Game Theory in Economics - Chapter 2: Decisionmakers

L. Shapley and M. Shubik

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SUMMARY

This chapter deals with the concepts of "player" and "coalition" in game theory, with special reference to economic applications. Two examples, a "duel" and a "truel", are analyzed in some detail, in order to reveal the sharp change in viewpoint required in passing from an inessential two-person game to an essential three-person game. Techniques for increasing the number of players in a model by replication or fragmentation (disaggregation) are described, and the literature on infinite-person games is briefly surveyed. Finally, some of the modeling problems associated with representative types of economic "player" are considered from a practical standpoint.
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CHAPTER 2
DECISIONMAKERS

1. THE CONCEPT OF A PLAYER

Our general description of a game begins with, and is centered around, a specified set of decisionmakers, whom we call the players. Each player has some array of resources at his disposal; some spectrum of alternative courses of action (including attempts to communicate and collaborate), and some inherent system of preferences or utility concerning the possible outcomes. Thus, in an economic "game", the rules will include tastes and technology as well as laws, initial endowments, and distribution and information channels, and the rules must relate each of these features to the individual player.

Outside the game model, the players are all alike—the theory refuses to distinguish among them. Any and all differentiating properties ought to be imbedded in the description of the game. Admittedly, it is easier to preach than to practice this principle, which we call external symmetry. But even in difficult cases (e.g., a game whose "players" might be several firms and a labor union, or a regulatory board and a group of public utilities, or a bank and its customers), the very attempt to achieve external symmetry will be beneficial and should at least lead to a clarification of the limitations of the model.
1.1. Free Will and Behavioristic Assumptions

In our game models we shall always assume that the identified players are rational, conscious decisionmakers, having well-defined goals, and exercising freedom of choice within prescribed limits. What they do with that freedom is a question for the solution theories to answer, not the rules of the game. The answers, as we have already stressed, will generally fall far short of a deterministic prescription of behavior.*

Despite this essential element of free will, the rules of the game may nevertheless severely restrict a player's behavior. An extreme example, previously mentioned, is the consumer who is constrained to accept the prices as named by other players, and to maximize his welfare subject to that constraint. Thus he may retain no conscious control over his purchases in the market. Yet even such a "strategic dummy" (see below, Sec. 1.3) is not barred a priori from negotiations or other forms of "cooperative" activity, as contemplated in some of the solution theories.**

In a model that is not game-theoretically closed,*** there will be individuals who are represented not as players

*See Ch. 1, Sec. 1.2.

**When we come to examine games with strategic dummies, we shall find that most of the relevant solution theories--not surprisingly--predict the failure of the dummies to enter into coalitions or to profit from negotiations. But this is a result of the analysis, not an assumption of the model.

***See Ch. 1, Sec. 4.1.
but as automata, or as an aggregate mechanism, and their behavior may or may not be related to any conscious optimization process. For example, we might represent the demand for the commodity in a market by a function $q = f(p)$, where $q$ denotes the quantity that will be purchased at the price $p$. If we look behind this function we may find a purely behavioral statement, based on habit, tribal customs or socio-psychological considerations. On the other hand, we may find a summation of many separate welfare-maximizing decisions, which in a larger, closed model we would be able to account for on an individual basis.

Having adopted a pluralistic attitude toward the role of solutions in game theory, we wish also to stress that we do not propose that there should be only one model of the individual. The utilitarian, rationalistic model of political and economic man may be valid for much of the professional behavior of those individuals who are primarily politicians or economic agents. But as Dahl has noted,* few citizens can be considered as being professionally involved in politics, and the typical housewife, or business man on vacation, can scarcely be described as professionally involved in economic activity.

A cursory examination of decisionmaking in economic and political life, conditioned as it is by deadlines, costs

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N.B. An incomplete citation (usually author and date) means that the work in question is listed in the Bibliography.
of information and computation, lack of knowledge, lack of understanding of one's own desires, lethargy, and a host of other "frictions", shows that subconscious or semirational processes and routines account for a large part of human behavior—especially "nonprofessional" behavior. Behavioristic theories, though we shall shun them, are not totally in conflict with the game theory approach. Most of the above-named "frictions" are manifestations of limited information, either within the rules or about the rules. To the extent that they constrain or routinize behavior, they could be built into a game-like model* by formalizing the requisite feedback mechanisms for adjusting actions and aspirations, etc.** But so long as the behavioral constraints leave some element of free choice in the interpersonal activities of the players, the theory of games will have a vital role to play.

