model's inputs and outputs as well as a presentation of the mathematics in more detailed than that given in Section III.

II. CONCEPTS AND VOCABULARY

The progression, constraints, and grade limitations models are steady-state, Markovian, mathematical flow models. The USAF officer force is a hierarchical system which has characteristics that make it "susceptible to analysis by Markovian models: (1) entries into military systems are made at specific low-level entry points in the hierarchy; (2) recruits (either officer or enlisted) are essentially undifferentiated, with specialization occurring as a result of training and experience that takes place after entry into military life; and (3) because breaks in service are rarely allowed, the process of movement through the military personnel system becomes a close analogy to an actuarial, life/death model of life expectancy."* (A basic assumption of Markovian models is that movement depends only on where an individual is located in the system, and not where he has been.)

The "flow" of personnel through a system may be thought of as movement from state to state. "The utility of the concept depends upon the definition of meaningful subsets of the population under consideration whose members are differentiated from all other members of the population. Each member of the population must be a member of one, and only one, state of the system. . . ."*

UNDERLYING ASSUMPTIONS

Before getting into the more detailed discussion of the officer states and movements between them in the Officer Force Planning Models, it would be useful to list and discuss the assumptions used in the development of these models:

- Academy graduates are regular component officers throughout their active career. It is possible for Academy graduated officers to give up their regular status by separating,

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*J. W. Merck and K. Hall, *A Markovian Flow Model: The Analysis of Movement in Large-Scale Military Personnel Systems*, The Rand Corporation, R-514-PR, February 1971.

joining a reserve unit and being recalled to active duty, for example, but the possibility of an Academy graduate being other than a regular component officer is not considered in these models.

- ROTC, OTS, and other non-Academy graduates who become officers enter the Air Force as reserve lieutenants in their first year of service. In the past, distinguished military graduates have been awarded regular commissions and have entered active Air Force duty as regular component officers in their first year of service. However, this policy has been discontinued, and the models are constructed accordingly. Also, lateral entries of officers in their first year of active service with grade higher than lieutenant are not considered by the models.
- Net suspensions or grounding of pilots or navigators is zero, that is, it is assumed that the number of pilots (or navigators) leaving flight status exactly equals those returning to flight status after having been grounded or suspended.
- Production of pilots or navigators, i.e., undergraduate pilot or navigator training, is completed prior to the first year of service in which lieutenants may be promoted to the grade of captain. Therefore, in the models, only lieutenants are assigned to UPT (or UNT) to become pilots (or navigators).
- All officers enter the Air Force as nonrated officers and it takes approximately one year to obtain an aeronautical rating of pilot or navigator. From this and the prior assumption, the models do not permit officers with only one year of service to be pilots or navigators, nor do they allow captains or higher-grade nonrated officers to become pilots or navigators.
- All grades of officers that are considered in the models are temporary rather than permanent grades.

STATES

The following attributes and subdivisions are used to group Air Force line officers into states which are the cells or nodes that form subsets of the officer force.

1. Component (C)*: Reserve (1)† or Regular (2).
2. Grade (G): Lieutenant (first or second) (1), Captain (2), Major (3), Lieutenant Colonel (4), Colonel and higher grades combined (5).
3. Source of Commission (SC): Academy (1), ROTC (2), and OTS (formerly SMS-0)/all other (3).
4. Rating (R): Pilot (1), Navigator (2), or Support [nonrated] (3).
5. Year of Service (Y): 1, 2, ... 35 where "years of service (YOS)" is synonymous with years of total active federal commissioned service, and an officer is in his ith YOS when he has completed i - 1 but not i years of total active federal commissioned service. The time unit in the grade limitations model is one year. Inputs and outputs are in terms of yearly units.

Figure 1 depicts a schematic representation of the possible states for officers with ROTC as their source of commission. A similar schematic representation applies for officers with OTS‡ or Academy source of commission, except that Academy officers would not be in reserve states.

Note that in year of service 1, only reserve, nonrated lieutenants exist. In the second year of service, we see the first rated officers appear as well as the first regular component officers. The officer flows, i.e., the movement of officers from one state to another, are discussed below.

CATEGORIES

For the purposes of this volume we will use the word "category" to define a group of officers who have the same grade, rating, source of commission and component, but do not necessarily have the same years

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*Symbol representing attribute appears within parentheses.

†Number of parentheses is code for subdivision.

‡SMS-0, School of Military Science-Officers, has since been renamed OTS, Officer Training School, a name used for this school in earlier years.

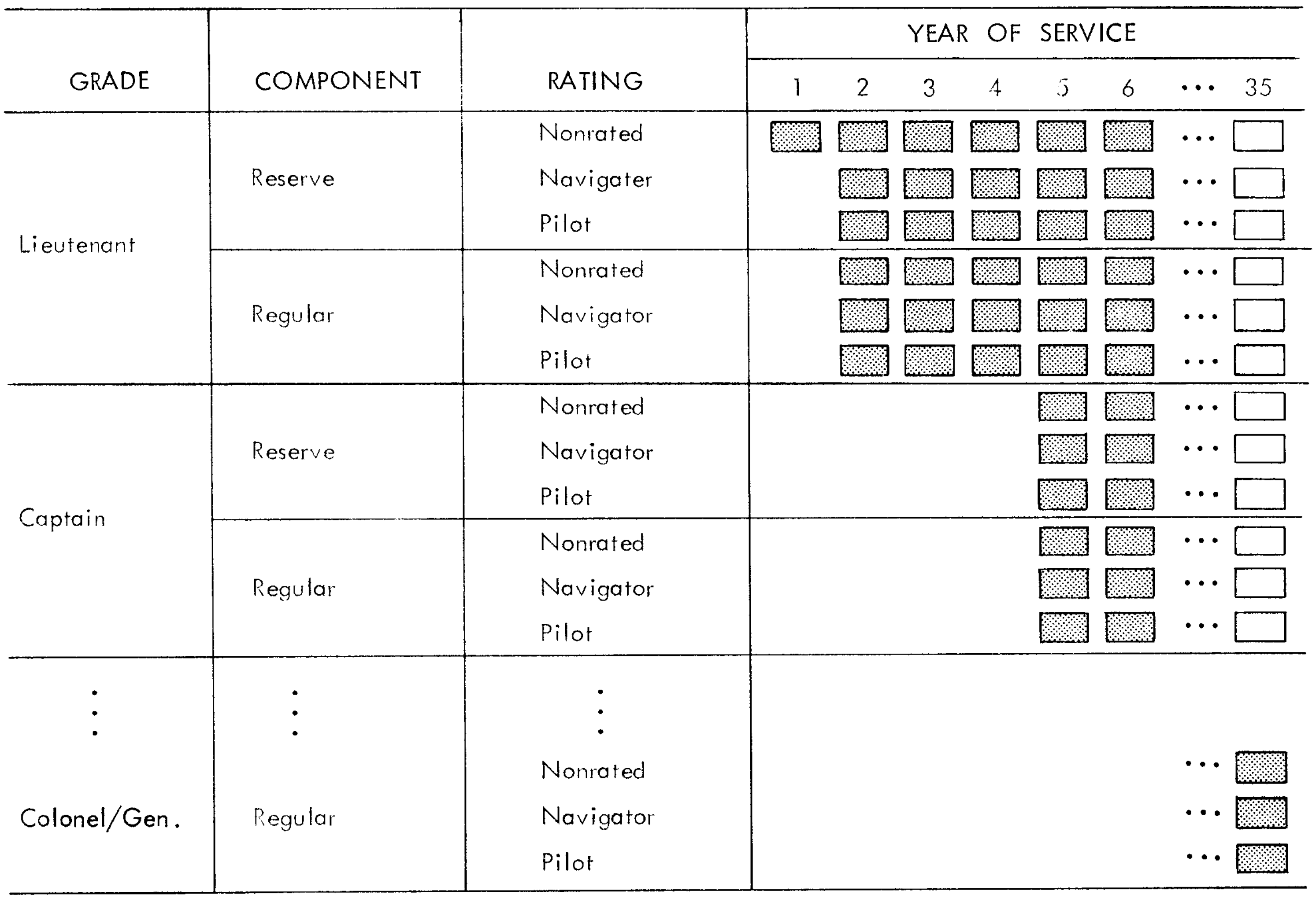

Fig. 1--Schematic representation of ROTC states

of service. In Fig. 1, for example, each row of states across the page is a category. All of the computations done by the grade limitations model are done with respect to categories. For example, each source of commission is treated entirely separately. The grade limitations model never looks at an <u>entire</u> grade all at once while performing its computations. It does, however, aggregate the outputs across attributes in any way desired by the user, as well as provide entirely unaggregated, and so, very detailed, outputs.

<u>FLOWS</u>

The grade limitations model and its companion models provide for the following flows through the Air Force officer system.

- Losses
- Rating transfers (nonrated to pilot via undergraduate pilot training (UPT) or nonrated to navigator via undergraduate navigator training (UNT) are the only rating transfers in the model)
- Augmentations (reserve to regular component)
- Rating transfer and augmentation (both in the same year)
- Promotions
- Promotions and augmentations (both in the same year)
- Lateral (officer ages one year and does not change category)

Figure 2 illustrates some of these flows for ROTC lieutenants with up to three years of service. In the first year of service all ROTC officers are reserve nonrated lieutenants (state nr_1). From this state several paths may be taken. The arrows leading nowhere from the upper right corners of the states indicate losses to the force. The horizontal arrows (for example, nr_1 to nr_2 or RT_2 to RT_3) indicate lateral flows. Flows such as nr_1 to NR_2 or rt_2 to RT_3, i.e., from reserve to regular, are augmentation flows. In the figure, rating transfers (nr_1 to rt_2) and rating transfer/augmentations (nr_1 to RT_2) are combined for pilots and navigators and are shown simply as transfers from nonrated to rated. In the models and all their inputs and outputs, such transfers are always considered separately for pilots and navigators.

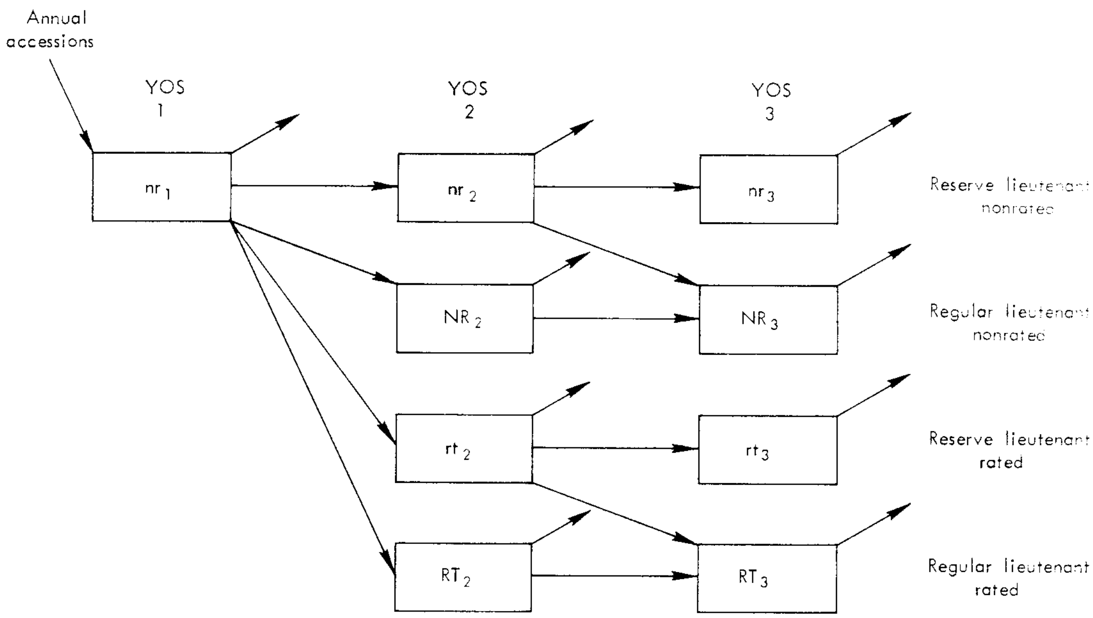

Fig. 2--Example of lieutenant states and flows in prepromotion years of service

The only types of flow not shown in the figure are promotions and promotion/augmentations. A promotion, of course, occurs when an officer advances from one grade to the next higher grade, say from lieutenant to captain. A promotion/augmentation, similar to a rating transfer/augmentation, occurs when an officer receives both a promotion and a regular commission in the same year of service, so that from one year to the next both his grade and his component change.

EQUILIBRIUM CONDITION

As was mentioned earlier, the grade limitations model is a steady-state, or static, personnel model. In a steady-state system the number flowing into a state equals the number previously in the state and also equals the number flowing out of the state; that is to say: the number in the state never varies.

Figure 3 illustrates the equilibrium condition. It shows flows into and out of a state of ROTC regular pilots and the quantity in the state.

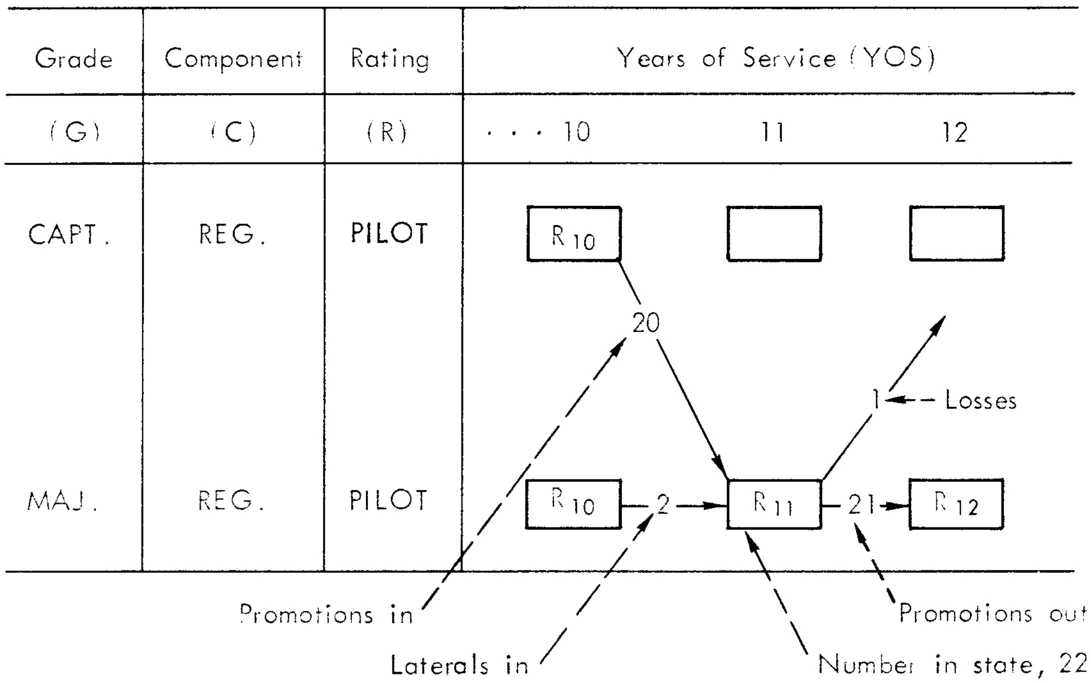

Fig. 3--Equilibrium conditions for regular pilot majors with 11 years of service: steady state quantity (22) = flows out = flows in. Source of commission, ROTC

III. MATHEMATICAL BASIS

The officer grade limitations model is sometimes called the "backward" model because of the order in which the computations are done relative to grade. All flows in the officer structure take place to fill vacancies created by losses or other flows. As a result, the model must compute the losses and flows out of a grade, say grade G, and so determine the vacancies to be filled, before it can compute the number of promotions to be made from grade G - 1 to G. The inputs to the model give it enough information to determine the number of losses from a grade. Since the only flows out of the highest grade (that of colonel)* are losses, the model can determine the vacancies in that grade directly from the inputs. The vacancies in colonel determine the promotion flows out of grade lieutenant colonel which, when combined with the inputs to compute losses from lieutenant colonel determine the vacancies in lieutenant colonel and so, the flows out of major. In this "backward" manner the model works all the way back to the lowest grade of lieutenant, with the yearly accessions (flows into lieutenant) being the last element of the officer flow structure computed.

In addition to computing, and outputting, the steady-state officer structure (that is, the number of officers in every state) and all the flows or movements of officers within that structure, the grade limitations model also computes the personnel policies and accessions necessary to maintain the officer structure with the given grade authorizations, loss rates, and other input parameters. Although Appendix C is a detailed discussion of the grade limitation inputs, it may be helpful to take a quick look at them before discussing the mathematics in more detail.

MODEL INPUTS

The distinguishing characteristic of the grade limitations model is, of course, that grade authorizations are given rather than computed.

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*General officers may be included in the colonel inventory.

Grade authorizations must be provided by component, rating, and source of commission as well as by grade. In fact, all of the inputs must be given separately for each officer attribute.

Besides the grade authorizations, the model needs:

- o Loss rates.
- o Flow ratios when two or more types* of flow can take place at the same time.
- o Flow distributions for each type of officer flow taking place.

Flow Ratios

We already know that the grade limitations model uses grade authorizations and loss rates to compute the total number of officers who should enter a given category each year. Once the model knows that X officers should become regular captains, how does it know how many of the total, X, should be promotions from lieutenant and how many should be augmentations from reserve captains? The flow ratio of augmentations to promotions for regular captains provides that information. For example, if X = 100 and the flow ratio is 1:3 (.333), the model knows that for each augmentation, three promotions should take place, i.e., the number of augmentations is 25; the number of promotions is 75, for a total of 100 entries into regular captaincies.

Flow Distributions

Now the model knows how many flows of each type should enter the given category. The flow distributions indicate how to divide these flows up by year of service, i.e., how many flows should take place in each year of service. To answer this question the model is given a set of flow distributions for each type of flow which enters the category. Take, for example, the 25 augmentations in the previous example. Figure 4 illustrates how a flow distribution of 0.80 in YOS 4 and 0.20 in YOS 5 results in 20 augmentations (80 percent of 25) taking place from

*The attribute which is changing determines the "type" of flow.

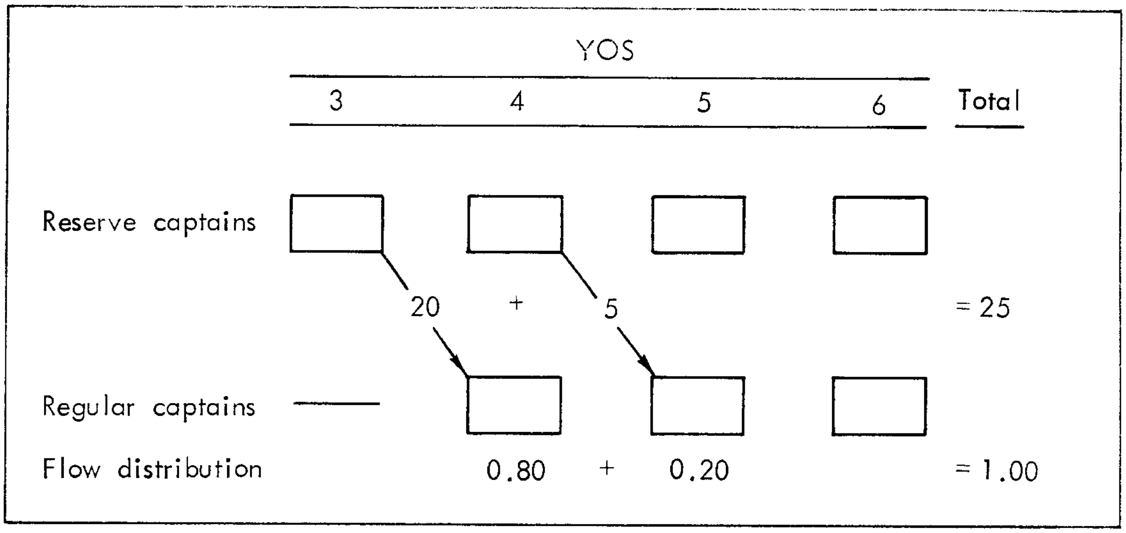


Fig. 4--Augmentations resulting from a flow distribution of 0.8 and 0.2 in service years 4 and 5

year 3 to year 4 and the other 5 augmentations (20 percent of 25) going from year 4 to year 5. The sum of each set of flow distributions must always be 1.00, since a flow distribution actually indicates what percentage of the flow takes place in a given YOS.

It may seem that coming up with some of these inputs can be rather difficult, i.e., that the user may have to settle for a poor guess at what flow ratios and flow distributions would give a desirable force structure. As was mentioned earlier, because of the interactive capabilities of these models, either of the officer force progression models can be used in coming up with initial inputs to the grade limitations model.

A SIMPLE EXAMPLE

Before showing the mathematics of the computation method in detail, we present a simplified example of how officer flows are computed in order to give the reader an idea of the basic concepts employed. We show a force which has only two grades (lieutenants and captains) and six years of service. The only types of flows are accessions, promotions, and losses. For simplicity, losses occur only in the sixth year of service for each grade.

Since accessions all enter in the first year of service and losses all leave in the last, the only flow distribution we need is one for promotions from lieutenant to captain. No flow ratios are needed since all flows into captain are promotions and all flows into lieutenant are accessions. The only other input needed is grade limitations. For the example, the inputs are as follows:

- Grade limitations
  - 3995 lieutenants
  - 805 captains
- Promotion distribution to captain (percent)
  - YOS 2 - 1.8
  - YOS 3 - 7.3
  - YOS 4 - 72.7
  - YOS 5 - 18.2

Figure 5 illustrates the situation before we have determined the number of officers in any state. It is known what percentage of the total promotions to captain (p) takes place in each year of service. As a result, the model knows how many officers are in each state of the captain grade with respect to p. The only problem, then, is to solve for p. Note that the number of officers in each state is the sum of the flows in, and that the sum of the steady states is the grade limitations for captains (805). It is quite easy to arrive at the conclusion that p = 275.

Figure 6 shows the next two steps of the computation of the officer force structure. The top of that figure shows how the force looks now that the captains have been structured. To complete the force, we need only compute the grade of lieutenants, and with captains distributed, we have all the necessary information for doing so. We know that all of the accessions (a) enter in year one and we know how many officers leave each state of the lieutenant grade as promotions to captain. With this information we can compute the number of officers in each lieutenant state with respect to a; i.e., we know, at each point, how many of the original, a, accessions are left

INPUTS:

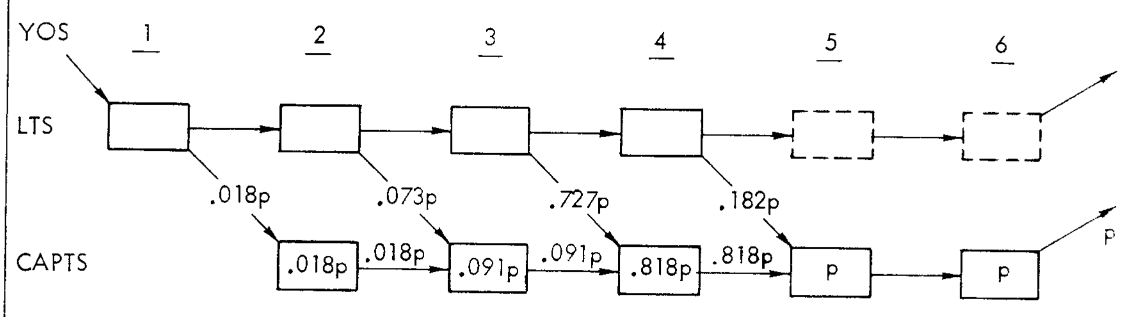
Grade limitations

Lieutenants	3995
Captains	805

Promotion distribution (percent)

YOS 2	1.8
YOS 3	7.3
YOS 4	72.7
YOS 5	18.2

COMPUTATIONS

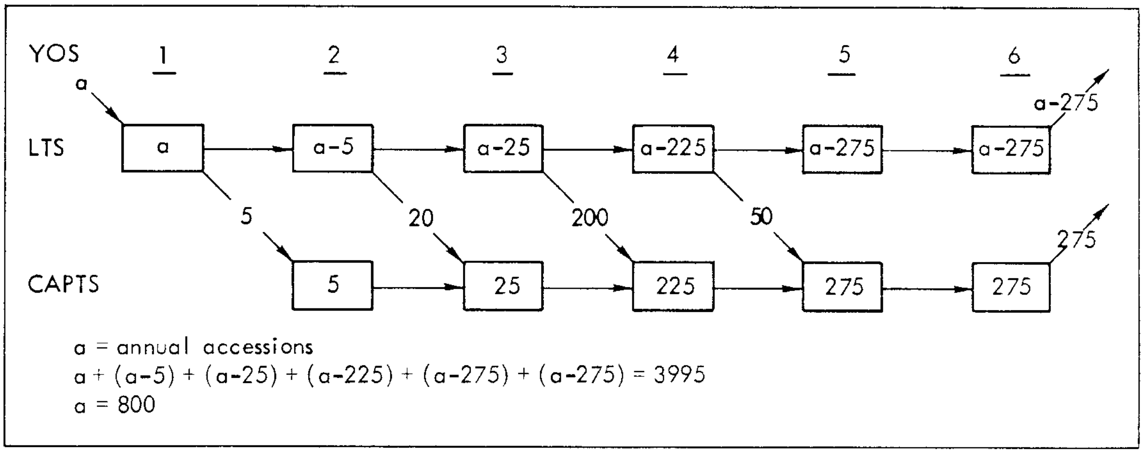


p = Total promotions from lieutenant to captain

$.018p + .091p + .818p + p + p = 2.927p = 805$

$p = 275$

Fig. 5--Grade limitation inputs and computations



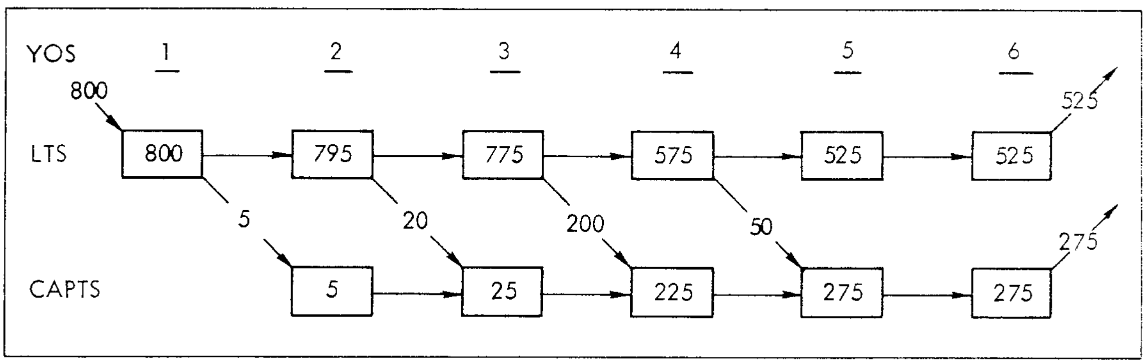


Fig. 6--Grade limitation computations: final phase

as lieutenants after promotions are taken out. Now, since the sum of all the lieutenant states is the grade limitation for lieutenants (3995), we can easily determine that a = 800. By substituting 800 for a in the officer structure we arrive at the final structure shown at the bottom of Fig. 6.

