Charging for Local Telephone Calls

Price Elasticity Estimates from the GTE Illinois Experiment

Rolla Edward Park, Bruce M. Wetzel, Bridger M. Mitchell
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This report describes research undertaken as part of a larger study, funded by a grant to The Rand Corporation from the National Science Foundation, of the allocative and distributive effects of usage-sensitive pricing of local residential telephone service. A preliminary version of the report was presented at the Eighth Annual Telecommunications Policy Research Conference, Annapolis, Maryland, and at the GTE Measured Service Demand Workshop, Milford, Connecticut, both during April 1980. A slightly abridged version of this report is forthcoming in *Econometrica*.
Price elasticities are estimated for local telephone calls and minutes of conversation using data from a pricing experiment in central Illinois conducted by General Telephone and Electronics. The experiment charges separately for calls and for minutes. Using a model that is consistent with the theory of telephone demand, the authors estimate the effects of both prices. The nonlinear generalized least squares estimates of the elasticities are fairly small—about 0.1 or less in absolute value at experimental price levels—but they are estimated with high precision. The report briefly considers the application of these results to predict the effects of introducing measured service telephone rates in other cities.
ACKNOWLEDGMENTS

The authors would like to acknowledge the financial support of the National Science Foundation; the multi-faceted cooperation and encouragement of Jim Alleman, Jerry Cohen, and others at GTE Service Corporation; and the very helpful comments and suggestions of Hans Kraepelien, Will Manning, Angus Deaton, and three referees for *Econometrica*. Of course, none of them is responsible for any of our remaining errors.
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I. INTRODUCTION AND SUMMARY

A. INTRODUCTION

Residential telephone subscribers in the United States typically pay a flat monthly rate for local service, with no extra charge for calls within the local area. The alternative of explicitly charging for local calls—commonly referred to as "usage-sensitive pricing" or "local measured service"—is of increasing interest to U.S. telephone companies and regulatory commissions (Cosgrove and Linhart, 1979; Garfinkel and Linhart, 1979; Baude, ed., 1979).\(^1\) Charging for calls that are now free clearly holds promise of increasing economic efficiency (Alleman, 1977; Mitchell, 1978). However, because there is some cost of measuring and billing local calls, the efficiency gains may not be realizable in practice. The potential gains depend critically on how elastic the demand for local telephone use is.\(^2\)

General Telephone and Electronics is conducting an experiment in central Illinois to determine the effects of charging for local calls. In this report we estimate residential price elasticities for local calls and minutes based on the GTE experimental data. All of the elasticities are fairly small—0.1 or less in absolute value—but they are estimated with surprising precision.

Previous empirical studies of the effects of charging for local telephone calls have looked at the introduction of a local measured-service tariff as a monolithic "treatment" administered to an experimental population (Pavarini, 1979; Jensik, 1979; Park et al., 1981; Wilkinson, 1981). In this analysis, we account for details of what can in practice be quite complex pricing structures. For example, the tariff that we deal with (the experimental tariff

\(^1\)Usage-sensitive pricing of local telephone calls has long been common in other countries; see Kraepelien (1976) and Mitchell (1979a, b).

\(^2\)In a related market, Daly and Mayor (1980) estimate a high elasticity and consequently large efficiency gains to charging for directory assistance calls.
described in detail in Sec. III.A below) levies separate charges for call set-up (initiation) and for call duration. We estimate the separate effects of these charges per call and per minute, on both the number of calls and the number of minutes of telephone use.

Also, the experimental tariff applies only to single-party customers; multi-party service is still offered at a flat monthly rate. This fact opens up certain substitution possibilities that may not be present in other situations. We must take these into account if our estimates are to be useful elsewhere.

In Sec. II, we review the theory of telephone demand and extend it to account for the special features of the GTE experiment. Section III describes the experiment and the experimental data that we analyze. In Sec. IV, we specify a nonlinear econometric model of telephone use with complex error covariance structure. Section V presents generalized nonlinear least squares estimates of the various price effects and compares our results with those of earlier studies. Section VI sets out some considerations and cautions regarding the application of our results to predict the effects of local measured-service tariffs everywhere.

B. SUMMARY

Theory of Telephone Demand

The theory of telephone demand distinguishes between the demand for use (calls) and the demand for access (subscriptions); we focus on the demand for use. A usage price has two effects on telephone use—a "direct effect" on use by a fixed set of subscribers and a "subscriber effect" due to the addition or deletion of marginal subscribers. In contrast, access price affects use only through subscriber effects.

In the GTE experiment, the existence of a flat-rate multi-party option to measured-rate single-party service complicates both direct and subscriber effects. The direct effects of a single-party usage charge can include increased multi-party use as well as reduced single-party use, as multi-party customers originate a larger
The proportion of conversations with their single-party friends. Subscriber effects on use can result from movement among three possible states: nonsubscriber, single-party subscriber, and multi-party subscriber. Subscriber effects of the GTE tariff, like direct effects, will tend to decrease single-party use and increase multi-party use. For both direct and subscriber effects, we call the reduction in single-party use that is offset by an increase in multi-party use "substitution." the remainder we call "repression."

The GTE tariff has separate prices for the number of calls and for minutes of conversation. Theory suggests that both prices will affect both measures of use.

The Experimental Data

The single-party usage charges in the GTE experiment vary sufficiently over time and cross-sectionally among the three communities in the experiment that we can estimate their effects on telephone use. In contrast, access prices and multi-party usage charges do not exhibit sufficient independent variation to allow estimates of their effects.

The number of single-party subscribers has grown substantially over time, whereas the number of multi-party subscribers has declined. We take account of these changes in the subscriber population by analyzing average telephone use per subscriber and by incorporating trend terms in our usage model.

Our data consist of monthly time series of calls per subscriber and minutes per subscriber, for both single-party and multi-party service in three communities. Plots of average use reveal several features of the data that we must allow for in our model--seasonality, differences between subscriber classes, and others.

Data on the six possible transitions among the three subscriber states (nonsubscriber, single-party, and multi-party) establish which of the theoretically possible subscriber effects are in fact important. All of the transitions are taking place continually regardless of price; thus, their influence on average telephone use will be captured by the trend terms in our model. The only substantial
discontinuity in any of the six transition processes is switching by more than 100 subscribers from single-party to multi-party service at the time the experimental tariff first took effect. This abrupt switching by relatively high-use customers reduces single-party use and increases multi-party use. It is the one subscriber effect that influences the estimated price effects.

Model Specification

Our model includes shift, dummy, and trend variables. The shift variables, which are functions of usage prices, pick up the changes in use due to the introduction or modification of usage charges. The dummy variables and trend variables control for cross-sectional differences and smooth changes in use over time that are of lesser interest.

Single-party use is specified as the product of five factors. Monthly reference level use allows for seasonal differences in telephone use. An exchange size factor allows for differences in use among exchanges. A growth factor allows for any smoothly trended influences. Repression and substitution factors measure the price effects that are our primary interest.

Multi-party use is the product of five similar factors. Monthly reference level use, the exchange size factor, and a growth factor are similar to those specified for single-party use. A multi-party factor allows for systematically lower levels of use by multi-party subscribers, compared with single party. A substitution factor, which is a function of single-party usage prices, measures the increase in multi-party use when single-party prices are increased.

We specify a complex error covariance matrix that allows for month-to-month heteroscedasticity, contemporaneous heteroscedasticity among different city/class of service combinations, contemporaneous covariance across cities and classes of service, and first-order autocorrelation within each city/class of service combination.

Demand Elasticity Estimates

Nonlinear generalized least squares estimates of the various
price effects (coefficients) all have the correct sign, reasonable magnitudes, and surprisingly small standard errors; see Table 5 in Section V.A. The per-call charge has about the same effect on number of minutes as on number of calls, indicating that the calls forgone because of the per-call charge are of average duration. The per-minute charge has a larger effect per penny on the number of calls than does the per-call charge, a reasonable result as most calls last more than one minute. The effect of the per-minute charge on minutes is larger still; thus a per-minute charge reduces average call duration. The estimated repression effects are substantially larger than the substitution effects.

Table 1 gives the estimates expressed as elasticities when the charge per call is 2.5 cents and the charge per minute is 1 cent. Elasticities vary with price in our model; they would be higher for higher prices.

Table 1
ESTIMATED PRICE ELASTICITIES WHEN THE PER-CALL CHARGE EQUALS 2.5 CENTS AND THE PER-MINUTE CHARGE EQUALS 1 CENT

<table>
<thead>
<tr>
<th></th>
<th>Repression Elasticity</th>
<th>Substitution Elasticity</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Calls     Minutes</td>
<td>Calls     Minutes</td>
</tr>
<tr>
<td>Per-call charge</td>
<td>-.076      -.086</td>
<td>-.002      -.005</td>
</tr>
<tr>
<td></td>
<td>(6.0)      (3.9)</td>
<td>(.6)       (1.2)</td>
</tr>
<tr>
<td>Per-minute charge</td>
<td>-.055      -.109</td>
<td>-.014      -.034</td>
</tr>
<tr>
<td></td>
<td>(4.8)      (5.8)</td>
<td>(6.8)      (10.0)</td>
</tr>
</tbody>
</table>

NOTE: Estimated t-statistics in parentheses.
SOURCE: Table 6, Section V.A.

Our estimates of the total reductions in single-party telephone use due to the GTE experimental tariffs, including both repression and substitution, are about 12 percent for calls and 20 percent for minutes. Substitution is a small part of the total—less than 2 percent for calls and 4 percent for minutes.

We compare these estimates with a number of others to establish
their reasonableness. First, an extraneous estimate of the effects of subscriber substitution alone is smaller than our estimates of the full substitution effects; the balance is direct substitution—increased use by continuing multi-party subscribers.

Second, we estimate two variants of our model, one without trend terms and one without price constraints. The estimates are not much affected by these changes.

Third, estimates by other researchers using the GTE experimental data (Jensik, 1979; Wilkinson, 1981; Park et al., 1981) tend to bracket our present estimates.

A fourth comparison is with theory, not other empirical estimates. A modification of Mitchell's (1978) model of telephone demand implies relationships among elasticities and other quantities that are not entirely consistent with our estimates. However, this is not too surprising, since our modification of the theory embodies somewhat unrealistic assumptions.

**Application Elsewhere**

Although our estimates provide some basis for predicting the effects of proposed local measured-service plans, they should not be used indiscriminately. Effects may differ from those in the GTE experiment for a number of reasons that wise telephone planners will consider.

First, there are some details of the experimental tariff that we have not taken into account. The GTE tariff sets a $19 per month ceiling on usage charges, and it offers discounts of 20 percent for evening use and 50 percent for nights. Other measured-service tariffs may differ in such details, affecting the response to usage prices.

Second, the response to prices may differ in other locations because of differences in economic and demographic factors and for other unmeasured reasons.

Third, substitution opportunities will differ. In locations with mandatory local measured service (and no flat-rate multi-party service), substitution will be more limited than in the GTE experiment.
Even there, however, one expects to find some substitution of personal calls placed from business telephones for personal calls from home. In locations with optional measured service, the extent of substitution will depend on the number of subscribers who choose measured service, compared with the number who choose to remain on flat rates.
II. THE THEORY OF TELEPHONE DEMAND

A. CURRENT THEORY

The theory of telephone demand (Alleman, 1977; Mitchell, 1978; Taylor, 1980) distinguishes between the demand for use of the telephone system (calls) and the demand for access to the system (subscriptions). The GTE experimental data allow us to estimate the demand for use, but they are not well suited for estimating the demand for access. However, access does affect use, so we must account for it in our model (specified in Sec. IV below).

The theory treats an individual household's telephone demand as the result of a two-stage process. In the first stage, the household determines how many calls it would make at a given price per call $P_C$, assuming it were to subscribe. Associated with the optimal number of calls is a potential value of consumer surplus $S$. In the second stage, the household compares $S$ with the price of access $P_A$ to determine whether or not to subscribe. Thus, the demand for access is derived from the demand for calls.

Both the market demand for access and the market demand for calls are influenced by both prices $P_A$ and $P_C$. If $P_A$ is decreased with $P_C$ constant, additional households will subscribe, increasing total calling. Calling by initial subscribers is unaffected. The average number of calls per subscribing household may increase or decrease, depending on whether new subscribers use their phones more or less than initial subscribers.

If $P_C$ is decreased with $P_A$ constant, initial subscribers will make more calls, and some additional households will subscribe. The number of subscribers and the number of calls both increase. The

---

1This neglects the income effect of a change in $P_A$ on the number of calls. The income effect will usually be negligible in practice, because $P_A$ is almost always small compared with total income.
average number of calls per subscribing household will almost certainly increase as well.\footnote{There is a theoretical possibility that the average number of calls per household could decline. This would occur if the increase in calls by existing subscribers were small, the number of new subscribers were large compared with the number of initial subscribers, and the new subscribers used their phones much less than initial subscribers did. In practice, such an outcome is unlikely.}

In this section, we focus on the effects of prices on total local telephone use by subscribing households. As suggested above, price changes can have two kinds of effects on total use:

- **Direct effects** on use by continuing subscribers---the fixed set of households that subscribe both before and after the price change.
- **Subscriber effects** on use that result from the addition or deletion of marginal subscribers.

Changes in \( P_A \) have only subscriber effects; changes in \( P_C \) have both direct effects and subscriber effects.

The GTE experiment differs in a number of ways from the simple theoretical situation outlined above. We discuss two important differences---the availability of a second class of service and the separate pricing of calls and minutes of conversation---and sketch the extensions to the theory necessary to take each of them into account.

\section*{B. More Than One Class of Service}

In many parts of the country, multi-party ("party-line") service is a thing of the past; not so in central Illinois. There, residential telephone subscribers can still choose between single-party and multi-party service.

A price change for one class of service will in general affect use by both classes. To be concrete, we shall deal with the effects of the type of price change that occurred in the Illinois experiment.
Single-party rates were changed from an initial flat-rate tariff \((P_a = P_{A1}, P_c = P_{C1} = 0)\) to a measured-rate tariff \((P_{A2} < P_{A1}, P_{C2} > 0)\). Multi-party customers have been billed at flat rates \((P^*_a = P^*_{A1}, P^*_c = P^*_{C1} = 0)\) throughout the experiment. The existence of multi-party service opens up certain substitution possibilities that would not otherwise be available. For example, because single-party calls are now billed while multi-party calls are still free, some calls originated by multi-party subscribers are substituted for calls originated by single-party subscribers. This substitution could result from explicit signaling arrangements, such as: When a single-party customer wants to talk to his multi-party friend, he lets the phone ring once and hangs up. His friend calls right back for free.\(^3\) Or it could result from less formal shifts in calling patterns, with multi-party customers originating a larger proportion of conversations with their single-party friends.

Such a substitution opportunity may not be available in other situations, for example, where there is mandatory application of local measured-service rates to all residential subscribers. Thus we must pay separate attention to two components of the reduction in single-party telephone use in the experimental setting. We adopt the following terminology:

- **Total decrease** in single-party use is the full reduction in single-party calls due to the price change.
- **Substitution** is that part of the total decrease that is offset by the increase in multi-party use.
- **Repression** is the remainder—that part of the decrease in single-party use that is not offset by the increase in multi-party use.

Taken together with our earlier distinction between direct and subscriber effects, these definitions make possible a useful two-way

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\(^3\) The telephone industry term for this is "code calling."
classification of the theoretically possible effects of the experimental price change, as shown in Table 2. We describe the entries in Table 2 in the next few paragraphs. We use the following notation:

\( n_{ij} \) is the number of households that were in service classification \( i \) before the price change and in service classification \( j \) after the price change; \( i, j = 0 \) for nonsubscribers; \( i, j = 1 \) for single-party service; \( i, j = 2 \) for multi-party service.

\( u_{ij}^k \) is average telephone use (calls per subscriber per month) by subscribers in group \( ij \) when subject to tariff \( k \); \( k = F \) before the price change (flat rates in effect for single-party service), and \( k = M \) after the price change (measured rates in effect for single-party service).