1.2. Economic Units, "Players", and People

The player is perforce the basic decision unit of the game. He is also the basic evaluation unit. In economic models he may be an individual consumer, a household, a firm or financial institution, the manager thereof, a labor union, a government agency, or even a whole nation

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*But possibly with incomplete information; see Ch. 1, Sec. 3.2.

**See Cyert and March (1963), esp. Ch. 6.
or its leader. A fuller discussion of the major player-types to be encountered in these volumes will be found in Sec. 4 of this chapter.

Specification of the players in a model carries with it tacit assumptions. For example, if the players are firms, then any internal organization problems are assumed to have been accounted for, so that the firm "speaks with one voice". In particular, the problem of defining the internal social welfare function, for the aggregate of individuals (or impersonal interests) that make up the firm, is assumed to have been solved. Similar remarks apply to other types of aggregated players.

The modeling of aggregates as a single player always presents difficulties, precisely because the group-decision and group-welfare problems are, in a certain fundamental sense, unsolvable.* In international politics, to take a glaring example, the representation of a country as a single individual can be dangerously misleading. The analogies between national and individual psychology may be convenient and superficially persuasive, yet they are inherently unsound.**

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*See Ch. 1, Sec. 1.2.

**How often, even in mature political commentary, one encounters dubious anthropomorphisms: "Egypt is friendly to ...", "France desires ...", "the Kremlin denies ...", "Peking believes ...", giving human shape, as in a cartoon, to nonhuman entities.

The problem of how to aggregate psychological attributes is certainly not outside the purview of game theory;
1.3. Dummies

A "dummy" is a player that is not a player, in some vital respect. He may appear in the model for the sake of formal completeness, or as a limiting case when parameters are varied. A strategic dummy is one who is so constricted by the rules that he has no choice in his actions. He may nevertheless have a stake in the outcome, and this may have an effect on the solution, or on our interpretation of the solution.*

Sometimes, a player, not a strategic dummy under the rules, becomes equivalent to one after later analysis has eliminated all but one of his strategies. A warning: the model-builder should beware anticipating such eliminations too far in advance—e.g., replacing a potential player by a mechanism because "he has only one reasonable course of action". Domination arguments to reduce the number of alternatives, which are so useful in decision theory (and in two-person zero-sum game theory), must be used with great circumspection in n-person game theory. (The "Shooting Match" to be analyzed in Sec. 2.2 below is a good

witness the progress made toward mathematization of other concepts, such as bluff, threat, toughness (in bargaining), trust, etc., that were once thought to be ineluctably psychological in nature.

*The effect is likely to be significant only in the case of very weak solution-concepts, like Pareto-optimality, or in the branch of game theory in which side payments can be made outside the strategic structure of the game. For an example, imagine a three-person game with a $100 prize, in which B and C are strategic dummies while A must choose whether the prize goes to B or to C. (Cf. von Neumann and Morgenstern's discussion of "removable" players (1944; pp. 533-535).)
example.) Deliberately "unreasonable" behavior, especially in "deterrence" situations, but in other contexts as well, can be a significant factor and should not be dismissed out of hand.

Two other kinds of dummy should be mentioned. The first, a formal, "fictitious player", is sometimes invoked as a device to balance the books in a non-zero-sum game.* He is a strategic and coalitional dummy, but has nevertheless a well-defined payoff function, equal to the negative of the sum of the real players' payoffs. This formal device may be contrasted with yet another type of dummy, the statisticians' opponent in a "game against Nature"; he (she?) has strategic choice but no motivation or payoff.** We shall have little occasion to refer to either of these devices in the present volume.

1.4. Coalitions

The term "coalition" will ordinarily be used in a neutral sense, without structural or institutional implications. A coalition is a mathematical figment; it is not the analogue of a firm or a trade union, where there would be codified rules limiting the actions of the officers and members. In the formal theory, a coalition can be any subset of players (even the all-player set),

*See von Neumann and Morgenstern (1944), p. 505ff.

**Milnor (1954), Blackwell and Girshick (1954).
considered in the context of their possible collaboration. Thus, we shall ordinarily not speak of coalitions "forming" during the game, or being forbidden, etc.; indeed, it could be said that all coalitions are "in being" simultaneously.