DERIVATION OF FLOWS IN

Since the grade limitations model always computes flows into, and never computes flows out of, any category, the officer force structure must be builtup or computed in the opposite direction from the direction of flows. This means that all nonlieutenant flows must be computed before the lieutenant flows or the model will have no way of knowing how many officers to promote to captain. Regular officers must be distributed before reserves so that the model will know how many reserve officers to augment to regular, and rated lieutenants must be done before nonrated so that the model will know how many rating transfers to give nonrated lieutenants. Before the computations for a given category are performed, all the flows out of that category are already known.

When computing the flows into a category, the grade limitations model deals with six separate cases which are distinguished by differences in flows-in:

1. For non-Academy* Regular officers with grade above lieutenant the flows in are promotions, promotion/augmentations, or augmentations.
2. For Reserve or Academy officers with grade above lieutenant the only flow in is through promotions.
3. For non-Academy Regular rated lieutenants flows in may be rating transfers, rating transfer/augmentations, or augmentations.
4. For Reserve and Academy rated lieutenants the only flows in are rating transfers.
5. For non-Academy Regular nonrated lieutenants the only flows in are augmentations.

*Academy officers are treated differently from other regular officers because all Academy officers begin with regular commissions, so no augmentations take place within the Academy source of commission.

6. For Reserve and Academy nonrated lieutenants the only flows-in are accessions.

The mathematics for computing the flows-in for these six cases is very similar. In fact, cases 1 and 3, which both have three types of flows-in, are identical if promotions are simply called rating transfers in case 3; and the remaining 4 cases share the same mathematics.

In the following equations let:

TPR = total promotions into the category

TPA = total promotion/augmentations into the category

TAU = total augmentations into the category

$FDPR_Y$ = flow distribution for promotions into service year Y

$FDPA_Y$ = flow distributions for promotion/augmentation into service year Y

$FDAU_Y$ = flow distribution for augmentations into service year Y

S_Y = steady-state number of officers in service year Y

PR_Y* = promotions out of service year Y

B_Y = loss rate for service year Y

$1 - B_Y$ = retention rate for service year Y. This is the complement of the loss rate.

GRAU = grade authorization for the category

RPA = flow ratio of promotion/augmentation to promotions for the category

RAU = flow ratio of augmentations to promotions for the category.

Cases 1 and 3. In case 1 we are solving for the three numbers TPR, TPA, and TAU. This is done with three simultaneous equations. Table 1 shows what the knowns and unknowns are at the beginning of the computation step for a category in case 1. Note that promotions are

*Note that the flow distribution arrays refer to flows into the node while the actual flow arrays refer to flows out of the node.

Table 1

TERMS IN EQUATIONS

Knowns: Inputs	Knowns: Previously Computed	Unknowns
FDPR, FDPA, FDAU } Flow distributions	PR - promotions out of each state	TPR - total promotions into the category
B - Loss rate		TPA - total promotion/augmentations into the category
GRAU - Grade authorization		TAU - total augmentations into the category
RPA, RAU } Flow ratios		S_Y - number of officers in the given category with Y years of service, i.e., number in state of that category

the only flows out and they were previously computed. The number of officers flowing out of a category is known before computation for that category begins. We begin with the number in steady state for the lowest service year in the category. Note that this is not necessarily the first year of service. In this service year the number in any given state is simply the number of officers that flow into that state, which is just the total number of flows into the category multiplied by the flow distribution factor for the service year in question.

For Y = first service year for a given category:

$$S_Y = FDPR_Y \cdot TPR + FDPA_Y \cdot TPA + FDAU_Y \cdot TAU \qquad (1)$$

Then, for the second service year of the category we have:

$$S_{Y+1} = S_Y \cdot [1 - B_Y] - PR_Y + FDPR_{Y+1} \cdot TPR + FDPA_{Y+1} \cdot TPA + FDAU_{Y+1} \cdot TAU \qquad (2)$$

which is simply all officers not lost to the Air Force from service year Y ($S_Y \cdot [1 - B_Y]$), minus those promoted out of service year Y (PR_Y), plus the three flows into service year Y + 1. Remember that all flows out of the category were previously computed or were equal to zero (for the colonel category) so that PR_Y is a known quantity. Equation (2) is valid for all values of Y for which we have some officers in the category, not just for the second year of the category. Now, if we substitute the right-hand side of Eq. (1) for S_Y in Eq. (2), the result is that every term in the right-hand side of Eq. (2) will be known except TPR, TPA, and TAU. By continuing to substitute the equation for S_Y into the equation for S_{Y+1} for all Y, and collecting terms as we go, we get an equation of the form:

$$S_Y = TERM1_Y \cdot TPR + TERM2_Y \cdot TPA + TERM3_Y \cdot TAU + GG_Y \qquad (3)$$

for each year Y in which the category exists, where $TERM1_Y$, $TERM2_Y$ and $TERM3_Y$ are recursive functions of the flow distributions $FDPR_Y$, $FDPA_Y$ and $FDAU_Y$, and GG_Y is a function of the flows out of the state.

The grade authorization, GRAU, is the total number of officers in the category, so GRAU is the sum of the steady-state numbers.

$$GRAU = \sum_Y S_Y$$

$$= \sum_Y (TERM1_Y \cdot TPR + TERM2_Y \cdot TPA + TERM3_Y \cdot TAU + GG_Y) \qquad (4)$$

Collecting terms:

$$GRAU = TPR \cdot \sum_{Y} TERM1_{Y} + TPA \cdot \sum_{Y} TERM2_{Y}$$
$$+ TAU \cdot \sum_{Y} TERM3_{Y} + \sum_{Y} GG_{Y} \quad (5)$$

where GRAU, $\sum_{Y} TERM1_{Y}$, $\sum_{Y} TERM2_{Y}$, $\sum_{Y} TERM3_{Y}$ and $\sum_{Y} GG_{Y}$ are all known terms. This is the first of the three simultaneous equations. The other two equations are taken directly from the definition of the flow ratios. They are:

$$RPA = TPA/TPR \quad (6)$$

and

$$RAU = TAU/TPR \quad (7)$$

We now solve Eqs. (5), (6), and (7) for TPR, TPA, and TAU, and we have the flows-in for case 1. Case 3 is exactly the same except that we are solving for rating transfers instead of promotions and for rating transfer/augmentations instead of promotion/augmentations. As for the mathematics of the problem, this is simply a change in terminology.

Appendix A presents a more detailed explanation of the mathematics shown here. It shows what the $TERM1_{Y}$'s, $TERM2_{Y}$'s, $TERM3_{Y}$'s and GG_{Y}'s are and gives the intermediate steps in the derivation of Eq. (5), which were not shown here.

The Remaining Four Cases. The method used to solve cases 1 and 3 is used for cases 2, 4, 5, and 6. Since these cases each have only one flow-in, we do not need three simultaneous equations. Instead, one equation is derived and solved directly for the desired flow total. In this equation there are more flows-out to be considered than in Eq. (5), but these are known already and so amount to nothing more than constants. The derivation for these equations is given in Appendix A.

COMPLETING THE OFFICER STRUCTURE

Now that the total flows into the category have been determined, it is a very easy matter to complete the description of the officer force by computing all the flows-in (and out) per service year, and the steady-state numbers. The flows-out were, of course, computed previously, and

in fact, were already used to compute total flows-in. All the flows coming in from another category are computed by multiplying the total flows of that type coming into the category by the correct flow distribution for the year and type of flow in question.

All that is left to compute is the steady-state numbers, the losses, and the lateral flows. To compute the steady-state number for service year Y, we need to know how many officers there were in year Y - 1. This is no problem since the number in the lowest numbered year is simply the sum of the flows-in, which are already known. We then use Eq. (2) (or one similar to it, depending on the case) to solve for the rest of the steady-state numbers. That is, take the number retained in the Air Force from the next lower year of service, subtract those who went to a new category, and add the flows into the present year. With the steady-state numbers known, the losses are simply the steady state multiplied by the loss rate, i.e.:

$$L_Y = S_Y \cdot B_Y \tag{8}$$

The lateral flows are:

For reserve:

$$\begin{aligned} LAT_Y = S_Y &\cdot [1 - B_Y] \\ &- PR_Y - AU_Y \\ &- PA_Y \end{aligned} \tag{9}$$

For regular:

$$\begin{aligned} LAT_Y = S_Y &\cdot [1 - B_Y] \\ &- PR_Y \end{aligned} \tag{10}$$

These equations are simply the number retained ($S_Y \cdot [1 - B_Y]$) minus the flows-out. Note that for nonrated lieutenants we would also subtract out rating transfers and rating transfer/augmentations.

Now that the entire officer structure and the movements within it have been pieced together, the model prints out detailed and summary

reports showing what the force looks like and what flows take place from category to category. In addition, using its knowledge of the officer force structure and flows, the model computes and reports the resulting personnel policies, such as promotion opportunity, augmentation rates, accessions, and rating transfer rates. The planner now knows in the long range and under steady-state conditions, what sort of personnel policies and accessions will result in a force with the desired grade authorizations.

INFEASIBILITIES

The grade limitations model is designed so that for every possible set of inputs a unique solution is computed by the model. That is, no two sets of inputs will generate the same officer force structure. Because of this it is possible to come up with infeasible solutions once in a while. An infeasible solution is one in which one or more states of the officer force structure has a negative number of officers in it. This occurs, for example, when the total number of officers flowing out of a state is computed to be more than the number of officers in that state or when the number of vacancies to be filled in a category exceeds the number of officers available to fill them. Since each input deck has its own unique solution, the model does not do anything to watch out for or prevent infeasibilities, but instead just prints the officer force derived from the given inputs. The user may then adjust the inputs in a manner which removes the infeasibility but minimally disturbs the inputs originally desired. This may be, depending upon the magnitude and location of the infeasibility, impossible, which means that the desired inputs are simply unworkable or mutually inconsistent and that the user should try a different approach. It should be noted that infeasibilities are not common. No realistic or reasonable set of inputs will result in an infeasibility.

In general, we have used three basic methods of removing infeasibilities. There are, of course, other approaches as well as combinations and variations of these three. Which method to use may depend upon the type of infeasibility and its magnitude and location. However, since any of the methods will usually work, the main consideration

should be the choice of the input that can most readily be altered without changing the user's objective.

The easiest of the three methods has to do with the flow ratios, and so does not apply when only one type of flow (in addition to loss flows and laterals) is taking place in the vicinity of the infeasibility. We will explain this method by describing a simple example. The top of Fig. 7 is an officer flow table for reserve lieutenants with rating pilot and source of commission OTS. Under "current officer state" we see that there are 322 officers in YOS 4. If we continue reading across the table for year 4, we see that 5 officers are lost to the force ("attrition"), 4 are added to the regular force, 112 are added to the regular force and given promotions, and 206 are promoted. This adds up to more than 322 and leaves an infeasibility of -3 as the current officer state in year 5.*

The bottom of the figure shows the officer flow table for regular captains who are OTS pilots. On the left side of the table (under "Flows Into the Current Officer State") we see the 112 officers who were added to the regular force with promotions from the reserve lieutenants at the end of year 4 entering the captain category at the beginning of year 5. We also see 67 promotions coming in in the same year. The ratio of promotion/augmentations to promotions in this example was 1.7.

One way to remove the infeasibility in reserve lieutenants is to have fewer promotion/augmentations to regular captains. This drop in flows to regular captains is made up for by increasing the number of promotions from regular lieutenants. The only change in the inputs is a decrease in the ratio of promotion/augmentations to promotions. In the example shown we decreased the ratio from 1.7 to 1.2. The top of Fig. 8 shows how this lowered the total number of promotion/augmentations out of reserve lieutenants from 114 to 97. In year 4 the number dropped from 112 to 96 and cleared up the infeasibility. The bottom of the figure shows the 96 promotion/augmentations coming

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*Note that the numbers on the table do not add up correctly. This is due entirely to round-off error. Typically, 1 or 2 officers are lost or gained due to rounding-off.

OFFICER FLOWS

COMPONENT RES GRADE LT RATING PIL SOURCE OF COMMISSION OTS

YOS	FLOWS INTO THE CURRENT OFFICER STATE: LATERAL FLOW	RATING TRANSFERS: ONLY	RATING TRANSFERS: WITH AUG	AUGMENTATIONS: ONLY	AUGMENTATIONS: WITH PRO	PROMOS ONLY	CURRENT OFFICER STATE	FLOWS OUT OF THE CURRENT OFFICER STATE: ATTRITION	RATING TRANSFERS: ONLY	RATING TRANSFERS: WITH AUG	AUGMENTATIONS: ONLY	AUGMENTATIONS: WITH PRO	PROMOS ONLY	LATERAL FLOW
2		430					430	6						423
3	423						423	6			88	2	4	322
4	322						322	5			4	112	206	-3
5	-3						-3	-3						0
6	0						0	0						
TOT	741.	430.	0.	0.	0.	0.	1171.	14.	0.	0.	92.	114.	210.	741.

TOTAL RATING TRANSFERS 430 IN 0 OUT TOTAL AUGMENTATIONS 0 IN 206 OUT TOTAL PROMOTIONS 0 IN 324 OUT

COMPONENT REG GRADE CAP RATING PIL SOURCE OF COMMISSION OTS

YOS	FLOWS INTO THE CURRENT OFFICER STATE: LATERAL FLOW	RATING TRANSFERS: ONLY	RATING TRANSFERS: WITH AUG	AUGMENTATIONS: ONLY	AUGMENTATIONS: WITH PRO	PROMOS ONLY	CURRENT OFFICER STATE	FLOWS OUT OF THE CURRENT OFFICER STATE: ATTRITION	RATING TRANSFERS: ONLY	RATING TRANSFERS: WITH AUG	AUGMENTATIONS: ONLY	AUGMENTATIONS: WITH PRO	PROMOS ONLY	LATERAL FLOW
4					2		2	0						
5	2			1	112	67	182	3						180
6	180			19			198	3						195
7	195						195	4					2	189
8	189						189	4					7	178
9	178						178	4					138	36
10	36						36	1						36
11	36						36	36						
TOT	816.	0.	0.	19.	114.	67.	1017.	53.	0.	0.	0.	0.	148.	816.

TOTAL RATING TRANSFERS 0 IN 0 OUT TOTAL AUGMENTATIONS 134 IN 0 OUT TOTAL PROMOTIONS 182 IN 148 OUT

Fig. 7—Officer grade limitations model: Case 1, infeasibility

OFFICER FLOWS

COMPONENT RES GRADE LT RATING PIL SOURCE OF COMMISSION OTS

YOS	FLOWS INTO THE CURRENT OFFICER STATE: LATERAL FLOW	RATING TRANSFERS: ONLY	RATING TRANSFERS: WITH AUG	AUGMENTATIONS: ONLY	AUGMENTATIONS: WITH PRO	PROMOS ONLY	CURRENT OFFICER STATE	FLOWS OUT OF THE CURRENT OFFICER STATE: ATTRITION	RATING TRANSFERS: ONLY	RATING TRANSFERS: WITH AUG	AUGMENTATIONS: ONLY	AUGMENTATIONS: WITH PRO	PROMOS ONLY	LATERAL FLOW
2		429					429	6						423
3	423						423	6			95	2	4	316
4	316						316	5			4	96	208	3
5	3						3	3						0
6	0						0	0						
TOT	742.1	429.	0.	0.	0.	0.	1171.	21.	0.	0.	99.	97.	212.	742.

TOTAL RATING TRANSFERS 428 IN 0 OUT TOTAL AUGMENTATIONS 0 IN 196 OUT TOTAL PROMOTIONS 0 IN 309 OUT

COMPONENT REG GRADE CAP RATING PIL SOURCE OF COMMISSION OTS

YOS	FLOWS INTO THE CURRENT OFFICER STATE: LATERAL FLOW	RATING TRANSFERS: ONLY	RATING TRANSFERS: WITH AUG	AUGMENTATIONS: ONLY	AUGMENTATIONS: WITH PRO	PROMOS ONLY	CURRENT OFFICER STATE	FLOWS OUT OF THE CURRENT OFFICER STATE: ATTRITION	RATING TRANSFERS: ONLY	RATING TRANSFERS: WITH AUG	AUGMENTATIONS: ONLY	AUGMENTATIONS: WITH PRO	PROMOS ONLY	LATERAL FLOW
4					2		2	0						2
5	2			1	96	81	179	3						176
6	176			22			199	3						196
7	196						196	4					2	190
8	190						190	4					7	179
9	179						179	4					138	37
10	37						37	1						36
11	36						36	36						
TOT	815.	0.	0.	23.	97.	81.	1017.	54.	0.	0.	0.	0.	148.	815

TOTAL RATING TRANSFERS 0 IN 0 OUT TOTAL AUGMENTATIONS 121 IN 0 OUT TOTAL PROMOTIONS 178 IN 148 OUT

Fig. 8—Officer grade limitations model: Case 1, infeasibility removed

into regular captains in year 5. We also see in this part of the figure that the promotions increased from 67 to 81 to balance out the loss in promotion/augmentations.

The second way of handling an infeasibility is to adjust the flow distributions at the point where the infeasibility takes place. Figure 9 illustrates a case for which this method may be used. The top half of the figure shows a flow table for reserve navigator lieutenants whose source of commission is ROTC in which an infeasibility of -26 appears in year of service 5. The bottom half of the figure shows the same table with the infeasibility removed. By looking at the flows-out of the top table in the figure, we see that all flows-out, from this category to another, flow out of years 3 and 4, i.e., into years 4 and 5 of the next category. There are just a few augmentation/promotions and promotions taking place out of year 3, while 50 percent of those officers receiving only augmentations come from year 3. This leaves 531 officers left over to remain in the category in year 4. From this 531, the model loses 8, awards regular commissions to 98, awards regular commissions to, <u>and</u> promotes, 100, and promotes 353. But this is more than 531 officers leaving the cell, so an infeasibility is created.

There is an equation from queueing theory, which always holds in the grade limitations model and which says:

mean number of officers in a category

= (arrival rate) × (mean time in the category).*

The category referred to is the one into which the officers are flowing, i.e., regular lieutenant navigators, so if we change the flow distributions so that augmentation flows are taking place in lower years of service, we <u>increase</u> the mean time that an officer spends in the new category (since we are not in any way affecting loss rates). Since the number of officers in the regular lieutenant navigator category is

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*An outline of the proof may be found in Ref. 5, Appendix B, p. 136; the complete proof is given in Ref. 3.

OFFICER FLOWS

COMPONENT RES GRADE LT RATING NAV SOURCE OF COMMISSION ROTC

YOS	IN: LATERAL FLOW	IN: RATING TRANSFERS ONLY	IN: RATING TRANSFERS WITH AUG	IN: AUGMENTATIONS ONLY	IN: AUGMENTATIONS WITH PRO	IN: PROMOS ONLY	CURRENT OFFICER STATE	OUT: ATTRITION	OUT: RATING TRANSFERS ONLY	OUT: RATING TRANSFERS WITH AUG	OUT: AUGMENTATIONS ONLY	OUT: AUGMENTATIONS WITH PRO	OUT: PROMOS ONLY	OUT: LATERAL FLOW
2		657					657	10						647
3	647						647	10			98	2	6	531
4	531						531	8			98	100	353	-26
5	-26						-26	-26						0
6	0						0	0						
TOT	1150.	657	0.	0.	0.	0.	1807.	0.	0.	0.	195.	102.	359.	1150.

TOTAL RATING TRANSFERS 657 IN 0 OUT TOTAL AUGMENTATIONS 0 IN 297 OUT TOTAL PROMOTIONS 0 IN 461 OUT

COMPONENT RES GRADE LT RATING NAV SOURCE OF COMMISSION ROTC

YOS	IN: LATERAL FLOW	IN: RATING TRANSFERS ONLY	IN: RATING TRANSFERS WITH AUG	IN: AUGMENTATIONS ONLY	IN: AUGMENTATIONS WITH PRO	IN: PROMOS ONLY	CURRENT OFFICER STATE	OUT: ATTRITION	OUT: RATING TRANSFERS ONLY	OUT: RATING TRANSFERS WITH AUG	OUT: AUGMENTATIONS ONLY	OUT: AUGMENTATIONS WITH PRO	OUT: PROMOS ONLY	OUT: LATERAL FLOW
2		657					657	10						647
3	647						647	10			144	2	6	484
4	484						484	7			5	100	353	19
5	19						19	19						0
6	0						0	0						
TOT	1150.	657.	0.	0.	0.	0.	1807.	46.	0.	0.	150.	102.	359.	1150.

TOTAL RATING TRANSFERS 657 IN 0 OUT TOTAL AUGMENTATIONS 0 IN 252 OUT TOTAL PROMOTIONS 0 IN 461 OUT

Fig. 9—Officer grade limitations model: Case 2, infeasibility (above); infeasibility removed (below)

held constant, as a grade authorization, the equation tells us that the arrival rate (or total number of officers flowing into the category) must decrease. Therefore, the augmentation flows out of the infeasible category must also decrease. This is the basic idea behind the method of removing infeasibilities by altering flow distributions.

In the case shown, the infeasibility was removed by changing the flow distributions for augmentations out of the category. In the top table, with 98 augmentations taking place in both year 3 and year 4, the flow distribution was 0.5 into year 4 and 0.5 into year 5. By changing this to 0.96 into year 4 and 0.04 into year 5, we decreased the total number of augmentations from 195 to 150. The difference is 45 officers which is precisely the difference between -26 (the number of officers in year 5 with the infeasibility) and 19 (the number of officers in year 5 in the bottom table).

Note that the infeasibility might also have been removed by adjusting either the promotion/augmentation distribution or the promotion distribution. In fact, if the user wishes, he can make small adjustments to two or even all three of the possible flows rather than making a larger adjustment to just one flow distribution. There may also be cases where it would be desirable to have more officers flowing <u>into</u> the infeasible category, rather than fewer flowing <u>out</u> of it. In these cases the user can adjust flows-in to take place later rather than adjusting flows-out to take place earlier, as we did.