Table 2
THEORETICAL EFFECTS OF THE PRICE CHANGES IN THE GTE EXPERIMENT

<table>
<thead>
<tr>
<th></th>
<th>Total Decrease in Single-Party Use (1)</th>
<th>Substitution = Increase in Multi-Party Use (2)</th>
<th>Repression = Total Decrease Minus Substitution (3) = (1) - (2)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Direct effects</td>
<td>( n_{11}(u_{11}^F - u_{11}^M) )</td>
<td>( n_{22}(u_{22}^M - u_{22}^F) )</td>
<td>( n_{11}(u_{11}^F - u_{11}^M) - n_{22}(u_{22}^M - u_{22}^F) )</td>
</tr>
<tr>
<td>Subscriber effects</td>
<td>( n_{10}u_{10}^F - n_{01}u_{01}^M )</td>
<td>( n_{12}u_{12}^M - n_{21}u_{21}^F )</td>
<td>( n_{10}u_{10}^F - n_{01}u_{01}^M + n_{12}u_{12}^M - n_{21}u_{21}^F )</td>
</tr>
<tr>
<td></td>
<td>+ ( n_{12}u_{12}^F - n_{21}u_{21}^M )</td>
<td></td>
<td>- ( n_{21}(u_{21}^M - u_{21}^F) )</td>
</tr>
</tbody>
</table>
Direct Effects

The direct effect of the experimental prices on telephone use by the \( n_{11} \) continuing single-party subscribers is to reduce their average number of telephone calls from \( u_{11}^F \) to \( u_{11}^M \) per month. There are two reasons for the reduction. First, because single-party calls are priced while multi-party use is still free, some multi-party calls are substituted for single-party calls, as discussed above. Substitution leads to an increase in average use by the \( n_{22} \) continuing multi-party subscribers, from \( u_{22}^F \) to \( u_{22}^M \) calls per month. Second, even if the substitution opportunity did not exist, the single-party per-call charge would reduce single-party use.

The direct repression effect is the difference between the total reduction in use by continuing single-party subscribers and the increase in use by continuing multi-party subscribers.

As in the simpler case discussed at the beginning of this section, the direct effect on average use \( u_{11} \) and \( u_{22} \) in Table 2 will be functions of usage prices \( P_C \) and \( P_C^\star \) only. However, the subscriber effects, to be discussed next, will be functions of access prices \( P_A \) and \( P_A^\star \) as well.

Subscriber Effects

Subscriber effects on use may result from moves among three possible states:

- Nonsubscriber
- Single-party subscriber
- Multi-party subscriber

The experimental price change may cause moves in either or both directions along arrows 1 and 2; because nonsubscriber and multi-party

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4 Again, this neglects the generally negligible income effect of access prices.
prices are unchanged, there should be no movement along arrow 3.

Specifically, previous nonsubscribers might sign up for single-party service because of the new low access price. Some single-party subscribers who use their phones a lot might switch to multi-party service to escape the new per-call charge, and some low-use multi-party subscribers might switch to single-party service to take advantage of the lower access price. The subscriber effect on total calls by single-party subscribers is the net result of all such moves, that is, the sum of calls formerly made at flat rates by the \( n_{10} + n_{12} \) former single-party subscribers less calls under measured rates by the \( n_{01} + n_{21} \) new single-party subscribers.

The subscriber substitution effect is the increase in multi-party calling due to changes in subscriber classification along arrow 2: the calls made after the price change by the \( n_{12} \) subscribers who switch from single- to multi-party service, less the calls made before the price change by the \( n_{21} \) subscribers who switch in the other direction.

The subscriber repression effect on single-party calling is the total subscriber effect less the subscriber substitution effect.

In Sec. III.D, we will examine data on transitions from one service classification to another to see which of the theoretically possible subscriber effects in Table 2 are in fact important in the GTE experiment.

C. PER-CALL AND PER-MINUTE CHARGES

So far in this section, we have treated telephone use as though it could be measured simply by the number of calls. In fact, both the number of calls and the number of minutes are important, in that capacity must be provided for each. Also, calls and minutes are billed separately in the GTE experiment and in local measured-service tariffs in other areas.

The theory of telephone demand can be easily extended to accommodate separate charges for calls \( (P_C) \) and for minutes \( (P_M) \). When one realizes that \( P_C \) is really an access charge--for access to minutes of conversation time--it is apparent from the discussion
above that $P_C$ and $P_M$ will each affect both the total number of calls and the total number of minutes. The model that we specify in Sec. IV allows for both effects.
III. THE EXPERIMENTAL DATA

A. EXPERIMENTAL TARIFFS

Our data come from the General Telephone and Electronics local measured service experiment in three small cities in central Illinois—Jacksonville, Clinton, and Tuscola. Since May 1975, GTE has recorded telephone use (both the number of calls and minutes of calling) for each telephone subscriber. Flat-rate tariffs were in effect until September 1, 1977, at which time the first experimental measured-service tariffs were introduced. A second measured-service tariff superseded the first on June 1, 1979, and remained in effect at the end of the period spanned by our data (December 1979).\(^1\) In contrast to some measured-service plans offered elsewhere, the experimental tariffs are nonoptional\(^2\) and include no allowance of free calls.

The tariffs are shown in Table 3. During the first 21 months that measured-service rates were in effect, single-party subscribers in Jacksonville, the largest of the three cities, paid \(P_C = 2\) cents per call plus \(P_M = 1\) cent per minute; those in Clinton and Tuscola had no call setup charge but paid 1.5 cents per minute. Thus, in each city, a typical four-minute call cost 6 cents. Under the subsequent tariff, measured rates were identical in all three cities: 2.5 cents per call plus 1 cent per minute.\(^3\) The combination of independent cross-sectional and time-series variation in the usage charges holds out hope that we can estimate the separate effects of per-call and per-minute charges.

\(^1\)For a more complete description of the experiment, its history, and its objectives, see Cohen (1977).

\(^2\)Multi-party service is not, however, subject to usage charges; it might be thought of as a lower-quality flat-rate option to single-party measured service.

\(^3\)We use prices in constant dollars to estimate our equations. The deflator is the consumer price index (all items) for urban wage earners and clerical workers in cities with population less than 75,000 in the north central region, using December 1978 as base.
<table>
<thead>
<tr>
<th></th>
<th>September '75-December '76</th>
<th>January '77-August '77</th>
<th>September '77-May '79</th>
<th>June '79-December '79</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>J</td>
<td>C</td>
<td>T</td>
<td>J</td>
</tr>
<tr>
<td><strong>Single-party service</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Monthly service charge</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Urban areas</td>
<td>$7.95</td>
<td>$6.20</td>
<td>$5.90</td>
<td>$8.60</td>
</tr>
<tr>
<td>Suburban areas</td>
<td>10.50</td>
<td>8.80</td>
<td>8.45</td>
<td>11.30</td>
</tr>
<tr>
<td>Price per call, $P_C$</td>
<td>--</td>
<td>--</td>
<td>--</td>
<td>--</td>
</tr>
<tr>
<td>Price per minute, $P_M$</td>
<td>--</td>
<td>--</td>
<td>--</td>
<td>--</td>
</tr>
<tr>
<td><strong>Multi-party service</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Monthly service charge</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Urban areas (2-party)</td>
<td>6.70</td>
<td>5.10</td>
<td>4.85</td>
<td>7.25</td>
</tr>
<tr>
<td>Suburban areas (4-party)</td>
<td>8.80</td>
<td>7.10</td>
<td>6.75</td>
<td>9.50</td>
</tr>
</tbody>
</table>

**NOTES:** J, C, and T indicate Jacksonville, Clinton, and Tuscola.
There is a 20 percent discount on usage charges $P_C$ and $P_M$ evenings (5-11 p.m.) and Sunday and holidays (8 a.m.-11 p.m.);
there is a 50 percent discount nights (11 p.m.-8 a.m.).
There is a $19 per month ceiling on usage charges.
Multi-party service has remained on flat rate throughout the experiment.
Business service has been measured since September 1977, with the same usage charges as for residential service, but
higher monthly charges.
In contrast with $P_C$ and $P_M$, there is insufficient variation in the other prices to identify their effects on use. The prices for multi-party service ($P^*_A$, $P^*_C$, $P^*_M$) have remained nearly constant throughout the period spanned by our data; it is not possible to estimate the effects of changing them. The single-party access price $P_A$ was reduced substantially in September 1977, but the reduction coincided with the introduction of usage charges $P_C$ and $P_M$. Thus the effect of $P_A$ is inextricably confounded with the effect of $P_C$ and $P_M$. However, theory tells us that $P_A$ has only subscriber effects, and we shall see below that total subscriber effects are small. Thus, although we are unable to estimate the separate effects of $P_A$, we can obtain fairly uncontaminated estimates of the effects of $P_C$ and $P_M$.

Other features of the experimental tariff may also influence telephone use. Evening calls benefit from an off-peak discount on usage charges, and hence the experimental tariff causes less reduction in total telephone use than if the daytime usage charges applied around the clock. Total usage charges are limited to $19, so marginal prices are zero for some customers who use their telephones a great deal. The effects of these tariff features are discussed in more detail in Sec. VI.A.

B. SUBSCRIBER TRENDS

A look at the subscriber data suggests the need for caution in applying the model of Sec. II. Figure 1 plots total subscribers in Jacksonville. There is a steady upward trend over our data period.\(^4\) Subscriber growth will increase the total number of calls and minutes of telephone use, in the absence of other changes. To compensate for exogenous growth in subscribers, we shall analyze telephone use on a

\(^4\) This statement is based on an F-test of the single-trend line shown versus individual-trend lines for the flat-rate and measured-rate tariff periods (September 1975-August 1977 and September 1977-December 1979). The other two exchanges, in contrast, show a significant decline in total subscriber growth rate after the introduction of measured rates.
per-subscriber basis (average calls and average minutes per main station).\(^5\)

In the presence of subscriber effects such as those shown in Table 2, estimates based on average use per subscriber will differ slightly from estimates based on total use by a class of subscribers. Consider, for example, the ratio of single-party use under measured rates to single-party use under flat rates (one measure of the effect of usage prices). Using Table 2's notation, the ratio calculated on the basis of total use is

\[
R_{\text{total}} = \frac{n_1^{\text{M}} u_1^{\text{M}} + n_0^{\text{M}} u_0^{\text{M}} + n_2^{\text{M}} u_2^{\text{M}}}{n_1^{\text{F}} u_1^{\text{F}} + n_1^{\text{F}} u_0^{\text{F}} + n_1^{\text{F}} u_2^{\text{F}}}
\]

\(^5\) "Main stations" exclude extension phones but include second lines (as, for example, when a family has a separate children's phone). Thus, the number of main stations is slightly greater than the number of households subscribing to telephone service.
In contrast, the ratio calculated on the basis of average use is

\[ R_{\text{average}} = \frac{n_{11} + n_{10} + n_{12}}{n_{11} + n_{01} + n_{21}} R_{\text{total}}. \]

In the GTE experiment, the difference between the two measures is negligible.

The steady increase in total subscribers shown in Fig. 1 conceals opposing trends in the number of single-party and multi-party subscribers; Fig. 2 shows the separate trends for Jacksonville. These opposing trends indicate that the growth in total subscribers is not simply scaling-up. The change in the proportion of single-to multi-party subscribers may well affect average telephone use as each subscriber population changes composition. To account for gradual changes in subscriber populations (and other gradually changing influences on telephone use), we shall include trend terms in our model (Sec. IV). The gaps in the trend lines upon introduction of measured-service tariffs in September 1977 are associated with subscriber effects of the price change and are discussed further below (Sec. III.D).

C. EXPERIMENTAL USAGE DATA

We seek to explain telephone use as a function of experimental prices and other factors. Specifically, "telephone use" consists of 12 time series:

- two measures of use (calls per main station and minutes per main station),
- in each of three exchanges (Jacksonville, Clinton, and Tuscola),

---

6This figure, as well as the following Figs. 4, 5, and 6, are obtained by fitting least squares trend lines to the plotted data. The graphs for the other two exchanges are qualitatively similar.
Fig. 2 — Single-party subscribers (top graph) and multi-party subscribers (bottom graph) in Jacksonville, Illinois
for each of two residential service categories (single-party and multi-party).

The time series span 52 months, from September 1975 through December 1979.\textsuperscript{7}

Figure 3 is a plot of two of the 12 time series: calls per main station in Jacksonville by single- and multi-party customers.\textsuperscript{8} Several features of the data are apparent.

1. There is a marked seasonal pattern. Telephone use is higher in the winter than in the summer; use is typically lowest in September.

2. Prior to introduction of the measured-service tariff, average use by multi-party customers was consistently lower than average use by single-party customers.

3. Usage levels, and the relationship between single- and multi-party use, differ among exchanges. (This is apparent from a comparison of Fig. 3 with similar plots for the other two exchanges.)

4. Single-party use decreased abruptly in September 1977, presumably in response to the usage charges that went into effect then.

5. Multi-party use increased beginning in September 1977, presumably as a substitution effect in response to the single-party price change.

The model specified in Sec. IV is designed to account for the above observed features of these data.

\textsuperscript{7} We dropped four months, May through August 1975, because there appear to have been some start-up problems in counting the number of main stations. Data subsequent to December 1979 were not available when we began our analysis.

\textsuperscript{8} Complete data are available from Dr. James H. Alleman, Economic Regulatory Research Group, GTE Service Corporation, One Stamford Forum, Stamford, Connecticut 06902.
Fig. 3 — Time series of telephone calls per main station in Jacksonville, Illinois
D. SUBSCRIBER EFFECTS OF PRICE CHANGES

As discussed in Sec. II.B, subscriber effects on telephone use occur because of price-induced transitions among the three possible subscriber states: nonsubscriber (0), single-party subscriber (1), and multi-party subscriber (2). Figures 4, 5, and 6 show the cumulative number of households in each of the six possible transitions since September 1975.

The graphs reveal three important things. First, the transitions are on-going processes, even in the absence of price changes. Their effects will be captured by trend terms in our model. Second, the rate at which the processes occur appears to be influenced by prices. In all cases, there are statistically significant differences in the slopes before and after the introduction of measured-service rates. In the case of switching between single- and multi-party service, the differences in trend are fairly large. We will allow for the effects of these differences in trend by specifying separate trend rates for telephone use before and after the introduction of measured service. Third, the only process that exhibits a substantial discrete jump or gap upon introduction of measured-service rates is switching from single- to multi-party service (presumably to avoid usage charges). Based on the gap in the linear trend lines, 106 subscribers switched service at the time measured rates were introduced. This is the only subscriber effect that will be measured as a discrete change in use in our model.

Table 4 is a modification of Table 2; it omits all of the theoretically possible subscriber effects that are not in fact present in our data (that is, \( n_{10} = n_{01} = n_{21} = 0 \)). Table 4 thus summarizes the effects that we must allow for when we specify our model in the next section. As discussed there, these effects will be captured by

---

9 As the individual data points plotted in Fig. 6 show, higher than normal switching was spread over a period of several months, starting before and ending after the introduction of measured service in September 1977. The value 106 is the size of the gap between the two linear trend lines that approximate the more complex actual pattern.
Fig. 4 — New subscriptions and terminations: single-party service, all three cities combined

Fig. 5 — New subscriptions and terminations: multi-party service, all three cities combined
shift terms in our model; other, smoothly trended, subscriber effects will be captured by trend terms.
Table 4

ACTUAL COMPARATIVE STATICS EFFECTS OF PRICE CHANGES IN THE GTE EXPERIMENT

<table>
<thead>
<tr>
<th></th>
<th>Total Decrease in Single-Party Use (1)</th>
<th>Substitution = Increase in Multi-Party Use (2)</th>
<th>Repression = Total Decrease Minus Substitution (3) = (1) - (2)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Direct effects</td>
<td>$n_{11}(u_{11}^F - u_{11}^M)$</td>
<td>$n_{22}(u_{22}^M - u_{22}^F)$</td>
<td>$n_{11}(u_{11}^F - u_{11}^M)$ - $n_{22}(u_{22}^M - u_{22}^F)$</td>
</tr>
<tr>
<td>Subscriber effects</td>
<td>$n_{12}u_{12}^F$</td>
<td>$n_{12}u_{12}^M$</td>
<td>$n_{12}(u_{12}^F - u_{12}^M)$</td>
</tr>
</tbody>
</table>
IV. MODEL SPECIFICATION

A. OVERVIEW

The demand for telephone use is a function of many variables in addition to access and usage prices $P_A$, $P_C$, and $P_M$; for example, the prices of other goods, real income, and the number of subscribers (reflecting external benefits of system size) (Taylor, 1980, p. 36). Other variables may also be important, including demographic characteristics such as household size, general levels of economic and social activity, and so on (Brandon, 1981; Infosino, 1980; Park et al., 1981).