Conversely, if a firm, cartel, union, etc., is represented in the model in such detail that the members of the group are players in the game, then it will not suffice to say merely that these players constitute a coalition. The fact that they are organized in some way, and are perhaps not entirely free agents, should be represented by stated institutional costs, commitments, constraints, etc., included among the rules of the game. Their corporate existence can sometimes, in fact, be modeled in the form of an additional player—a manager, let us say, who is narrowly restricted in his actions and whose preferences are directly tied to the fortunes of the organization.*

It should be pointed out, however, that the internal workings of organizations and institutions are difficult to treat adequately with the predominantly static tools of this book. Accordingly, we shall generally try to avoid such questions in our models, despite their high game-theoretic content.**

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*Shapley and Shubik (1967).

**Two empirical studies of political coalition-forming, utilizing game-theoretic models, are Riker (1959) and Leiserson (1968).
2. ILLUSTRATING THE "n-PERSON PROBLEM"

In a game model, the very number of players may have a decisive effect on the nature of the game, on the nature of the analysis to be employed, and on the nature of the conclusions to be expected. Von Neumann and Morgenstern have remarked on the repeated appearance of qualitatively new phenomena with every increase in the number of players.* We have already had occasion to emphasize the inherent difficulties in extending ordinary rationality concepts into multi-person situations.** A wide gulf separates the well-regulated domain of "inessential" two-person games from the wilderness area of "essential" n-person games. Since the pervasive "n-person problem" exerts such an influence over the theory as a whole, we have chosen for our first, simple excursion into mathematical analysis a pair of similar games situated on opposite sides of that gulf.

In the first example, there are two players, and their interests point in directly opposite directions, and so in a useful sense of the word, are parallel. Because of this, straightforward "rational" arguments lead us convincingly to a satisfactory solution of the game,
using the "minimax" principle.* But in the second example, with three players competing for the prize, there is no parallelism, and the same kind of "rational" arguments lead only to a treacherous sort of equilibrium, which on closer inspection turns out to be surprisingly unreasonable.

2.1. A Two-Person Example

A DART DUEL. Players A and B approach balloons marked "B" and "A" respectively, each with a dart to throw when he wishes. The player whose balloon is first hit, loses.**

To analyze this game, let the accuracy functions (hit probabilities) be \(a(t)\) and \(b(t)\) respectively; both functions (let us assume) increase continuously from 0 to 1. If your opponent throws and misses, then you should obviously hold your dart until you have a sure hit. But if he throws and hits, then the game is over. The only question, then, is how long you should wait before throwing your dart. It is obviously bad to throw too soon, but it is also risky to wait too long.

*Since economic interests are rarely parallel (in any sense), minimax theory has found little application in economics despite the occasional chapter given over to it in some modern textbooks.

**This is an example of a "noisy duel", a class of games first treated by Blackwell and others in 1949 (see Ch. 3, Sec. 3.8). Burger (1959) has suggested the following economic scenario: Two merchants compete for a customer. Each "makes his pitch" at a point in time of his choice. The customer (who is not a player) makes at most one purchase, and he makes it with a probability, \(a(t)\) or \(b(t)\), that depends on when he is approached and by whom.
Let us assume that the players have decided to throw at times $t = x$ and $t = y$ respectively. A's probability of winning is then $a(x)$ if he gets to throw first, or $1 - b(y)$ if B throws first. If we suppose that simultaneous-hit and no-hit games are decided by lot, then the following "payoff function" describes A's prospects:

$$
P_A(x, y) = \begin{cases} 
a(x) & \text{if } x < y, \\
1 - b(y) & \text{if } x > y, \\
\frac{1}{2} [a(x) + 1 - b(y)] & \text{if } x = y. 
\end{cases}
$$

Since someone always wins, we have

$$
P_B(x, y) = 1 - P_A(x, y).
$$

Now suppose that A assumes, cautiously, that his decision "x" will be guessed in advance by B, or perhaps deduced on theoretical grounds. He can then hope for no better than the following payoff:

$$
Q_A(x) = \min [a(x), 1 - b(x)],
$$

since B can choose either to wait until $t = 1$ or to throw just before $t = x$. To make the best of it, A must try to maximize $Q_A(x)$. But this means choosing $x = t_o$, where $t_o$ is defined by the equation $a(t_o) = 1 - b(t_o)$, or
(2-1) \hspace{1cm} a(t_o) + b(t_o) = 1.

(See Fig. 1.) By throwing his dart at this time (if B has not yet thrown), A can assure a win probability of at least $a(t_o)$ regardless of what B knows or does.