The third, and last, type of infeasibility removal that we will discuss is that of changing the grade authorizations. Figure 10 shows an officer inventory table for nonrated OTS officers with an infeasibility of -19 in regular lieutenants. Figure 11 is the same table with the infeasibility removed. To remove infeasibilities using authorizations we either put more officers into the infeasible category or decrease the number of officers in those categories whose flows-in come from the infeasible category or both. Usually we do both. We use the following algorithm to approximate the number of officers which should be moved: for each category which has an infeasibility take the size of the largest infeasibility (for example, in Fig. 10, for regular lieutenants it would be 19) and multiply this by the number of years

OFFICER FORCE GRADE DISTRIBUTION
RATING NR SOURCE OF COMMISSION OTS

YEAR	RESERVE COMPONENT LIEUT	CAPT	MAJOR	LTCOL	CL/GN	TOTAL	REGULAR COMPONENT LIEUT	CAPT	MAJOR	LTCOL	CL/GN	TOTAL	BOTH RESERVE AND REGULAR COMPONENTS LIEUT	CAPT	MAJOR	LTCOL	CL/GN	TOTAL
1	3848					3848							3848					3848
2	2793					2793							2793					2793
3	2487					2487	264					264	2751					2751
4	2082	16				2098	606	6				611	2688	22				2710
5	57	858				914	-19	652				632	37	1510				1546
6	1	721				722	0	759				758	0	1480				1480
7		699				699		736				736		1435				1435
8		109	1			110		706	8			714		814	9			824
9		93	4			97		660	34			694		753	37			790
10		17	66			84		149	533			683		167	600			767
11		16	1			17		146	582			728		162	582			745
12									570			570			570			570
13									559			559			559			559
14									518	30		547			518	30		547
15									482	54		536			482	54		536
16									161	364		526			161	364		526
17									158	357		515			158	357		515
18									155	350		505			155	350		505
19									152	343		495			152	343		495
20									149	296	40	485			149	296	40	485
21										267	46	313				267	46	313
22										148	158	306				148	158	306
23										145	155	300				145	155	300
24										135	144	279				135	144	279
25										111	133	243				111	133	243
26										89	113	201				89	113	201
27											103	103					103	103
28											93	93					93	93
29											84	84					84	84
30											76	76					76	76
TOTAL	11266.	2529.	72.	0.	0.	13867.	850.	3814.	4060.	2689.	1143.	12556.	12116.	6343.	4132.	2689.	1143.	26423.
AVERAGE YEAR OF SERVICE																		
	2.26	6.18	9.93	0.0	0.0	3.02	3.66	7.26	13.45	19.48	25.00	13.25	2.36	6.83	13.39	19.48	25.00	7.88

Fig. 10—Officer grade limitations model: Case 3, infeasibility

OFFICER FORCE GRADE DISTRIBUTION
RATING NR SOURCE OF COMMISSION OTS

YEAR	RESERVE COMPONENT						REGULAR COMPONENT						BOTH RESERVE AND REGULAR COMPONENTS					
	LIEUT	CAPT	MAJOR	LTCOL	CL/GN	TOTAL	LIEUT	CAPT	MAJOR	LTCOL	CL/GN	TOTAL	LIEUT	CAPT	MAJOR	LTCOL	CL/GN	TOTAL
1	3855					3855							3855					3855
2	2800					2800							2800					2800
3	2486					2486	272					272	2758					2758
4	2071	16				2088	624	5				629	2695	22				2717
5	53	857				910	4	646				650	57	1502				1559
6	1	721				722	0	751				751	1	1472				1473
7		700				700		728				728		1428				1428
8		109	1			110		699	8			707		807	9			816
9		93	4			97		853	34			687		746	37			783
10		18	66			84		143	533			676		160	600			760
11		16	1			17		140	582			721		155	582			738
12									570			570			570			570
13									559			559			559			559
14									518	30		547			518	30		547
15									482	54		536			482	54		536
16									161	364		526			161	364		526
17									158	357		515			158	357		515
18									155	350		505			155	350		505
19									152	343		495			152	343		495
20									149	296	40	485			149	296	40	485
21										267	46	313				267	46	313
22										148	158	306				148	158	306
23										145	155	300				145	155	300
24										135	144	279				135	144	279
25										111	133	243				111	133	243
26										89	113	201				89	113	201
27											103	103					103	103
28											93	93					93	93
29											84	84					84	84
30											76	76					76	76
TOTAL	11266.	2529.	72.	0.	0.	13867.	900.	3764.	4060.	2689.	1143.	12556.	12166.	6293.	4132.	2689.	1143.	26423.
AVERAGE YEAR OF SERVICE																		
	2.26	6.18	9.93	0.0	0.0	3.02	3.70	7.25	13.45	19.48	25.00	13.23	2.37	6.82	13.39	19.48	25.00	7.87

Fig. 11—Officer grade limitations model: Case 3, infeasibility removed

in which that category exists up to and including the year with that infeasibility. For regular lieutenants, in our example, take 19 and multiply by 3 for the two years before the infeasibility plus the year of the infeasibility. This gives 57, which we consider to be approximately 50. The number calculated is simply a rough estimate of the change which should be made. Often a few more officers will have to be moved to remove the infeasibility, and sometimes (as in our example) a few less are sufficient. The algorithm given will keep the planner from having to make blind guesses by giving a "ballpark" estimate.

Since regular lieutenants flow into regular captains, we can solve the infeasibility either by increasing the number of lieutenants or decreasing the number of captains. In the example we did both by moving 50 officers from regular captains to regular lieutenants. As can be seen in Fig. 11, this increased the number of lieutenants by enough to remove the infeasibility.

After having experimented with the grade limitations for a while, the user should become more and more familiar with these and possibly other methods of removing infeasibilities. He will develop a "feeling" for this so that he will know almost intuitively which method will work best for a given problem and how large an adjustment he should make.

IV. THE GOODNESS MEASURE PACKAGE

Once the grade limitations model has completed the officer structure and shown all the flows taking place within it, the user can make judgments about the desirability of having such a force structure. The number of officers in each grade is, of course, that which was specified in the inputs, but it is also important that the number of accessions calculated be a reasonable number and that the age of the force and the distribution of the officers be acceptable. However, in addition to requirements about the force size and shape, there are other attributes which need to be considered when making judgments about the desirability of "goodness" of a particular force structure. One example of these is promotion opportunity. It may not be desirable, however nice the shape of the force, to have very low promotion opportunities or unequal promotion opportunities for different categories within the same grade. Promotion opportunity may, in this context, be called a "goodness measure", that is, a numerical parameter of the officer force which aids the planner in making qualitative judgments about the force structure computed.

The "Goodness Measure Package" is a set of routines added to the grade limitations model which computes and prints certain goodness measures. Basically, there are two types of computations done by these routines, but they are done for various categories of officers and for each type of flow out of the category in question.

The first of these computations is the probability of leaving the category via a given type of exit. For example, an officer can leave the category of regular captains only by being promoted to major or by leaving the Air Force entirely. The goodness measure package calculates the probability that he will leave that category by being promoted and the probability that he will leave it by being lost to the Air Force. All of these probabilities are conditional on having made it to the category in question. In our example, the probability of being promoted is described as follows: given that an officer has just become a regular captain, what is the probability that he will become a regular

major?* The probability of loss answers the question: given that an officer has just become a regular captain, what is the probability that he will be lost to the Air Force while still a regular captain? Note that since there are no other ways of changing one's status from regular captain, the sum of these two probabilities is one. For each category, the sum of all probabilities of leaving is, of course one.

After computing the probabilities of exit, the program calculates the mean times to each of these exits. These parameters indicate the mean time from entering the category to leaving it by the specified type of exit; that is, the mean time spent in the category by officers who leave in the prescribed manner. Going back to the example of regular captains, the program determines, for those officers who are promoted to regular major, the mean time between their entry into regular captain and their promotion to major. The same type of thing is computed for the officers who are lost as regular captains.

COMPUTING THE GOODNESS MEASURE

Figure 12 shows a typical category for which goodness measures can be computed. Each arrow in the figure is a flow. Note that any given state may have more than one flow leading out of it, for example, a

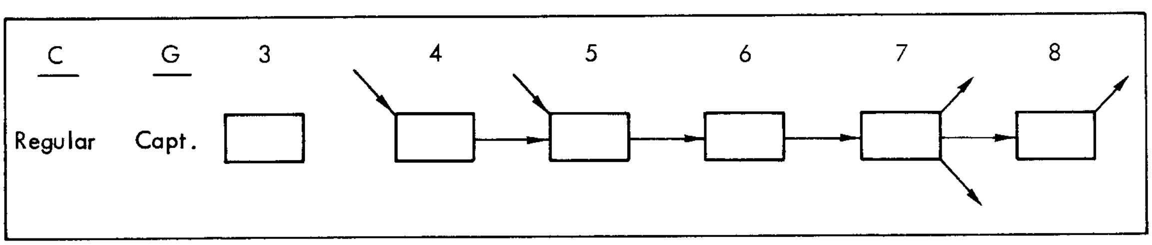

Fig. 12--Example of flows computed for the goodness measures

*This differs from promotion opportunity in that promotion opportunity is the probability that an officer will be promoted given that he has made it to the phase point year.

promotion flow, a loss flow, and a lateral flow (YOS 7 in Fig. 12). Since the officer force structure has already been completed, the goodness measure program knows how many officers there are in each state and how many take each flow from the state. By dividing the number of officers taking a given flow by the total number in the state the program determines the probability of leaving the state via the flow in question. This is known as the "flow probability." Since there is no way to leave a state without taking one of the flows leading out of it, the sum of the flow probabilities of any given state is always one. Figure 13 is a numerical example based on Fig. 12. The flow probabilities are shown in parenthesis for each flow in the figure. Note that it does not matter how officers enter the category; i.e., via promotion or augmentation, etc. The goodness measure package needs to know how many officers enter each state of the category, but where they come from is unimportant.

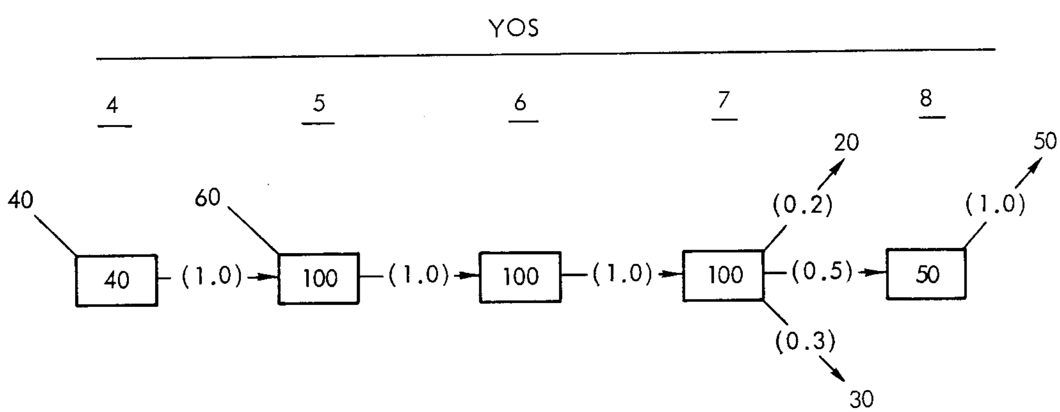

Fig. 13--Numerical example of flows computed for goodness measures

A "path" is a series of flows leading through a category and out of it. Each officer in a category follows a path through that category which begins when he first enters the category and ends when he leaves the category. In Figs. 12 and 13, the flows shown form six paths for

officers who become regular captains in years 4 or 5:

- o enter year 4, promoted year 7
- o enter year 4, lost year 7
- o enter year 4, lost year 8
- o enter year 5, promoted year 7
- o enter year 5, lost year 7
- o enter year 5, lost year 8

Note that all paths are a series of lateral flows beginning at a point of entry to the category and ending with one flow which leads out of the category entirely. The probability of taking any path through the category, given that the officer enters the category at the point where that path begins, is known as the "path probability." It is equal to the product of the flow probabilities for all the flows forming the path.* Note that these probabilities are conditional on knowing where the officer entered the category. They are not "the probability of his taking any given path," but are "the probability of taking any path <u>that begins where he entered the category.</u>" Table 2 shows the path probabilities for each of the six paths of Fig. 13. Note that the path probabilities for a given entry point sum to 1.0.

Now, to get the first goodness measure, i.e., the probability of leaving the category via a given type of exit, we need first to know how many of the officers who entered a given state left by each path which begins at that state. This is equal to the number who entered the state times the path probabilities for each path beginning there. Table 2 shows the number of officers taking each of the six paths. Next take all of those paths which begin in the state <u>and</u> end with the type of exit in mind, (for example all those which end in a promotion) and add together the number of officers who took these paths. If we also add together the number of officers who left via promotion for each entry state, we get the total number who leave the category by

---

*This is derived from the fact that the probability of several independent events occurring together is equal to the product of the probabilities of each of the individual events occurring separately.

Table 2

PATH PROBABILITIES FOR THE
SIX PATHS OF FIG. 13

Entry Point	Flow Out	Path Probability	No. of Officers Taking Path	Time to Exit
YOS 4	Promotion YOS 7	0.3	12	4 Years
	Lost YOS 7	0.2	8	4 Years
	Lost YOS 8	0.5	20	5 Years
YOS 5	Promotion YOS 7	0.3	18	3 Years
	Lost YOS 7	0.2	12	3 Years
	Lost YOS 8	0.5	30	4 Years

promotion. For example, in Fig. 13, there are two paths which end in promotion: one from year 4 to year 7, and one from year 5 to year 7. From the table we see that the numbers of officers taking these paths are 12 and 18 respectively, so the total number of officers who leave the category via promotion is 30. Finally, we compute the probability of leaving the category via promotion by dividing the number of officers promoted out by the total number of officers who entered the category. In our example, 100 officers altogether entered the category (40 + 60), and 30 left via promotion. So the probability of leaving this category via promotion is 30/100 = 0.3.

Since the paths taken and the number of officers who take each of them are known, the number of years taken by each officer to get through the category is also known (see the last column of Table 2). To calculate the second goodness measure, i.e., the mean time to a given exit, add together the man-years taken by all the officers who exit in that manner and divide by the number of officers who took that exit, i.e., take a weighted average of the number of years. For example, if we

continue looking at officers promoted in Fig. 13, we see (from Table 2) that 12 officers took 4 years to be promoted and the other 18 took 3 years. The total man-years taken by the first 12 is 12 x 4 = 48, and the other 18 took 18 x 3 = 54. So the mean time to promotion (or average number of years an officer must wait before being promoted) is (48 + 54)/30 = 3.4.

The reader may have noticed that our method of calculating the probability of taking a certain type of exit amounts, in the end, to simply dividing the number of officers who take that exit by the number of officers who enter the category each year; and these numbers can be readily picked out from the force structure by looking at flows into and out of the category. The more complicated method of computing and adding subprobabilities and subtotals was chosen because we need to know all the possible paths through a category and the number of officers who take each one in order to calculate the mean times to each type of exit.

In the goodness measure package there is an option by which the user can specify a minimum number of years that must pass between two consecutive promotions awarded to the same person, i.e., the minimum time in grade required before being eligible for promotion to the next grade. This type of policy is often used in real life; although it affects only the goodness measures and does not change the basic officer force computations, the option is included because it helps make the goodness measures more realistic. If the minimum amount of time is specified to be more than one year, some of the officers (those who have not waited the minimum amount of time since their last promotion) in certain states are not eligible for promotion. This increases the amount of time they must wait before being promoted. The model is now distinguishing officers eligible for promotion and those not eligible into two groups, each with its own probabilities of leaving a state in a given manner. This, of course, affects the mean time to exit, which is a number that varies depending on the minimum number of years specified by the user.*

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*A more detailed account of the mathematics used in the goodness measure routines is provided in Appendix B. Appendix D on the model's output, shows examples of the reports generated by these routines.

In specifying the minimum number of years between consecutive promotions, the user should be aware of two things. First, since an officer cannot be promoted twice in one year, the specified waiting period between promotions must equal or exceed one year. In the model, all promotions which take place in a given year take place at the same time. That is, the model looks only at where an officer ends one year of service and where he begins the next. All changes in status take place at the end of each year of service, and the difference from one year to the next in an officer's rank cannot exceed one grade level. The other thing to watch out for is that the minimum waiting period not exceed the number of years between the first promotion into a category and the first promotion out of it. If it did, there would be no officers eligible to receive promotions out of the category in the first year in which the inputs permit it. If the minimum waiting period were such that not enough officers were eligible for promotions which actually took place in the officer force structure, then the goodness measure statistics computed would be inconsistent with the officer force structure.

V. CONCLUSIONS

As has been mentioned earlier, the grade limitations model can be used as an aid in finding solutions to problems not commonly addressed by typical steady-state models built in the past. This model was built with grade-size related problems in mind, and we have found that it can be used to study many such problems. For example, it is clear that changes in grade limitations will effect promotion opportunity; but what happens if grade limitations and augmentation parameters are both changed, or if phase points* and grade limitations are changed? What kind of tradeoffs can be made between different policy parameters? What are the long-term effects on the force of having fewer or more field grade officers? All of these questions can be studied with the grade limitations model and many of them already have been.†

Used in conjunction with the progression, constraints, and behavioral models, the grade limitations model's power can be increased considerably. The models were designed with interaction in mind and were made to share several attributes. These include similar input formats (as much as possible considering that the inputs to the various models are quite different), the same array structure within the models, and by far the most important feature, the same output package. The sharing of the output package gives the progression and constraints models the capability of computing, printing, and punching the inputs to the grade limitations model, which would then replicate the structures produced by the progression or constraints model. It has already been mentioned that the grade limitations model can produce corresponding inputs for the progression model. There are many applications of the models where it is necessary to iterate between the grade limitations model and one of the progression models. In these applications the models' abilities to punch each other's inputs become extremely useful.

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*The phase point, for a given grade, is that year of service during which most of the promotions to the grade are made.

†Details will be included in *A System of U.S. Air Force Officer Personnel Planning Models: An Overview* (to be published).

Another advantage of the officer force models over previously developed models is their ability to present the officer structure in terms of officer inventories and flows in great detail or in summary form. Thus the user of the model may obtain a broad view of the officer force structured by the model; or he may examine, in detail, the inventories and flows for a group of officers with the same component, grade, rating, and source of commission (and even year of service, if he wants such detail); or he may look at intermediate levels such as the pilot force or all academy graduates.

All of these features, as well as the others mentioned throughout this report, contribute to a tool for studying long-term Air Force personnel problems which allows the planner to take any of several views of the force in both highly detailed or highly aggregated forms. As a result, the planner can tailor the use of these models to specifically handle the type of problem he is interested in.

Authorized users may obtain the grade limitations model as well as the other officer force models upon written request to Rand. A magnetic tape should be included. A program distribution package will be returned that provides detailed instructions on how to install the models. Every attempt has been made to avoid the use of nonstandard FORTRAN or IBM 370-dependent features in the officer force models. Included in the program distribution package are suggestions for the removal of such features.

Appendix A
MATHEMATICAL BACKGROUND

NOTATION AND SYMBOLS USED

In the text of this report, array subscripting was limited to year of service for ease in reading. Since the purpose of this and the next appendix is to show complete mathematical detail, all five officer attribute subscripts will be used. They are symbolized as follows, with their possible values shown in parenthesis:

C = component subscript (1 = reserve, 2 = regular)
G = grade subscript (1 = Lieutenant, 2 = Captain, 3 = Major, 4 = Lt. Col., 5 = Cols. and higher)
R = rating subscript (1 = pilot, 2 = navigator, 3 = support)
SC = source of commission subscript (1 = AFA, 2 = ROTC, 3 = OTS/other)
Y = year of service (values are 1 to 35)

We will employ the following output arrays as the result of the computations. A set of five subscripts (C, G, R, SC, Y), refers to a state of the officer force.

S(C,G,R,SC,Y)	= Number of officers in state, i.e., steady state number.
L(C,G,R,SC,Y)	= Losses from state.
LAT(C,G,R,SC,Y)	= Lateral flows out of state (C,G,R,SC,Y) to (C,G,R,SC,Y+1).
PR(C,G,R,SC,Y)	= Number of promotions from (C,G,R,SC,Y) to (C,G+1,R,SC,Y+1).
PA(1,G,R,SC,Y)	= Number of promotion/augmentations from (1,G,R,SC,Y) to (2,G+1,R,SC,Y+1).
AU(1,G,R,SC,Y)	= Number of augmentations from (1,G,R,SC,Y) to (2,G,R,SC,Y+1).

Note that in the above arrays the direction of flow is from Y to Y + 1, i.e., the state referred to by the array subscripts is the state from which flows come. Because of programming considerations the direction of flow in the following arrays is the reverse. Flows are from Y - 1 to Y, i.e., the state referred to by the array subscripts is the state into which flows go.

RT(C,1,R,SC,Y) = Number of rating transfers from (C,1,R,SC,Y-1) to (C,1,R,SC,Y), R=1 or 2.

RA(2,1,R,SC,Y) = Number of rating transfer/augmentations from (1,1,3,SC,Y-1) to (2,1,R,SC,Y), R=1 or 2.

For the grade limitations input arrays, we use these symbols:

B(C,G,R,SC,Y) = Loss rate for state.

APR(C,G,R,SC,Y) = Promotion flow distribution for promotions into (C,G,R,SC,Y) from (C,G-1,R,SC,Y-1).

APA(2,G,R,SC,Y) = Promotion/augmentation flow distributions for flows into (2,G,R,SC,Y) from (1,G-1,R,SC,Y-1).

AAU(2,G,R,SC,Y) = Augmentation flow distributions for augmentations into (2,G,R,SC,Y) from (1,G,R,SC,Y-1).

GRAU(C,G,R,SC) = Number of officers authorized to be in category (C,G,R,SC).

$$UPRPA(G,R,SC) \begin{cases} = \sum_{y} PA(1,G-1,R,SC,Y)/\sum_{y} PR(2,G-1,R,SC,Y) \text{ if } G \neq 1 \\ = \sum_{y} RA(2,G,R,SC,Y)/\sum_{y} AU(1,G,R,SC,Y) \text{ if } G = 1 \text{ and} \\ \quad R = 1 \text{ or } 2 \end{cases}$$

= Flow ratio for combination flows

$$UPRAU(G,R,SC) \begin{cases} = \sum_{y} AU(1,G,R,SC,Y)/\sum_{y} PR(2,G-1,R,SC,Y) \text{ if } G \neq 1 \\ = \sum_{y} RT(2,G,R,SC,Y)/\sum_{y} AU(1,G,R,SC,Y) \text{ if } G = 1 \text{ and} \\ \quad R = 1 \text{ or } 2 \end{cases}$$

= Flow ratio for rating transfers and augmentations.

ART(C,1,R,SC,Y) = Flow distributions for rating transfers into (C,1,R,SC,Y), R = 1 or 2 from (C,1,3,SC,Y-1).

ARA(2,1,R,SC,Y) = Flow distributions for rating transfer/augmentations into (2,1,R,SC,Y), R = 1 or 2 from (1,1,3,SC,Y-1).

In addition, the following symbols are used in the equations included in this appendix:

TOTPR(C,G,R,SC) $= \sum_{y}$ PR(C,G-1,R,SC,Y)

= Total promotions into the category (C,G,R,SC).

TOTPA(2,G,R,SC) $= \sum_{y}$ PA(1,G-1,R,SC,Y)

= Total promotion/augmentations into category (2,G,R,SC).

TOTAU(2,G,R,SC) $= \sum_{y}$ AU(1,G,R,SC,Y)

= Total augmentations into category (2,G,R,SC).

TOTRT(C,1,R,SC) $= \sum_{y}$ RT(C,1,R,SC,Y)

= Total rating transfers into category (C,1,R,SC), R = 1 or 2.

TOTRA(2,1,R,SC) $= \sum_{y}$ RA(2,1,R,SC,Y)

= Total rating transfer/augmentations into category (2,1,R,SC), R = 1 or 2.

STEADY STATE AND FLOW QUANTITIES

Although the grade limitations model computes the total flows into a category before computing the steady-state numbers and flow quantities, we present the steady state and flow equations first, for purposes of explanation. Later, the equations for deriving total flows into a category are shown. For the moment, assume that TOTPR, TOTPA, TOTAU, TOTRT, and TOTRA are known. Then, for all grades except lieutenant (i.e., G $\neq$ 1) we have the following.