To obtain unbiased estimates of the effects of usage prices on telephone use, we must control for the other influential factors as well. Data with sufficient independent variation are not available to estimate separately the effects of each potentially influential factor. However, we can specify a model that controls for all such factors. To do so, we specify the systematic portion of our model to include shift, dummy, and trend variables. The shift variables, which are functions of usage prices, will pick up the changes in use due to the introduction of usage charges. The dummy variables and trend variables will control for other effects of lesser interest.

The dummy variables allow for cross-sectional differences in use between exchanges and classes of service. Additional dummy variables allow for seasonal differences in use between different months of the year. The cross-sectional differences may arise for many reasons, including initial differences in income, in the number of subscribers in each service class in each exchange, in demographic characteristics,

---

1 For example, average income is very nearly the same in all three communities. Our analysis of telephone survey data collected by Opinion Research, Inc. (1980) shows the following mean income for households surveyed during April 1978:

- Jacksonville: $18,989
- Clinton: 20,151
- Tuscola: 18,734.
and in economic and social activity. They may also arise because of such idiosyncratic factors as the construction of a nuclear power plant in one of the cities (Clinton). The dummy variables control for all of these influences, not just the one or two for which dis-aggregated data might be available.

The trend terms allow for smooth changes over time in any of the cross-sectional influences (such as income). They will also capture most of the subscriber effects of the transitions plotted in Figs. 4 through 6. Separate trend terms for the periods before and after the introduction of measured service allow for changes in the rates of the subscriber transitions.

The shift terms, which are functions of usage prices, will pick up the sudden changes in use that occurred when the measured-service tariff was introduced (September 1977) or changed (June 1979). They will also capture the subscriber effect of the abrupt switch from single-party to multi-party service by more than 100 subscribers around September 1977 (Fig. 6).

Figure 7 illustrates the role of trend and shift terms in our model. The plotted points represent telephone use during three months before and three months after the introduction of measured service. Our model with trend terms will fit the solid trend lines to the observed points and attribute the shift or gap A to the effect of the price change. A model without trend terms would fit the horizontal dashed lines and estimate the smaller price effect represented by gap B. (The price effect estimated without trends may be either smaller or larger than the with-trend estimate, depending on the relationship between the slopes before and after the price change.)

If the trend after the introduction of measured service is mostly due to changes in exogenous factors such as income, then shift A is clearly the appropriate estimate of the price effect. However, to the extent that the trend is itself a result of the price change (perhaps due to a lagged response, or a price effect that decays over time), the trend terms in the model will capture a portion of the price effect as well. To guard against this kind
of misspecification, we estimate the model both with and without trend terms (Sec. V), and find that the results are about the same in both cases.

B. DETAILED SPECIFICATION: SYSTEMATIC EFFECTS

We now specify a model that accounts for the observed features of the data and that lets us estimate the effects of usage price changes on telephone use, defined either as calls per month per main station or minutes per month per main station. Two regressions, one with calls and one with minutes as dependent variable, yield separate coefficient estimates for a calls equation and a minutes equation.

We specify telephone use to be the product of several factors that modify monthly "reference level" use. Different sets of factors determine single-party and multi-party use. We discuss the two sets of factors separately, and then join the single-party and multi-party parts of the equation to form a single estimating equation.
Single-Party Use

Single-party use =

\[
\begin{bmatrix}
\text{Monthly reference level use} \\
\text{factor}
\end{bmatrix}
\begin{bmatrix}
\text{Exchange size factor} \\
\text{factor}
\end{bmatrix}
\begin{bmatrix}
\text{Growth factor} \\
\text{factor}
\end{bmatrix}
\begin{bmatrix}
\text{Repression factor} \\
\text{factor}
\end{bmatrix}
\begin{bmatrix}
\text{Substitution factor} \\
\text{factor}
\end{bmatrix}
\text{+ error}
\]

\[\mu_t \cdot a_i \cdot y_{it} \cdot \eta_{it} \cdot \delta_{it} + \varepsilon_{it}. \quad (1)\]

Monthly reference level use \(\mu_t\) is allowed to differ from one calendar month to another to account for seasonal influences. It is an estimate of the long-run average detrended level of use in each calendar month under a flat-rate tariff, by single-party customers in the largest exchange (Jacksonville). Notation is somewhat complicated by the need to distinguish between 52 monthly observations in our data set, and 12 calendar months, each with a different reference level of use. We use \(t = 1, \ldots, 52\) to index observations, and \(m(t), m = 1, \ldots, 12\) to index the corresponding calendar months.\(^2\) The first term of the equation is then

\[\mu_t = \mu^M_{m(t)} \quad (2)\]

where \(\mu\) is a 12-element row vector of monthly reference levels of use to be estimated, and \(M_{m(t)}\) is a 12-element column vector, all elements of which are zero except the \(m(t)\)th, which is one.

Exchange size factor \(a_i\) reflects the fact that local telephone use tends to be lower in smaller exchanges, presumably because there are fewer people in the local area to call.\(^3\) Systematic differences in telephone use among exchanges may also result from other economic, demographic, and unmeasurable differences.

---

\(^2\) The first observation, for example, is for September, so \(m(1) = 9\).

\(^3\) This is the basis for the common telephone company practice of charging more for flat-rate service in larger local calling areas.
\[ a_i = a D_i \]  

where \( a \) is the three-element row vector \([1 \ a_C \ a_T]\) and \( D_i \) is a three-element column vector with all elements zero except that the first equals one if the observation is for Jacksonville, the second equals one if the observation is for Clinton, and the third equals one if the observation is for Tuscola. Thus \( a_j = 1 \) (for the largest exchange), and \( a_C \) and \( a_T \) are to be estimated (for the two smaller exchanges).

Growth factor \( \gamma_{it} \) accounts for any smoothly trended effects. We specify each growth factor as a linear trend line with an initial value of 1 on September 1, 1975, and a change in slope at the introduction of single-party measured service on September 1, 1977. The trends may differ in each of the three cities. Analytically, the growth factor is specified as

\[
\gamma_{it} = 1 + F_t [\gamma_1 D_i (t - .5)] + (1 - F_t) [24 \gamma_1 D_i + \gamma_2 D_i (t - 24.5)],
\]

where \( \gamma_1 \) and \( \gamma_2 \) are three-element row vectors \([\gamma_{J1} \gamma_{C1} \gamma_{T1}]\) and \([\gamma_{J2} \gamma_{C2} \gamma_{T2}]\) of growth rates in each of the three exchanges during flat-rate months and measured-rate months, respectively; \( D_i \) is the column vector of exchange dummies used above; and \( F_t \) is a dummy variable equal to 1 during flat-rate months \((t = 1, \ldots, 24)\) and 0 otherwise.

Repression factor \( n_{it} \) captures the reduction in use due to the experimental usage charges, excluding the reduction that is due to the substitution of multi-party for single-party use. We expect both the per-call charge \( P_C \) and the per-minute charge \( P_M \) to affect both calls per main station and minutes per main station. We

---

The .5s appear in Eq. (4) because each monthly observation represents the middle of the month, whereas the trend line starts at the beginning of September 1975.
specify the repression factor as a negative exponential function of the prices:

\[ \eta_{it} = \exp(-\eta_{c,C, it} - \eta_{p,P, it}) \]  

(5)

In this formulation, a given absolute change in price results in a constant percentage change in use, a relationship that is illustrated by the \( Q = \exp(-\eta P) \) line in Fig. 8. In our case, this formulation makes more sense than do two other commonly specified functional forms. The constant elasticity relationship \( Q = P^{-\eta} \) is ruled out because it implies infinite use at a zero price (flat rates). The linear relationship \( Q = 1 - \eta P \) is numerically very close to our negative exponential specification in the range of observed reductions in use (20 percent or less). The linear relationship is, however, probably less realistic than the negative exponential for extrapolating the effects of usage charges substantially higher.
than those used in the Illinois experiment.\footnote{For example, a 29-cent charge per call would drive the number of calls to zero in a linear model. In a negative exponential model, it would cut the number of calls to 37 percent of the number under flat rate. The latter seems more realistic. (These values are based on regression results reported in Sec. V.A below.)}

Substitution factor $\delta_{it}$ is specified to be the same sort of function of usage charges as is the repression factor:

$$
\delta_{it} = \exp(-\delta_P C_{it} - \delta_M M_{it}).
$$

(6)

Note that the substitution factor and the repression factor are not separately identified in the single-party part of the equation because they are identical functions of the same variables $P_C$ and $P_M$. The total reduction in single-party use is estimable, but it can be ascribed in arbitrary proportions to repression and to substitution. We need an extraneous estimate of substitution to obtain a separate estimate of repression. We get the extraneous estimate from the multi-party part of the equation, which is specified below.

**Multi-Party Use**

Multi-party use =

$$
\begin{bmatrix}
\text{Monthly reference level use} \\
\text{Exchange size factor}
\end{bmatrix}
\begin{bmatrix}
\text{Multi-party factor} \\
\text{Growth factor}
\end{bmatrix}
\begin{bmatrix}
\text{Substitution factor}
\end{bmatrix}
+ \text{error}
$$

$$
= \mu_t \cdot \alpha_i \cdot \beta_i \cdot \gamma_{it} \cdot \delta_{it}^{*} + \varepsilon_{it}.
$$

(7)

Monthly reference level use $\mu_t$ and exchange size factor $\alpha_i$ are exactly the same as in the single-party part of the equation.

Multi-party factor $\beta_i$ accounts for the fact that multi-party use is observed to be less than single-party use when both groups of subscribers are on a flat-rate tariff. We allow the usage ratio to differ among exchanges, and specify
\[ \beta_i = \beta D_i \]  

(8)

where \( \beta \) is the three-element row vector \([\beta_J \beta_C \beta_T]\) and \( D_i \) is the column vector of exchange dummies used in the specification of the smaller exchange factor.

**Growth factor** \( \gamma_i^* \) is specified to have the same form as the single-party growth factor \( \gamma_{it} \); however, the multi-party slopes \( \gamma_i^* = [\gamma_{i1}^*, \gamma_{i2}^*, \gamma_{iT}^*] \) and \( \gamma_i^* = [\gamma_{i1}^*, \gamma_{i2}^*, \gamma_{iT}^*] \) may differ from the single-party values.

\[
\gamma_{it}^* = 1 + F_t[\gamma_{i1}^* D_i(t - .5)] \\
+ (1 - F_t)[2\gamma_{i2}^* D_i + \gamma_{iT}^* D_i(t - 24.5)] .
\]  

(9)

**Substitution factor** \( \delta_{it}^* \) is identified in the multi-party part of the equation. The increase in multi-party use after imposition of a measured-service tariff on single-party use is attributed entirely to substitution. This increase can be used to identify the decrease in single-party use due to substitution \( \delta_{it} \) by using the following:

**Identifying Assumption:** The decrease due to substitution in the total number of calls (or total minutes of calling) by single-party customers is equal to the increase due to substitution in the total number of calls (or total minutes of calling) by multi-party customers.

To write the identifying assumption in terms of the factors in the model, we multiply the change in single- and multi-party calls per main station by the number of single- and multi-party main stations, \( N_{it} \) and \( N_{it}^* \), respectively. Then the assumption is

\[
N_{it} \alpha_i \gamma_{it} (1 - \delta_{it}) = N_{it}^* \alpha_i \beta_i \gamma_{it}^* (\delta_{it}^* - 1) .
\]

Solving, we find
\[ \delta_{it}^* = 1 + (N_{it}/N^*)\delta_{it}^* \gamma_{it}/\gamma^*_{it} (\eta_{it}/\beta_{i})(1 - \delta_{it}) \] (10)

to complete the specification of the multi-party part of the equation.

**The Complete Estimating Equation**

We combine Eqs. (1) and (7) (single-party use and multi-party use) into a single estimating equation with the help of a class-of-service dummy variable \( C_j \) which equals 0 if the observation is for single-party use \((j = 1)\) and 1 if for multi-party use \((j = 2)\). Then the estimating equation, including the additive error \( \varepsilon_{ijt} \), is

\[ \text{Use}_{ijt} = (1-C_j)\mu_{it}^\alpha_i\gamma_{it}^\gamma_{it}^* \delta_{it}^* + C_j \mu_{it}^\alpha_i\gamma_{it}^\gamma_{it}^* \delta_{it}^* + \varepsilon_{ijt}, \] (11)

where \( i = 1, 2, 3 \) exchanges
\( j = 1, 2 \) classes of service
\( t = 1, \ldots, 52 \) months.

When we write out the full specification of each of the factors, substituting Eqs. (2) through (6) and (8) through (10) in (11), the equation is

\[ \text{Use}_{ijt} = (1-C_j)\mu_{m(t)}^M_{it}^p \gamma_{it}^\gamma_{it}^* \delta_{it}^* [1 + F_t(\gamma_{it}^\gamma_{it}^* \delta_{it}^* + (1-F_t)[24\gamma_{it}^\gamma_{it}^* \delta_{it}^* + \varepsilon_{ijt} \right] \right] \]

\[ + \exp(-n_{C,C,It} - n_{M,M,It}) \exp(-\delta_{C,C,It} - \delta_{M,M,It}) \]

\( \cdot \exp(-n_{C,C,It} - n_{M,M,It}) \exp(-\delta_{C,C,It} - \delta_{M,M,It}) \]

\[ + C_j \mu_{m(t)}^M_{it}^p \gamma_{it}^\gamma_{it}^* [1 + F_t(\gamma_{it}^\gamma_{it}^* \delta_{it}^* + (1-F_t)[24\gamma_{it}^\gamma_{it}^* \delta_{it}^* + \varepsilon_{ijt} \right] \right] \]

\[ + \exp(-n_{C,C,It} - n_{M,M,It}) [1 - \exp(-\delta_{C,C,It} - \delta_{M,M,It})] \]

\[ + \varepsilon_{ijt}. \] (12)

---

Greek letters are coefficients or vectors of coefficients to be estimated; roman letters are "data," many of them dummy variables. Note that \( F_{C,It} \) and \( F_{M,It} \) are the usage charges for single-party.
C. DETAILED SPECIFICATION: ERROR STRUCTURE

We have specified the systematic portion of a nonlinear model

\[ y_{ijt} = f(x_{ijt}; \theta) + \varepsilon_{ijt}, \quad (13) \]

where \( y \) is telephone use (either calls or minutes), \( x \) is a set of independent variables, and \( \theta \) is a set of coefficients to be estimated. We shall now specify a structure for the error covariance matrix, \( E(\varepsilon \varepsilon') = \Sigma \). For this purpose it is convenient to order our observations by month, within months by exchange, and within exchanges by class of service.\(^7\) We can then replace the exchange index \( i = 1, 2, 3 \) and the class of service index \( j = 1, 2 \) with a single index, \( i = 1, \ldots, 6 \), of exchange/class-of-service combinations.\(^8\) Thus our data are structured as \( T = 52 \) monthly groups of \( N = 6 \) observations.

