RESIDENTIAL ENERGY USE: AN ECONOMETRIC ANALYSIS

PREPARED FOR THE NATIONAL SCIENCE FOUNDATION

KENT P. ANDERSON

R-1297-NSF
OCTOBER 1973

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PREFACE

This report is part of the Rand research program on *The Growing Demand for Energy* for the National Science Foundation. Deane N. Morris is the Principal Investigator. The general focus of the program is on the underlying cause of the conflict between energy and the environment--the rapid increase in demand for all forms of energy--and the implications of this conflict for governmental policymaking.

This report, on residential energy use, is an extension of the econometric research described in Kent P. Anderson, *Residential Demand for Electricity: Econometric Estimates for California and the United States*, R-905-NSF, January 1972; and *Some Implications of Policies to Slow the Growth of Electricity Demand in California*, R-990-NSF/CSA, December 1972, Appendix B. It is directed to a narrow audience: economists or other analysts interested in the application of econometrics to energy demand estimation. Although the results obtained in this study are relevant to policymaking, no attempt has been made to discuss policy issues or applications. These are being addressed as part of Rand's policy reserach in the energy conservation field, funded by an NSF grant.

SUMMARY

The price elasticity estimates obtained in several recent studies of residential gas and electricity demands leave one in doubt as to the nature of responses to price and, in particular, to the role of inter-fuel substitution compared with the role of alterations in usage rates or the size, efficiency, and features of new and renovated equipment. The own- and cross-price elasticities obtained in these studies imply that large equal percentage increases in the prices of all types of energy sources would eventually lead either to marked increases in energy conservation or to a shift from utility gas and electricity to other sources of energy. The latter interpretation is intuitively implausible for price increases that would apply to all types of energy. But the alternative interpretation--that there is a quantitatively significant link between energy price and the degree of energy conservation--is not well supported by casual observation.

The difficulties of interpretation just noted are due to two persistent weaknesses: (1) The interdependencies between household demand for one type of energy source and household demand for competing energy sources are not adequately accounted for in the specification of the models and in the estimation procedures. (2) The list of competing energy price variables included in a given energy demand equation is usually restricted to a single energy type, despite the possible importance of other such prices. The study described here eliminates these weaknesses and helps to clarify the nature of consumers' responses to energy prices.

The equations estimated in the study fall into two classes: those for predicting stocks of energy-using equipment and those for predicting energy consumption. The analysis of equipment stocks yields information about the importance of fuel choice relative to rates of energy use in residential energy consumption.

Both static and dynamic relationships are estimated using cross-sectional data for 50 states for the years 1960 and 1970. The results of the two approaches differ little; only those for the static case

are discussed below. The explanatory variables include energy prices (coal, utility gas, bottled gas, fuel oil, electricity, and kerosene), appliance prices, household income, household size, urbanity, housing structure, winter temperature, and summer temperature. (Each explanatory variable appears only in those equations for which it is appropriate; for example, winter temperature is not included in the air conditioning equations.)

The energy equations estimate average consumption per household of utility gas and electricity. (Suitable data for fuel oil and bottled gas are not available.) The two equations are estimated both independently and simultaneously with a constraint upon the cross-elasticity coefficients for gas and electricity prices.

The stock equations for a given function (for example, heating) are estimated simultaneously for all types of energy sources (including none) and are consistent with the constraint that

$$\sum_{i=1}^{m} s_{ik} = 1 \ , \tag{1}$$

where s_{ik} is the fraction of households using energy type i to perform function k. (In some cases s_{ik} refers not to fuel type but to other characteristics such as automatic or wringer washers and room or central air conditioning.)

The stock equations are used to obtain indirect estimates of the own- and cross-price elasticities of energy consumption. Average per household consumption of energy type i, q_i, may be expressed as follows:

$$q_i = \sum_{k=1}^{n} e_{ik} \cdot s_{ik} \tag{2}$$

where e_{ik} is the average amount of energy type i used for household function k. The s_{ik} are the dependent variables of the stock equations. The e_{ik} reflect the effects of installation size, efficiency, and operating conditions upon average consumption per installation. Assuming the e_{ik} to be given constants, one has

$$\frac{\partial q_i}{\partial p_j} \cdot \frac{p_j}{q_i} = \sum_{k=1}^{n} \left(\frac{e_{ik} s_{ik}}{\sum_i e_{ik} s_{ik}} \cdot \frac{\partial s_{ik}}{\partial p_j} \cdot \frac{p_j}{s_{ik}} \right) . \qquad (3)$$

That is to say, the elasticity of consumption of energy type i with respect to the price of energy type j is a weighted sum of the elasticities of installation of units using energy type i with respect to the price of energy type j. Since estimates of the e_{ik} are available, Eq. (3) can be used to calculate elasticities of consumption at the sample mean values of the s_{ik}. These can then be compared with the elasticities estimated directly in the energy consumption equations.

Generally, one would expect the indirectly computed elasticities to be biased toward zero, since they assume the effects of price upon the e_{ik} to be zero. Such bias should be quite small in the case of cross-price effects, since it is difficult to imagine why the price of, say, gas or oil should measurably affect the size, features, or average operating levels of electrical equipment. The bias in the own-price effect should be notable, however, since some, if not all, of the e_{ik} are likely to show a response to differing prices for the fuels actually used.