For C = 2 (the regular force),

$$
\begin{aligned}
S(2,G,R,SC,Y) = {} & [1-B(2,G,R,SC,Y-1)] \cdot S(2,G,R,SC,Y-1) \\
& - PR(2,G,R,SC,Y-1) \\
& + APR(2,G,R,SC,Y) \cdot TOTPR(2,G,R,SC) \\
& + APA(2,G,R,SC,Y) \cdot TOTPA(2,G,R,SC) \\
& + AAU(2,G,R,SC,Y) \cdot TOTAU(2,G,R,SC) \qquad (1)
\end{aligned}
$$

The recursive nature of this equation presents no problem since when Y = 1, S(2,G,R,SC,Y-1) and PR(2,G,R,SC,Y-1) are both zero. Note that since B(2,G,R,SC,Y) is the loss rate, its complement 1-B(2,G,R,SC,Y), is the retention rate for the state.

For C = 1 (the reserve force),

$$
\begin{aligned}
S(1,G,R,SC,Y) = {} & [1-B(1,G,R,SC,Y-1)] \cdot S(1,G,R,SC,Y-1) \\
& - PR(1,G,R,SC,Y-1) - AU(1,G,R,SC,Y-1) \\
& - PA(1,G,R,SC,Y-1) \\
& + APR(1,G,R,SC,Y) \cdot TOTPR(1,G,R,SC) \qquad (2)
\end{aligned}
$$

Both of the above equations amount to the number of officers not lost from the previous state, minus all flows leaving the category from the previous state, plus all flows entering the present state. Recall that once we are ready to calculate a given steady state number, all flows out of that state are already known, while flows in have yet to be calculated (the "backward" concept)--hence the different notation for flows in and flows out.

When we compute steady-state numbers for lieutenants (G = 1) we have to consider the rating transfer flows. For nonrated (R = 3) lieutenants we have the following.

When C = 2 (the regular force):

$$
\begin{aligned}
S(2,1,3,SC,Y) = {} & [1-B(2,1,3,SC,Y-1)] \cdot S(2,1,3,SC,Y-1) \\
& - RT(2,1,1,SC,Y) - RT(2,1,2,SC,Y) \\
& - PR(2,1,3,SC,Y-1) \\
& + AAU(2,1,3,SC,Y) \cdot TOTAU(2,1,3,SC) \qquad (3)
\end{aligned}
$$

and for C = 1 (reserve force):

$$\begin{aligned} S(1,1,3,SC,Y) = & [1-B(1,1,3,SC,Y-1)] \cdot S(1,1,3,SC,Y-1) \\ & - PR(1,1,3,SC,Y-1) - RT(1,1,1,SC,Y) \\ & - RT(1,1,2,SC,Y) - RA(2,1,1,SC,Y) \\ & - RA(2,1,2,SC,Y) - AU(1,1,3,SC,Y-1) \\ & - PA(1,1,3,SC,Y-1) \end{aligned} \tag{4}$$

Note that there are no flows into reserve nonrated lieutenants in the above equation. These officers enter the force only via annual accessions. The steady-state equations for rated (R = 1 or 2) lieutenants are:

for C = 2 (regular),

$$\begin{aligned} S(2,1,R,SC,Y) = & [1-B(2,1,R,SC,Y-1)] \cdot S(2,1,R,SC,Y-1) \\ & - PR(2,1,R,SC,Y-1) \\ & + AAU(2,1,R,SC,Y) \cdot TOTAU(2,1,R,SC) \\ & + ART(2,1,R,SC,Y) \cdot TOTRT(2,1,R,SC) \\ & + ARA(2,1,R,SC,Y) \cdot TOTRA(2,1,R,SC) \end{aligned} \tag{5}$$

and for C = 1 (reserve),

$$\begin{aligned} S(1,1,R,SC,Y) = & [1-B(1,1,R,SC,Y-1)] \cdot S(1,1,R,SC,Y-1) \\ & - PR(1,1,R,SC,Y-1) - PA(1,1,R,SC,Y-1) \\ & - AU(1,1,R,SC,Y-1) \\ & + ART(1,1,R,SC,Y) \cdot TOTRT(1,1,R,SC) \end{aligned} \tag{6}$$

Now that all the steady-state numbers have been computed, we can complete the flows for the officer structure. For all (C,G,R,SC,Y) the flows in are:

$$PR(C,G-1,R,SC,Y-1) = APR(C,G,R,SC,Y) \cdot TOTPR(C,G,R,SC) \tag{7}$$

$$PA(1,G-1,R,SC,Y-1) = APA(2,G,R,SC,Y) \cdot TOTPA(2,G,R,SC) \tag{8}$$

$$AU(2,G,R,SC,Y-1) = AAU(2,G,R,SC,Y) \cdot TOTAU(2,G,R,SC) \quad (9)$$

and, for those categories which apply (i.e., when G = 1 and R = 1 or 2):

$$RT(C,1,R,SC,Y) = ART(C,1,R,SC,Y) \cdot TOTRT(C,1,R,SC) \quad (10)$$

$$RA(2,1,R,SC,Y) = ARA(2,1,R,SC,Y) \cdot TOTRA(2,1,R,SC) \quad (11)$$

The losses, of course, are always:

$$L(C,G,R,SC,Y) = B(C,G,R,SC,Y) \cdot S(C,G,R,SC,Y) \quad (12)$$

All that is needed to complete all the flows is lateral movements. For C = 1 (reserve):

$$\begin{aligned} LAT(1,G,R,SC,Y) = {} & [1-B(1,G,R,SC,Y)] \cdot S(1,G,R,SC,Y) \\ & - PR(1,G,R,SC,Y) - AU(1,G,R,SC,Y) \\ & - PA(1,G,R,SC,Y) \end{aligned} \quad (13)$$

For C = 2 (regular):

$$\begin{aligned} LAT(2,G,R,SC,Y) = {} & [1-B(2,G,R,SC,Y)] \cdot S(2,G,R,SC,Y) \\ & - PR(2,G,R,SC,Y) \end{aligned} \quad (14)$$

Equations (13) and (14) are simply the officers retained minus the flows out of the category. Note that, for nonrated lieutenants, rating transfers and rating transfer/augmentations would also have to be subtracted.

Now, let us see how the total flows into the category were computed.

TOTAL FLOWS IN

Recall, that for these calculations we divided the officer force into six cases according to the flows into the categories. The six cases were:

1. Non-Academy regular officers with grade above lieutenant.
2. Reserve or Academy officers with grade higher than lieutenant.
3. Non-Academy regular rated lieutenants.
4. Reserve and Academy rated lieutenants.
5. Non-Academy regular nonrated lieutenants.
6. Reserve and Academy nonrated lieutenants.

As was explained in Section IV, the mathematics for cases 1 and 3 is identical, except that case 1 deals with promotions, whereas case 3 deals with rating transfers. The same statement may be made about cases 2 and 4.

Cases 1 and 3

These cases include all non-Academy regular officers except nonrated lieutenants. We begin with the equations for the steady-state number in the first year in which grade G exists; call it Y_1.

C = 1 for regular officers.

$$\begin{aligned} S(2,G,R,SC,Y_1) = {} & APR(2,G,R,SC,Y_1) \cdot TOTPR(2,G,R,SC) \\ & + APA(2,G,R,SC,Y_1) \cdot TOTPA(2,G,R,SC) \\ & + AAU(2,G,R,SC,Y_1) \cdot TOTAU(2,G,R,SC) \\ & + GG(2,G,R,SC,Y_1) \end{aligned} \tag{15}$$

where $GG(2,G,R,SC,Y_1)$ is zero by definition. For the second year of grade G, $(Y_2 = Y_1 + 1)$, we have:

$$\begin{aligned} S(2,G,R,SC,Y_2) = {} & [1-B(2,G,R,SC,Y_1)] \cdot S(2,G,R,SC,Y_1) \\ & - PR(2,G,R,SC,Y_1) \\ & + APR(2,G,R,SC,Y_2) \cdot TOTPR(2,G,R,SC) \\ & + APA(2,G,R,SC,Y_2) \cdot TOTPA(2,G,R,SC) \\ & + AAU(2,G,R,SC,Y_2) \cdot TOTAU(2,G,R,SC) \end{aligned} \tag{16}$$

Now, if we plug the right-hand side of Eq. (15) into Eq. (16) and collect terms, we get:

$$\begin{aligned} S(2,G,R,SC,Y_2) = {} & TOTPR(2,G,R,SC) \cdot \{[1-B(2,G,R,SC,Y_1)] \\ & \cdot APR(2,G,R,SC,Y_1) + APR(2,G,R,SC,Y_2)\} \\ & + TOTPA(2,G,R,SC) \cdot \{[1-B(2,G,R,SC,Y_1)] \\ & - APA(2,G,R,SC,Y_1) + APA(2,G,R,SC,Y_2)\} \\ & + TOTAU(2,G,R,SC) \cdot \{[1-B(2,G,R,SC,Y_1)] \\ & \cdot AAU(2,G,R,SC,Y_1) + AAU(2,G,R,SC,Y_2)\} \\ & + [1-B(2,G,R,SC,Y_1)] \cdot GG(2,G,R,SC,Y_1) \\ & - PR(2,G,R,SC,Y_1) \end{aligned} \quad (17)$$

To simplify this equation and make it more readable, let:

$$\begin{aligned} S(2,G,R,SC,Y_2) = {} & TOTPR(2,G,R,SC) \cdot TERM1(2,G,R,SC,Y_2) \\ & + TOTPA(2,G,R,SC) \cdot TERM2(2,G,R,SC,Y_2) \\ & + TOTAU(2,G,R,SC) \cdot TERM3(2,G,R,SC,Y_2) \\ & + GG(2,G,R,SC,Y_2) \end{aligned} \quad (18)$$

where for all $Y \geq Y_2$

$$\begin{aligned} TERM1(2,G,R,SC,Y) = {} & [1-B(2,G,R,SC,Y-1)] \cdot TERM1(2,G,R,SC,Y-1) \\ & + APR(2,G,R,SC,Y) \end{aligned} \quad (19)$$

$$\begin{aligned} TERM2(2,G,R,SC,Y) = {} & [1-B(2,G,R,SC,Y-1)] \cdot TERM2(2,G,R,SC,Y-1) \\ & + APA(2,G,R,SC,Y) \end{aligned} \quad (20)$$

$$\begin{aligned} TERM3(2,G,R,SC,Y) = {} & [1-B(2,G,R,SC,Y-1)] \cdot TERM3(2,G,R,SC,Y-1) \\ & + APA(2,G,R,SC,Y) \end{aligned} \quad (21)$$

and,

$$\begin{aligned} GG(2,G,R,SC,Y) = {} & [1-B(2,G,R,SC,Y-1)] \cdot GG(2,G,R,SC,Y-1) \\ & - PR(G,R,SC,Y-1) \end{aligned} \quad (22)$$

Now, recall from definitions of our inputs that for $G \neq 1$, UPRPA is the ratio of promotion/augmentations to promotions,

$$UPRPA(G,R,SC) = TOTPA(2,G,R,SC)/TOTPR(2,G,R,SC) \quad (23)$$

and UPRAU is the ratio of augmentations to promotions,

$$UPRAU(G,R,SC) = TOTAU(2,G,R,SC)/TOTPR(2,G,R,SC) \quad (24)$$

and GRAU is the grade authorization,

$$GRAU(2,G,R,SC) = \sum_{Y} S(C,G,R,SC,Y) \quad (25)$$

Note that Eq. (18) is valid for all values of Y, not just for Y_2, so that if we sum Eq. (18) over all values of Y we have:

$$\begin{aligned} GRAU(2,G,R,SC) &= TOTPR(2,G,R,SC) \cdot \sum_{Y} TERM1(2,G,R,SC,Y) \\ &\quad + TOTPA(2,G,R,SC) \cdot \sum_{Y} TERM2(2,G,R,SC,Y) \\ &\quad + TOTAU(2,G,R,SC) \cdot \sum_{Y} TERM3(2,G,R,SC,Y) \\ &\quad + \sum_{Y} GG(2,G,R,SC,Y) \\ &= TOTPR(2,G,R,SC) \cdot \sum_{Y} TERM1(2,G,R,SC,Y) \\ &\quad + TOTPR(2,G,R,SC) \cdot UPRPA(G,R,SC) \\ &\quad \cdot \sum_{Y} TERM2(2,G,R,SC,Y) \\ &\quad + TOTPR(2,G,R,SC) \cdot UPRAU(G,R,SC) \\ &\quad \cdot \sum_{Y} TERM3(2,G,R,SC,Y) \\ &\quad + \sum_{Y} GG(2,G,R,SC,Y) \end{aligned} \quad (26)$$

The only unknown in Eq. (26) is TOTPR(2,G,R,SC), so solving for it we get:

$$\begin{aligned} TOTPR(2,G,R,SC) &= [GRAU(2,G,R,SC) - \sum_{Y} GG(2,G,R,SC,Y)]/ \\ &\quad [\sum_{Y} TERM1(2,G,R,SC,Y) + \sum_{Y} TERM2(2,G,R,SC,Y) \\ &\quad \cdot UPRPA(G,R,SC) + \sum_{Y} TERM3(2,G,R,SC,Y) \\ &\quad \cdot UPRAU(G,R,SC)] \end{aligned} \quad (27)$$

Then, again, using the flow ratio definitions,

$$TOTAU(2,G,R,SC) = UPRAU(G,R,SC) \cdot TOTPR(2,G,R,SC) \quad (28)$$

$$TOTPA(2,G,R,SC) = UPRPA(G,R,SC) \cdot TOTPR(2,G,R,SC) \quad (29)$$

We now know the total flows in for case 1. Case 3 is solved in exactly the same way, except that TOTRT is substituted for TOTPR everywhere, and TOTRA is used instead of TOTPA, since rating transfers are taking place instead of promotions. Note that we would take the definitions of the flow ratios which hold when G = 1, so Eqs. (23) and (24) would become

$$UPRPA(1,R,SC) = TOTRA(2,G,R,SC)/TOTAU(2,G,R,SC) \quad (30)$$

and

$$UPRAU(1,R,SC) = TOTRT(2,G,R,SC)/TOTAU(2,G,R,SC) \quad (31)$$

Since TOTAU (total augmentations) is the denominator here, we would solve for it first and then use Eqs. (30) and (31) to solve for TOTRA and TOTRT (rating transfer/augmentations and rating transfers).

Cases 2, 4 and 5

These cases include all reserve and academy officers except nonrated lieutenants plus regular nonrated lieutenants. We again start by looking at the equation for the steady-state number in the first year of the category. The only flow into these two cases is promotions.

$$S(C,G,R,SC,Y_1) = APR(C,G,R,SC,Y_1) \cdot TOTPR(C,G,R,SC) + GG(C,G,R,SC,Y_1) \quad (32)$$

where $GG(C,G,R,SC,Y_1) = 0.0$ by definition and C = 1 except for Academy categories.

In year $Y_2 = Y_1 + 1$ and in all subsequent years in which the category exists, the steady-state equation is:

$$\begin{aligned} S(C,G,R,SC,Y) = {} & S(C,G,R,SC,Y-1) \cdot [1-B(C,G,R,SC,Y-1)] \\ & + APR(C,G,R,SC,Y) \cdot TOTPR(C,G,R,SC) \\ & - PR(C,G,R,SC,Y-1) - AU(1,G,R,SC,Y-1) \\ & - PA(1,G,R,SC,Y-1) \end{aligned} \tag{33}$$

Now if we let $Y = Y_2$ in Eq. (33) and substitute Eq. (32) for S(C,G,R,SC,Y-1) we get:

$$\begin{aligned} S(C,G,R,SC,Y_2) = {} & TOTPR(C,G,R,SC) \cdot \{[1-B(C,G,R,SC,Y_1)] \\ & \cdot APR(C,G,R,SC,Y_1) + APR(C,G,R,SC,Y_2)\} \\ & + [1-B(C,G,R,SC,Y_1)] \cdot GG(C,G,R,SC,Y_1) \\ & - PR(C,G,R,SC,Y_1) - AU(1,G,R,SC,Y_1) \\ & - PA(1,G,R,SC,Y_1) \end{aligned} \tag{34}$$

We can simplify the equation by letting:

$$\begin{aligned} S(C,G,R,SC,Y_2) = {} & TERM1(C,G,R,SC,Y_2) \cdot TOTPR(C,G,R,SC) \\ & + GG(C,G,R,SC,Y_2) \end{aligned} \tag{35}$$

where for all $Y \geq Y_2$:

$$\begin{aligned} TERM1(C,G,R,SC,Y) = {} & [1-B(C,G,R,SC,Y-1)] \cdot TERM1(C,G,R,SC,Y-1) \\ & + APR(C,G,R,SC,Y) \end{aligned} \tag{36}$$

and

$$\begin{aligned} GG(C,G,R,SC,Y) = {} & [1-B(C,G,R,SC,Y-1)] \cdot GG(C,G,R,SC,Y-1) \\ & - PR(C,G,R,SC,Y-1) - AU(1,G,R,SC,Y-1) \\ & - PA(1,G,R,SC,Y-1) \end{aligned} \tag{37}$$

We then use the fact that the grade authorization is the sum of the steady-state numbers and sum over Eq. (35) to get:

$$\begin{aligned} GRAU(C,G,R,SC) &= \sum_{Y} S(C,G,R,SC,Y) \\ &= TOTPR(C,G,R,SC) \cdot \sum_{Y} TERM1(C,G,R,SC,Y) \\ &\quad + \sum_{Y} GG(C,G,R,SC,Y) \end{aligned} \tag{38}$$

Then, solving for TOTPR(C,G,R,SC), we get:

$$\begin{aligned} TOTPR(C,G,R,SC) = {} & [GRAU(C,G,R,SC) - \sum_{Y} GG(C,G,R,SC,Y)] / \\ & [\sum_{Y} TERM1(C,G,R,SC,Y)] \end{aligned} \tag{39}$$

and we are done computing flows in for case 2. As with cases 1 and 3, the difference between cases 2 and 4 can be resolved by simply substituting TOTRT for TOTPR everywhere it appears between Eqs. (32) and (39). Case 5 is also like case 2. TOTAU would be substituted for TOTPR and GG(C,G,R,SC,Y) would be

$$\begin{aligned} GG(2,1,3,SC,Y) = {} & [1-B(2,1,3,SC,Y-1)] \cdot GG(2,1,3,SC,Y-1) \\ & - PR(2,1,3,SC,Y-1) - RT(2,1,1,SC,Y) \\ & - RT(2,1,2,SC,Y) \end{aligned} \tag{40}$$

since flows out in case 5 are rating transfers and promotions rather than augmentations, promotions and promotion/augmentations.

Case 6

Case 6 deals with Academy and reserve nonrated lieutenants, where the only flows in are accessions. We begin, again with the equations for steady-state numbers. In year Y_1 (which is, in this case, year of service 1) the number in steady state is simply the accessions.

(R = 3 for nonrated),

$$S(C,1,3,SC,1) = ACCESS(C,1,3,SC) + GG(C,1,3,SC,1) \tag{41}$$

where ACCESS(C,1,3,SC) is the total accessions for the category and GG(C,1,3,SC,1) is 0.0 by definition. For YOS 2 the equations are

slightly different for the two components. When C = 1 (reserve) we have:

$$\begin{aligned} S(1,1,3,SC,2) = {} & [1-B(1,1,3,SC,1)] \cdot S(1,1,3,SC,1) \\ & - PR(1,1,3,SC,1) - RT(1,1,1,SC,2) \\ & - RT(1,1,2,SC,2) - AU(1,1,3,SC,1) \\ & - PA(1,1,3,SC,1) - RA(2,1,1,SC,2) \\ & - RA(2,1,2,SC,2) \end{aligned} \qquad (42)$$

and when C = 2 (regular, Academy only applies, i.e., SC = 1):

$$\begin{aligned} S(2,1,3,1,2) = {} & [1-B(2,1,3,1,1,)] \cdot S(2,1,3,1,1) \\ & - PR(2,1,3,1,1) - RT(2,1,1,1,2) \\ & - RT(2,1,2,1,2) \end{aligned} \qquad (43)$$

The difference is that augmentation and all the combination flows flow out of the reserve categories, but not out of the Academy category. We continue as we did for the other cases, substituting Eq. (41) into Eqs. (42) and (43), and get the following equation, which applies to both components.

$$\begin{aligned} S(C,1,3,SC,Y) = {} & TERM1(C,1,3,SC,Y) \cdot ACCESS(C,1,3,SC) \\ & + GG(C,1,3,SC,Y) \end{aligned} \qquad (44)$$

where:

$$TERM1(C,1,3,SC,Y) = [1-B(C,1,3,SC,Y-1)] \cdot TERM1(C,1,3,SC,Y-1) \qquad (45)$$

and, for C = 1 (reserve)

$$\begin{aligned} GG(1,1,3,SC,Y) = {} & [1-B(1,1,3,SC,Y-1)] \cdot GG(1,1,3,SC,Y-1) \\ & - PR(1,1,3,SC,Y-1) - RT(1,1,1,SC,Y) \\ & - RT(1,1,2,SC,Y) - AU(1,1,3,SC,Y-1) \\ & - RA(2,1,2,SC,Y) \end{aligned} \qquad (46)$$

and, for C = 2 (regular)

$$GG(2,1,3,SC,Y) = [1-B(2,1,3,SC,Y-1)] \cdot GG(2,1,3,SC,Y-1) - PR(2,1,3,SC,Y-1) - RT(2,1,1,SC,Y) - RT(2,1,2,SC,Y) \quad (47)$$

So, summing over the steady-state number, as we did before, we get:

$$GRAU(C,1,3,SC) = \sum_{Y} TERM1(C,1,3,SC,Y) \cdot ACCESS(C,1,3,SC) + \sum_{Y} GG(C,1,3,SC,Y) \quad (48)$$

Which, solving for ACCESS(C,1,3,SC) gives us:

$$ACCESS(C,1,3,SC) = [GRAU(C,1,3,SC) - \sum_{Y} GG(C,1,3,SC,Y)] / [\sum_{Y} TERM1(C,1,3,SC,Y)] \quad (49)$$

and we are completely done.

Appendix B

THE GOODNESS MEASURE MATHEMATICAL BACKGROUND

In computing the goodness measures, the officer force is divided into the same six cases that were used in the force structure computations (see Appendix A). Since the mathematical differences between these cases are, again, only minor, we will show detailed mathematics for only one case and very briefly discuss the changes which would have to be made to solve the other cases. The case we will look at is the first one: non-Academy regular officers with grade above lieutenant. This is the most basic case, and so, best exemplifies the method used with a minimum amount of case-specific adjustments.

The symbols and notation used in the preceding appendix continue to be used here. New symbols will be defined as we go along; however, first we must define the variable MIN:

MIN = the minimum amount of time which must pass between two promotions awarded to the same person. MIN is given in years and must be an integer greater than zero and less than the shortest time between the first promotion into a category and the first promotion out of it (see Sec. IV). Only one value of MIN is given and it applies to all categories in the officer force structure.

COMPUTATION FOR CASE 1

We begin the computations by computing the number of officers eligible for promotion from each year of the category in question:

$$C = 2 \text{ (for regular)}$$

$$\begin{aligned} ELG(2,G,R,SC,Y) &= S(2,G,R,SC,Y) - PR(2,G-1,R,SC,Y-1) \\ &\quad - PA(1,G-1,R,SC,Y-1) - \left\{ \sum_{i=3}^{MIN} \left[\prod_{k=2}^{i=1} [1-B(2,G,R,SC,Y-k)] \right] \right. \\ &\quad \left. \cdot [PR(2,G-1,R,SC,Y-i) + PA(1,G-1,R,SC,Y-i)] \right\} \end{aligned}$$

= number of officers eligible for promotion from (2,G,R,SC,Y) to (2,G+1,R,SC,Y+1) . (1)

The whole of Eq. 1 applies only when MIN is three or greater. If MIN is 1, the eligibles equal the number in state, i.e., everyone is eligible. If MIN is two, the number of eligibles is the steady-state number less those just promoted into the state (this is the first three terms in Eq. 1). If MIN is greater than two, we must subtract out those officers who were promoted within the minimum waiting period. The summation in Eq. 1 does this by subtracting out officers who were promoted or promoted/augmented within the waiting period. Before the officers are subtracted out, a retention rate (that is, the complement of the loss rate) is applied for each year of service since they were promoted to account for those who were lost to the Air Force before becoming eligible for another promotion.

Now that the number of eligibles is known, we can divide the flows out of the state between the eligibles and the noneligibles. We make the assumption that losses and lateral flows are divided proportionately between eligibles and noneligibles.