We make the following assumptions about the error terms \( \varepsilon_{it} \):

* Zero mean: \( E(\varepsilon_{it}) = 0 \), for all \( i, t \).

* Monthly heteroscedasticity: We expect the error variance to differ from one calendar month to another. Winter months, with generally higher telephone use and more variable and more extreme weather, are likely to have larger error variance than summer months. Thus we specify \( E(\varepsilon_{it} \varepsilon_{jt}') = \kappa m(t) \varepsilon_{ij} \), for all \( i, j, t \).

* Contemporaneous heteroscedasticity: Use by different exchange/class-of-service categories may have different variances. Thus we allow \( \sigma_{ii} \) and \( \sigma_{jj} \) to differ, all \( i \neq j \).

* Contemporaneous covariance: Different exchange/class-of-service categories are subject to some of the same stochastic influences. This is particularly true for single- and multi-party service within the service in exchange \( i \) during month \( t \). Thus they may take on positive values in the multi-party part of the equation, despite the fact that multi-party usage charges are always zero in our data.

\(^7\)Within each month, observations are ordered as follows: Jacksonville single-party, Jacksonville multi-party, Clinton single-party, Clinton multi-party, Tuscola single-party, Tuscola multi-party.

\(^8\)For the remainder of Sec. IV.C, the subscripts \( i, j, k, \) and \( t \) are alternative indexes of the six exchange/class-of-service combinations.
same exchange. Since all three exchanges are within about 100 miles of one another, it is probably true between exchanges as well; the effects of weather, for example, will be similar in all three exchanges. We thus allow $\sigma_{ij}$ to be nonzero for $i \neq j$, and to differ from $\sigma_{k1}$, $ij \neq k2$.

First-order autocorrelation: There is no reason to suppose that stochastic influences on telephone use respect the boundaries between calendar months. To allow for correlation of residuals over time, we specify a simple first-order autocorrelated structure:

$$E(e_i t e_j s) = \rho \left| t-s \right| \kappa^{1/2}_m(t) \kappa^{1/2}_m(s) \sigma_{ij}.$$  \hspace{1cm} (14)

These assumptions about the error structure imply the following error covariance matrix:

$$\Sigma = \sigma^2_e K^{1/2}_R K^{1/2} \Theta V$$

where

$$K^{1/2}_{(T \times T)} = \begin{bmatrix}
\kappa^{1/2}_m(1) & 0 & 0 & \cdots & 0 \\
0 & \kappa^{1/2}_m(2) & 0 & \cdots & \\
& \kappa^{1/2}_m(3) & \cdots & 0 & \\
\text{diagonal} & & & & \\
& & & \kappa^{1/2}_m(T) & \\
\end{bmatrix}$$

$$R^{(T \times T)} = \begin{bmatrix}
1 & \rho & \rho^2 & \cdots & \rho^{T-1} \\
1 & \rho & \cdots & \rho^{T-2} & \\
1 & \cdots & \rho^{T-3} & \\
\text{symmetric} & & & & \\
& & & 1 & \\
\end{bmatrix}$$


\[ V_{(N \times N)} = \begin{bmatrix} \sigma_{11} & \sigma_{12} & \sigma_{13} & \cdots & \sigma_{1N} \\ \sigma_{21} & \sigma_{22} & \sigma_{23} & \cdots & \sigma_{2N} \\ \sigma_{31} & \sigma_{32} & \sigma_{33} & \cdots & \sigma_{3N} \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ \sigma_{NN} & & & & \end{bmatrix} \]

and we normalize so that \( \text{tr}(K^{\frac{1}{2}}K^{\frac{1}{2}}) = T \) and \( \text{tr}(V) = N \).

Nonlinear generalized least squares (GLS) estimates of the coefficients \( \theta \) are found by minimizing

\[ (y - f(x; \theta))'(\Sigma^{-1})(y - f(x; \theta)) \]

As in the linear case, it is computationally convenient to find a transformation matrix \( H \) such that \( H'\Sigma H = \Sigma^{-1} \). We have \( \Sigma^{-1} = (1/\sigma^2)(K^{\frac{1}{2}})^{-1}R^{-1}(K^{\frac{1}{2}})^{-1} \theta V^{-1} \). If we define \( K^{-\frac{1}{2}} = (K^{\frac{1}{2}})^{-1} \), \( P'P/(1-\rho^2) = R^{-1} \), and \( (V^{-\frac{1}{2}})'V^{-\frac{1}{2}} = v^{-1} \), that is:

\[
K^{-\frac{1}{2}} = \begin{bmatrix}
\kappa_{m(1)}^{-\frac{1}{2}} & 0 & 0 & \cdots & 0 \\
0 & \kappa_{m(2)}^{-\frac{1}{2}} & 0 & \cdots & 0 \\
0 & 0 & \kappa_{m(3)}^{-\frac{1}{2}} & \cdots & 0 \\
\vdots & \vdots & \vdots & \ddots & \vdots \\
0 & 0 & 0 & \cdots & \kappa_{m(T)}^{-\frac{1}{2}}
\end{bmatrix}
\]

\[
P = \begin{bmatrix}
(1-\rho^2)^{-\frac{1}{2}} & 0 & 0 & \cdots & 0 \\
-\rho & 1 & 0 & \cdots & 0 \\
0 & -\rho & 1 & \cdots & 0 \\
\vdots & \vdots & \vdots & \ddots & \vdots \\
0 & 0 & 0 & \cdots & 1
\end{bmatrix}
\]

\[
V^{-\frac{1}{2}} = \text{Cholesky decomposition of } V^{-1},
\]
then we have

\[ \Sigma^{-1} = \left[ \frac{1}{\sigma^2(1-\rho^2)} \right] \left( P K^{-1/2} \otimes V^{-1/2} \right)' \left( P K^{-1/2} \otimes V^{-1/2} \right). \]  

(15)

Our transformation matrix (up to a scalar multiple) is

\[ H = P K^{-1/2} \otimes V^{1/2} \]
\[ = P K^{-1/2} I_T \otimes I_N \otimes V^{1/2} \]
\[ = (P \otimes I_N)(K^{-1/2} \otimes I_N)(I_T \otimes V^{1/2}) \]
\[ = (P \otimes I_N)(I_T \otimes V^{1/2})(K^{-1/2} \otimes I_N). \]  

(16)

Thus we can do the transformation in steps, transforming first for monthly heteroscedasticity \( K^{-1/2} \), then for monthly covariance \( V^{1/2} \), and last for autocorrelation \( P \).

We can then find the generalized least squares estimates of the coefficients \( \theta \) by a nonlinear ordinary least squares (OLS) regression of the transformed variables \( Hy \) on the transformed function values \( Hf(x; \theta) \). This is fairly easily accomplished using a standard nonlinear regression package, for example, the SAS procedure NLIN.\(^9\) Unlike the linear case, the transformation does not apply to the independent variables individually. Instead, the transformation applies "outside" the nonlinear function. Our SAS program to do this is in Appendix B.

Of course, we do not know \( \Sigma \). We substitute consistent estimates of its elements \( \kappa_m(t), \sigma_{ij} \), and \( \rho \) based on residuals \( e \) from a first-stage ordinary nonlinear regression. We choose estimators that should be reasonably efficient given the nature of our data.

Our procedure is first to obtain consistent estimators of monthly heteroscedasticity \( K \), second to use these in the consistent estimation of contemporaneous covariance \( V \), and finally to use both in estimating autocorrelation \( R \).

First we estimate monthly heteroscedasticity factors

\[ \hat{\kappa}_m = \frac{1}{N T_m} \sum_{t=1}^{T} \sum_{i=1}^{2} e_{it}^2, \quad \tau = (t:m(t) = m) \] (17)

where \( T_m \) is the number of times the \( m \)th month recurs in our data.\(^{10}\)

We then normalize so that

\[ \sum_{t=1}^{T} \hat{\kappa}_m(t) = T. \]

We find that the variance does indeed differ greatly from month to month. For the minutes model, the highest \( \kappa \) is over 3 in January, and the lowest is under .3 in July.\(^{11}\) The range is even greater for the calls model. The complete set of estimates for \( \kappa \) and other elements of the covariance matrix is in Table 5.

To estimate the contemporaneous covariance matrix \( V \), we first weight the OLS residuals to account for monthly heteroscedasticity. Substituting the estimated \( \kappa \) in \( K^{-\frac{1}{2}} \), we calculate transformed residuals \( e^* = (K^{-\frac{1}{2}} \otimes I_N) e \). We use these to estimate

\[ \hat{\sigma}_{ij} = \frac{1}{T} \sum_{t=1}^{T} e_{it}^* e_{jt}^* , \] (18)

then rescale the estimates so that

\[ \sum_{i=1}^{N} \hat{\sigma}_{ii} = N. \]

---

\(^{10}\) For example, September recurs five times, so \( T_9 = 5 \).

\(^{11}\) Residential telephone use shoots up when severe weather keeps everyone at home. The high variance for the winter months results from the random occurrence of a small number of especially bad storms.
Table 5
ESTIMATES OF ELEMENTS OF COVARIANCE MATRIX I

<table>
<thead>
<tr>
<th>Month</th>
<th>Calls Model</th>
<th>Minutes Model</th>
</tr>
</thead>
<tbody>
<tr>
<td>January</td>
<td>4.320</td>
<td>3.602</td>
</tr>
<tr>
<td>February</td>
<td>.736</td>
<td>.586</td>
</tr>
<tr>
<td>March</td>
<td>2.308</td>
<td>1.703</td>
</tr>
<tr>
<td>April</td>
<td>.289</td>
<td>.194</td>
</tr>
<tr>
<td>May</td>
<td>.195</td>
<td>.902</td>
</tr>
<tr>
<td>June</td>
<td>.269</td>
<td>.400</td>
</tr>
<tr>
<td>July</td>
<td>.219</td>
<td>.292</td>
</tr>
<tr>
<td>August</td>
<td>.338</td>
<td>.437</td>
</tr>
<tr>
<td>September</td>
<td>.774</td>
<td>.928</td>
</tr>
<tr>
<td>October</td>
<td>.213</td>
<td>.353</td>
</tr>
<tr>
<td>November</td>
<td>.799</td>
<td>.682</td>
</tr>
<tr>
<td>December</td>
<td>1.673</td>
<td>1.943</td>
</tr>
</tbody>
</table>

Contemporaneous Covariance Matrix V

<table>
<thead>
<tr>
<th></th>
<th>Jacksonville Single-party (1)</th>
<th>Multi-party (2)</th>
<th>Clinton Single-party (3)</th>
<th>Multi-party (4)</th>
<th>Tuscola Single-party (5)</th>
<th>Multi-party (6)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Calls Model</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>(1)</td>
<td>.800</td>
<td>.776</td>
<td>.306</td>
<td>.066</td>
<td>.393</td>
<td>.542</td>
</tr>
<tr>
<td>(2)</td>
<td></td>
<td></td>
<td>.341</td>
<td>.303</td>
<td>.263</td>
<td>.512</td>
</tr>
<tr>
<td>(3)</td>
<td></td>
<td></td>
<td>.1025</td>
<td>.754</td>
<td>.203</td>
<td>.423</td>
</tr>
<tr>
<td>(4)</td>
<td></td>
<td></td>
<td></td>
<td>.281</td>
<td>.018</td>
<td>.378</td>
</tr>
<tr>
<td>(5)</td>
<td>symmetric</td>
<td></td>
<td></td>
<td></td>
<td>.712</td>
<td>.511</td>
</tr>
<tr>
<td>(6)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>.994</td>
</tr>
<tr>
<td>Minutes Model</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>(1)</td>
<td>.733</td>
<td>.579</td>
<td>.341</td>
<td>.083</td>
<td>.320</td>
<td>.312</td>
</tr>
<tr>
<td>(2)</td>
<td></td>
<td></td>
<td>.881</td>
<td>.498</td>
<td>.246</td>
<td>.330</td>
</tr>
<tr>
<td>(3)</td>
<td></td>
<td></td>
<td>.772</td>
<td>.642</td>
<td>.422</td>
<td>.453</td>
</tr>
<tr>
<td>(4)</td>
<td></td>
<td></td>
<td></td>
<td>.1387</td>
<td>.261</td>
<td>.346</td>
</tr>
<tr>
<td>(5)</td>
<td>symmetric</td>
<td></td>
<td></td>
<td></td>
<td>.680</td>
<td>.615</td>
</tr>
<tr>
<td>(6)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>1.547</td>
</tr>
</tbody>
</table>

First-Order Autocorrelation \( \rho \) and Variance \( \sigma^2 \)

<table>
<thead>
<tr>
<th></th>
<th>Calls Model</th>
<th>Minutes Model</th>
</tr>
</thead>
<tbody>
<tr>
<td>Autocorrelation</td>
<td>.193</td>
<td>.258</td>
</tr>
<tr>
<td>Variance</td>
<td>19.119</td>
<td>383.312</td>
</tr>
</tbody>
</table>

NOTE: All estimates are based on residuals from nonlinear OLS (first-stage) regressions. Variance is estimated as mean squared error; other estimates are described in the text.
An unweighted estimate of \( V \) would be much less efficient because it would be dominated by high-variance residuals from a few winter months. We do find some contemporaneous heteroscedasticity. In the minutes model, single-party variance terms in the three exchanges are closely grouped between .68 and .77, but multi-party variance terms range from .88 to 1.55. The within-exchange covariance terms average .61; the between-exchange covariances are smaller, as expected, averaging .33.

Although the effect of contemporaneous covariance is less important than the effect of monthly heteroscedasticity, we can increase the precision of estimated \( \rho \) somewhat by correcting for estimated \( V \) as well. The further transformed residuals are \( e^{**} = (I_T \otimes V^{-\frac{1}{2}})e^* \). These yield an estimate of

\[
\hat{\rho} = \frac{1}{N(T-1)} \sum_{t=1}^{T-1} \sum_{i=1}^{N} e^{**}_{it} e^{**}_{i,t+1} \sqrt{NT} \sum_{t=1}^{T} \sum_{i=1}^{N} e^{**}_{it} e^{**}_{it} \cdot \tag{19}
\]

For the minutes model, \( \hat{\rho} = .26 \).

We use these consistent estimates of the elements of \( \Sigma \) in our transformation matrix \( H \). The resulting nonlinear generalized least squares estimates of \( \vartheta \) are consistent and asymptotically efficient. Presumably they are also more efficient in finite samples than are ordinary least squares estimates. The coefficient estimates are presented in the following section.
V. DEMAND ELASTICITY ESTIMATES

A. ESTIMATED PRICE EFFECTS

Nonlinear generalized least squares estimates of the price effects in Eq. (12) are shown in Table 6.\(^1\) The estimated price effects all have the correct sign (price increases reduce use), and their relative magnitudes seem generally reasonable. The per-call charge \(P_C\) has about the same effect on the number of minutes as it does on the number of calls, indicating that the calls forgone because of \(P_C\) are of about average duration. The per-minute charge \(P_M\) has a larger effect on the number of calls than does \(P_C\), which seems reasonable in that most calls last longer than one minute. The effect of \(P_M\) on the number of minutes is larger still; thus a per-minute charge reduces average call duration. The asymptotic t-statistics (in parentheses) are all quite high.\(^2\)

Our estimates partition the reduction in single-party use into a repression effect and a substitution effect. Because there is no usage charge for calls from multi-party phones, some multi-party use gets substituted for single-party use. The repression effect accounts for the remainder of the reduction. The estimated repression effects are substantially larger than the substitution effects. However, at least for the per-minute charge \(P_M\), the substitution effects are too large to be neglected.

The estimated price effects do not change very much when the model is estimated without trends; see Table 9, Sec. V.B. Thus we can be confident that inclusion of the trend terms does not bias the estimated price effects.