By the same token, a cautious B, by solving the same equation (2-1) and taking $y = t_o$, can assure a win probability of at least $b(t_o)$. But these two win probabilities add up to 1. There is no room for improvement over the "cautious" strategies. They are therefore optimal strategies, and the game is solved. The rule for correct play for either player can be stated very simply: "Throw your dart as soon as the sum of the accuracies reaches 1."

For a numerical example, let $a(t) = t$, $b(t) = t^2$, as in Fig. 1, making A a uniformly better marksman. Then, by elementary algebra, both players should throw at time $t_o = (\sqrt{5} - 1)/2 = .618$. A's probability of winning is .618; B's is .382.

*An early throw is always incorrect, but a decision to throw later than $t_o$ will not matter, unless the other player, perhaps anticipating the mistake, also delays his throw but by a smaller amount.
Fig. 1
2.2. A Three-Person Example

A SHOOTING MATCH. Contestants A, B, and C fire at each other's balloons with pistols, from fixed positions. At the beginning, and after each shot, the players with unbroken balloons decide by lot who is to shoot next. The surviving balloon determines the winner.*

This game does not correspond exactly to the preceding. Here the problem is not when to fire, but at what. Also, we do not know how long the game will last. As soon as one player is eliminated, however, the other two players become strategic dummies, and direct calculation can be made.

Thus, suppose that C is out, and that A and B have fixed accuracies a and b respectively. The next shot will result in one of the following events: an immediate win for A (probability a/2); an immediate win for B (probability b/2); or a miss and a repetition of the status quo (probability 1 - a/2 - b/2). Writing $P_{A,B}$ for the probability that A will ultimately win when pitted against B alone, we have

$$P_{A,B} = \frac{a}{2} + (1 - \frac{a}{2} - \frac{b}{2})P_{A,B}. \tag{2-2}$$

*Kinnaird (1946, p. 246), Larsen (1948), Shubik (1954), Gardner (1961), Boot (1967), and doubtless others. We have added the lottery feature in this "truel" to eliminate sequential effects.

Again an economic setting can be created, if we try hard enough: Three merchants compete for a sale. Contacts with the buyer occur in a random order. At each contact,
Solving this equation gives \( P_{A,B} = \frac{a}{a+b} \). In similar fashion, we find that \( P_{A,C} = \frac{a}{a+c} \), and so on.

Now suppose that the players are ranked in skill by \( a > b > c \). Then it seems clear whose balloon A should attack in the first part of the match. His chances of an immediate hit are the same, but he would definitely prefer to shoot it out later with C, rather than B, since \( P_{A,C} > P_{A,B} \). Therefore, he aims at B's balloon. For exactly analogous reasons, B and C both aim at A's balloon. *

The strategic question settled, we can complete the calculations. Write \( P_A \) for A's initial chance of winning. Considering just the first round, we have

\[
P_A = \left( \frac{2}{3} \right) P_{A,C} + \left( 1 - \frac{a}{3} - \frac{b}{3} - \frac{c}{3} \right) P_A,
\]

in analogy to (2-2). The equations for the other two players are similar in form. After a little pencil work, we obtain the expressions

\[
\begin{align*}
P_A &= \frac{a^2}{(a + b + c)(a + c)} \\
P_B &= \frac{b}{a + b + c} \\
P_C &= \frac{c(2a + c)}{(a + b + c)(a + c)}
\end{align*}
\]

*This is the "domination argument" mentioned in Sec. 1.3.
These add up to 1, since an infinitely protracted match has probability 0.

Apparently the game has been solved. We have deduced a rule for rational play: "Shoot first at the balloon of your stronger opponent", and we have found formulas for the resulting expectations. Everything seems in order. "What is all the shooting about?" the reader may wonder.

2.3. Critique of the Solution

For reply, we insert some numerical values into our "solution" (2-3), say \( a = .8, \ b = .6, \ c = .4 \). Some further pencil work then reveals a paradox:

\[
P_A = .296, \ P_B = .333, \ P_C = .370.
\]

The order of skill has been reversed! The poorest shot has the best chance to win!

A little reflection shows what has happened. If \( A \) and \( B \) foolishly insist on being "rational", they end up cutting each other's throats, to the great benefit of \( C \).* Another calculation can be made to show the extent of their folly. Had they somehow settled on the "irrational" policy of first shooting down \( C \)'s balloon, then their winning chances would be \( .444 \), and \( .465 \), respectively—a striking improvement over (2-4). It is hard to see how they

could be content with the solution given above if there were any way for them to agree, openly or covertly, on that "irrational" course of action.