NELG(2,G,R,SC,Y) = S(2,G,R,SC,Y) - ELG(2,G,R,SC,Y)
= number of officers in state (2,G,R,SC,Y) who are not eligible for promotion to the state (2,G+1,R,SC,Y+1) (2)

ELGCON(2,G,R,SC,Y) = LAT(2,G,R,SC,Y) - {NELG(2,G,R,SC,Y) · [1-B(2,G,R,SC,Y)]}
= The number of officers eligible for promotion who continue in Grade G from YOS Y to Y+1. (3)

PROM(2,G,R,SC,Y) = PR(2,G,R,SC,Y)/ELG(2,G,R,SC,Y)
= The fraction of officers eligible for promotion who are promoted. (4)

CONT(2,G,R,SC,Y) = ELGCON(2,G,R,SC,Y)/ELG(2,G,R,SC,Y)

= The fraction of officers eligible for promotion who continue in grade G in YOS Y + 1 (5)

REG(2,G,R,SC,Y) = (L(2,G,R,SC,Y) - NELG(2,G,R,SC,Y) · B(2,G,R,SC,Y))/ELG(2,G,R,SC,Y)

= The fraction of officers eligible for promotion who retire in YOS Y. (6)

Note that the numerator of Eq. 6 is the number of eligible officers who retire and is the total losses minus the number of noneligible officers lost. Also, note that since we are dealing with regular nonlieutenants, the only way to leave a state is by promotion, loss, or lateral flow. Therefore, since PROM, RET and CONT tell us the fraction of eligible officers who take each of these exits,

PROM(2,G,R,SC,Y) + CONT(2,G,R,SC,Y) + RET(2,G,R,SC,Y) = 1 . (7)

For the noneligibles we have:

AR(2,G,R,SC,Y) = (L(2,G,R,SC,Y) - ELG(2,G,R,SC,Y) · B(2,G,R,SC,Y))/NELG(2,G,R,SC,Y)

= fraction of officers not eligible for promotion who retire in YOS Y. (8)

CF(2,G,R,SC,Y) = 1-AR(2,G,R,SC,Y)

= fraction of officers not eligible for promotion who continue in grade G in YOS Y + 1. (9)

Having calculated the fractions of both eligible and noneligible officers who take each flow from the state, we know the flow probabilities mentioned in Sec. IV. The path probabilities will also be viewed as two distinct sets; one for eligibles and one for noneligibles. In the arrays which follow (G contains path probabilities for exiting via promotion, and H contains path probabilities for exiting via retirement), the fourth subscript indicates how the officers originally entered the state (2,G,R,SC,Y):

> If the 4th subscript is 1, they entered via promotion or promotion/augmentation and so are ineligible for promotion for MIN number of years.
>
> If the 4th subscript is 2 they entered via augmentation and so are always eligible for promotion.

$$
G(Y,G,I,1) \begin{cases} = 0 \text{ if } I < Y + MIN \\[2ex] = \left[\prod_{D=Y}^{Y+MIN-1} CF(2,G,R,SC,D) \right] \cdot PROM(2,G,R,SC,I) & \text{if } I = Y + MIN \\[2ex] = \left[\prod_{D=Y}^{Y+MIN-1} CF(2,G,R,SC,D) \right] \left[\prod_{D=Y+MIN}^{I-1} CONT(2,G,R,SC,D) \right] \\ \quad \cdot PROM(2,G,R,SC,I) & \text{if } I > Y + MIN \end{cases}
$$

= the probability that an officer promoted into grade G in YOS Y will be promoted to grade G+1 in YOS I. (10)

$$
G(Y,G,I,2) \begin{cases} = PROM(2,G,R,SC,I) & \text{if } I = Y \\[2ex] = \left[\prod_{D=Y}^{I-1} CONT(2,G,R,SC,D) \right] \cdot PROM(2,G,R,SC,I) & \text{if } I > Y \end{cases}
$$

= the probability that an officer who had an entry other than promotion into grade G in YOS Y will be promoted to grade G+1 in YOS I. (11)

$$
H(Y,G,I,1)\begin{cases}
= AR(2,G,R,SC,I) & \text{if } I = Y \\
= \left[\prod_{D=Y}^{I-1} CF(2,G,R,SC,D)\right] \cdot AR(2,G,R,SC,I) & \text{if } Y < I < Y + MIN \\
= \left[\prod_{D=Y}^{Y+MIN-1} CF(2,G,R,SC,D)\right] \cdot RET(2,G,R,SC,I) & \text{if } I = Y + MIN \\
= \left[\prod_{D+Y}^{Y+MIN-1} CF(2,G,R,SC,D)\right] \cdot \left[\prod_{D=Y+MIN}^{I-1} CONT(2,G,R,SC,D)\right] \cdot RET(2,G,R,SC,I) & \text{if } I > Y + MIN
\end{cases}
$$

= the probability that an officer promoted to grade G in YOS Y will be lost from grade G in YOS I. (12)

$$
H(Y,G,I,2)\begin{cases}
= RET(2,G,R,SC,I) & \text{if } I = Y \\
= \left[\prod_{D=Y}^{I-1} CONT(2,G,R,SC,D)\right] \cdot RET(2,G,R,SC,I) & \text{if } I > Y
\end{cases}
$$

= the probability that an officer who had an entry other than promotion into grade G in YOS Y will be lost from grade G in YOS I. (13)

With the path probabilities we can compute the number of officers who took each of the two possible exits from the grade (or category). In the following equations MAXYOS refers to the highest year of service for which the grade (or category) exits.

$$NP(G,Y) = \sum_{D=Y}^{MAXYOS} [PR(2,G-1,R,SC,Y-1) + PA(1,G-1,R,SC,Y-1)] \cdot G(Y,G,D,1) + \sum_{D=Y}^{MAXYOS} AU(1,G,R,SC,Y-1) \cdot G(Y,G,D,2)$$

= the number of officers who entered grade G in YOS Y and were promoted out of grade G to grade G + 1. (14)

$$NR(G,Y) = \sum_{D=Y}^{MAXYOS} H(Y,G,D,1) \cdot [PR(2,G-1,R,SC,Y-1) + PA(1,G-1,R,SC,Y-1)] + \sum_{D=Y}^{MAXYOS} H(Y,G,D,2) \cdot AU(1,G,R,SC,Y-1)$$

= the number of officers who entered grade G in YOS Y and are lost or retired from the Air Force while still in grade G. (15)

We are finally ready to calculate the first goodness measure, the probability of leaving the category via a given exit. In this case, there were only two possible exits: promotion and retirement.

$$PPROB(G) = \sum_{J=1}^{MAXYOS} NP(G,J) / \sum_{D=1}^{MAXYOS} [PR(2,G-1,R,SC,D) + PA(1,G-1,R,SC,D) + AU(1,G,R,SC,D)]$$

= the probability that an officer will be promoted to grade G + 1 given that he reached category (2,G,R,SC). (Note that the denominator is the total number of officers entering category (2,G,R,SC).) (16)

$$RPROB(G) = \sum_{J=1}^{MAXYOS} NR(G,J) / \sum_{D=1}^{MAXYOS} [PR(2,G-1,R,SC,D) + PA(1,G-1,R,SC,D) + AU(1,G,R,SC,D)]$$

= the probability that an officer will retire from category (2,G,R,SC) given that he entered the category. (17)

To calculate the mean times to each exit we need to know the number of years between an officer's entrance into and exit from the category.

$$PYRS(G,Y) = \sum_{D=Y}^{MAXYOS} G(Y,G,D,1) \cdot (D-Y+1) \cdot [PR(2,G-1,R,SC,Y-1) + PA(1,G-1,R,SC,Y-1)] + \sum_{D=Y}^{MAXYOS} G(Y,G,D,2) \cdot (D-Y+1) \cdot AU(1,G,R,SC,Y-1)$$

= the number of years officers who entered grade G in YOS Y spend in grade G before being promoted to grade G + 1. (18)

$$RYRS(G,Y) = \sum_{D=Y}^{MAXYOS} H(Y,G,D,1) \cdot (D-Y+1)$$

$$\cdot [PR(2,G-1,R,SC,Y-1) + PA(1,G-1,R,SC,Y-1)]$$

$$+ \sum_{D=Y}^{MAXYOS} H(Y,G,D,2) \cdot (D-Y+1) \cdot AU(1,G,R,SC,Y-1)$$

= the number of years officers who entered grade G in YOS Y spend in grade G before retiring from Grade G. (19)

The mean times can now be computed as:

$$MTTP(GP) = \sum_{J=1}^{MAXYOS} PYRS(G,J) / \sum_{J=1}^{MAXYOS} PR(2,G,R,SC,J)$$

= mean time between entering the category and being promoted out of it. (20)

$$MTTR(G) = \sum_{J=1}^{MAXYOS} RYRS(G,J) / \sum_{J=1}^{MAXYOS} L(2,G,R,SC,J)$$

= mean time between entering the category and being lost from the service before leaving the category. (21)

ADJUSTMENTS FOR THE OTHER CASES

The main difference between the mathematics for case 1 and for the other cases is that the flows in and out differ from case to case. When dealing with reserve officers, fractions of eligibles and noneligibles who leave a state to augment the regular force must be computed as well as the fractions promoted and retired; and it must be remembered that only eligibles may receive promotion/augmentations. In determining the number of reserve eligibles, it is necessary to keep in mind that some of the officers who were promoted within the waiting period (and so would not yet be eligible) were made regular before they became eligible. The equation for finding the number of reserve nonlieutenant eligibles when MIN is greater than 2 is:

$$ELG(1,G,R,SC,Y) = S(1,G,R,SC,Y) - PR(1,G-1,R,SC,Y-1) - \left\{ \sum_{i=3}^{MIN} \left[\prod_{k=2}^{i=1} [1-B(1,G,R,SC,Y-k) - AUG(1,G,R,SC,Y-k)] \right] \cdot PR(1,G-1,R,SC,Y-i) \right\} \quad (22)$$

where

$$AUG(1,G,R,SC,Y) = AU(1,G,R,SC,Y)/S(1,G,R,SC,Y)$$

= augmentation rate, i.e., the fraction of officers in state (1,G,R,SC,Y) who are augmented to state (2,G,R,SC,Y). This is comparable to the loss rate and is used similarly. (23)

Also for the reserves, there are more ways of leaving the category, so that in addition to goodness measures for loss and promotion, goodness measures for promotion/augmentation and augmentation need to be computed. On the other hand, all officers enter the category via promotion so that the path probability arrays do not need the fourth subscript used in Eqs. (10) to (13) to distinguish between officers who entered via promotion and those who entered by some other means. This does not mean that there are fewer possible paths, since the increase in exits balances the decrease in entrances. The path probabilities are computed as before, that is, by multiplying all possible combinations of flow probabilities for each combination of year of entrance and year of exit.

For lieutenants, the mathematics are simplified by the fact that all lieutenants are eligible for promotion at all times, so it is never necessary to divide the officers in any cell into two groups with differing path probabilities. On the other hand, rating transfers to pilot and navigator, and rating transfer/augmentations to pilot and navigator must be accounted for either as additional ways of entering rated lieutenants (in which case, of course, we only deal with the

transfer to the appropriate rating) or as additional ways of exiting from nonrated categories (in which case we have an additional four types of exits for which to compute goodness measures).

Essentially the adjustments which need to be made to the mathematics are simply adding or removing terms from the equations to account for all the flows into or out of a state or category. Once all the route probabilities have been computed the manner of computing the individual goodness measures is identical to that for case 1.

Appendix C
INPUT DATA

After the two header cards (which will be discussed later) that begin the input data deck for the grade limitation model, there are nine types of data which can be used by the model. Each type of data and its input format will be discussed in detail. In general, the nine types are as follows:

1. Grade authorizations.
2. Flow ratios* of augmentations to promotions (for nonlieutenants) or rating transfers to augmentations (for lieutenants).
3. Flow ratios* of promotion/augmentations to promotions or rating transfer/augmentations to augmentations.
4. Flow distributions* for rating transfers.
5. Flow distributions* for rating transfer/augmentations.
6. Flow distributions* for augmentations.
7. Flow distributions* for promotion/augmentations.
8. Flow distributions* for promotions.
9. Loss rates.

All of the above data may be entered in great detail, with specific data given for each category, or in some cases, even for each state, of the officer structure. If, however, such great flexibility is not desired, the data can also be aggregated by component, source of commission, or rating. Because of these options it is necessary that each data card indicate what subset of the officer force it applies to. Unless otherwise indicated, Columns 1 to 20 of each input card should contain the following information.

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*See Section III, Model Inputs for definitions of these data types.

Columns 1 to 4	Type of Data
	GRAU - Grade Authorizations PRP - Augmentation to Promotion Ratios PRA - Promotion/Augmentation to Promotion Ratios TRAN - Rating Transfer Distributions RTAU - Rating Transfer/Augmentation Distributions AUG - Augmentation Distributions PRAU - Promotion/Augmentation Distributions PROM - Promotion Distributions LOSS - Loss Rates
Columns 5 to 8	Component
	REG - Regular RES - Reserve ALL - RES and REG
Columns 9 to 12*	Grade
	LT - Lieutenant CAP - Captain MAJ - Major LTC - Lieutenant Colonel COL - Colonel and General
Columns 13 to 16*	Rating
	PIL - Pilot NAV - Navigator NR } SUP } - Nonrated or Support RAT - PIL and NAV ALL - PIL, NAV and NR
Columns 17 to 20*	Source of Commission
	AFA - Academies ROTC - Reserve Officer Training Corps SMSO } OTS } - SMS-O or Officer Training School RES - ROTC and SMSO ALL - AFA, ROTC, and SMSO

*It will be noted that the order of columns is somewhat different on the Grade Authorization Data format and Rating Transfer Data (Figs. 26 to 28).

In addition, all the cards for a given type of data should be grouped together, preceded by a card stating the type of data in Columns 1 to 4, and followed by a blank card. Finally, Columns 73 to 80 of each card may be used for comments or sequencing of the input deck. The nine groups of data cards, one for each data type, may be arranged in any order in the input deck. Also, if a certain type of data will not be used in a given run, the group of cards for it may be left out entirely. For example, if a run is to be made in which no rating transfer augmentations are to take place, it is not necessary to include any input cards that refer to the flow distributions for rating transfer augmentations. The input deck will have only eight data types in it.

Grade Authorizations (GRAU)

These data indicate the number of officers in each category of the officer structure. Figure 14 illustrates the card format for the data.

Note that if the first card in the GRAU input deck had ALL in the component field rather than RES, this would not mean that there are a total of 3681 ROTC pilot lieutenants. It would mean that there are 3681 regular ROTC pilot lieutenants and 3681 reserve ROTC pilot lieutenants. This is true for all the possible aggregations for each type of data. When we aggregate the data we are not summing over the categories or failing to distinguish between categories; we are just saying that the various categories included in the aggregation have the same data. This saves the user from having to enter the same data more than once if they are shared by several categories.

Flow Ratios (PRP and PRA)

Figures 15 and 16 show the card formats for the two types of flow ratio data. Note that Columns 5 to 8 (usually the component identifier) are left blank. This is because a flow ratio is needed only when two (or more) different types of flows come into any one category. This happens only when augmentations or some combination flow including augmentations takes place. Therefore, it occurs only for flows into the regular component, so it is not necessary to specify which component the data apply to. Columns 5 to 8 must be left blank; do not put "REG" in them.

GRADE AUTHORIZATION DATA FORMAT

FORM 20
8 FEB 66

Type of Data: GRAU	Com-ponent (Reg or Res) or all	Rating	Source of Commis-sion	Data: Grade	Data: Number of Men Authorized	Data: Grade	Data: Number of Men Authorized	Data: Grade	Data: Number of Men Authorized	Data: Grade	Data: Number of Men Authorized	Data: Grade	Data: Number of Men Authorized		Optional Sequencing Field
01 02 03 04	05 06 07 08	09 10 11 12	13 14 15 16	17 18 19 20	21 22 23 24 25	26 27 28 29	30 31 32 33 34	35 36 37 38	39 40 41 42 43	44 45 46 47	48 49 50 51 52	53 54 55 56	57 58 59 60 61	62 63 64 65 66 67 68 69 70 71 72	73 74 75 76 77 78 79 80
GRAU															
GRAU	RES	PIL	ROTC	LT	3681	CAP		MAJ		LTC		COL			

Fig. 14--Grade authorization data format

RATIO OF AUGMENTATIONS TO PROMOTIONS OR (FOR LT) OF RATING TRANSFERS TO AUGMENTATIONS

FORM 20
8 FEB 66

Type of Data	X	Rating	Source of Commis-sion	Data: Grade	Data: Ratio	Data: Grade	Data: Ratio	Data: Grade	Data: Ratio	Data: Grade	Data: Ratio	Data: Grade	Data: Ratio		Optional Sequencing Field
01-04	05-08	09-12	13-16	17-20	21-25	26-29	30-34	35-38	39-43	44-47	48-52	53-56	57-61	62-72	73-80
PRP															
PRP		PIL	ROTC			CAP	.815								

Fig. 15--Ratio of augmentations to promotions or (for lieutenant) of rating transfers to augmentations

RATIO OF PROMOTION AUGMENTATIONS TO PROMOTIONS OR (FOR LT) OF RATING TRANSFER-AUGMENTATIONS TO AUGMENTATIONS

FORM 20
8 FEB 66

Type of Data	X	Rating	Source of Commis-sion	Data: Grade	Data: Ratio	Data: Grade	Data: Ratio	Data: Grade	Data: Ratio	Data: Grade	Data: Ratio	Data: Grade	Data: Ratio		Optional Sequencing Field
01-04	05-08	09-12	13-16	17-20	21-25	26-29	30-34	35-38	39-43	44-47	48-52	53-56	57-61	62-72	73-80
PRA															
PRA		PIL	ROTC			CAP	.288								

Fig. 16--Ratio of promotion/augmentations to promotions or (for lieutenant) of rating-transfer/augmentations to augmentations

Flow ratios always refer to flows into a category, never out of it. Since all grades, except lieutenant, always have promotions flowing in, we use promotions as the denominator of the flow ratios for those grades. Lieutenants, however, have no promotions flowing in, so the base of the ratios for lieutenants is augmentations. Note also that since rating transfers flow only into rated categories, there will be no flow ratios for nonrated lieutenants.

The data cards can each hold five data items, but it is not necessary to use up the whole card. If the user wishes, he may put only one data item on each card and leave the rest of the fields blank. But if a data item is equal to zero, it is not necessary to enter it at all, although the user may if he wants to. These options are true for all nine types of data.

Flow Distributions (TRAN, RTAU, AUG, PRAU, PROM)

Figures 17 thru 21 show the card format for the five types of flow distribution data. As usual, these all refer to flows into the category identified in Columns 1 to 20. The year of service is the year that the flow ends in. For example, Fig. 17 shows that rating transfers into Academy pilots flow from the first year of service to the second year of service. Since the flow distribution for that year is 1.00 (meaning 100 percent of these rating transfers occur into year 2), no rating transfers to Academy pilots take place at any other time. Since rating transfers apply only to lieutenants, the grade identifier in Columns 9 to 12 is left blank. Further, for rating transfer/augmentations, the component identifier also (Columns 5 to 8) is left blank, since only the regular component receives augmentation flows. The component is not specified for augmentations and promotion/augmentations for the same reason. Note that only those years of service for which the flow distribution is not zero need be included on the data cards.

Loss Rates (LOSS)

These data indicate the rates of attrition (due to death, disability, retirement, separation, etc.) by component, grade, rating, source of commission, and year of service. Figure 22 illustrates the input deck's

RATING TRANSFER FLOW PROPORTIONS

FORM 20
8 FEB 66

Type of Data	Com-ponent (Reg, Res, or All)		Rating	Source of Commis-sion	Data: Year of Service	Data: Flow Proportions	Data: Year of Service	Data: Flow Proportions	Data: Year of Service	Data: Flow Proportions	Data: Year of Service	Data: Flow Proportions	Data: Year of Service	Data: Flow Proportions	Data: Year of Service	Data: Flow Proportions		Optional Sequencing Field
01 02 03 04	05 06 07 08	09 10 11 12	13 14 15 16	17 18 19 20	21 22 23	24 25 26 27 28	29 30 31	32 33 34 35 36	37 38 39	40 41 42 43 44	45 46 47	48 49 50 51 52	53 54 55	56 57 58 59 60	61 62 63	64 65 66 67 68	69 70 71 72	73 74 75 76 77 78 79 80
TRAN																		
TRAN	REG		PIL	AFA	2	1.00												

Fig. 17--Rating transfer flow proportions

RATING TRANSFER-AUGMENTATION FLOW PROPORTIONS

FORM 20
8 FEB 66

Type of Data		Rating	Source of Commis-sion	Data: Year of Service	Data: Flow Proportion	Data: Year of Service	Data: Flow Proportion	Data: Year of Service	Data: Flow Proportion	Data: Year of Service	Data: Flow Proportion	Data: Year of Service	Data: Flow Proportion	Data: Year of Service	Data: Flow Proportion		Optional Sequencing Field
01 02 03 04	05 06 07 08 09 10 11 12	13 14 15 16	17 18 19 20	21 22 23	24 25 26 27 28	29 30 31	32 33 34 35 36	37 38 39	40 41 42 43 44	45 46 47	48 49 50 51 52	53 54 55	56 57 58 59 60	61 62 63	64 65 66 67 68	69 70 71 72	73 74 75 76 77 78 79 80
RTAU																	
RTAU		PIL	ROTC	2	1.000												

Fig. 18--Rating-transfer/augmentation flow proportions

AUGMENTATION FLOW PROPORTIONS

FORM 20
8 FEB 66

Type of Data		Grade	Rating	Source of Commission	Data: Year of Service	Data: Flow Proportions	Data: Year of Service	Data: Flow Proportions	Data: Year of Service	Data: Flow Proportions	Data: Year of Service	Data: Flow Proportions	Data: Year of Service	Data: Flow Proportions	Data: Year of Service	Data: Flow Proportions		Optional Sequencing Field
01–04	05–08	09–12	13–16	17–20	21–23	24–28	29–31	32–36	37–39	40–44	45–47	48–52	53–55	56–60	61–63	64–68	69–72	73–80
AUG																		
AUG		LT	PIL	ROTC	4	.961	5	.039										

Fig. 19--Augmentation flow proportions

PROMOTION-AUGMENTATION FLOW PROPORTIONS

FORM 20
8 FEB 66

Type of Data		Grade	Rating	Source of Commission	Data: Year of Service	Data: Flow Proportion	Data: Year of Service	Data: Flow Proportion	Data: Year of Service	Data: Flow Proportion	Data: Year of Service	Data: Flow Proportion	Data: Year of Service	Data: Flow Proportion	Data: Year of Service	Data: Flow Proportion		Optional Sequencing Field
01–04	05–08	09–12	13–16	17–20	21–23	24–28	29–31	32–36	37–39	40–44	45–47	48–52	53–55	56–60	61–63	64–68	69–72	73–80
PRAU																		
PRAU		CAP	PIL	ROTC	4	.018	5	.982										

Fig. 20--Promotion/augmentation flow proportions

PROMOTION FLOW PROPORTIONS

FORM 20
8 FEB 66

Type of Data	Component	Grade	Rating	Source of Commission	Data		Data		Data		Data		Data		Data			Optional Sequencing Field
					Year of Service	Flow Proportion	Year of Service	Flow Proportion	Year of Service	Flow Proportion	Year of Service	Flow Proportion	Year of Service	Flow Proportion	Year of Service	Flow Proportion		
01 02 03 04	05 06 07 08	09 10 11 12	13 14 15 16	17 18 19 20	21 22 23	24 25 26 27 28	29 30 31	32 33 34 35 36	37 38 39	40 41 42 43 44	45 46 47	48 49 50 51 52	53 54 55	56 57 58 59 60	61 62 63	64 65 66 67 68	69 70 71 72	73 74 75 76 77 78 79 80
PROM																		
PROM	REG	MAJ	PIL	AFA	8	.016	9	.047	10	.937								

Fig. 21--Promotion flow proportions

format. Each data entry indicates the fraction of officers in the state that leave the officer force during or at the conclusion of the year of service.

Two Header Cards

In addition to the nine types of input data, two cards are required at the beginning of the input deck. On the first card, the user may enter a run title which will appear on each page of output produced by the model. This title may include the entire 80 columns of the card.

The second card is an "option card" which allows the user to specify which sets of output, from either the regular output package or the goodness measure package, he wishes to have produced. Since the entire outputs, aggregated and unaggregated, may run to hundreds of pages, the user may specify that he wishes only aggregated outputs or only some of the unaggregated outputs. The format of this card is shown in Fig. 23. Blanks indicate that the option is not desired, e.g., if Columns 1 and 2 are left blank, officer flow reports for each component will not be produced. If anything other than blanks are shown in an option field, the user will get the indicated output. The fields are:

Columns 1 to 2:	Leave blank unless officer flow reports are wanted for each component.
Columns 3 to 4:	Leave blank unless officer flow reports are wanted for each rating.
Columns 5 to 6:	Leave blank unless officer flow reports are wanted for each source of commission.
Columns 7 to 10:	Leave blank if implied forward computation inputs are not wanted.
Columns 11 to 12:	Leave blank if no goodness measure outputs are wanted.
Columns 13 to 16:	Specify the minimum waiting period (in years) between promotions if wanted. This must be 1 or more and defaults to 2. (Goodness measure package only).