---

\(^1\)Estimates of the other (nuisance) coefficients are tabulated in Appendix A, along with the first-stage OLS estimates. The OLS estimates are similar in pattern to the GLS estimates. There are two anomalous signs on insignificant OLS coefficients. The GLS estimated standard errors are mostly smaller than those estimated by OLS.

\(^2\)The estimated t-statistics may overstate the true significance of the estimated coefficients. See, for example, Park and Mitchell (1980) or Gallant and Goebel (1976).
Table 6
ESTIMATED PRICE EFFECTS

<table>
<thead>
<tr>
<th>Resulting from a 1-Cent Increase in</th>
<th>Repression</th>
<th>Substitution</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Calls</td>
<td>Minutes</td>
</tr>
<tr>
<td>Per-call charge, ( P_C )</td>
<td>-3.0</td>
<td>-3.4</td>
</tr>
<tr>
<td></td>
<td>(6.0)</td>
<td>(3.9)</td>
</tr>
<tr>
<td>Per-minute charge, ( P_M )</td>
<td>-5.5</td>
<td>-10.9</td>
</tr>
<tr>
<td></td>
<td>(4.8)</td>
<td>(5.8)</td>
</tr>
</tbody>
</table>

NOTES: The repression effects are estimates of \( \eta_C \) and \( \eta_M \) in Eq. (12) (expressed as negative percent). The substitution effects are estimates of \( \delta_C \) and \( \delta_M \). Effects on calls and effects on minutes are from separate regressions with, respectively, calls per main station and minutes per main station as dependent variables. Estimated asymptotic t-statistics are in parentheses. Estimates of the other (nuisance) coefficients in the estimating Eq. (12) are tabulated in Appendix A. Appendix A also shows one more digit for the price effects than does this table. Values from Appendix A are used to compute values shown in subsequent tables in this section.

Estimated Price Elasticities

The estimated price effects in Table 6 are not themselves elasticities. In our model, the elasticities vary with price. Specifically, the elasticity is equal to the price effect (\( \eta_C \) or \( \delta_C \), for example) multiplied by the corresponding price (\( P_C \) in this case). Elasticities at the experimental prices in effect during December 1979 (\( P_C = 2.5, P_M = 1 \)) are shown in Table 7. The elasticities are all quite small; none much exceeds .1 in absolute value. They would, however, be proportionately higher at higher prices.³

³This is qualitatively consistent with the higher elasticities estimated by others for (higher-priced) toll calls, although clearly local and toll calls are not the same commodity; see Taylor (1980, pp. 97-117, 170).
Table 7
ESTIMATED PRICE ELASTICITIES AT DECEMBER 1979 EXPERIMENTAL PRICES

<table>
<thead>
<tr>
<th>Charge</th>
<th>Repression Elasticity</th>
<th>Substitution Elasticity</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Calls</td>
<td>Minutes</td>
</tr>
<tr>
<td>Per-call charge, $P_C$</td>
<td>-.076</td>
<td>-.086</td>
</tr>
<tr>
<td></td>
<td>(6.0)</td>
<td>(3.9)</td>
</tr>
<tr>
<td>Per-minute charge, $P_M$</td>
<td>-.055</td>
<td>-.109</td>
</tr>
<tr>
<td></td>
<td>(4.8)</td>
<td>(5.8)</td>
</tr>
</tbody>
</table>

NOTE: Estimated asymptotic t-statistics are in parentheses.

a December 1979 experimental prices are $P_C = 2.5c$ and $P_M = 1c$.

b Elasticities with respect to $P_C$ and $P_M$ are calculated as $-P_C \eta_C$ and $-P_M \eta_M$, respectively. Values for $\eta_C$ and $\eta_M$ are from Appendix A.

c Elasticities with respect to $P_C$ and $P_M$ are calculated as $-P_C \delta_C$ and $-P_M \delta_M$, respectively. Values for $\delta_C$ and $\delta_M$ are from Appendix A.

Estimated Reductions in Use Due to Experimental and Hypothetical Tariffs

The estimated price effects in Table 6 may be used in the estimating Eq. (12) to calculate percentage reductions in single-party use due to experimental and other tariffs. The results are shown in Table 8. Both the total reductions in use due to the experimental tariffs and the change due to substitution alone are consistent with other estimates, as discussed in Sec. V.B.

The hypothetical tariffs are included to illustrate the separate effects of per-call and per-minute charges by setting one of the prices equal to zero and keeping the other at its current experimental value. Setting $P_C = 0$ cuts the reduction in calls to about 55 percent of the reduction caused by the current tariff. Setting $P_M = 0$ cuts the

---

4For example, with $P_C = 2$ and $P_M = 1$, the repression factor for calls is $\exp(-.0302 \times 2 - .0555 \times 1) = .891$. The percentage change is $(.891 - 1) \times 100 = -10.9$. The values for the price effects are from Appendix A.
Table 8
ESTIMATED REDUCTION IN SINGLE-PARTY TELEPHONE USE DUE TO VARIOUS TARIFFS

<table>
<thead>
<tr>
<th>Change from Flat Rate to</th>
<th>Percent Reduction Due To</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Repression Calls Minutes</td>
</tr>
<tr>
<td>Experimental tariffs</td>
<td></td>
</tr>
<tr>
<td>$P_C = 2, P_M = 1^a$</td>
<td>10.9 16.3</td>
</tr>
<tr>
<td>$P_C = 0, P_M = 1.5^b$</td>
<td>8.0 15.1</td>
</tr>
<tr>
<td>$P_C = 2.5, P_M = 1^c$</td>
<td>12.3 17.7</td>
</tr>
<tr>
<td>Hypothetical tariffs</td>
<td></td>
</tr>
<tr>
<td>$P_C = 0, P_M = 1$</td>
<td>5.4 10.3</td>
</tr>
<tr>
<td>$P_C = 2.5, P_M = 0$</td>
<td>7.3 8.2</td>
</tr>
</tbody>
</table>

**NOTE:** The substitution effects are calculated as the difference between the total and the repression effects.


$^b$Experimental tariff in Clinton and Tuscola from September 1977 through May 1979.

$^c$Experimental tariff in all three exchanges from June 1979 through December 1979.

Reduction in minutes to about 40 percent of the reduction caused by the current tariff.

**B. COMPARISON WITH OTHER ESTIMATES**

We will next compare the above estimates with a number of others, to check on their robustness and reasonableness. The comparisons are with other results that fall in four categories:
An extraneous estimate of the effects of subscriber substitution, to assure that the econometrically estimated substitution effect is reasonable;

Two variants of our model, to assure that our results are not unduly sensitive to particular assumptions;

Other researchers' estimates of the total reductions in use due to the experimental tariffs; and

Mitchell's (1978) theoretical model of telephone demand.

An Extraneous Estimate of Subscriber Substitution

The substitution effects in Table 8 lump together what we have termed direct substitution and subscriber substitution. In the model, there is no way to identify separately the two kinds of substitution, but we can use extraneous information to estimate subscriber substitution separately. From Table 4, the subscriber substitution effect equals \( n_{12} u_{12}^M \)--the number of switchers from single- to multi-party service times their average telephone use under measured rates. We estimate \( n_{12} \) from Fig. 6 to be 106 households.

To estimate \( u_{12}^M \), we use a sample of 1839 telephone numbers from all three experimental exchanges.\(^5\) Among those 1839 households, there were 31 that downgraded from single- to multi-party service between July 1977 and October 1979. We have data on their telephone use while they were single-party customers; thus June of 1977 is the last month during which we have data for all of them. Their use during that month was approximately double that of the average single-party subscriber, as shown in the following table.\(^6\)

\(^5\) These are all of the numbers interviewed by Opinion Research, Inc., under contract to GTE, during the first three of four survey waves (September-October 1975, December 1975, and April 1978). See Opinion Research (1980) for details of the survey.

\(^6\) There is no significant difference between use by customers who downgraded their class of service during the first year following July 1977, and use by customers who downgraded later.
June 1977 Telephone Use

<table>
<thead>
<tr>
<th>Sample of 31 switchers</th>
<th>Calls</th>
<th>Minutes</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mean</td>
<td>169.0</td>
<td>730.0</td>
</tr>
<tr>
<td>Standard error of mean</td>
<td>(16.5)</td>
<td>(74.7)</td>
</tr>
</tbody>
</table>

All single-party subscribers\(^7\)

| Mean                  | 90.5  | 335.3  |

Using these households' flat-rate use during June 1977 to estimate \(u_{12}^M\), we can calculate the subscriber substitution effect as \(n_{12}u_{12}^M = 106 \times 169.0 = 17,900\) calls and \(106 \times 730.0 = 77,400\) minutes.

For comparison with Table 8 estimates, we can put the subscriber substitution effect in percentage terms as follows: During June 1977, all 12,310 single-party subscribers made an average of 90.5 calls and talked for an average of 335.3 minutes,\(^8\) for a total of 1,114,000 calls and 4,128,000 minutes. Switching would reduce the average calls to \((1,114,000 - 17,900)/(12,310 - 106) = 89.8\) per subscriber and average minutes to \((4,128,000 - 72,400)/(12,310 - 106) = 331.9\). That is an 8 percent reduction in calls and a 1.0 percent reduction in minutes.

Comparing these figures to the total substitution effects in Table 8, we see that subscriber substitution accounts for about one-quarter (in the case of minutes) to one-half (in the case of calls) of the full substitution effect. The remainder is direct substitution—increased use by continuing multi-party subscribers.

Variants of Our Model

We give results from two variants of our model: a model without trend terms and a model without price constraints. The trend terms in our model are included to control for gradual changes in subscriber

\(^7\)Calculated using data tables available from GTE (see Sec. III.C).

\(^8\)Source: Data tables available from GTE (Sec. III.C).
populations, economic conditions, and other nonprice factors that may affect telephone use. It is conceivable that the trend terms might be overcontrolling, and capturing some price effects as well. For example, assume that telephone use is gradually creeping back to higher levels following an initial drop due to measured rates. A model with trend terms would estimate repression based on the initial drop, whereas a model without trend terms would estimate repression averaged over a longer period. The latter might be more interesting.

No trend terms. We check the sensitivity of our estimates to the presence of trend terms by estimating a model without them. The results, shown in Table 9, differ little from those for the model with trends in Table 6. The repression effects in the minutes equation are slightly smaller. Significance levels are higher, because estimated price effects and estimated trends are correlated in the full model. Substitution effects are higher because they include some of the effects of continued switching from single-party to multi-party service subsequent to the jump in September 1977.

Table 10 compares total reductions in single-party telephone use estimated in the models with and without trends. The estimated reductions in calls are very close together. The reductions in minutes differ somewhat more, but are still within about two percentage points of each other.

### Table 9

ESTIMATED PRICE EFFECTS IN A MODEL WITHOUT TREND TERMS

<table>
<thead>
<tr>
<th>Resulting from a 1¢ Increase in</th>
<th>Percent Change Due To</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Repression Calls</td>
</tr>
<tr>
<td>Per-call charge, $p_c$</td>
<td>-3.0 (7.7)</td>
</tr>
<tr>
<td>Per-minute charge, $p_m$</td>
<td>-5.2 (7.3)</td>
</tr>
</tbody>
</table>

**NOTE:** See Table 6.
Table 10
COMPARISON OF TOTAL PERCENTAGE REDUCTION IN SINGLE-PARTY TELEPHONE USE 
ESTIMATED USING DIFFERENT MODEL SPECIFICATIONS

<table>
<thead>
<tr>
<th>Type of Estimate</th>
<th>Initial Measured-Rate Tariff(^a) (September 1977-May 1979)</th>
<th>Subsequent Measured-Rate Tariff (June 1979-December 1979)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>J</td>
<td>C</td>
</tr>
<tr>
<td>Calls</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Price equation with trends(^b)</td>
<td>12.3</td>
<td>9.8</td>
</tr>
<tr>
<td>Price equation without trends</td>
<td>12.9</td>
<td>9.8</td>
</tr>
<tr>
<td>Direct estimate without trends</td>
<td>12.4</td>
<td>10.0</td>
</tr>
<tr>
<td>Standard error</td>
<td>(0.9)</td>
<td>(1.2)</td>
</tr>
<tr>
<td>Minutes</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Price equation with trends(^b)</td>
<td>19.3</td>
<td>19.3</td>
</tr>
<tr>
<td>Price equation without trends</td>
<td>17.4</td>
<td>17.5</td>
</tr>
<tr>
<td>Direct estimate without trends</td>
<td>16.4</td>
<td>17.7</td>
</tr>
<tr>
<td>Standard error</td>
<td>(1.3)</td>
<td>(1.5)</td>
</tr>
</tbody>
</table>

\(^a\)J denotes Jacksonville, C denotes Clinton, T denotes Tuscola.
\(^b\)From Table 8.
No price constraints. Our model specification imposes certain constraints on estimates of reductions in use. For example, because we have specified the repression and substitution factors to be functions of usage prices, the percentage reduction in use is constrained to be equal in exchanges with equal prices (Clinton and Tuscola from September 1977 to May 1979, or all three exchanges since June 1979). To check on how constraining our specification is, we also made "direct" estimates of the changes in use due to measured service. For the direct estimates, we replaced both the repression factor and the substitution factor in the single-party part of the equation with a single use-reduction factor that is allowed to take on different values in each exchange during each measured service tariff period. (We included a similar set of use-increase factors in the multi-party part of the equation, with no constraint connecting these to the single-party factors.)

Table 10 also compares the direct estimates with those based on the price equations. The two sets of estimates are remarkably close. All of the estimates based on the price equations without trends fall within two standard errors of the corresponding direct estimates. We conclude that our specification is not unduly constraining.

Other Researchers' Estimates of Total Reductions in Single-Party Telephone Use

Jensik (1979) also used time series data from the GTE experiment. He fit a Box-Jenkins model to data for the aggregate of all three exchanges from May 1975 through December 1978 and estimated a 12 percent reduction in calls and a 19 percent reduction in minutes. Jensik's estimates are very close to ours for the initial tariff period, as shown in Table 11.

Wilkinson (1981) fit a somewhat more complex Box-Jenkins model to GTE data from May 1975 to September 1980. His estimated reductions are generally larger than Jensik's and ours, as shown in Table 11. Wilkinson speculates that part of the reduction he estimates for the most recent tariff period is due to a slowdown in economic activity; this probably accounts for part of the difference from our estimates.
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<td>Jensik (1979)</td>
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</table>

\(^a\)A denotes the aggregate of all three cities, J denotes Jacksonville, C denotes Clinton, and T denotes Tuscola.

\(^b\)From Table 8.
Park et al. (1981, p. 6) calculate reductions in calls made by 641 surveyed households during June, July, and August 1977 (flat rate) and again during June, July, and August 1978 (initial measured rate). These reductions, smaller than the other estimates, are also shown in Table 11. Part of the reason is that all 641 sample households were single-party customers throughout the period June 1977 through August 1978. Thus there was no subscriber substitution in the sample, so reductions should be somewhat less than our estimates.

**Mitchell's Theoretical Model of Demand for Telephone Calls**

The final comparison is with theory, not with other empirical estimates. Mitchell (1978) derives some relationships among price effects in a model of demand for access to telephone service (subscriptions) and for telephone calls. If one thinks of the initiation of a telephone call as "access" to minutes of conversation, Mitchell's model can be applied *mutatis mutandis* to the demand for calls and for minutes.