There is another way in which A might make his superiority felt, which requires no collusion or "gentlemen's agreement" with B. Let A openly commit himself to shoot at C's balloon if ever C shoots at his (and misses!), but otherwise let him shoot at B's balloon as in his original "rational" strategy. Given this commitment on A's part, B will still find it advantageous to shoot at A's balloon. But C will now find it better to defer to the threat of retaliation and to attack B instead of A.* The resulting expectations come to .444, .200, and .356, respectively, with B definitely getting the worst of it. The key to this solution, which seems intuitively not unreasonable, is the presumption that A can commit himself to an "irrational" act of revenge, which will never actually be carried out--in short, that he can employ a strategy of deterrence.

We do not assert that the original solution (2-3) is necessarily wrong, or that it could not occur in practice among intelligent players. The moral of the example

*Unless B too can commit himself to retaliate. Curiously, C would prefer most of all to shoot into the air, if the rules would permit it. This is because his only goal in the first stage of the game, so long as no one is attacking him, is to increase the chance of having to face B rather than A in the second stage. If he could "pass", the expectations would be .381, .257, and .362. On the other hand, A could perhaps modify his threat, to enjoin C from passing.
is rather, that multiperson games cannot be properly analyzed or solved until adequate information is provided about the social climate—in particular, about the possibilities for communication, compensation, commitment, and trust. Given this sociological information, one can proceed to the selection of a suitable solution concept.

EXERCISES. 1. Suppose the accuracy functions in the "Dart Duel" have the form indicated in Fig. 2. What is the rule for optimal play?

2. In the "Shooting Match" suppose that $a > b = c > 0$. What is the analogue of the "rational" solution described in Sec. 2.2?
Fig. 2
3. INCREASING THE NUMBER OF PLAYERS

In the applications of game theory to economic po-

titical or social problems, it is frequently important
to be able to vary the numbers of players, and especially
to consider the effect of letting the number of players
increase without bound. The behavior manifested by mass
markets or political systems with many millions of voters
may differ considerably from the behavior of similar sys-
tems with only a handful of participants, and an elucida-
tion of these differences is one of the major goals of
game-theoretic analysis.

To illustrate the modeling techniques required, sup-
pose we begin with a market having just two traders, each
with a large amount of goods. We may wish to compare the
outcome from trade in this market with the outcomes in
similar bilateral markets, having four, six, or more
traders. One way to do this is to replicate the market,
i.e., to add more traders identical to the existing traders
in every way (i.e., having the same preferences, endowments
and strategic possibilities). In this construction, it is
as though the spatial boundaries of the market were expanded,
to encompass additional markets identical to the first.*

Another approach is to fracture the original system,
by "breaking the players in half", or into smaller pieces,
and endowing the resulting new traders with suitably scaled-
down versions of the attributes that described the original

*Debreu and Scarf (1963), Shapley (1964), Shapley and
traders. In this construction, it is as though the original traders were aggregates, who are now being disaggregated.*

For many purposes, these two ways of generating games with larger numbers of players are mathematically equivalent, but this is not always the case. For one thing, if we pass to the limit the replication method leads most naturally to a countable infinity of players, while the fracturing method leads most naturally to a game in which the players form an uncountable continuum.** (See Sec. 4.1 below.) There are of course many other, less symmetrical ways of enlarging the player set while preserving other features of the model. For example, one or two large firms might hold fixed shares of the productive capacity of the industry while the others are fractured into progressively smaller units.

In economics, although many verbal accounts of market behavior, from Adam Smith on, pay lip service to the importance of the number of competitors, in most of the mathematical models of large systems the properties of mass behavior are implicitly or explicitly assumed a priori (e.g., via a demand function), rather than deduced as limiting properties of individual behavior in small markets. The game theory approach, beginning in this case with Edgeworth (1881) but lying dormant for three-quarters of a century, enables a careful distinction to be made between

*Shapley and Shubik (1972).

**Shapley and Shapiro (1960), Kannai (1966).
assumptions and deduction in this regard. Indeed, a major portion of current research in economic game theory is devoted to just this question: investigating the limiting behavior of various game-theoretic solutions and studying the extent to which these limits agree with other models of mass decision-making.