LOSS RATES DATA FORMAT

FORM 20
8 FEB 66

Type of Data	Component	Grade	Rating	Source of Commission	Data: Year of Service	Data: Loss Rate	Data: Year of Service	Data: Loss Rate	Data: Year of Service	Data: Loss Rate	Data: Year of Service	Data: Loss Rate	Data: Year of Service	Data: Loss Rate	Data: Year of Service	Data: Loss Rate		Optional Sequencing Field
01-04	05-08	09-12	13-16	17-20	21-23	24-28	29-31	32-36	37-39	40-44	45-47	48-52	53-55	56-60	61-63	64-68	69-72	73-80
LOSS																		
LOSS	RES	LT	PIL	ROTC	2	.015	3	.015	4	.015	5	.990	6	1.000				

Fig. 22--Loss rate data format

OPTIONS CARD (2nd CARD IN INPUT DECK)

FORM 20
8 FEB 66

Officer Flow Report Details?: Component	Officer Flow Report Details?: Rating	Officer Flow Report Details?: SOC	Implied Forward Com-putation Flows ?	Goodness Measures: Desired ?	Goodness Measures: Minimum Years Between Promo-tions	Goodness Measures: Aggregations: SOC	Goodness Measures: Aggregations: SOC & Comp	Goodness Measures: Aggregations: Comp	Goodness Measures: Aggregations: Rating	Punch Implied Input Decks = 1: OFPM 2: OGLA 3: BOTH	Maximum Iterations (Constr. Model Only)	YOS Range for AUG Opportunity Report: Low YOS	YOS Range for AUG Opportunity Report: High YOS		Debugging Flags: 1	2	3	4	5	6	7	8	9	10	
01-02	03-04	05-06	07-10	11-12	13-16	17-18	19-20	21-22	23-24	25-28	29-32	33-36	37-40	41-52	53-54	55-56	57-58	59-60	61-62	63-64	65-66	67-68	69-70	71-72	73-80

Fig. 23--Option card

Columns 17 to 18: Leave blank if goodness measures are not to be aggregated by source of commission.

Columns 19 to 20: Leave blank if goodness measures are not to be aggregated by source and component.

Columns 21 to 22: Leave blank if goodness measure are not to be aggregated by component only.

Columns 23 to 24: Leave blank if goodness measures are not to be aggregated by rating.

Columns 25 to 28: This contains: 1 if the user wants to have an implied input deck for the progression model punched out; 2 if he wants an implied input deck for the grade limitations model; 3 if he wants both.*

Columns 33 to 40: Year of service range for the augmentation opportunity report. The user must tell the model the first and last years in which augmentation takes place. The default is 3 to 7.

---

*After the grade limitations model computes the officer structure and the associated flows, it determines what the inputs must have been in order to generate the officer structure and associated flows. This is done without referring back to the original inputs used to generate the officer force structure. In addition, the model determines what the progression model inputs would have to be in order to generate an identical officer structure and associated officer flows. These newly determined inputs can then be punched as input decks, if the user so requests. The ability of the model to generate such input decks is useful when the grade limitations model and the progression model are used together.

Appendix D
SAMPLE OUTPUT

This appendix contains excerpts from a computer printout of the grade limitations model. The first 8 pages shown (pages 2 to 8 and 10 of the printout) show summary printouts of the model's inputs. Page 15 of the printout shows which types of data were in the input deck. We see there were only 8 data types read, and data type RTAU was not included. This means that no rating transfer/augmentations took place in this particular run. The remaining pages show sample output reports from the same computer run. For the purposes of illustration we will show only ROTC reports, although similar reports are given for OTS and Academy in all cases.

OFFICER INVENTORIES

Output pages 21 to 25 show the officer inventory, or grade distribution, tables for the ROTC source of commission. For example, page 21 shows how ROTC pilots are distributed by component over grades and years of service. Page 22 does the same for navigators. On page 23 (nonrated ROTC officers), under reserve lieutenants in year of service 1, we see that the yearly ROTC accessions for this run are 4500 officers. Accessions are shown more explicitly later. Page 24 shows the ROTC officer distribution for rated officers, i.e., the pilot and navigator distributions added together. At the bottom of each officer inventory report, there is a line which gives the average year of service for the officers in each column of the report. For example, by looking at the right-most column of page 25 (all ratings), we see that there are a total of 46,739 ROTC officers in the force, and they have an average year of service of 8.86 years.

Page 40 of the output gives the grade distribution across the entire force. We see, by looking at the sum of the right-most column, that the force size is 97,849. Under year 1 for reserve and regular lieutenants, we see that a total of 9,359 yearly accessions are needed to support this force size.

OFFICER GRADE LIMITATIONS MODEL SAMPLE RUN

GRADE AUTHORIZATIONS

	COMPONENT	RATING	SOURCE OF COMMISSION	GRADE	NUMBER OF MEN	GRADE	NUMBER OF MEN	GRADE	NUMBER OF MEN	GRADE	NUMBER OF MEN	GRADE	NUMBER OF MEN
GRAU	RES	PIL	ROTC	LT	3955.	CAP	1722.	MAJ	25.	LTC	0.	COL	0.
GRAU	RES	NAV	ROTC	LT	1820.	CAP	848.	MAJ	12.	LTC	0.	COL	0.
GRAU	RES	NR	ROTC	LT	10493.	CAP	2052.	MAJ	58.	LTC	0.	COL	0.
GRAU	RES	PIL	SMSO	LT	1146.	CAP	499.	MAJ	7.	LTC	0.	COL	0.
GRAU	RES	NAV	SMSO	LT	1436.	CAP	668.	MAJ	9.	LTC	0.	COL	0.
GRAU	RES	NR	SMSO	LT	11489.	CAP	2659.	MAJ	84.	LTC	0.	COL	0.
GRAU	REG	PIL	AFA	LT	1817.	CAP	2485.	MAJ	2219.	LTC	1455.	COL	638.
GRAU	REG	NAV	AFA	LT	281.	CAP	385.	MAJ	344.	LTC	223.	COL	92.
GRAU	REG	NR	AFA	LT	1660.	CAP	931.	MAJ	863.	LTC	563.	COL	240.
GRAU	REG	PIL	ROTC	LT	378.	CAP	3424.	MAJ	3625.	LTC	2393.	COL	1048.
GRAU	REG	NAV	ROTC	LT	158.	CAP	1446.	MAJ	1557.	LTC	1016.	COL	420.
GRAU	REG	NR	ROTC	LT	783.	CAP	3012.	MAJ	3340.	LTC	2212.	COL	942.
GRAU	REG	PIL	SMSO	LT	110.	CAP	993.	MAJ	1052.	LTC	695.	COL	304.
GRAU	REG	NAV	SMSO	LT	125.	CAP	1141.	MAJ	1229.	LTC	802.	COL	331.
GRAU	REG	NR	SMSO	LT	916.	CAP	3528.	MAJ	3956.	LTC	2621.	COL	1115.

Fig. 24--Printout page 2

OFFICER GRADE LIMITATIONS MODEL SAMPLE RUN

RATIO OF PROMOTION-AUGMENTATIONS TO PROMOTIONS
(FOR GRADE=LT RATIO IS RATING TRANSFER-AUGMENTATIONS TO AUGMENTATIONS)

	RATING	SOURCE OF COMMISSION	GRADE	RATIO	GRADE	RATIO	GRADE	RATIO	GRADE	RATIO	GRADE	RATIO
PRP	PIL	ROTC		0.0	CAP	0.815	MAJ	0.0	LTC	0.0	COL	0.0
PRP	NAV	ROTC		0.0	CAP	0.754	MAJ	0.0	LTC	0.0	COL	0.0
PRP	NR	ROTC		0.0	CAP	0.051	MAJ	0.0	LTC	0.0	COL	0.0
PRP	PIL	SMSO		0.0	CAP	0.815	MAJ	0.0	LTC	0.0	COL	0.0
PRP	NAV	SMSO		0.0	CAP	0.757	MAJ	0.0	LTC	0.0	COL	0.0
PRP	NR	SMSO		0.0	CAP	0.051	MAJ	0.0	LTC	0.0	COL	0.0
PRP	PIL	ROTC	LT	0.0		0.0		0.0		0.0		0.0
PRP	NAV	ROTC	LT	0.0		0.0		0.0		0.0		0.0
PRP	PIL	SMSO	LT	0.0		0.0		0.0		0.0		0.0
PRP	NAV	SMSO	LT	0.0		0.0		0.0		0.0		0.0
				0.0		0.0		0.0		0.0		0.0

Fig. 25--Printout page 3

OFFICER GRADE LIMITATIONS MODEL SAMPLE RUN

PAGE 4

RATIO OF AUGMENTATIONS TO PROMOTIONS
(FOR GRADE=LT RATIO IS RATING TRANSFERS TO AUGMENTATIONS)

	RATING	SOURCE OF COMMISSION	GRADE	RATIO	GRADE	RATIO	GRADE	RATIO	GRADE	RATIO	GRADE	RATIO
PRA	PIL	ROTC		0.0	CAP	0.288	MAJ	0.044	LTC	0.0	COL	0.0
PRA	NAV	ROTC		0.0	CAP	0.386	MAJ	0.049	LTC	0.0	COL	0.0
PRA	NR	ROTC		0.0	CAP	0.193	MAJ	0.119	LTC	0.0	COL	0.0
PRA	PIL	SMSO		0.0	CAP	0.290	MAJ	0.044	LTC	0.0	COL	0.0
PRA	NAV	SMSO		0.0	CAP	0.385	MAJ	0.050	LTC	0.0	COL	0.0
PRA	NR	SMSO		0.0	CAP	0.194	MAJ	0.119	LTC	0.0	COL	0.0
PRA	PIL	ROTC	LT	0.0		0.0		0.0		0.0		0.0
PRA	NAV	ROTC	LT	0.0		0.0		0.0		0.0		0.0
PRA	PIL	SMSO	LT	0.0		0.0		0.0		0.0		0.0
PRA	NAV	SMSO	LT	0.0		0.0		0.0		0.0		0.0
				0.0		0.0		0.0		0.0		0.0

Fig. 26--Printout page 4

OFFICER GRADE LIMITATIONS MODEL SAMPLE RUN PAGE 5

RATING TRANSFER DISTRIBUTIONS

COMPONENT	RATING	SOURCE OF COMMISSION	YOS	FLOW RATE	YOS	FLOW RATE	YOS	FLOW RATE	YOS	FLOW RATE	YOS	FLOW RATE	YOS	FLOW RATE
TRAN REG	PIL	AFA	2	1.000	0	0.0	0	0.0	0	0.0	0	0.0	0	0.0
TRAN REG	NAV	AFA	2	1.000	0	0.0	0	0.0	0	0.0	0	0.0	0	0.0
TRAN RES	PIL	ROTC	2	1.000	0	0.0	0	0.0	0	0.0	0	0.0	0	0.0
TRAN RES	NAV	ROTC	2	1.000	0	0.0	0	0.0	0	0.0	0	0.0	0	0.0
TRAN RES	PIL	SMSO	2	1.000	0	0.0	0	0.0	0	0.0	0	0.0	0	0.0
TRAN RES	NAV	SMSO	2	1.000	0	0.0	0	0.0	0	0.0	0	0.0	0	0.0

Fig. 27--Printout page 5

OFFICER GRADE LIMITATIONS MODEL SAMPLE RUN

AUGMENTATION DISTRIBUTIONS

	GRADE	RATING	SOURCE OF COMMISSION	YOS	FLOW RATE	YOS	FLOW RATE	YOS	FLOW RATE	YOS	FLOW RATE	YOS	FLOW RATE	YOS	FLOW RATE
AUG	LT	PIL	ROTC	4	0.962	5	0.038	0	0.0	0	0.0	0	0.0	0	0.0
AUG	CAP	PIL	ROTC	5	0.040	6	0.960	0	0.0	0	0.0	0	0.0	0	0.0
AUG	MAJ	PIL	ROTC	9	0.018	10	0.051	11	0.931	0	0.0	0	0.0	0	0.0
AUG	LT	NAV	ROTC	4	0.965	5	0.035	0	0.0	0	0.0	0	0.0	0	0.0
AUG	CAP	NAV	ROTC	5	0.027	6	0.878	7	0.095	0	0.0	0	0.0	0	0.0
AUG	MAJ	NAV	ROTC	9	0.018	10	0.051	11	0.931	0	0.0	0	0.0	0	0.0
AUG	LT	NR	ROTC	3	0.429	4	0.569	5	0.002	0	0.0	0	0.0	0	0.0
AUG	CAP	NR	ROTC	5	0.002	6	0.998	0	0.0	0	0.0	0	0.0	0	0.0
AUG	MAJ	NR	ROTC	9	0.018	10	0.051	11	0.931	0	0.0	0	0.0	0	0.0
AUG	LT	PIL	SMSO	4	0.962	5	0.038	0	0.0	0	0.0	0	0.0	0	0.0
AUG	CAP	PIL	SMSO	5	0.039	6	0.961	0	0.0	0	0.0	0	0.0	0	0.0
AUG	MAJ	PIL	SMSO	9	0.018	10	0.051	11	0.931	0	0.0	0	0.0	0	0.0
AUG	LT	NAV	SMSO	4	0.964	5	0.036	0	0.0	0	0.0	0	0.0	0	0.0
AUG	CAP	NAV	SMSO	5	0.027	6	0.878	7	0.095	0	0.0	0	0.0	0	0.0
AUG	MAJ	NAV	SMSO	9	0.018	10	0.051	11	0.931	0	0.0	0	0.0	0	0.0
AUG	LT	NR	SMSO	3	0.431	4	0.567	5	0.002	0	0.0	0	0.0	0	0.0
AUG	CAP	NR	SMSO	5	0.002	6	0.998	0	0.0	0	0.0	0	0.0	0	0.0
AUG	MAJ	NR	SMSO	9	0.018	10	0.051	11	0.931	0	0.0	0	0.0	0	0.0

Fig. 28--Printout page 6

OFFICER GRADE LIMITATIONS MODEL SAMPLE RUN PAGE 7

PROMOTION-AUGMENTATION DISTRIBUTIONS

	GRADE	RATING	SOURCE OF COMMISSION	YOS	FLOW RATE	YOS	FLOW RATE	YOS	FLOW RATE	YOS	FLOW RATE	YOS	FLOW RATE	YOS	FLOW RATE
PRAU	CAP	PIL	ROTC	4	0.017	5	0.983	0	0.0	0	0.0	0	0.0	0	0.0
PRAU	CAP	NAV	ROTC	4	0.019	5	0.981	0	0.0	0	0.0	0	0.0	0	0.0
PRAU	CAP	NR	ROTC	4	0.094	5	0.906	0	0.0	0	0.0	0	0.0	0	0.0
PRAU	CAP	PIL	SMSO	4	0.017	5	0.983	0	0.0	0	0.0	0	0.0	0	0.0
PRAU	CAP	NAV	SMSO	4	0.019	5	0.981	0	0.0	0	0.0	0	0.0	0	0.0
PRAU	CAP	NR	SMSO	4	0.092	5	0.908	0	0.0	0	0.0	0	0.0	0	0.0

Fig. 29--Printout page 7

OFFICER GRADE LIMITATIONS MODEL SAMPLE RUN

PROMOTION DISTRIBUTIONS

	COMPONENT	GRADE	RATING	SOURCE OF COMMISSION	YOS	FLOW RATE	YOS	FLOW RATE	YOS	FLOW RATE	YOS	FLOW RATE	YOS	FLOW RATE	YOS	FLOW RATE
PROM	REG	CAP	PIL	AFA	4	0.014	5	0.986	6	0.0	0	0.0	0	0.0	0	0.0
PROM	REG	MAJ	PIL	AFA	8	0.016	9	0.047	10	0.937	11	0.0	0	0.0	0	0.0
PROM	REG	LTC	PIL	AFA	14	0.081	15	0.069	16	0.850	17	0.0	0	0.0	0	0.0
PROM	REG	COL	PIL	AFA	20	0.246	21	0.054	22	0.700	23	0.0	0	0.0	0	0.0
PROM	REG	CAP	NAV	AFA	4	0.014	5	0.986	6	0.0	0	0.0	0	0.0	0	0.0
PROM	REG	MAJ	NAV	AFA	8	0.016	9	0.047	10	0.937	11	0.0	0	0.0	0	0.0
PROM	REG	LTC	NAV	AFA	14	0.081	15	0.069	16	0.850	17	0.0	0	0.0	0	0.0
PROM	REG	COL	NAV	AFA	20	0.246	21	0.054	22	0.700	23	0.0	0	0.0	0	0.0
PROM	REG	CAP	NR	AFA	4	0.014	5	0.986	6	0.0	0	0.0	0	0.0	0	0.0
PROM	REG	MAJ	NR	AFA	8	0.016	9	0.047	10	0.937	11	0.0	0	0.0	0	0.0
PROM	REG	LTC	NR	AFA	14	0.081	15	0.069	16	0.850	17	0.0	0	0.0	0	0.0
PROM	REG	COL	NR	AFA	20	0.246	21	0.054	22	0.700	23	0.0	0	0.0	0	0.0
PROM	RES	CAP	PIL	ROTC	4	0.019	5	0.981	6	0.0	0	0.0	0	0.0	0	0.0
PROM	REG	CAP	PIL	ROTC	5	1.000	6	0.0	0	0.0	0	0.0	0	0.0	0	0.0
PROM	RES	MAJ	PIL	ROTC	8	0.019	9	0.051	10	0.930	11	0.0	0	0.0	0	0.0
PROM	REG	MAJ	PIL	ROTC	8	0.016	9	0.047	10	0.937	11	0.0	0	0.0	0	0.0
PROM	REG	LTC	PIL	ROTC	14	0.081	15	0.069	16	0.850	17	0.0	0	0.0	0	0.0
PROM	REG	COL	PIL	ROTC	20	0.246	21	0.054	22	0.700	23	0.0	0	0.0	0	0.0
PROM	RES	CAP	NAV	ROTC	4	0.018	5	0.982	6	0.0	0	0.0	0	0.0	0	0.0
PROM	REG	CAP	NAV	ROTC	5	1.000	6	0.0	0	0.0	0	0.0	0	0.0	0	0.0
PROM	RES	MAJ	NAV	ROTC	8	0.019	9	0.051	10	0.930	11	0.0	0	0.0	0	0.0
PROM	REG	MAJ	NAV	ROTC	8	0.016	9	0.047	10	0.937	11	0.0	0	0.0	0	0.0
PROM	REG	LTC	NAV	ROTC	14	0.081	15	0.069	16	0.850	17	0.0	0	0.0	0	0.0
PROM	REG	COL	NAV	ROTC	20	0.246	21	0.054	22	0.700	23	0.0	0	0.0	0	0.0
PROM	RES	CAP	NR	ROTC	4	0.019	5	0.981	6	0.0	0	0.0	0	0.0	0	0.0
PROM	REG	CAP	NR	ROTC	4	0.004	5	0.996	6	0.0	0	0.0	0	0.0	0	0.0
PROM	RES	MAJ	NR	ROTC	8	0.019	9	0.051	10	0.931	11	0.0	0	0.0	0	0.0

Fig. 30--Printout page 8

OFFICER GRADE LIMITATIONS MODEL SAMPLE RUN

LOSS RATES

	COMPONENT	GRADE	RATING	SOURCE OF COMMISSION	YOS	FLOW RATE	YOS	FLOW RATE	YOS	FLOW RATE	YOS	FLOW RATE	YOS	FLOW RATE	YOS	FLOW RATE
LOSS	RES	LT	PIL	ROTC	2	0.015	3	0.015	4	0.015	5	0.990	6	1.000	0	0.0
LOSS	RES	CAP	PIL	ROTC	4	0.015	5	0.015	6	0.550	7	0.865	8	0.100	9	0.100
LOSS	RES	CAP	PIL	ROTC	10	0.100	11	1.000	0	0.0	0	0.0	0	0.0	0	0.0
LOSS	RES	MAJ	PIL	ROTC	8	0.100	9	0.100	10	0.100	11	1.000	0	0.0	0	0.0
LOSS	RES	LT	NAV	ROTC	2	0.015	3	0.015	4	0.015	5	0.990	6	1.000	0	0.0
LOSS	RES	CAP	NAV	ROTC	4	0.015	5	0.015	6	0.550	7	0.865	8	0.100	9	0.100
LOSS	RES	CAP	NAV	ROTC	10	0.100	11	1.000	0	0.0	0	0.0	0	0.0	0	0.0
LOSS	RES	MAJ	NAV	ROTC	8	0.100	9	0.100	10	0.100	11	1.000	0	0.0	0	0.0
LOSS	RES	LT	NR	ROTC	1	0.025	2	0.015	3	0.015	4	0.550	5	0.990	6	1.000
LOSS	RES	CAP	NR	ROTC	4	0.550	5	0.020	6	0.030	7	0.843	8	0.110	9	0.100
LOSS	RES	CAP	NR	ROTC	10	0.100	11	1.000	0	0.0	0	0.0	0	0.0	0	0.0
LOSS	RES	MAJ	NR	ROTC	8	0.110	9	0.100	10	0.100	11	1.000	0	0.0	0	0.0
LOSS	RES	LT	PIL	SMSO	2	0.015	3	0.015	4	0.015	5	0.990	6	1.000	0	0.0
LOSS	RES	CAP	PIL	SMSO	4	0.015	5	0.015	6	0.550	7	0.865	8	0.100	9	0.100
LOSS	RES	CAP	PIL	SMSO	10	0.100	11	1.000	0	0.0	0	0.0	0	0.0	0	0.0
LOSS	RES	MAJ	PIL	SMSO	8	0.100	9	0.100	10	0.100	11	1.000	0	0.0	0	0.0
LOSS	RES	LT	NAV	SMSO	2	0.015	3	0.015	4	0.015	5	0.990	6	1.000	0	0.0
LOSS	RES	CAP	NAV	SMSO	4	0.015	5	0.015	6	0.550	7	0.865	8	0.100	9	0.100
LOSS	RES	CAP	NAV	SMSO	10	0.100	11	1.000	0	0.0	0	0.0	0	0.0	0	0.0
LOSS	RES	MAJ	NAV	SMSO	8	0.100	9	0.100	10	0.100	11	1.000	0	0.0	0	0.0
LOSS	RES	LT	NR	SMSO	1	0.025	2	0.015	3	0.015	4	0.550	5	0.990	6	1.000
LOSS	RES	CAP	NR	SMSO	4	0.550	5	0.020	6	0.030	7	0.843	8	0.110	9	0.100
LOSS	RES	CAP	NR	SMSO	10	0.100	11	1.000	0	0.0	0	0.0	0	0.0	0	0.0
LOSS	RES	MAJ	NR	SMSO	8	0.110	9	0.100	10	0.100	11	1.000	0	0.0	0	0.0
LOSS	REG	LT	PIL	AFA	2	0.015	3	0.015	4	0.015	5	0.990	6	1.000	0	0.0
LOSS	REG	CAP	PIL	AFA	4	0.015	5	0.015	6	0.200	7	0.050	8	0.020	9	0.020
LOSS	REG	CAP	PIL	AFA	10	0.020	11	1.000	0	0.0	0	0.0	0	0.0	0	0.0