In our notation, the repression effect of a per-call charge $P_C$ is measured by the coefficient $\eta_C$. If we append superscripts to distinguish between the effect on calls $\eta_C^C$ (estimated in the calls equation) and the effect on minutes $\eta_C^M$, and similarly for the effects of the per-minute charge $\eta_M^C$ and $\eta_M^M$, we can write Mitchell's restrictions as

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<th>Change in Minutes</th>
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<td>$\eta_C^M = (\bar{D}/\bar{D}) \eta_C^C$</td>
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<tr>
<td>Per-minute charge $P_M$</td>
<td>$\eta_M^C = \bar{D} \eta_C^C$</td>
<td>$\eta_M^M = \eta_M^D + (\bar{D}/\bar{D}) \eta_C^C$</td>
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</table>

where the new symbols are:
Denote the duration of a marginal call, that is, a call that is just worth making at prevailing levels of \( P_C \) and \( P_M \),

\[
\bar{D} = \text{the mean duration of all calls, that is, the total number of minutes divided by the total number of calls, and}
\]

\[
\eta_M^{Di} = \text{the repressing effect of } P_M \text{ on the duration of an individual call.}
\]

We estimate the four price effects \( \eta_C^C, \eta_M^C, \eta_C^M, \text{ and } \eta_M^M \). Are our estimates consistent with Mitchell's restrictions? Since Mitchell's restrictions also involve four parameters \( (\eta_C^C, \bar{D}, \bar{D}, \text{ and } \eta_M^{Di}) \), we can solve for the values of those parameters that are implied by our estimates. The implied values are

\[
\bar{D} = 1.8 ,
\]

\[
\bar{D} = 1.6 , \text{ and}
\]

\[
\eta_M^{Di} = .063 .
\]

We do not have any check on \( \bar{D} \) or \( \eta_M^{Di} \), but \( \bar{D} \) is directly observable; in our data, \( \bar{D} \) is about 3.5 minutes, which is more than twice as long as the implied value.

The conclusion, then, is that our estimates are not entirely consistent with Mitchell's restrictions as we have modified them to apply to calls and minutes of calling. Since the theoretical restrictions are based on some not-necessarily-realistic assumptions,

---

9 Using estimated values from Appendix A, the restrictions on \( \eta_M^C \) implies \( \bar{D} = \eta_M^C/\eta_C^C = .0555/.0302 = 1.84 \). Then the restriction on \( \eta_M^M \) implies \( \bar{D} = \eta_M^M/\eta_C^M = 1.84 \times .0302/.0342 = 1.62 \). Finally, the restriction on \( \eta_M^M \) implies \( \eta_M^{Di} = \eta_M^M - \bar{D}^2 \eta_C^C/\bar{D} = .1092 - (1.84)^2 \times .0302/1.62 = .063 .

10 The assumptions in our modification of Mitchell's model include: (a) The set of potential calls is fixed and finite (corresponding to the set of potential subscribers in Mitchell's model). This rules out the possibility, for example, that two or more calls
the inconsistency suggests that the theory needs elaboration for this application.

might be combined in response to the imposition of a call set-up charge. (b) Each potential call is characterized by a demand curve giving the value of a marginal minute of call duration, and the demand curves for all potential calls are non-intersecting. The non-intersection assumption, in particular, is clearly unrealistic. Certainly many brief calls are higher-valued than many long calls; contrast, for example, an emergency call to the fire department with a long, desultory chat with a boring acquaintance.
VI. APPLICATION ELSEWHERE

Our results go beyond previous work to provide precise estimates of repression and substitution effects of both per-call and per-minute charges in the GTE local measured-service experiment. We find that usage prices have a small but significant effect on local telephone use, with price elasticities ranging up to about 0.1 in absolute value. Our estimates are not, however, the complete answer to every telephone forecaster's prayers. The forecasting problem is to predict the effects that local measured service in other areas will have on capacity requirements and revenues. Our results may be an important input to this task. However, adjustments and judgments are required to account for differences in tariff details, behavior, and substitution opportunities.

A. EFFECTS OF TARIFF DETAILS

There are two details of the GTE experimental measured-service tariff that affect the response in Illinois and that may differ in other measured-service applications: the $19 ceiling on usage charges, and the evening and night discounts.

The experimental tariff limits usage charges to $19 per month per telephone line. In a typical month, a little over 1 percent of single-party customers exceed this ceiling. Marginal prices are zero for customers above the ceiling, so those customers presumably do not reduce their telephone use at all because of the experimental tariff.\(^1\) Under flat rates, the top 1 percent of users each month make about 5 percent of all calls. Moreover, under measured rates, the very largest users reduce their calling about twice as much as the average user (Park et al., 1981). Thus, if there were no ceiling, we would

---

\(^1\)However, if such customers are uncertain whether they will exceed (or have exceeded) the ceiling during a given month, they will reduce their use.
expect to observe about 10 percent more repression than was actually observed (say 16.5 percent rather than 15 percent).²

The experimental tariff includes discounts of 20 percent evenings and Sundays, and 50 percent nights. A different discount structure could alter the amount of repression, both during the peak period and overall. For example, if cross-price elasticities between tariff periods were large, greater off-peak discounts would result in more peak-period repression.

Other measured-service applications may have their own peculiarities that are not represented in the Illinois tariff, such as usage allowances (for example, the first 30 calls each month are free). Allowances will reduce the response to measured service. The effect of modest allowances will be small, however, because they only affect telephone use by the smallest users, who contribute little to the total reduction in use.

B. EFFECTS OF BEHAVIORAL DIFFERENCES

Park et al. (1981) find that repression by individual households within the experimental exchanges is systematically related to certain household characteristics. Based on those results, we would expect to find larger percentage reductions due to measured service in areas where telephone use exceeds that in the three Illinois cities, and in areas with larger (many-person) households. Smaller reductions would be expected in very-high-income areas. Use of our estimates for forecasting requires adjustments for these and other factors.

C. DIFFERENT SUBSTITUTION OPPORTUNITIES

The existence of a flat-rate multi-party option to single-party measured service in the GTE experiment accounts for part of the observed reduction in single-party telephone use. Some single-party

²This is a rough calculation. A more precise calculation would take into account the substantial differences in distributions of telephone use in the three different exchanges.
reduction is offset by an increase in multi-party use as some high-use single-party households switch to multi-party service, and others now originate a smaller fraction of their conversations with multi-party friends.

Substitution opportunities in other applications of measured service will vary. If measured service is mandatory for all residential subscribers, there will be no such opportunities. Then our repression estimate (appropriately adjusted for tariff details and behavioral differences) should predict total residential response.

Even with mandatory measured service, however, the forecaster must account for another kind of substitution. When residential service is changed from flat rate to measured, there is surely some increase in the number of personal calls placed from business telephones. This increase will offset some of the decrease in residential use, and must be accounted for in capacity and revenue forecasting.

If measured service is optional, then a large proportion of customers may choose to remain on flat rate. In that case, the substitution opportunities may exceed those in the Illinois experiment, where flat-rate multi-party customers are a small fraction of the total. The resultant substitution of unpriced flat-rate calls for priced measured-rate calls should be accounted for in forecasting revenues. Mitchell and Park (1981) find that total repression under optional measured service will be much less than in the GTE experiment, because households that choose measured rates tend to make fewer than the average number of calls and to make smaller reductions in use.
Appendix A

DETAILED TABLES OF ESTIMATES

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### Table A.1

**PRICE MODEL WITH TRENDS: COEFFICIENT ESTIMATES**

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NOTE: Asymptotic standard errors in parentheses.
Table A.2

PRICE MODEL WITHOUT TRENDS: COEFFICIENT ESTIMATES

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NOTE: Asymptotic standard errors in parentheses.
Table A.3
PRICE MODEL WITHOUT TRENDS: ESTIMATES OF ELEMENTS OF COVARIANCE MATRIX $\mathbf{\Gamma}$

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<th>Month</th>
<th>Calls Model</th>
<th>Minutes Model</th>
</tr>
</thead>
<tbody>
<tr>
<td>January</td>
<td>4.635</td>
<td>4.139</td>
</tr>
<tr>
<td>February</td>
<td>.822</td>
<td>.768</td>
</tr>
<tr>
<td>March</td>
<td>2.143</td>
<td>1.262</td>
</tr>
<tr>
<td>April</td>
<td>.250</td>
<td>.459</td>
</tr>
<tr>
<td>May</td>
<td>.211</td>
<td>.479</td>
</tr>
<tr>
<td>June</td>
<td>.235</td>
<td>.434</td>
</tr>
<tr>
<td>July</td>
<td>.241</td>
<td>.354</td>
</tr>
<tr>
<td>August</td>
<td>.302</td>
<td>.489</td>
</tr>
<tr>
<td>September</td>
<td>.704</td>
<td>.933</td>
</tr>
<tr>
<td>October</td>
<td>.267</td>
<td>.543</td>
</tr>
<tr>
<td>November</td>
<td>.785</td>
<td>.924</td>
</tr>
<tr>
<td>December</td>
<td>1.574</td>
<td>1.293</td>
</tr>
</tbody>
</table>

### Contemporaneous Covariance Matrix $V$

<table>
<thead>
<tr>
<th></th>
<th>Jacksonville</th>
<th>Clinton</th>
<th>Tuscola</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Single-party</td>
<td>Multi-party</td>
<td>Single-party</td>
</tr>
<tr>
<td></td>
<td>(1)</td>
<td>(2)</td>
<td>(3)</td>
</tr>
<tr>
<td>Calls Model</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>(1)</td>
<td>.751</td>
<td>.697</td>
<td>.267</td>
</tr>
<tr>
<td>(2)</td>
<td>.394</td>
<td>.465</td>
<td>.176</td>
</tr>
<tr>
<td>(3)</td>
<td>.991</td>
<td>.739</td>
<td>.144</td>
</tr>
<tr>
<td>(4)</td>
<td></td>
<td>1.239</td>
<td>.075</td>
</tr>
<tr>
<td>(5)</td>
<td>symmetric</td>
<td></td>
<td>.689</td>
</tr>
<tr>
<td>(6)</td>
<td></td>
<td></td>
<td>1.138</td>
</tr>
<tr>
<td>Minutes Model</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>(1)</td>
<td>.672</td>
<td>.544</td>
<td>.363</td>
</tr>
<tr>
<td>(2)</td>
<td>.947</td>
<td>.411</td>
<td>.612</td>
</tr>
<tr>
<td>(3)</td>
<td>.878</td>
<td>.546</td>
<td>.382</td>
</tr>
<tr>
<td>(4)</td>
<td></td>
<td>1.270</td>
<td>.343</td>
</tr>
<tr>
<td>(5)</td>
<td></td>
<td></td>
<td>.687</td>
</tr>
<tr>
<td>(6)</td>
<td></td>
<td></td>
<td>1.747</td>
</tr>
</tbody>
</table>

### First-Order Autocorrelation $\rho$ and Variance $\sigma^2$

<table>
<thead>
<tr>
<th></th>
<th>Calls Model</th>
<th>Minutes Model</th>
</tr>
</thead>
<tbody>
<tr>
<td>Autocorrelation</td>
<td>.253</td>
<td>.347</td>
</tr>
<tr>
<td>Variance</td>
<td>20.204</td>
<td>432.658</td>
</tr>
</tbody>
</table>

NOTES: All estimates are based on residuals from nonlinear OLS (first-stage) regressions. Variance is estimated as mean squared error; other estimates are described in the text.
Table A.4
NON-PRICE MODEL WITHOUT TRENDS: COEFFICIENT ESTIMATES

<table>
<thead>
<tr>
<th>Coefficient</th>
<th>Calls Model</th>
<th>Minutes Model</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>OLS</td>
<td>GLS</td>
</tr>
<tr>
<td>$\lambda_{J1}$</td>
<td>-.1262 (.0134)</td>
<td>-.1240 (.0089)</td>
</tr>
<tr>
<td>$\lambda_{C1}$</td>
<td>-.0850 (.0149)</td>
<td>-.0998 (.0118)</td>
</tr>
<tr>
<td>$\lambda_{T1}$</td>
<td>-.0989 (.0179)</td>
<td>-.0992 (.0115)</td>
</tr>
<tr>
<td>$\lambda_{J2}$</td>
<td>-.1583 (.0207)</td>
<td>-.1471 (.0128)</td>
</tr>
<tr>
<td>$\lambda_{C2}$</td>
<td>-.1489 (.0224)</td>
<td>-.1497 (.0168)</td>
</tr>
<tr>
<td>$\lambda_{T2}$</td>
<td>-.1543 (.0271)</td>
<td>-.1372 (.0164)</td>
</tr>
<tr>
<td>$\lambda_{J1}$</td>
<td>.1647 (.0186)</td>
<td>.1660 (.0172)</td>
</tr>
<tr>
<td>$\lambda_{C1}$</td>
<td>.1984 (.0219)</td>
<td>.1646 (.0205)</td>
</tr>
<tr>
<td>$\lambda_{T1}$</td>
<td>.2063 (.0230)</td>
<td>.2005 (.0206)</td>
</tr>
<tr>
<td>$\lambda_{J2}$</td>
<td>.1527 (.0278)</td>
<td>.1733 (.0234)</td>
</tr>
<tr>
<td>$\lambda_{C2}$</td>
<td>.1771 (.0326)</td>
<td>.1630 (.0277)</td>
</tr>
<tr>
<td>$\lambda_{T2}$</td>
<td>.1658 (.0343)</td>
<td>.1753 (.0275)</td>
</tr>
<tr>
<td>$\alpha_{C}$</td>
<td>.9216 (.0133)</td>
<td>.9269 (.0076)</td>
</tr>
<tr>
<td>$\alpha_{T}$</td>
<td>.7595 (.0122)</td>
<td>.7584 (.0056)</td>
</tr>
<tr>
<td>$\beta_{J}$</td>
<td>.7696 (.0123)</td>
<td>.7634 (.0052)</td>
</tr>
<tr>
<td>$\beta_{C}$</td>
<td>.7095 (.0130)</td>
<td>.7075 (.0066)</td>
</tr>
<tr>
<td>$\beta_{T}$</td>
<td>.8198 (.0166)</td>
<td>.8216 (.0082)</td>
</tr>
</tbody>
</table>
Table A.4—continued

<table>
<thead>
<tr>
<th>Coefficient</th>
<th>Calls Model</th>
<th>Minutes Model</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>OLS</td>
<td>GLS</td>
</tr>
<tr>
<td>$\mu_1$</td>
<td>117.7</td>
<td>114.9</td>
</tr>
<tr>
<td></td>
<td>(1.6)</td>
<td>(3.6)</td>
</tr>
<tr>
<td>$\mu_2$</td>
<td>95.2</td>
<td>95.1</td>
</tr>
<tr>
<td></td>
<td>(1.5)</td>
<td>(1.6)</td>
</tr>
<tr>
<td>$\mu_3$</td>
<td>106.0</td>
<td>105.2</td>
</tr>
<tr>
<td></td>
<td>(1.5)</td>
<td>(2.4)</td>
</tr>
<tr>
<td>$\mu_4$</td>
<td>97.6</td>
<td>97.7</td>
</tr>
<tr>
<td></td>
<td>(1.5)</td>
<td>(0.9)</td>
</tr>
<tr>
<td>$\mu_5$</td>
<td>98.5</td>
<td>98.7</td>
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<tr>
<td></td>
<td>(1.5)</td>
<td>(0.9)</td>
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<tr>
<td>$\mu_6$</td>
<td>94.8</td>
<td>95.9</td>
</tr>
<tr>
<td></td>
<td>(1.5)</td>
<td>(1.0)</td>
</tr>
<tr>
<td>$\mu_7$</td>
<td>94.0</td>
<td>93.6</td>
</tr>
<tr>
<td></td>
<td>(1.5)</td>
<td>(1.0)</td>
</tr>
<tr>
<td>$\mu_8$</td>
<td>94.2</td>
<td>93.5</td>
</tr>
<tr>
<td></td>
<td>(1.5)</td>
<td>(1.0)</td>
</tr>
<tr>
<td>$\mu_9$</td>
<td>85.7</td>
<td>84.8</td>
</tr>
<tr>
<td></td>
<td>(1.3)</td>
<td>(1.3)</td>
</tr>
<tr>
<td>$\mu_{10}$</td>
<td>93.6</td>
<td>94.4</td>
</tr>
<tr>
<td></td>
<td>(1.4)</td>
<td>(1.0)</td>
</tr>
<tr>
<td>$\mu_{11}$</td>
<td>91.4</td>
<td>92.2</td>
</tr>
<tr>
<td></td>
<td>(1.4)</td>
<td>(1.5)</td>
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<tr>
<td>$\mu_{12}$</td>
<td>106.8</td>
<td>107.8</td>
</tr>
<tr>
<td></td>
<td>(1.5)</td>
<td>(1.8)</td>
</tr>
</tbody>
</table>