3.1. Infinite Player Sets

Games with infinite sets of players have been attracting increasing attention as models for mass phenomena, both economic and political. While unrealistic on their face--even absurd, such models are similar in purpose to the ideal constructs commonly found in the physical sciences, as for example when the large number of discrete particles that make up a fluid are represented by a continuous medium.

The presumption is that the behavior of the infinite, continuous model, as analyzed by the methods of differential calculus, is indicative of the behavior that would be approached in the limit if a series of increasingly refined but increasingly complex finite models were considered instead.

Several different kinds of infinite-person games are found in the mathematical literature. In the countable case, the set of players can be identified with the sequence of natural numbers, \[1, 2, 3, \ldots\]. While each player can have a perceptible effect on the course of play, the rules
are usually defined so that this effect dwindles to next-to-nothing for players who are far out in the sequence.

A simple illustration is the infinite-person voting game, in which the \( n \)-th player casts the fraction \( 1/2^n \) of the total vote:

\[
  w_1 = \frac{1}{2}, \quad w_2 = \frac{1}{4}, \quad w_3 = \frac{1}{8}, \quad w_4 = \frac{1}{16}, \quad \ldots.
\]

If a two-thirds majority is needed to win, then every player can be shown to have some influence. This may be seen for player 4, for example, from the fact that \( \{1, 3, 4\} \) is a winning combination, but \( \{1, 3\} \) is not.*

In the uncountable or continuous case, the players may be like the real numbers in an interval, or the points in a multidimensional region. The possibility then arises of a nonatomic game, where none of the players have any influence as individuals, or even in finite coalitions.**

There is a strong tie here with measure theory, and with the idea that finite sets of points, or other suitably sparse point-sets, can have "measure zero" (i.e., zero

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*Shapley (1962). Curiously, if the majority required to win is set at \( 5/8 \) instead of \( 2/3 \), then player 4 and those following him are essentially powerless.

**See Kannai (1966), Aumann and Shapley (1968, 1969, 1970ab, 1971), Rosenmüller (1971b). The term "atom" denotes a point of positive measure; hence in the present discussion an atom is not a midget, but a giant.
weight, length, area, probability, etc.), and can therefore be disregarded. Nonatomic games have recently been pressed into service as models of large exchange or exchange-and-production economies.*

Continuously-infinite games need not be nonatomic, however. Indeed, among the first such games to be studied were the so-called *oceanic games*: a favored few "major" players have individual voting power, which they wield amidst a continuum of "minor" players whose infinitesimal votes count only in the mass. The conceptual archetype of this "mixed" atomic-continuous model is a corporation with a small coterie of major stockholders, the other holdings being of insignificant size but very numerous.**

There are still other kinds of games with uncountably many players. One simple example is the game of "unanimous consent," which, though a voting game, has no ocean of minor players since each individual is a "giant", capable of upsetting the applecart.***

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4. FIVE DECISIONMAKING TYPES

In the light of our belief in the need for many models of economic, political and social man, and in preparation for the mathematical models of the individual which will appear in subsequent chapters, we shall attempt to delineate in this section five contrasting roles for the individual decisionmaker. Taxonomies can quickly become tedious, especially when the use for the various subdivisions is not clear. Our present typology is meant only to be suggestive, and is neither exhaustive nor exclusive; people and professions can readily be found in economic and political life that do not fit neatly into the categories suggested.

Our five types are: the citizen (as consumer, laborer, and voter), the industrialist or businessman, the financier, the politician, and the administrator or bureaucrat. Our eventual concern will be primarily with the first two.

The Citizen

The citizen is considered in his roles as consumer, supplier of labor, small tradesman or producer, and participant in political life. For the most part he is (and regards himself as) strategically powerless. As a consumer he is almost always a price-taker, though in some markets he may bargain. If he is somewhat more sophisticated than most, he may do some price-forecasting and
stockpiling. As a producer most individuals are wage earners, individually employed professionals, or small farmers or businessmen. In the ranks of the middle and upper executives and professionals there may be more leeway for bilateral bargaining, as there is for organized labor; otherwise most labor services are sold by price-takers.

Financially, most citizens may be more correctly described as savers rather than investors. Beyond their investment in their houses and consumer durables they buy financial claims rather than real assets. In this role they again are price-takers.

Few individuals are politically active, and even among those who are, few are active on more than one or two issues. This being the case, a politico-economic model describing voting in behavioristic terms could provide a good approximation of the political activity of most citizens in "the game of politics".