Fig. 31--Printout page 10

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OFFICER GRADE LIMITATIONS MODEL SAMPLE RUN                PAGE  15

              DATA GROUPS READ

8 GROUPS OF DATA WERE READ.  THEY ARE AS FOLLOWS:

          1     GROUP(S) OF TYPE    GRAU

          1     GROUP(S) OF TYPE    PRP

          1     GROUP(S) OF TYPE    PRA

          1     GROUP(S) OF TYPE    PROM

          1     GROUP(S) OF TYPE    AUG

          1     GROUP(S) OF TYPE    PRAU

          1     GROUP(S) OF TYPE    TRAN

          0     GROUP(S) OF TYPE    PTAU

          1     GROUP(S) OF TYPE    LOSS
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Fig. 32--Printout page 15

OFFICER GRADE LIMITATIONS MODEL SAMPLE RUN

OFFICER FORCE GRADE DISTRIBUTION
RATING PIL SOURCE OF COMMISSION ROTC

	RESERVE COMPONENT						REGULAR COMPONENT						BOTH RESERVE AND REGULAR COMPONENTS					
YEAR	LIEUT	CAPT	MAJOR	LTCOL	CL/GN	TOTAL	LIEUT	CAPT	MAJOR	LTCOL	CL/GN	TOTAL	LIEUT	CAPT	MAJOR	LTCOL	CL/GN	TOTAL
2	1449					1449							1449					1449
3	1427					1427							1427					1427
4	1041	14				1054	347	4				351	1388	18				1406
5	38	724				761	31	593				624	68	1316				1385
6	0	623				623	0	674				674	1	1297				1297
7		280				280		663				663		944				944
8		37	0			38		642	8			650		680	9			688
9		32	1			34		606	32			638		638	33			671
10		6	23			29		122	504			626		128	527			655
11		5	0			6		120	514			634		125	514			640
12									504			504			504			504
13									494			494			494			494
14									458	26		484			458	26		484
15									426	48		474			426	48		474
16									143	322		465			143	322		465
17									140	316		456			140	316		456
18									137	310		446			137	310		446
19									134	303		438			134	303		438
20									131	262	36	429			131	262	36	429
21										238	41	280				238	41	280
22										132	142	274				132	142	274
23										130	139	268				130	139	268
24										122	130	252				122	130	252
25										101	121	222				101	121	222
26										82	104	186				82	104	186
27											96	96					96	96
28											88	88					88	88
29											79	79					79	79
30											71	71					71	71
TOTAL	3955.	1722.	25.	0.	0.	5702.	378.	3424.	3625.	2393.	1048.	10868.	4333.	5146.	3650.	2393.	1048.	16570.
AVERAGE YEAR OF SERVICE																		
	2.92	5.86	9.92	0.0	0.0	3.83	4.08	7.24	13.41	19.51	25.06	13.61	3.02	6.78	13.39	19.51	25.06	10.25

Fig. 33--Printout page 21

OFFICER GRADE LIMITATIONS MODEL SAMPLE RUN

OFFICER FORCE GRADE DISTRIBUTION
RATING NAV   SOURCE OF COMMISSION ROTC

	RESERVE COMPONENT						REGULAR COMPONENT						BOTH RESERVE AND REGULAR COMPONENTS					
YEAR	LIEUT	CAPT	MAJOR	LTCOL	CL/GN	TOTAL	LIEUT	CAPT	MAJOR	LTCOL	CL/GN	TOTAL	LIEUT	CAPT	MAJOR	LTCOL	CL/GN	TOTAL
2	661					661							661					661
3	651					651							651					651
4	488	7				494	146	2				147	633	8				642
5	19	360				379	12	240				253	32	601				632
6	0	309				309	0	283				283	0	592				592
7		134				134		284				284		418				418
8		18	0			18		275	3			278		292	4			296
9		15	1			16		259	14			273		274	14			289
10		3	11			14		52	216			268		55	227			282
11		3	0			3		51	221			272		54	221			275
12									217			217			217			217
13									212			212			212			212
14									197	11		208			197	11		208
15									183	21		204			183	21		204
16									61	139		200			61	139		200
17									60	136		196			60	136		196
18									59	133		192			59	133		192
19									58	130		188			58	130		188
20									57	113	15	184			57	113	15	184
21										100	17	118				100	17	118
22										56	60	115				56	60	115
23										55	58	113				55	58	113
24										50	54	104				50	54	104
25										41	49	90				41	49	90
26										32	41	73				32	41	73
27											37	37					37	37
28											33	33					33	33
29											30	30					30	30
30											26	26					26	26
TOTAL	1820.	848.	12.	0.	0.	2680.	158.	1446.	1557.	1016.	420.	4597.	1978.	2294.	1569.	1016.	420.	7277.
AVERAGE YEAR OF SERVICE																		
	2.93	5.84	9.92	0.0	0.0	3.88	4.08	7.27	13.41	19.45	24.93	13.55	3.02	6.74	13.39	19.45	24.93	9.99

Fig. 34--Printout page 22

OFFICER GRADE LIMITATIONS MODEL SAMPLE RUN

OFFICER FORCE GRADE DISTRIBUTION
RATING NR SOURCE OF COMMISSION ROTC

	RESERVE COMPONENT						REGULAR COMPONENT						BOTH RESERVE AND REGULAR COMPONENTS					
YEAR	LIEUT	CAPT	MAJOR	LTCOL	CL/GN	TOTAL	LIEUT	CAPT	MAJOR	LTCOL	CL/GN	TOTAL	LIEUT	CAPT	MAJOR	LTCOL	CL/GN	TOTAL
1	4500					4500							4500					4500
2	2277					2277							2277					2277
3	2014					2014	229					229	2243					2243
4	1665	13				1678	527	4				531	2192	18				2209
5	36	695				731	27	520				547	64	1215				1278
6	0	586				586	0	605				605	1	1190				1191
7		568				568		587				587		1155				1155
8		88	1			89		562	7			569		650	8			658
9		75	3			78		525	28			553		600	31			631
10		14	54			68		106	439			545		120	493			612
11		13	0			13		104	478			582		116	478			595
12									469			469			469			469
13									459			459			459			459
14									426	24		450			426	24		450
15									397	45		441			397	45		441
16									133	300		432			133	300		432
17									130	294		424			130	294		424
18									127	288		415			127	288		415
19									125	282		407			125	282		407
20									122	244	33	399			122	244	33	399
21										219	38	257				219	38	257
22										122	130	252				122	130	252
23										119	128	247				119	128	247
24										111	119	230				111	119	230
25										91	109	200				91	109	200
26										73	93	166				73	93	166
27											84	84					84	84
28											77	77					77	77
29											69	69					69	69
30											62	62					62	62
TOTAL	10493.	2052.	58.	0.	0.	12603.	783.	3012.	3340.	2212.	942.	10289.	11276.	5064.	3398.	2212.	942.	22892.
AVERAGE YEAR OF SERVICE																		
	2.09	6.18	9.91	0.0	0.0	2.79	3.74	7.23	13.45	19.48	25.00	13.24	2.21	6.80	13.39	19.48	25.00	7.49

Fig. 35--Printout page 23

OFFICER GRADE LIMITATIONS MODEL SAMPLE RUN

OFFICER FORCE GRADE DISTRIBUTION
RATING RAT SOURCE OF COMMISSION ROTC

	RESERVE COMPONENT						REGULAR COMPONENT						BOTH RESERVE AND REGULAR COMPONENTS					
YEAR	LIEUT	CAPT	MAJOR	LTCOL	CL/GN	TOTAL	LIEUT	CAPT	MAJOR	LTCOL	CL/GN	TOTAL	LIEUT	CAPT	MAJOR	LTCOL	CL/GN	TOTAL
2	2110					2110							2110					2110
3	2079					2079							2079					2079
4	1528	20				1549	492	6				499	2021	27				2048
5	57	1084				1141	43	833				876	100	1917				2017
6	1	932				932	0	957				957	1	1888				1889
7		414				414		947				947		1361				1361
8		55	1			56		917	11			928		972	12			984
9		48	2			50		865	46			910		912	48			960
10		9	34			43		174	719			894		183	753			937
11		8	0			8		171	735			906		179	736			915
12									721			721			721			721
13									706			706			706			706
14									655	38		692			655	38		692
15									610	69		678			610	69		678
16									204	461		665			204	461		665
17									200	452		651			200	452		651
18									196	443		638			196	443		638
19									192	434		626			192	434		626
20									188	375	51	613			188	375	51	613
21										339	58	397				339	58	397
22										188	201	389				188	201	389
23										184	197	381				184	197	381
24										172	184	356				172	184	356
25										142	170	312				142	170	312
26										114	145	259				114	145	259
27											133	133					133	133
28											122	122					122	122
29											109	109					109	109
30											98	98					98	98
TOTAL	5775.	2570.	37.	0.	0.	8382.	536.	4870.	5182.	3409.	1468.	15465.	6311.	7440.	5219.	3409.	1468.	23847.
AVERAGE YEAR OF SERVICE																		
	2.92	5.85	9.92	0.0	0.0	3.85	4.08	7.25	13.41	19.49	25.02	13.59	3.02	6.77	13.39	19.49	25.02	10.17

Fig. 36--Printout page 24

OFFICER GRADE LIMITATIONS MODEL SAMPLE RUN

OFFICER FORCE GRADE DISTRIBUTION
RATING ALL SOURCE OF COMMISSION ROTC

YEAR	RESERVE COMPONENT LIEUT	CAPT	MAJOR	LTCOL	CL/GN	TOTAL	REGULAR COMPONENT LIEUT	CAPT	MAJOR	LTCOL	CL/GN	TOTAL	BOTH RESERVE AND REGULAR COMPONENTS LIEUT	CAPT	MAJOR	LTCOL	CL/GN	TOTAL
1	4500					4500							4500					4500
2	4388					4388							4388					4388
3	4093					4093	229					229	4322					4322
4	3193	34				3227	1019	11				1030	4212	44				4257
5	93	1778				1872	70	1353				1424	164	3132				3295
6	1	1517				1518	1	1561				1562	2	3079				3080
7		982				982		1534				1534		2516				2516
8		143	2			145		1479	18			1497		1622	20			1642
9		123	5			128		1389	74			1463		1513	79			1591
10		23	88			111		280	1158			1438		303	1246			1549
11		21	0			21		274	1214			1488		295	1214			1509
12									1189			1189			1189			1189
13									1166			1166			1166			1166
14									1080	62		1142			1080	62		1142
15									1006	113		1120			1006	113		1120
16									336	761		1097			336	761		1097
17									330	746		1075			330	746		1075
18									323	731		1054			323	731		1054
19									317	716		1033			317	716		1033
20									310	618	83	1012			310	618	83	1012
21										558	96	654				558	96	654
22										310	331	641				310	331	641
23										304	325	628				304	325	628
24										283	303	586				283	303	586
25										233	279	512				233	279	512
26										187	238	425				187	238	425
27											217	217					217	217
28											198	198					198	198
29											178	178					178	178
30											160	160					160	160
TOTAL	16268.	4622.	95.	0.	0	20985.	1319.	7882.	8522.	5621.	2410.	25754.	17587.	12504.	8617.	5621.	2410.	45739.
AVERAGE YEAR OF SERVICE																		
	2.38	6.00	9.91	0.0	0.0	3.21	3.88	7.24	13.43	19.49	25.01	13.45	2.50	6.78	13.39	19.49	25.01	8.86

Fig. 37--Printout page 25

OFFICER GRADE LIMITATIONS MODEL SAMPLE RUN

OFFICER FORCE GRADE DISTRIBUTION
RATING ALL SOURCE OF COMMISSION ALL

	RESERVE COMPONENT						REGULAR COMPONENT						BOTH RESERVE AND REGULAR COMPONENTS					
YEAR	LIEUT	CAPT	MAJOR	LTCOL	CL/GN	TOTAL	LIEUT	CAPT	MAJOR	LTCOL	CL/GN	TOTAL	LIEUT	CAPT	MAJOR	LTCOL	CL/GN	TOTAL
1	8399					8399	960					960	9359					9359
2	8189					8189	936					936	9125					9125
3	7568					7568	1419					1419	8988					8988
4	6015	60				6075	2747	31				2778	8762	91				8853
5	167	3165				3331	164	3175				3339	331	6340				6670
6	2	2703				2705	2	3496				3498	3	6200				6203
7		1909				1909		3309				3309		5218				5218
8		283	4			286		3174	40			3213		3457	43			3500
9		243	10			253		2980	158			3138		3223	168			3391
10		45	174			219		596	2487			3083		641	2662			3303
11		40	7			48		584	2587			3172		625	2594			3219
12									2536			2536			2536			2536
13									2485			2485			2485			2485
14									2303	132		2435			2303	132		2435
15									2145	242		2386			2145	242		2386
16									717	1622		2339			717	1622		2339
17									703	1589		2292			703	1589		2292
18									689	1557		2246			689	1557		2246
19									675	1526		2201			675	1526		2201
20									661	1318	178	2157			661	1318	178	2157
21										1190	205	1394				1190	205	1394
22										660	706	1366				660	706	1366
23										647	692	1339				647	692	1339
24										603	645	1248				603	645	1248
25										496	595	1091				496	595	1091
26										398	507	905				398	507	905
27											462	462					462	462
28											422	422					422	422
29											379	379					379	379
30											340	340					340	340
TOTAL	30339.	8448.	195.	0.	0.	38982.	6228.	17345.	18185.	11980.	5130.	58868.	36567.	25793.	18380.	11980.	5130.	97849.
AVERAGE YEAR OF SERVICE																		
	2.39	6.04	9.95	0.0	0.0	3.21	3.04	7.19	13.42	19.49	25.01	12.73	2.50	6.81	13.39	19.49	25.01	8.94

Fig. 38--Printout page 40

Pages 43 and 44 illustrate the rating distribution tables given for lieutenant colonel and lower grades, and for colonel and higher grades.

OFFICER FLOWS

Pages 53 to 55 of the printout illustrate the officer flow reports. These tables show officer flows into and out of each state by component, grade, rating, source of commission, and year of service. The ROTC flows aggregated by component and rating are shown. At the bottom of page 53 we see the flows for all ROTC lieutenants. Note that in YOS 1 the current officer state is 4500 lieutenants, which is the number of ROTC accessions. Of these 4500, 113 are lost to the Air Force (attrition), 2110 receive rating transfers, and 2277 remain nonrated lieutenants. Looking at flows out of year 4 we see that 384 officers receive augmentations and are promoted to captain and 2711 officers are simply promoted to captain. If we move to page 54 now and look at flows into ROTC captains in service year 5, we see the 384 promotion/augmentations and 2711 promotions that flowed out of year 4 as lieutenants. Pages 54 and 55 show the flows for majors, lieutenant colonels, and colonels/generals. Note the summary lines following each flow table.

PROGRESSION MODEL INPUTS AND OFFICER FLOWS

Rates and Flows

The next report is illustrated on output page 64; it shows how the officers move through the various phases of the progression model computation process as they flow from one year of service to the next. The Officer Force Progression Model moves officers from one year of service to the next in the following steps:

- Attrition
- Rating transfers (if Lieutenant)
- Augmentation
- Promotion

OFFICER GRADE LIMITATIONS MODEL SAMPLE RUN

OFFICER FORCE RATING DISTRIBUTION
LIEUTENANT COLONEL AND BELOW
SOURCE OF COMMISSION ROTC

YEAR OF SERVICE	PILOT			NAVIGATOR			NONRATED			TOTAL OVER ALL RATINGS		
	RESERVE	REGULAR	TOTAL	RESERVE	REGULAR	TOTAL	RESERVE	REGULAR	TOTAL	RESERVE	REGULAR	TOTAL
1							4500		4500	4500		4500
2	1449		1449	661		661	2277		2277	4388		4388
3	1427		1427	651		651	2014	229	2243	4093	229	4322
4	1054	351	1406	494	147	642	1678	531	2209	3227	1030	4257
5	761	624	1385	379	253	632	731	547	1278	1872	1424	3295
6	623	674	1297	309	283	592	586	605	1191	1518	1562	3080
7	280	663	944	134	284	418	568	587	1155	982	1534	2516
8	38	650	688	18	278	296	89	569	658	145	1497	1642
9	34	638	671	16	273	289	78	553	631	128	1463	1591
10	29	626	655	14	268	282	68	545	612	111	1438	1549
11	6	634	640	3	272	275	13	582	595	21	1488	1509
12		504	504		217	217		469	469		1189	1189
13		494	494		212	212		459	459		1166	1166
14		484	484		208	208		450	450		1142	1142
15		474	474		204	204		441	441		1120	1120
16		465	465		200	200		432	432		1097	1097
17		456	456		196	196		424	424		1075	1075
18		446	446		192	192		415	415		1054	1054
19		438	438		188	188		407	407		1033	1033
20		393	393		169	169		366	366		929	929
21		238	238		100	100		219	219		558	558
22		132	132		56	56		122	122		310	310
23		130	130		55	55		119	119		304	304
24		122	122		50	50		111	111		283	283
25		101	101		41	41		91	91		233	233
26		82	82		32	32		73	73		187	187
TOTAL	5702.	9820.	15522.	2680.	4177.	6857.	12603.	9347.	21950.	20985.	23344.	44329.
AVERAGE YEAR OF SERVICE	3.83	12.39	9.25	3.88	12.40	9.07	2.79	12.06	6.74	3.21	12.26	7.98

Fig. 39--Printout page 43

OFFICER GRADE LIMITATIONS MODEL SAMPLE RUN PAGE 44

OFFICER FORCE RATING DISTRIBUTION
COLONEL/GENERALS
SOURCE OF COMMISSION ROTC

YEAR OF SERVICE	PILOT			NAVIGATOR			NONRATED			TOTAL OVER ALL RATINGS		
	RESERVE	REGULAR	TOTAL	RESERVE	REGULAR	TOTAL	RESERVE	REGULAR	TOTAL	RESERVE	REGULAR	TOTAL
21		41	41		17	17		38	38		96	96
22		142	142		60	60		130	130		331	331
23		139	139		58	58		128	128		325	325
24		130	130		54	54		119	119		303	303
25		121	121		49	49		109	109		279	279
26		104	104		41	41		93	93		238	238
27		96	96		37	37		84	84		217	217
28		88	88		33	33		77	77		198	198
29		79	79		30	30		69	69		178	178
30		71	71		26	26		62	62		160	160
TOTAL	0.	1012.	1012.	0.	405.	405.	0.	909.	909.	0.	2327.	2327.
AVERAGE YEAR OF SERVICE												
	0.0	25.24	25.24	0.0	25.12	25.12	0.0	25.18	25.18	0.0	25.19	25.19

Fig. 40--Printout page 44

OFFICER GRADE LIMITATIONS MODEL SAMPLE RUN PAGE 53

OFFICER FLOWS

COMPONENT ALL GRADE COL RATING ALL SOURCE OF COMMISSION AFA

YOS	FLOWS INTO THE CURRENT OFFICER STATE: LATERAL FLOW	RATING TRANSFERS ONLY	RATING TRANSFERS WITH AUG	AUGMENTATIONS ONLY	AUGMENTATIONS WITH PRO	PROMOS ONLY	CURRENT OFFICER STATE	FLOWS OUT OF THE CURRENT OFFICER STATE: ATTRITION	RATING TRANSFERS ONLY	RATING TRANSFERS WITH AUG	AUGMENTATIONS ONLY	AUGMENTATIONS WITH PRO	PROMOS ONLY	LATERAL FLOW
20						33	33	2						31
21	31					7	38	1						38
22	38					95	132	3						130
23	130						130	8						121
24	121						121	9						112
25	112						112	16						96
26	96						96	8						88
27	88						88	7						81
28	81						81	8						72
29	72						72	7						65
30	65						65	65						
TOT	835.	0.	0.	0.	0.	135.	970.	135.	0.	0.	0.	0.	0.	835.

TOTAL RATING TRANSFERS 0 IN 0 OUT TOTAL AUGMENTATIONS 0 IN 0 OUT TOTAL PROMOTIONS 135 IN 0 OUT

COMPONENT ALL GRADE LT RATING ALL SOURCE OF COMMISSION ROTC

YOS	FLOWS INTO THE CURRENT OFFICER STATE: LATERAL FLOW	RATING TRANSFERS ONLY	RATING TRANSFERS WITH AUG	AUGMENTATIONS ONLY	AUGMENTATIONS WITH PRO	PROMOS ONLY	CURRENT OFFICER STATE	FLOWS OUT OF THE CURRENT OFFICER STATE: ATTRITION	RATING TRANSFERS ONLY	RATING TRANSFERS WITH AUG	AUGMENTATIONS ONLY	AUGMENTATIONS WITH PRO	PROMOS ONLY	LATERAL FLOW
1	4500						4500	113	2110					2277
2	2277	2110					4388	66			229			4093
3	4093			229			4322	65			796	9	36	3417
4	3417			796			4212	954			20	384	2711	144
5	144			20			164	162						2
6	2						2	2						
TOT	14432.	2110.	0.	1045.	0.	0.	17587.	1361.	2110.	0.	1045.	392.	2747.	9932.

TOTAL RATING TRANSFERS 2110 IN 2110 OUT TOTAL AUGMENTATIONS 1045 IN 1437 OUT TOTAL PROMOTIONS 0 IN 3139 OUT

Fig. 41--Printout page 53

OFFICER GRADE LIMITATIONS MODEL SAMPLE RUN

OFFICER FLOWS

COMPONENT ALL GRADE CAP RATING ALL SOURCE OF COMMISSICN ROTC

YOS	FLOWS INTO THE CURRENT OFFICER STATE						CURRENT OFFICER STATE	FLOWS OUT OF THE CURRENT OFFICER STATE						
	LATERAL FLOW	RATING TRANSFERS		AUGMENTATIONS		PROMOS ONLY		ATTRITION	RATING TRANSFERS		AUGMENTATIONS		PROMOS ONLY	LATERAL FLOW
		ONLY	WITH AUG	ONLY	WITH PRO				ONLY	WITH AUG	ONLY	WITH PRO		
4					9	36	44	8			5			31
5	31			5	384	2711	3132	53			231			2847
6	2847			231			3079	562			5			2511
7	2511			5			2516	874					20	1622
8	1622						1622	50					59	1513
9	1513						1513	40					1169	303
10	303						303	8						295
11	295						295	295						
TOT	9123.	0.	0.	242.	392.	2747.	12504.	1891.	0.	0.	242.	0.	1249.	9123.

TOTAL RATING TRANSFERS 0 IN 0 OUT TOTAL AUGMENTATIONS 634 IN 242 OUT TOTAL PROMOTIONS 3139 IN 1249 OUT

COMPONENT ALL GRADE MAJ RATING ALL SOURCE OF COMMISSICN ROTC

YOS	FLOWS INTO THE CURRENT OFFICER STATE						CURRENT OFFICER STATE	FLOWS OUT OF THE CURRENT OFFICER STATE						
	LATERAL FLOW	RATING TRANSFERS		AUGMENTATIONS		PROMOS ONLY		ATTRITION	RATING TRANSFERS		AUGMENTATIONS		PROMOS CNLY	LATERAL FLOW
		ONLY	WITH AUG	ONLY	WITH PRO				ONLY	WITH AUG	ONLY	WITH PRO		
8						20	20	1			2			18
9	18			2		59	79	2			4			72
10	72			4		1169	1246	32			79			1135
11	1135			79			1214	25						1189
12	1189						1189	24						1166
13	1166						1166	23					62	1080
14	1080						1080	22					53	1006
15	1006						1006	20					650	336
16	336						336	7						330
17	330						330	7						323
18	323						323	6						317
19	317						317	6						310
20	310						310	310						
TOT	7284.	0.	0.	85.	0.	1249.	8617.	484.	0.	0.	85.	0.	764.	7284.

TOTAL RATING TRANSFERS 0 IN 0 OUT TOTAL AUGMENTATIONS 85 IN 85 OUT TOTAL PROMOTIONS 1249 IN 764 OUT

Fig. 42--Printout page 54

OFFICER GRADE LIMITATIONS MODEL SAMPLE RUN

OFFICER FLOWS

COMPONENT ALL GRADE LTC RATING ALL SOURCE OF COMMISSION ROTC

YOS	FLOWS INTO THE CURRENT OFFICER STATE: LATERAL FLOW	RATING TRANSFERS: ONLY	RATING TRANSFERS: WITH AUG	AUGMENTATIONS: ONLY	AUGMENTATIONS: WITH PRO	PROMOS ONLY	CURRENT OFFICER STATE	FLOWS OUT OF THE CURRENT OFFICER STATE: ATTRITION	RATING TRANSFERS: ONLY	RATING TRANSFERS: WITH AUG	AUGMENTATIONS: ONLY	AUGMENTATIONS: WITH PRO	PROMOS ONLY	LATERAL FLOW
14						62	62	1						61
15	61					53	113	2						111
16	111					650	761	15						746
17	746						746	15						731
18	731						731	15						716
19	716						716	14					83	618
20	618						618	42					18	558
21	558						558	11					237	310
22	310						310	6						304
23	304						304	21						283
24	283						283	50						233
25	233						233	46						187
26	187						187	187						
TOT	4857.	0.	0.	0.	0.	764.	5621.	425.	0.	0.	0.	0.	339.	4857.

TOTAL RATING TRANSFERS 0 IN 0 OUT TOTAL AUGMENTATIONS 0 IN 0 OUT TOTAL PROMOTIONS 764 IN 339 OUT

COMPONENT ALL GRADE COL RATING ALL SOURCE OF COMMISSION ROTC

YOS	FLOWS INTO THE CURRENT OFFICER STATE: LATERAL FLOW	RATING TRANSFERS: ONLY	RATING TRANSFERS: WITH AUG	AUGMENTATIONS: ONLY	AUGMENTATIONS: WITH PRO	PROMOS ONLY	CURRENT OFFICER STATE	FLOWS OUT OF THE CURRENT OFFICER STATE: ATTRITION	RATING TRANSFERS: ONLY	RATING TRANSFERS: WITH AUG	AUGMENTATIONS: ONLY	AUGMENTATIONS: WITH PRO	PROMOS ONLY	LATERAL FLOW
20						83	83	6						78
21	78					18	96	2						94
22	94					237	331	7						325
23	325						325	22						303
24	303						303	23						279
25	279						279	41						238
26	238						238	21						217
27	217						217	19						198
28	198						198	20						178
29	178						178	18						160
30	160						160	160						
TOT	2071.	0.	0.	0.	0.	339.	2410.	339.	0.	0.	0.	0.	0.	2071.