NOTE: Asymptotic standard errors in parentheses.
### Table A.5

**Non-Price Model Without Trends**

<table>
<thead>
<tr>
<th>Month</th>
<th>Calls Model</th>
<th>Minutes Model</th>
</tr>
</thead>
<tbody>
<tr>
<td>January</td>
<td>4.864</td>
<td>4.677</td>
</tr>
<tr>
<td>February</td>
<td>.858</td>
<td>.922</td>
</tr>
<tr>
<td>March</td>
<td>2.024</td>
<td>1.225</td>
</tr>
<tr>
<td>April</td>
<td>.228</td>
<td>.369</td>
</tr>
<tr>
<td>May</td>
<td>.214</td>
<td>.365</td>
</tr>
<tr>
<td>June</td>
<td>.255</td>
<td>.347</td>
</tr>
<tr>
<td>July</td>
<td>.273</td>
<td>.273</td>
</tr>
<tr>
<td>August</td>
<td>.298</td>
<td>.451</td>
</tr>
<tr>
<td>September</td>
<td>.675</td>
<td>.962</td>
</tr>
<tr>
<td>October</td>
<td>.308</td>
<td>.485</td>
</tr>
<tr>
<td>November</td>
<td>.863</td>
<td>.879</td>
</tr>
<tr>
<td>December</td>
<td>1.343</td>
<td>1.169</td>
</tr>
</tbody>
</table>

**Contemporaneous Covariance Matrix \( \Sigma \)**

<table>
<thead>
<tr>
<th></th>
<th>Jacksonville</th>
<th>Clinton</th>
<th>Tuscola</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Single-party</td>
<td>Multi-party</td>
<td>Single-party</td>
</tr>
<tr>
<td>(1)</td>
<td>.704</td>
<td>.737</td>
<td>.338</td>
</tr>
<tr>
<td>(2)</td>
<td>1.199</td>
<td>.445</td>
<td>.495</td>
</tr>
<tr>
<td>(3)</td>
<td>1.054</td>
<td>.789</td>
<td>.163</td>
</tr>
<tr>
<td>(4)</td>
<td>1.256</td>
<td>.054</td>
<td>.546</td>
</tr>
<tr>
<td>(5)</td>
<td>symmetric</td>
<td>.674</td>
<td>.540</td>
</tr>
<tr>
<td>(6)</td>
<td></td>
<td></td>
<td>1.113</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th></th>
<th>Calls Model</th>
<th>Minutes Model</th>
</tr>
</thead>
<tbody>
<tr>
<td>(1)</td>
<td>.749</td>
<td>.697</td>
</tr>
<tr>
<td>(2)</td>
<td>1.058</td>
<td>.451</td>
</tr>
<tr>
<td>(3)</td>
<td>.782</td>
<td>.630</td>
</tr>
<tr>
<td>(4)</td>
<td>1.118</td>
<td>.322</td>
</tr>
<tr>
<td>(5)</td>
<td>symmetric</td>
<td>.710</td>
</tr>
<tr>
<td>(6)</td>
<td></td>
<td>1.583</td>
</tr>
</tbody>
</table>

**First-Order Autocorrelation \( \rho \) and Variance \( \sigma^2 \)**

<table>
<thead>
<tr>
<th></th>
<th>Calls Model</th>
<th>Minutes Model</th>
</tr>
</thead>
<tbody>
<tr>
<td>Autocorrelation</td>
<td>.207</td>
<td>.251</td>
</tr>
<tr>
<td>Variance</td>
<td>20.000</td>
<td>414.232</td>
</tr>
</tbody>
</table>

**NOTES:** All estimates are based on residuals from nonlinear OLS (first-stage) regressions. Variance is estimated as mean squared error; other estimates are described in the text.
SAS NONLINEAR GENERALIZED LEAST SQUARES PROGRAM

DATA DATA1:
    INFILE PT20F001:
    INPUT TIME INDEX CALLS C01 C02 C03 C04 C05 C06
        M01 M02 M03 M04 M05 M06
        D1 D2 D3 D4 D5 D6 D7 D8 D9 D10 D11 D12 S
        PC PM NS NM RATIO1
        E1 PC1 PM1 NS1 NM1 RATIO3
        E2 PC2 PM2 NS2 NM2 RATIO2
        E3 PC3 PM3 NS3 NM3 RATIO3 (RB4.):;
    T=0.0:
    IF TIME >= 25.0 THEN T=1.0:
    LGT=0.0:
    IF TIME >= 26.0 THEN LGT=1.0:
    IF D1=1.0 THEN MONTH=1.0:
    IF D2=1.0 THEN MONTH=2.0:
    IF D3=1.0 THEN MONTH=3.0:
    IF D4=1.0 THEN MONTH=4.0:
    IF D5=1.0 THEN MONTH=5.0:
    IF D6=1.0 THEN MONTH=6.0:
    IF D7=1.0 THEN MONTH=7.0:
    IF D8=1.0 THEN MONTH=8.0:
    IF D9=1.0 THEN MONTH=9.0:
    IF D10=1.0 THEN MONTH=10.0:
    IF D11=1.0 THEN MONTH=11.0:
    IF D12=1.0 THEN MONTH=12.0:
    LGCALLS=CALLS:

NOTE: INFILE PT20F001 IS:
    DSNAMEM=W.W3.137.A3299.SAS.NLGLS.INPUT.
    UNIT=TEMP.VOL=SER=USE253.DISP=SHR.
    DCB=(BLKSIZE=3120.LRECL=200.RECFM=FB)

NOTE: 312 LINES WERE READ FROM INFILE PT20F001.
NOTE: DATA SET WORK.DAT1 HAS 312 OBSERVATIONS AND 56 VARIABLES. 42 OBS/TRK.
NOTE: THE DATA STATEMENT USED 0.40 SECONDS AND 180K.

DATA DATA2:
    INFILE PT24F001:
    INPUT PINDEX 8.3:
    "**** SCALE BY DEC. 1978 "PINDEX" VALUE ****":
    PINDEX = PINDEX / 110.2:
    OUTPUT DATA:
    OUTPUT DATA:
    OUTPUT DATA:
    OUTPUT DATA:
    OUTPUT DATA:

NOTE: 52 LINES WERE READ FROM INFILE PT24F001.
NOTE: DATA SET WORK.DAT2 HAS 312 OBSERVATIONS AND 1 VARIABLES. 1588 OBS/TRK.
NOTE: THE DATA STATEMENT USED 0.10 SECONDS AND 172K.

DATA DATA1:
38   MERGE DATA1 DATAX:
39     IF PINDEX > 0.0 THEN DO:
40       PC = PC/PINDEX;
41       PM = PM/PINDEX;
42       PC1 = PC1/PINDEX;
43       PC2 = PC2/PINDEX;
44       PC3 = PC3/PINDEX;
45       PM1 = PM1/PINDEX;
46       PM2 = PM2/PINDEX;
47       PM3 = PM3/PINDEX;
48     END:

NOTE: DATA SET WORK.DATA1 HAS 312 OBSERVATIONS AND 57 VARIABLES. 41 OBS/TRK.
NOTE: THE DATA STATEMENT USED 0.23 SECONDS AND 180K.

DATA RSIDMAT:
50   INFILE PT22F001:
51     INPUT (TIME INDEX RSID) (Y5 BY4. @13 BY4. @25 BY4.):

NOTE: INFILE PT22F001 IS:
55   DNAME=W3137.A3299.SAS.XOUTPUT1.
59   UNIT=TEMP.VOL=SER=USER.53.DISP=SER.     
60   DCB=(BLKSIZE=3136.LRECL=28.RECFM=F8)

NOTE: 312 LINES WERE READ FROM INFILE PT22F001.
NOTE: DATA SET WORK.RSIDMAT HAS 312 OBSERVATIONS AND 3 VARIABLES. 690 OBS/TRK.
NOTE: THE DATA STATEMENT USED 0.12 SECONDS AND 172K.

PROC MATRIX:
53   *<ESTIMATE K-FACTORS, V-MATRIX AND RHO. ALSO SETUP GLS TRANS. MAT.>:  
54     *:  
55     *:  
56     *<ESTIMATE K-FACTORS ON MONTHLY BASIS>:  
57     *:  
58     *:  
59     *:

60   FETCH D 312 DATA=DATA1(KEEP=D1 D2 D3 D4 D5 D6 D7 D8 D9 D10  
61     D11 D12):
62   FETCH RSID 312 DATA=RSIDMAT(KEEP=RSID):
63   MSSORSID = (RSID * RSID) / 52.0:
64   PRINT MSSORSID:  
65     *<NUMBER OF TIME PERIODS = T = 52>: 
66   RSIDSO = RSID#RSID:
67   KK = J(312,1,0.0):
68   DO IM=1 TO 12:
69     KMON = (D(,IM)** RSIDSO )#/MSSORSID:
70     SUMD = SUM(D(,IM)):
71     KMON = (KMON#6.0) / SUMD:
72     KK = KK + (D(,IM)#KMON):
73     KM = KM + KMON:  
74   END:
75   PRINT KM:
76   SUMKK = SUM(KK) / 6.0:
77   PRINT SUMKK:  
78   KK = KK##(0.5):
ZKK = 0.0;
DO IT=1 TO 52:
   IX = (IT - 1) * 5 + 1;
   ZKK = ZKK // KK(IX);
   KKZ = KKZ // KK(IX):
END:
KKZ = KKZ // 0.0:
*: *
*: 
  *<ESTIMATE V-MATRIX>:
  *<CONDITIONAL ON K-FACTORS>:
  *
  DO IT=1 TO 52:
    FETCH RHL DEL 6 DATA=SSIDMAT(KEEP=RSID):
    IX = (IT - 1) * 5 + 1:
    RHL = RHL #/ KK(IX):
    R = R || RHL:
END:
V = R * R':
TRCY = TRACE(V):
V = V**6.0:
  *< NUMBER OF CROSS-SECTIONS = N = 6>:
V = V/#TRCY:
  *< NORMALIZE V-MATRIX SO ITS TRACE = N>:
PRINT V:
  *< CHOLESKY SQUARE ROOT OF V-INVERSE>:
H = HALF(INV(V)):
PRINT H:
  *:
  *:
  *< ESTIMATE AUTO-REGRESSIVE PARAMETERS>:
  *<CONDITIONAL ON K-FACTORS AND V-MATRIX>:
  *
MSSORSID = 0.0:
RHO = 0.0:
B = H * B:
DO IN=1 TO 6:
   ZR = 0.0 || H(IN):
   HZ = H(IN) || 0.0:
   RHOIN = ZR * HZ:
   RHO = RHO + RHOIN:
   BHOS = BHOS || (RHOIN #/ (SSQ(ZR))) :
   MSSORSID = MSSORSID + SSQ(ZR):
END:
BHOS = BHOS #(52.0#/51.0):
PRINT BHOS:
MSSORSID = MSSORSID #/ 52.0:
RHO = RHO #/ 51.0:
PRINT RHO MSSORSID:
  *:
  *< SET UP GLS TRANSFORMATION MATRIX>:
  *
ONE = J(6,1,1.0):
A = ONE #(SORT(1.0-(RHO#BHOS))#/ KKZ(1):
LGA = ONE # ZKK(1):
FETCH CALLS 6 DATA=DATA1(KEEP=CALLS):
LGCALLS = J(6,1,0.0):
TRCALLS = H * (A#CALLS - LGA#LGCALLS):
Z = H || A || LGA || TRCALLS:
DO IT=1 TO 52:
FETCH CALLS 6 DATA=DATA1(KEEP=CALLS):
IF IT=1 THEN GO TO LAST:
A = ONE &/ KKE(IT):*
LGA = (ONE & RHG) &/ ZKK(IT):
FETCH LGCALLS 6 DATA=DATA1(KEEP=LGCALLS):
IF IT=2 THEN PRINT CALLS LGCALLS:
TRCALLS = H *(A#CALLS - LGA#LGCALLS):
Z = Z / (H # A # LGA # TRCALLS):
LAST:
END:
Z = Z # IK:
*< OUTPUT THE MATRiX FOR GLS TRANSFORMATION>:
OUTPUT Z OUT=DATA2 DROP=ROW RENAME=(COL1=H1 COL2=H2 COL3=H3
COL4=H4 COL5=H5 COL6=H6 COL7=A COL8=LGA COL9=TRCALLS
COL10=IK):
NOTE: DATA SET WORK.DATAZ HAS 312 OBSERVATIONS AND 10 VARIABLES. 226 OBS/TRK.
NOTE: THE PROCEDURE MATRIX USED 1.00 SECONDS AND 232K AND PRINTED PAGES 1 TO 2.
DATA DATA1:
MERGE DATA1 DATA2:
NOTE: DATA SET WORK.DATAZ HAS 312 OBSERVATIONS AND 67 VARIABLES. 35 OBS/TRK.
NOTE: THE DATA STATEMENT USED 0.22 SECONDS AND 180K.
PROC MATRIX:
*:
*< SET UP LAGGED VARIABLES>:
*:  
LG = J((6.21,0.0):
*:  
*:  *< TO INITIALIZE LGD9 SO WE DON'T DIVIDE BY 0.0
*:  IN COMPUTING DER.DI'S FOR FIRST OBSERVATIONS>:
*:  
LG(1,9) = 1.0:
*:  
LG(2,9) = 1.0:
*:  
LG(3,9) = 1.0:
*:  
LG(4,9) = 1.0:
*:  
LG(5,9) = 1.0:
*:  
LG(6,9) = 1.0:
*:  
DO IT=1 TO 51:
FETCH Y 6 DATA=DATA1(KEEP=D1 D2 D3 D4 D5 D6 D7 D8 D9
D10 D11 D12
PC1 PM1 RATIO1
PC2 PM2 RATIO2
PC3 PM3 RATIO3):
LG = LGY // Y:
END:
OUTPUT LGY OUT=LGDATA1 DROP=ROW RENAME=(COL1=LGD1 COL2=LGD2
COL3=LGD3 COL4=LGD4 COL5=LGD5 COL6=LGD6 COL7=LGD7
COL8=LGD8 COL9=LGD9 COL10=LGD10 COL11=LGD11 COL12=LGD12
COL13=LGPC1 COL14=LGPM1 COL15=LGPRATIO1
COL16=LGPC2 COL17=LGPM2 COL18=LGPRATIO2
COL19=LGPC3 COL20=LGPM3 COL21=LGPRATIO3):
NOTE: DATA SET WORK.LGDATA1 HAS 312 OBSERVATIONS AND 21 VARIABLES. 110 OBS/TRK.
NOTE: THE PROCEDURE MATRIX USED 0.47 SECONDS AND 240K AND PRINTED PAGE 3.
DATA DATASET:
MERGE DATA1 LGDATA1:
*

< NON-LINEAR-GENERALIZED-LEAST-SQUARES (NLGLS)
PARAMETER ESTIMATION>
*

NOTE: DATA SET WORK.DATASET HAS 312 OBSERVATIONS AND 98 VARIABLES. 26 OBS/TRK.
NOTE: THE DATA STATEMENT USED 0.26 SECONDS AND 188K.