The Industrialist and Businessman

This category includes those individuals who are officers of the larger business institutions of our society. Limiting ourselves to the large institutions appearing in the Fortune "500", we find around ten thousand such individuals in the United States.* If we went to a 90 percent level of control of assets, this group would include several hundred thousand decisionmakers.

*Poor's Register of Corporations, Directors and Executives, Standard and Poor's, New York, 1971.
For the most part, these businessmen are not direct owners, but the trustees of the funds of others. Although money may by no means be their only measure of incentive, monetary measurements and profits play an important part in the shaping of their activities. In most instances in the United States they are constrained by law from overt collusion or joint action with competing institutions.

Many of the large corporations or business institutions dealing directly with the public are in an oligopolistic situation which can be best described as a noncooperative game among the businesses, with the customers represented passively as price-takers. Many businesses are also heavily engaged in intercorporate trade, where oligopolistic-oligopsonistic relations may be encountered; this situation would require modeling as a cooperative (or at least quasicooperative) game. There is in addition the large sector of regulated industry, such as utilities and railroads, where the situation may again best be represented as a cooperative negotiatory game, with the government's actions subject to a voting mechanism.

Finally, there is a considerable amount of economic traffic between private firms and governmental institutions. On the business side of this market, noncooperative decision-making is a reasonable approximation, while on the other side, policy coordination as well as voting considerations may influence strategy. An example of this is provided by
noncooperative sealed bidding, conducted by government agencies after approval has been voted for a project.

The businessman will, in general, treat the individual citizen-customer-laborer-small stockholder as part of an aggregate whose behavior may be predicted.

The Financier

The financial decisionmakers include the leaders of banking institutions of various sorts, insurance companies, savings and loan societies, and other financial institutions. At the most, they comprise a few tens of thousands of individuals. For the most part they obtain funds as oligopsonists or competitors. They are again trustees of the funds of others. Profits and monetary measures of incentive are possibly even more important for this group than for other businessmen.

One of their major roles appears to be as outside evaluators and comparers of the money risk and money worth of enterprises. They are measurers of the levels of confidence, honesty, cooperation, and capability exhibited by other individuals and institutions.

The Politician

The politician is a trustee of the "power", and to a certain extent the assets, of the mass of citizens. Though he may view them as an aggregate behavioral mechanism with no strategic initiative, he recognizes that this
mechanism reflects the tastes, preferences and habits of real individuals. Far more than any other group, the political decisionmaker must consider interpersonal comparisons of welfare, both from the viewpoints of the needs of his constituents and of their power to affect his career.

Almost all political decisions are best described in cooperative terms. Voting at any level (if the individuals are not treated mechanistically) involves discussion, debate, communication, and coordinated action—all characteristic of cooperative games. An exception is provided by the analysis of party politics during an election campaign, with the parties as the players. In a two-party system the game may become inessential, and hence noncooperative.* With more than two parties, however, cooperative possibilities reappear.

The Administrator or Bureaucrat

The decisions of the politician, businessman or financier are implemented by bureaucracies of varying sizes. Most of them are sufficiently large that personal contacts between the top policy-makers and those in the implementing organization are relatively infrequent. There exist large cadres of high permanent civil servants, upper-middle executives, general managers, colonels, and so forth, who bear the main responsibility for implementation. As

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*Shubik (1968).
providers of continuity in organizations, they limit the strategic scope of the top decision-makers.

A completely satisfactory theory of welfare economics would include the effect of bureaucracies upon implementation of decisions. Possibly the appropriate model would be for each organization to be represented as a game, with constraints placed upon all members, but where each member of the organization is nevertheless a player.*

4.1. Conclusions

Much of micro-economic theory deals with the problems of optimal choice; the first two behavioral types noted are primarily "choosers." The next two types are more involved in the definition, generation and presentation of alternatives, and the last in the implementations of decisions once the choice has been made.

In these volumes our major concern will be with the first two types of decisionmakers. In some instances we may wish to consider a single individual playing two roles at the same time. The modeling of different roles for the individual decisionmaker is an ad hoc process; the special attributes of the real person and the special purposes of the model cannot be ignored. Even the most general economic theory should not be confused with a universal decision theory. This being the case, a certain amount of

*Shubik (1962); Shapley (1972).
explicit modeling of the differentiated roles of the individual in an economy helps both to narrow the field of inquiry and to give the models more structure and relevance.
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