TOTAL RATING TRANSFERS 0 IN 0 OUT TOTAL AUGMENTATIONS 0 IN 0 OUT TOTAL PROMOTIONS 339 IN 0 OUT

Fig. 43--Printout page 55

OFFICER GRADE LIMITATIONS MODEL SAMPLE RUN

OFFICER FLOWS AND IMPLIED FORWARD COMPUTATION RATES

COMPONENT RES GRADE LT RATING PIL SOURCE OF COMMISSION ROTC

YEAR	INITIAL OFFICER STATE	ATTRITION PHASE LOSSES	LOSS RATE	RESIDUE	RATING TRANSFER PHASE TRNSFRS IN	RESIDUE	AUGMENTATION PHASE AUGMNTS OUT	AUGMENT RATE	RESIDUE	PROMOTION PHASE PROMOS	PROMO RATE	RESIDUE
1					1449	1449			1449			1449
2	1449	22	0.0150	1427		1427			1427			1427
3	1427	21	0.0150	1406		1406	351	0.2499	1054	14 OUT	0.0131	1041
4	1041	16	0.0150	1025		1025	274	0.2670	751	714 OUT	0.9499	38
5	38	37	0.9900	0		0			0			0
6	0	0	1.0000									

COMPONENT RES GRADE CAP RATING PIL SOURCE OF COMMISSION ROTC

YEAR	INITIAL OFFICER STATE	ATTRITION PHASE LOSSES	LOSS RATE	RESIDUE	AUGMENTATION PHASE AUGMNTS OUT	AUGMENT RATE	RESIDUE	PROMOTION PHASE PROMOS	PROMO RATE	RESIDUE
4	14	0	0.0150	14	4	0.2746	10	714 IN		724
5	724	11	0.0150	713	90	0.1259	623			623
6	623	343	0.5500	280			280			280
7	280	242	0.8650	38			38	0 OUT	0.0124	37
8	37	4	0.1000	34			34	1 OUT	0.0376	32
9	32	3	0.1000	29			29	23 OUT	0.7910	6
10	6	1	0.1000	5			5			5
11	5	5	1.0000							

COMPONENT RES GRADE MAJ RATING PIL SOURCE OF COMMISSION ROTC

YEAR	INITIAL OFFICER STATE	ATTRITION PHASE LOSSES	LOSS RATE	RESIDUE	AUGMENTATION PHASE AUGMNTS OUT	AUGMENT RATE	RESIDUE	PROMOTION PHASE PROMOS	PROMO RATE	RESIDUE
8	0	0	0.1000	0	0	0.9402	0	1 IN		1
9	1	0	0.1000	1	1	0.9730	0	23 IN		23
10	23	2	0.1000	21	21	0.9922	0			0
11	0	0	1.0000							

Fig. 44--Printout page 64

The progression model does not handle combination flows (such as promotion/augmentation, etc.) separately, as does the grade limitations model. Instead, it can promote officers who received regular commissions in the augmentation step of the same cycle. This, of course, has the same effect. The reports illustrate the actions taken at each step of the cycle from the progression model point of view.

The examples shown refer to ROTC pilots holding reserve commissions, and each report deals with a different grade. At the top of page 64 we see the report for lieutenants. In YOS 1 there are no officers in the current state (the initial officer state), and so there are no losses. There are, however, 1449 rating transfers from nonrated to pilot; and the report shows this as a flow into the cycle. Continuing, no augmentations and no promotions take place thus yielding 1449 officers in YOS 2. This is precisely the number reported in the Officer Force Grade Distribution report on output page 21 and the officer Force Rating Distribution Report on output page 43.

For another example, note that there are 1041 officers in the initial state for YOS 4; 16 are lost, leaving 1025 and implying a loss rate of 0.015. No rating transfers take place, and 274 augmentations to regular take place leaving 751 officers in the state and implying an augmentation rate of 0.267. Finally we see 714 promotions to captain taking place, which leaves 38 and gives a promotion rate of 0.9499. Now, looking at the report for captains in the middle of page 64, we see 714 flows into the YOS 4 promotion phase. These are the 714 officers who were promoted out of lieutenant.

Recall that the rating transfer rates, augmentation rates, and loss rates just generated are given to the progression model, but that the promotion rates are not. The promotion opportunities and percentages needed are calculated and printed out separately by the grade limitations model.

Accessions

Page 90 shows the annual accessions of each source of commission, the number of rating transfers (UPT and UNT graduates), and the fraction of the accessions that receive each type of rating transfer.

OFFICER GRADE LIMITATIONS MODEL SAMPLE RUN PAGE 90

RATING TRANSFERS AS FRACTIONS OF ANNUAL ACCESSIONS

YEAR	SOURCE OF COMMISSION AFA ANNUAL ACCESSIONS 960				SOURCE OF COMMISSION ROTC ANNUAL ACCESSIONS 4500				SOURCE OF COMMISSION OTS ANNUAL ACCESSIONS 3899			
	PILOTS		NAVIGATORS		PILOTS		NAVIGATORS		PILOTS		NAVIGATORS	
	NUMBER OF UPT GRADS.	FRACTION OF ANNUAL ACCESSIONS	NUMBER OF UNT GRADS.	FRACTION OF ANNUAL ACCESSIONS	NUMBER OF UPT GRADS.	FRACTION OF ANNUAL ACCESSIONS	NUMBER OF UNT GRADS.	FRACTION OF ANNUAL ACCESSIONS	NUMBER OF UPT GRADS.	FRACTION OF ANNUAL ACCESSIONS	NUMBER OF UNT GRADS.	FRACTION OF ANNUAL ACCESSIONS
1	608.	0.63296	94.	0.09793	1449.	0.32200	661.	0.14698	420.	0.10772	522.	0.13383

Fig. 45--Printout page 90

Promotion Parameters

The progression model's promotion parameters for captains and majors, as calculated by the grade limitations model, are shown on page 91. In the progression model, all promotions into a given grade are constrained to take place in a four-year promotion "zone". This four-year zone is broken up into two subzones of two years each. The last two years of the promotion zone are known as the "primary zone" and the first two as "below-the-zone" years. The first year of the primary zone (i.e. the third year of the entire promotion zone) is known as the "phase point year". Going from left to right on page 91, we see the cumulative promotion opportunities for the 1st and 2nd years of the primary zone for the category identified, then the percentage of the total promotions which take place below the zone. The fourth column of numbers is the percentage of below-the-zone promotions which take place in the 1st year below-the-zone, and the fifth column is the phase point year. These five numbers are the inputs needed by the progression model; the eligibles and total promotions were included in the output for purposes of checking and to make it easier for the user to see what is going on.

AUGMENTATION OPPORTUNITIES

Although the progression models use augmentation rates, policy planners often need to know augmentation opportunities. Page 93 of the printout shows the augmentation opportunities for the sample run. Augmentation opportunity is defined to be the percentage of the total number of officers with a given grade and rating who hold regular commissions. In contrast the augmentation rate is the percentage of officers who are not lost to the Air Force and who do not receive rating transfers that hold regular commissions. The difference is that opportunity is calculated before losses and rating transfers are taken into account, while rates are calculated after losses and rating transfers are accounted for.

REGENERATION OF THE GRADE LIMITATIONS MODEL INPUTS

Pages 94, 96, and 99 show excerpts from the reports of the grade limitations model's regeneration of its own inputs. Page 94, at the top, shows the grade limitations or authorizations (summary of officer state) and, below, all the various flow ratios required. Note that,

OFFICER GRADE LIMITATIONS MODEL SAMPLE RUN

IMPLIED FORWARD COMPUTATION PROMOTION PARAMETERS

GRADE	RATING	SOURCE OF COMMISSION	CUMULATIVE PROMOTION OPPORTUNITY: FIRST YEAR OF PRIMARY ZONE	CUMULATIVE PROMOTION OPPORTUNITY: SECOND YEAR OF PRIMARY ZONE	BELOW-THE-ZONE PROMOTIONS: BELOW-THE-ZONE PROMOTION PCT.	BELOW-THE-ZONE PROMOTIONS: FIRST YEAR BELOW THE-ZONE PERCENT	PROMOTION PHASE POINT	ELIGIBLES	PROMOTIONS
CAP	PIL	AFA	95.05	95.05	1.40	0.0	5	581	552
CAP	PIL	ROTC	95.05	95.05	1.39	0.0	5	1385	1317
CAP	PIL	OTS	95.04	95.04	1.39	0.0	5	401	382
CAP	PIL	ALL	95.05	95.05	1.39	0.0	5	2367	2250
CAP	NAV	AFA	95.19	95.19	1.40	0.0	5	90	86
CAP	NAV	ROTC	95.02	95.02	1.41	0.0	5	632	601
CAP	NAV	OTS	94.92	94.92	1.41	0.0	5	499	473
CAP	NAV	ALL	94.99	94.99	1.41	0.0	5	1221	1160
CAP	NR	AFA	95.07	95.07	1.40	0.0	5	224	213
CAP	NR	ROTC	95.02	95.02	1.45	0.0	5	1278	1222
CAP	NR	OTS	95.09	95.09	1.47	0.0	5	1581	1513
CAP	NR	ALL	95.06	95.06	1.45	0.0	5	3083	2948
CAP	ALL	AFA	95.07	95.07	1.40	0.0	5	894	850
CAP	ALL	ROTC	95.03	95.03	1.42	0.0	5	3295	3139
CAP	ALL	OTS	95.05	95.05	1.44	0.0	5	2481	2368
CAP	ALL	ALL	95.04	95.04	1.42	0.0	5	6670	6358
MAJ	PIL	AFA	80.45	80.45	6.30	25.40	10	397	320
MAJ	PIL	ROTC	80.42	80.42	6.33	25.49	10	655	528
MAJ	PIL	OTS	80.40	80.40	6.33	25.49	10	190	153
MAJ	PIL	ALL	80.43	80.43	6.32	25.46	10	1242	1001
MAJ	NAV	AFA	80.49	80.49	6.30	25.40	10	61	50
MAJ	NAV	ROTC	80.44	80.44	6.34	25.50	10	282	227
MAJ	NAV	OTS	80.35	80.35	6.34	25.50	10	222	179
MAJ	NAV	ALL	80.41	80.41	6.33	25.49	10	565	455
MAJ	NR	AFA	80.47	80.47	6.30	25.40	10	154	124
MAJ	NR	ROTC	80.43	80.43	6.38	25.62	10	612	494
MAJ	NR	OTS	81.16	81.16	6.39	25.64	10	729	593
MAJ	NR	ALL	80.79	80.79	6.38	25.61	10	1495	1211
MAJ	ALL	AFA	80.46	80.46	6.30	25.40	10	613	494
MAJ	ALL	ROTC	80.43	80.43	6.35	25.54	10	1549	1249
MAJ	ALL	OTS	80.87	80.87	6.37	25.59	10	1141	925
MAJ	ALL	ALL	80.59	80.59	6.35	25.53	10	3303	2667

Fig. 46--Printout page 91

OFFICER GRADE LIMITATIONS MODEL SAMPLE RUN

AUGMENTATION OPPORTUNITIES

SOURCE OF COMMISSION ROTC

YEAR OF SERVICE	PILOT				NAVIGATOR				NONRATED			
	RESERVE	REGULAR	TOTAL	OPPORTUNITY	RESERVE	REGULAR	TOTAL	OPPORTUNITY	RESERVE	REGULAR	TOTAL	OPPORTUNITY
3	1427	0	1427	0	651	0	651	0	2014	229	2243	10
4	1054	351	1406	25	494	147	642	23	1678	531	2209	24
5	761	624	1385	45	379	253	632	40	731	547	1278	43
6	623	674	1297	52	309	283	592	48	586	605	1191	51
7	280	663	944	70	134	284	418	68	568	587	1155	51

SOURCE OF COMMISSION OTS

YEAR OF SERVICE	PILOT				NAVIGATOR				NONRATED			
	RESERVE	REGULAR	TOTAL	OPPORTUNITY	RESERVE	REGULAR	TOTAL	OPPORTUNITY	RESERVE	REGULAR	TOTAL	OPPORTUNITY
3	414	0	414	0	514	0	514	0	2548	269	2816	10
4	305	102	407	25	390	116	506	23	2153	621	2774	22
5	220	181	401	45	299	200	499	40	940	641	1581	41
6	180	196	376	52	243	223	467	48	763	711	1474	48
7	81	192	274	70	105	224	329	68	740	689	1429	48

SOURCE OF COMMISSION RES

YEAR OF SERVICE	PILOT				NAVIGATOR				NONRATED			
	RESERVE	REGULAR	TOTAL	OPPORTUNITY	RESERVE	REGULAR	TOTAL	OPPORTUNITY	RESERVE	REGULAR	TOTAL	OPPORTUNITY
3	1841	0	1841	0	1165	0	1165	0	4562	498	5059	10
4	1360	453	1813	25	884	264	1148	23	3831	1153	4984	23
5	982	804	1786	45	678	453	1131	40	1671	1188	2859	42
6	804	869	1673	52	552	506	1058	48	1349	1316	2665	49
7	361	856	1217	70	239	508	747	68	1308	1276	2584	49

Fig. 47--Printout page 93

IMPLIED BACKWARD COMPUTATION INPUTS

SUMMARY OF OFFICER STATE

COMPONENT	SOURCE OF COMMISSION	PILOT LIEUT.	PILOT CAPT.	PILOT MAJOR	PILOT LT COL	PILOT CL/GEN	NAVIGATOR LIEUT.	NAVIGATOR CAPT.	NAVIGATOR MAJOR	NAVIGATOR LT COL	NAVIGATOR CL/GEN	NONRATED LIEUT.	NONRATED CAPT.	NONRATED MAJOR	NONRATED LT COL	NONRATED CL/GEN
RES	ROTC	3955	1722	25	0	0	1820	848	12	0	0	10493	2052	58	0	0
RES	OTS	1146	499	7	0	0	1436	668	9	0	0	11489	2659	84	0	0
REG	AFA	1817	2485	2219	1455	638	281	385	344	223	92	1660	931	863	563	240
REG	ROTC	378	3424	3625	2393	1048	158	1446	1557	1016	420	783	3012	3340	2212	942
REG	OTS	110	993	1052	695	304	125	1141	1229	802	331	916	3528	3956	2621	1115

RATIO OF PROMOTION-AUGMENTATIONS TO REGULAR PROMOTIONS

COMPONENT	SOURCE OF COMMISSION	PILOT LIEUT.	PILOT CAPT.	PILOT MAJOR	PILOT LT COL	PILOT CL/GEN	NAVIGATOR LIEUT.	NAVIGATOR CAPT.	NAVIGATOR MAJOR	NAVIGATOR LT COL	NAVIGATOR CL/GEN	NONRATED LIEUT.	NONRATED CAPT.	NONRATED MAJOR	NONRATED LT COL	NONRATED CL/GEN
	ROTC		0.8150	0.0	0.0	0.0		0.7540	0.0	0.0	0.0		0.0510	0.0	0.0	0.0
	OTS		0.8150	0.0	0.0	0.0		0.7570	0.0	0.0	0.0		0.0510	0.0	0.0	0.0

RATIO OF AUGMENTATIONS IN GRADE TO REGULAR PROMOTIONS TO GRADE

COMPONENT	SOURCE OF COMMISSION	PILOT LIEUT.	PILOT CAPT.	PILOT MAJOR	PILOT LT COL	PILOT CL/GEN	NAVIGATOR LIEUT.	NAVIGATOR CAPT.	NAVIGATOR MAJOR	NAVIGATOR LT COL	NAVIGATOR CL/GEN	NONRATED LIEUT.	NONRATED CAPT.	NONRATED MAJOR	NONRATED LT COL	NONRATED CL/GEN
	ROTC		0.2880	0.0440	0.0	0.0		0.3860	0.0490	0.0	0.0		0.1930	0.1190	0.0	0.0
	OTS		0.2900	0.0440	0.0	0.0		0.3850	0.0500	0.0	0.0		0.1940	0.1190	0.0	0.0

RATING TRANSFER RATIOS

RESERVE OFFICER TRAINING CORPS				SCHOOL OF MILITARY SCIENCE - OFFICERS			
RATING TRANSFER-AUGMENTATIONS TO AUGMENTATIONS ONLY		REGULAR RATING TRANSFERS TO AUGMENTATIONS ONLY		RATING TRANSFER-AUGMENTATIONS TO AUGMENTATIONS ONLY		REGULAR RATING TRANSFERS TO AUGMENTATIONS ONLY	
PILOT	NAVIGATOR	PILOT	NAVIGATOR	PILOT	NAVIGATOR	PILOT	NAVIGATOR
0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0

Fig. 48--Printout page 94

OFFICER GRADE LIMITATIONS MODEL SAMPLE RUN PAGE 96

IMPLIED BACKWARD COMPUTATION INPUTS

AUGMENTATION DISTRIBUTIONS

SOURCE OF COMMISSION ROTC

PILOT

LIEUTENANT		CAPTAIN		MAJOR		LIEUT. COL.		COL/GEN	
YOS	FRACT.	YOS	FRACT.	YOS	FRACT.	YOS	FRACT.	YOS	FRACT.
4	0.9620	5	0.0400	9	0.0180				
5	0.0380	6	0.9600	10	0.0510				
				11	0.9310				

NAVIGATOR

LIEUTENANT		CAPTAIN		MAJOR		LIEUT. COL.		COL/GEN	
YOS	FRACT.	YOS	FRACT.	YOS	FRACT.	YOS	FRACT.	YOS	FRACT.
4	0.9650	5	0.0270	9	0.0180				
5	0.0350	6	0.8780	10	0.0510				
		7	0.0950	11	0.9310				

NONRATED

LIEUTENANT		CAPTAIN		MAJOR		LIEUT. COL.		COL/GEN	
YOS	FRACT.	YOS	FRACT.	YOS	FRACT.	YOS	FRACT.	YOS	FRACT.
3	0.4290	5	0.0020	9	0.0180				
4	0.5690	6	0.9980	10	0.0510				
5	0.0020			11	0.9310				

Fig. 49--Printout page 96

OFFICER GRADE LIMITATIONS MODEL SAMPLE RUN

IMPLIED BACKWARD COMPUTATION INPUTS

PROMOTION DISTRIBUTIONS

COMPONENT REG   SOURCE OF COMMISSION AFA

PILOT

CAPTAIN		MAJOR		LT. COL		COL/GEN	
YOS	FRACT	YOS	FRACT	YOS	FRACT	YOS	FRACT
4	0.014	8	0.016	14	0.081	20	0.246
5	0.986	9	0.047	15	0.069	21	0.054
		10	0.937	16	0.850	22	0.700

NAVIGATOR

CAPTAIN		MAJOR		LT. COL		COL/GEN	
YOS	FRACT	YOS	FRACT	YOS	FRACT	YOS	FRACT
4	0.014	8	0.016	14	0.081	20	0.246
5	0.986	9	0.047	15	0.069	21	0.054
		10	0.937	16	0.850	22	0.700

NONRATED

CAPTAIN		MAJOR		LT. COL		COL/GEN	
YOS	FRACT	YOS	FRACT	YOS	FRACT	YOS	FRACT
4	0.014	8	0.016	14	0.081	20	0.246
5	0.986	9	0.047	15	0.069	21	0.054
		10	0.937	16	0.850	22	0.700

PROMOTION DISTRIBUTIONS

COMPONENT RES   SOURCE OF COMMISSION ROTC

PILOT

CAPTAIN		MAJOR		LT. COL		COL/GEN	
YOS	FRACT	YOS	FRACT	YOS	FRACT	YOS	FRACT
4	0.019	8	0.019				
5	0.981	9	0.051				
		10	0.930				

NAVIGATOR

CAPTAIN		MAJOR		LT. COL		COL/GEN	
YOS	FRACT	YOS	FRACT	YOS	FRACT	YOS	FRACT
4	0.018	8	0.019				
5	0.982	9	0.051				
		10	0.930				

NONRATED

CAPTAIN		MAJOR		LT. COL		COL/GEN	
YOS	FRACT	YOS	FRACT	YOS	FRACT	YOS	FRACT
4	0.019	8	0.019				
5	0.981	9	0.051				
		10	0.930				

PROMOTION DISTRIBUTIONS

COMPONENT REG   SOURCE OF COMMISSION ROTC

PILOT

CAPTAIN		MAJOR		LT. COL		COL/GEN	
YOS	FRACT	YOS	FRACT	YOS	FRACT	YOS	FRACT
5	1.000	8	0.016	14	0.081	20	0.246
		9	0.047	15	0.069	21	0.054
		10	0.937	16	0.850	22	0.700

NAVIGATOR

CAPTAIN		MAJOR		LT. COL		COL/GEN	
YOS	FRACT	YOS	FRACT	YOS	FRACT	YOS	FRACT
5	1.000	8	0.016	14	0.081	20	0.246
		9	0.047	15	0.069	21	0.054
		10	0.937	16	0.850	22	0.700

NONRATED

CAPTAIN		MAJOR		LT. COL		COL/GEN	
YOS	FRACT	YOS	FRACT	YOS	FRACT	YOS	FRACT
4	0.004	8	0.016	14	0.081	20	0.246
5	0.996	9	0.047	15	0.069	21	0.054
		10	0.937	16	0.850	22	0.700

Fig. 50--Printout page 99

in this run, rating transfers and augmentations never took place at the same time, hence the lack of all rating transfer ratios. Page 96 shows augmentation distributions for ROTC officers as an example of how the flow distributions are displayed. Note that for each rating and grade the sum of the distributions is 1.00. Page 99 is a sample of how promotion distributions are displayed.

THE GOODNESS MEASURES

Pages 104 and 105 show the results of the goodness measure computations for ROTC pilots. Note that the only ways of leaving the category of regular ROTC pilot are via promotion or retirement (loss), but that reserve ROTC pilots may also receive regular commissions. The probabilities and mean times for nonrated officers would include various rating transfer flows.

OFFICER GRADE LIMITATIONS MODEL SAMPLE RUN PAGE 104

COMPONENT: REGULAR RATING: PILOT SOURCE OF COMMISSION: ROTC

PROBABILITIES OF POSSIBLE EXITS FROM GRADE

	LIS	CAPTAIN	MAJOR	LT COL	COL/GEN
PROMOTION	0.900	0.737	0.617	0.446	0.0
RETIREMENT	0.100	0.263	0.383	0.554	1.000
TOTAL	1.000	1.000	1.000	1.000	1.000

MEAN TIME TO POSSIBLE EXITS FROM GRADE

	LIS	CAPTAIN	MAJOR	LT COL	COL/GEN
PROMOTION	1.000	4.794	5.803	5.678	0.0
RETIREMENT	1.484	5.640	8.670	8.767	7.253

Fig. 51--Printout page 104

OFFICER GRADE LIMITATIONS MODEL SAMPLE RUN

COMPONENT: RESERVE   RATING: PILOT   SOURCE OF COMMISSION: ROTC

PROBABILITIES OF POSSIBLE EXITS FROM GRADE

	LTS	CAPTAIN	MAJOR	LT COL	COL/GEN
PROMOTION	0.502	0.034	0.0	0.0	0.0
RETIREMENT	0.067	0.837	0.107	0.0	0.0
PROMOTION-AUGMENTATION	0.183	0.0	0.0	0.0	0.0
AUGMENTATION	0.249	0.128	0.893	0.0	0.0
TOTAL	1.000	1.000	1.000	0.0	0.0

MEAN TIME TO POSSIBLE EXITS FROM GRADE

	LTS	CAPTAIN	MAJOR	LT COL	COL/GEN
PROMOTION	2.981	4.925	0.0	0.0	0.0
RETIREMENT	2.722	2.471	1.064	0.0	0.0
PROMOTION-AUGMENTATION	2.983	0.0	0.0	0.0	0.0
AUGMENTATION	2.038	1.013	1.002	0.0	0.0

Fig. 52--Printout page 105

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Sammis, Miller and Shukiar

THE OFFICER GRADE LIMITATIONS MODEL: A STEADY-STATE MATHEMATICAL MODEL OF THE U. S. AIR FORCE OFFICER STRUCTURE

R-1632-PR