PROC NLIN ITER=10 METHOD=MARQUARDT:
PARAMETERS K1=0.71 K2=0.77 K3=0.82 L1=0.07 L3=0.24
GSF1=0.0 GSF2=0.0 GSF3=0.0 GSF4=0.0 GSF5=0.0
GSM1=0.0 GSM2=0.0 GSM3=0.0 GSM4=0.0 GSM5=0.0
AC=0.93709 AM=0.06492 AC=0.002590 BM=0.01723
C1=115.712 C2=94.303 C3=104.999 C4=96.998 C5=97.007 C6=93.532
C7=92.278 C8=91.649 C9=92.932 C10=91.596 C11=90.213 C12=105.707:

GSM = GSM1*E1 + GSM2*E2 + GSM3*E3
GSM = GSM1*E1 + GSM2*E2 + GSM3*E3
GSM = GSM1*E1 + GSM2*E2 + GSM3*E3
GSM = GSM1*E1 + GSM2*E2 + GSM3*E3:

TS = 1.0 + GSF*(1-T)*(TIME-0.5)*GSM*(TIME-24.0)*GSM*(TIME-24.5):

TH = 1.0 + GSF*(1-T)*(TIME-0.5)*GSM*(TIME-24.0)*GSM*(TIME-24.5):

LT = 1.0 + GSF*GSM*GSM*(TIME-1.5)*GSM*(TIME-25.0)*GSM*(TIME-25.5):

LGTM = 1.0 + GSF*GSM*GSM*(TIME-1.5)*GSM*(TIME-25.0)*GSM*(TIME-25.5):

EA1 = EXP*(AC*PC1 + AN*PM1):
EA2 = EXP*(AC*PC2 + AN*PM2):
EA3 = EXP*(AC*PC3 + AN*PM3):
EB1 = EXP*(BC*PC1 + BM*PM1):
EB2 = EXP*(BC*PC2 + BM*PM2):
EB3 = EXP*(BC*PC3 + BM*PM3):

LGEM1 = EXP*(AC*LGPC1 + AN*LGPM1):
LGEM2 = EXP*(AC*LGPC2 + AN*LGPM2):
LGEM3 = EXP*(AC*LGPC3 + AN*LGPM3):
LEB1 = EXP*(BC*LGPC1 + BM*LGPM1):
LEB2 = EXP*(BC*LGPC2 + BM*LGPM2):
LEB3 = EXP*(BC*LGPC3 + BM*LGPM3):

SNC = (C1*D1) + (C2*D2) + (C3*D3) + (C4*D4) + (C5*D5) + (C6*D6) + (C7*D7) + (C8*D8)
+ (C9*D9) + (C10*D10) + (C11*D11) + (C12*D12):

LGNC = (C1*LGD1) + (C2*LGD2) + (C3*LGD3)
+ (C4*LGD4) + (C5*LGD5) + (C6*LGD6) + (C7*LGD7) + (C8*LGD8)
+ (C9*LGD9) + (C10*LGD10) + (C11*LGD11) + (C12*LGD12):

V1 = TS*SNC*E1*EB1*(1.-L1):
V2 = SM*C*T*K1 + TS*RATIO1*EA1*(1.-EB1) + (1.-L1):
V3 = TS*SNC*EA2*EB2:
V4 = TS*SNC*E1*EB2*(1.-EB2):
V5 = TS*SNC*EA3*EB3*(1.-L3):
V6 = SM*C*T*K3 + TS*RATIO3*EA3*(1.-EB3) + (1.-L3):

LG1 = LGT*LSM*LGCA1*LEB1*(1.-L1):
LG2 = LGM*LGMD*LGCA1*LEB1*(1.-L1):
LG3 = LGM*LSM*LGCA2*LEB2:
LG4 = LGM*LGMD*LGCA2*LEB2:
LG5 = LGM*LGMD*LGCA3*LEB3*(1.-L3):
LG6 = LGM*LGMD*LGCA3*LEB3*(1.-L3):

V2M = SM*C*T*K1*(1.-L1):
V4M = SM*C*T*K2:
V6M = SM*C*T*K3*(1.-L3):
LGWV = LGSMC * LGTM * K1 * (1 - L1)
LGTV = LGSMC * LGTM * K2
LGVM = LGSMC * LGTM * K3 * (1 - L3)
SMHV = H1 * V1 + H3 * V3 + H5 * V5
SMV = H2 * V2 + H4 * V4 + H6 * V6
SMVM = H2 * V2M + H4 * V4M + H6 * V6M
LSMHSV = H1 * LGY1 + H3 * LGY3 + H5 * LGY5
LSMV = H2 * LGY2 + H4 * LGY4 + H6 * LGY6
LSMVMM = H2 * LGV2M + H4 * LGV4M + H6 * LGV6M
LSMHSV = LGSMHSV + LGSMVVM
*: MODEL INCLUDES CROSS-SECTION & TIME-SERIES:
*: MODEL TRCALLS = A * SMHV - LGA * LGSNMV:
*: < C1, C2, ..., C12 ARE UNDISTURBED MONTHLY CALLING OR MINUTES OF CALLING FOR JACKSONVILLE>
*: DER. C1 = A * D1 * (SMHV / SMC) - LGA * LGD1 * (LGSNMV / LGSNC)
*: DER. C2 = A * D2 * (SMHV / SMC) - LGA * LGD2 * (LGSNMV / LGSNC)
*: DER. C3 = A * D3 * (SMHV / SMC) - LGA * LGD3 * (LGSNMV / LGSNC)
*: DER. C4 = A * D4 * (SMHV / SMC) - LGA * LGD4 * (LGSNMV / LGSNC)
*: DER. C5 = A * D5 * (SMHV / SMC) - LGA * LGD5 * (LGSNMV / LGSNC)
*: DER. C6 = A * D6 * (SMHV / SMC) - LGA * LGD6 * (LGSNMV / LGSNC)
*: DER. C7 = A * D7 * (SMHV / SMC) - LGA * LGD7 * (LGSNMV / LGSNC)
*: DER. C8 = A * D8 * (SMHV / SMC) - LGA * LGD8 * (LGSNMV / LGSNC)
*: DER. C9 = A * D9 * (SMHV / SMC) - LGA * LGD9 * (LGSNMV / LGSNC)
*: DER. C10 = A * D10 * (SMHV / SMC) - LGA * LGD10 * (LGSNMV / LGSNC)
*: DER. C11 = A * D11 * (SMHV / SMC) - LGA * LGD11 * (LGSNMV / LGSNC)
*: DER. C12 = A * D12 * (SMHV / SMC) - LGA * LGD12 * (LGSNMV / LGSNC)
*: < K1, K2, K3 ARE MULTI-PTY FACTORS FOR CLINTON JACKSONVILLE AND TUSCOLA RESP.>
*: DER. K1 = A * H2 * TSMC * (1 - L1)
*: DER. K2 = A * H4 * TSMC
*: DER. K3 = A * H6 * TSMC * (1 - L3)
*: DER. K1 = DER. K1 - LGA * H2 * LGTM * LGSMC * (1 - L1)
*: DER. K2 = DER. K2 - LGA * H4 * LGTM * LGSMC
*: DER. K3 = DER. K3 - LGA * H6 * LGTM * LGSMC * (1 - L3)
*: < L1 AND L3 ARE EXCHANGE FACTORS FOR CLINTON AND TUSCOLA>
*: DER. L1 = A * (H1 * V1 + H2 * V2) / (1 - L1)
*: DER. L3 = A * (H5 * V5 + H6 * V6) / (1 - L3)
*: < PRICE RESPONSE VARIABLES ---
*: SINGLE-PTY. EFFECTS:
*: AC ( % CHANGE IN CALLS PER $0.01)
*: AN ( % CHANGE IN MIN. PER $0.01)
*: MULTI-PTY. EFFECTS (CODE CALLING):
*: BC ( % CHANGE IN CALLS PER $0.01)
*: BM ( % CHANGE IN MIN. PER $0.01)
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DER. AC = A *( -a (H1*V1 + H2*V2)*PC1 + (H3*V3 + H4*V4)*PC2
+ (H5*V5 + H6*V6)*PC3 )
+ TM*MC* (H2*K1*(1-L1)*PC1 + H4*K2*PC2
+ H6*K3*(1-L3)*PC3 )
)

DER. AC = DER. AC-LGA* (-(H1*LV1+H2*LV2)*LGPC1 + (H3*LV3+H4*LV4)*LGPC2
+ (H5*LV5 + H6*LV6)*LGPC3 )
+ LGTM*LGSMC* (H2*K1*(1-L1)*LGPC1 + H4*K2*LGPC2
+ H6*K3*(1-L3)*LGPC3 )
)

DER. AM = A *( -a (H1*V1 + H2*V2)*PM1 + (H3*V3 + H4*V4)*PM2
+ (H5*V5 + H6*V6)*PM3 )
+ TM*MC* (H2*K1*(1-L1)*PM1 + H4*K2*PM2
+ H6*K3*(1-L3)*PM3 )
)

DER. AM = DER. AM-LGA* (-(H1*LV1+H2*LV2)*LGPMM1 + (H3*LV3+H4*LV4)*LGPMM2
+ (H5*LV5 + H6*LV6)*LGPMM3 )
+ LGTM*LGSMC* (H2*K1*(1-L1)*LGPMM1 + H4*K2*LGPMM2
+ H6*K3*(1-L3)*LGPMM3 )
)

DER. BC = A *( -(H1*V1*PC1 + H3*V3*PC2 + H5*V5*PC3 )
+ TS*MC* (H2*(1-L1)*RATIO1*EA1*EB1*PC1
+ H4* RATIO2*EA2*EB2*PC2
+ H6*(1-L3)*RATIO3*EA3*EB3*PC3 )
)

DER. BC = DER. BC-LGA* (-(H1*LV1+H2*LV2)*LGPB1 + (H3*LV3+H4*LV4)*LGPB2
+ (H5*LV5 + H6*LV6)*LGPB3 )
+ LGTS*LGSMC* (H2*(1-L1)*LGRTIO1*LEGA1*LEGB1*LGPC1
+ H4* LGRTIO2*LEGA2*LEGB2*LGPC2
+ H6*(1-L3)*LGRTIO3*LEGA3*LEGB3*LGPC3 )
)

DER. BM = A *( -(H1*V1*PM1 + H3*V3*PM2 + H5*V5*PM3 )
+ TS*MC* (H2*(1-L1)*RATIO1*EA1*EB1*PM1
+ H4* RATIO2*EA2*EB2*PM2
+ H6*(1-L3)*RATIO3*EA3*EB3*PM3 )
)

DER. BM = DER. BM-LGA* (-(H1*LV1+H2*LV2)*LGPBM1 + (H3*LV3+H4*LV4)*LGPBM2
+ (H5*LV5 + H6*LV6)*LGPBM3 )
+ LGTS*LGSMC* (H2*(1-L1)*LGRTIO1*LEGA1*LEGB1*LGPB1
+ H4* LGRTIO2*LEGA2*LEGB2*LGPB2
+ H6*(1-L3)*LGRTIO3*LEGA3*LEGB3*LGPB3 )
)

DER. GSF1 = (((1-T) *(TIME-0.5) *(T*24)) /TS) *A * (SMHV+SMHV-SMHVMM)
- ((1-LGT) *(TIME-1.5) *(LGT*24)) /LGT) *LGA
* (LGSMMV+LGSMMV-LGSMMVMM) ;

DER. GSF2 = DER. GSF1*B2 ;

DER. GSF3 = DER. GSF1*T3 ;

DER. GSF1 = DER. GSF1*B1 ;

DER. GMF1 = (((1-T) *(TIME-0.5) *(T*24)) /TM) *A * (SMHVMM)
- ((1-LGT) *(TIME-1.5) *(LGT*24)) /LGT) *LGA
* (LGSMMVMM) ;

DER. GMF2 = DER. GMF1*B2 ;

DER. GMF3 = DER. GMF1*T3 ;

DER. GMF1 = DER. GMF1*B1 ;

DER. GMM1 = T*(TIME-24.5)/TS) /A * (SMHV+SMHV-MMHVMM)
- LGT* (TIME-25.5) /LGT) *LGA*(LGSMMV+LGSMMV-LGSMMVMM) ;

DER. GMM2 = DER. GMM1*B2 ;

DER. GMM3 = DER. GMM1*T3 ;

DER. GMM1 = DER. GMM1*B1 ;

DER. GMM1 = T*(TIME-24.5)/TM) /A * (SMHVMM)
- LGT* (TIME-25.5) /LGT) *LGA*(LGSMMVMM) ;

DER. GMM2 = DER. GMM1*B2 ;

DER. GMM3 = DER. GMM1*T3 ;

DER. GMM1 = DER. GMM1*B1 ;

OUTPUT OUT=DATA1 PREDICTED=TRPBN RESIDUAL=TSRESID;
TO TRY IT. JUST OMIT THE DSB. STATEMENTS.
NOTE: DATA SET WORK.DATAR HAS 312 OBSERVATIONS AND 1 VARIABLES. 1588 OBS/TRK.
NOTE: THE PROCEDURE MATRIX USED 0.50 SECONDS AND 192K AND PRINTED PAGE 9.

DATA DATA1:
MERGE DATA1 DATAR:

NOTE: DATA SET WORK.DATAR HAS 312 OBSERVATIONS AND 91 VARIABLES. 26 OBS/TRK.
NOTE: THE DATA STATEMENT USED 0.25 SECONDS AND 188K.

DATA COMB:
KEEP TIME BSID1 BSID2 BSID3 BSID4 BSID5 BSID6 BSID1 BSID2 BSID3 BSID4 BSID5 BSID6;
SET DATA1:
RETAIN BSID1 BSID2 BSID3 BSID4 BSID5 BSID6 BSID1 BSID2 BSID3 BSID4 BSID5 BSID6;
IF INDEX=1.0 THEN BSID1=TBRSID;
IF INDEX=2.0 THEN BSID2=TBRSID;
IF INDEX=3.0 THEN BSID3=TBRSID;
IF INDEX=4.0 THEN BSID4=TBRSID;
IF INDEX=5.0 THEN BSID5=TBRSID;
IF INDEX=6.0 THEN BSID6=TBRSID;
IF INDEX=1.0 THEN BRSID1=BSID1;
IF INDEX=2.0 THEN BRSID2=BSID2;

396 IF INDEX=3.0 THEN BRSID3=BRSID;
397 IF INDEX=4.0 THEN BRSID4=BRSID;
398 IF INDEX=5.0 THEN BRSID5=BRSID;
399 IF INDEX=6.0 THEN BRSID6=BRSID;
400 IF INDEX = 6.0 THEN DELETE:

NOTE: DATA SET WORK.COMB HAS 52 OBSERVATIONS AND 13 VARIABLES. 176 OBS/TRK.
NOTE: THE DATA STATEMENT USED 0.22 SECONDS AND 199K.

401 PROC PRINT DATA=COMB:
402 VAR RSID1 RSID2 RSID3 RSID4 RSID5 RSID6
403 BRSID1 BRSID2 BRSID3 BRSID4 BRSID5 BRSID6:

NOTE: THE PROCEDURE PRINT USED 0.19 SECONDS AND 172K AND PRINTED PAGE 9.

404 PROC MEANS DATA=DATA1 MEAN STD VAR CSS SKEWNESS KURTOSIS:
405 VAR CALLS TECALLS TEPDN TERSID BRSID:

NOTE: THE PROCEDURE MEANS USED 0.26 SECONDS AND 188K AND PRINTED PAGE 10.

406 DATA DATA1:
407 SET DATA1:
408 FILE PT21F001:
409 PUT (TIME TECALLS CALLS $INDEX MONTH TEPDN TERSID BRSID)
410 (RBW.):

NOTE: FILE PT21F001 IS:
DSNAME=W.W3137.A3299.WLGLS.XCTREND.
UNIT=TEMP_VOL=SER=USER53_DISP=NEW.
DCB=(BLKSIZE=3132_LRECL=36_RECFT=FB)

NOTE: 312 LINES WERE WRITTEN TO FILE PT21F001.
NOTE: DATA SET WORK.DATA1 HAS 312 OBSERVATIONS AND 91 VARIABLES. 26 OBS/TRK.
NOTE: THE DATA STATEMENT USED 0.34 SECONDS AND 188K.

NOTE: SAS INSTITUTE INC.
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REFERENCES