Given the hypothesis that the directly estimated elasticities are unbiased estimators of the true sample mean elasticities, then the indirectly estimated own-price elasticities should be of lower absolute magnitude than the former. In addition, the directly and indirectly estimated cross elasticities should all be positive (assuming substitutes), with the indirect estimates being slightly smaller than the direct. However, the results show that a number of the directly estimated cross elasticities are negative whereas all indirectly estimated ones are positive. Even where the cross elasticities estimated by the two approaches are both positive, the indirect estimates are not always smaller. The own-price elasticities appear to be consistent with the above hypothesis, since the calculated own-price elasticities for gas and electricity are substantially lower in absolute value than their counterparts obtained from the energy consumption equations. The most appealing explanation for the difference has already been noted: Both

estimates are "accurate." The difference reflects the influence of price upon the average rate of consumption per installation. An alternative hypothesis, namely that price has no effect upon the rate of consumption per installation and hence that only one of the two coefficients is correctly estimated, can be rejected on the basis of the results obtained.

One interpretation of the cross-elasticity results is that the indirectly computed values are close to the true values, while the direct estimates have a large error component due primarily to inappropriate specification of the energy consumption equations. Energy equations require aggregation of demands across individual household energy-using functions and cannot be readily adapted to account for the dependency of price effects upon the varying extents of installation of competing types of equipment. Moreover, the rather large standard errors and negative signs of many of the directly estimated cross elasticities do not easily lend themselves to any other interpretation.

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I. INTRODUCTION

Recent studies of residential gas and electricity demands[1] have a number of persistent weaknesses: (1) The interdependencies between household demand for one type of energy source and that for competing energy sources have not been adequately accounted for in the specification of the models and in the estimation procedures. (2) The list of competing energy price variables included in a given energy demand equation is usually restricted to a single energy type, despite the possible importance of other such prices. (3) The price elasticity estimates obtained leave one in doubt as to the nature of responses to price and, in particular, to the role of inter-fuel substitution compared with the role of alterations in usage rates or the size, efficiency, and features of new and renovated equipment.

Table 1 displays long-run price elasticity estimates reported in several recent studies of electricity demand. All of these estimates are roughly comparable.[2] Table 2 does the same for gas, although the degree of comparability is not as high. Balestra [7], for example, estimates combined commercial and residential demand; and McAvoy and Pindyck [9] estimate a derived wholesale demand, the demand for gas for resale to all types of customers by utility companies. Excepting the latter case and the third example from Balestra, which constrains the elasticity with respect to competing-fuel price to be equal to the own-price elasticity, the results in both tables indicate that the own-price elasticity of demand is substantially greater in absolute terms

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[1] See Tables 1 and 2.

[2] Noteworthy differences among the studies include choice of electricity price variable, type of equation estimated (demand vs. reduced-form consumption), and estimation procedure (two-stage vs. ordinary least squares). Despite important theoretical differences the practical distinctions in the results are much diminished by (1) the likelihood of considerable imperfection in the ways residential consumers obtain and process information on energy prices and (2), perhaps more important, the relative insensitivity of the marginal price of electricity to changes in consumption at the mean consumption levels of the recent past. (See Table 3 and accompanying discussion.)

Table 1

LONG-RUN PRICE ELASTICITIES OF ELECTRICITY DEMAND OBTAINED IN RECENT STUDIES[a]

Study and Data Used	Dependent Variable	Long-Run Elasticities of Demand with Respect to Electricity Price[b]	Long-Run Elasticities of Demand with Respect to Utility Gas Price[b]
Wilson [1] 77 cities, 1966	EPC_{66}	-1.33[c]	0.31[c]
		--	--
Anderson [2] 50 states, 1969	EPC_{69}	-0.91[d]	0.13
		(0.10)	(0.07)
same	$\frac{QE_{69}-0.9QE_{68}}{CE_{69}-0.9CE_{68}}$	-0.88[d]	0.17
		(0.12)	(0.09)
Halvorsen [3] 48 states, 1969	EPC_{69}	-1.25	0.05
		(0.12)	(0.05)
same	EPC_{69}	-1.01[e]	0.07[e]
		(0.09)	(0.04)
same 48 states, 1961-69	EPC_t	-1.14	0.05
		(0.04)	(0.02)
Mount et al. [4] 47 states, 1947-70	QE_t	-1.21[f]	0.21[f]
		--	--

[a]All coefficients are directly estimated through log-linear specifications with the exception of those from [4]. Standard errors are given in parentheses. The dependent variables are defined in Table 13. Subscripts indicate time periods.

[b]All prices are average prices except where otherwise noted.

[c]The electricity price variable is the typical electric bill for 500 kWh per month. Standard errors are not available. The coefficients for electricity and gas prices are significant at the .001 and .01 levels respectively.

[d]Electricity price is measured by the difference between typical electric bills for 1000 and 500 kWh per month.

[e]Explanatory variables are weighted averages of 1961-69 values. The weighting follows an "inverse V" pattern.

[f]Elasticities are calculated from a lagged dynamic model. The estimated coefficients and t-ratios for electricity and gas prices and for the lagged consumption term are as follows:

PELEC	-.1385 (-12.5)
PGAS	.0238 (6.7)
QE_{t-1}	.8859 (136.5)

The formula for calculating the long-run elasticity is $E_i = \beta_i(1 - \lambda)$, $i = e,g$, where λ is the coefficient of QE_{t-1} and the β_i are the price coefficients.

Table 2

PRICE COEFFICIENTS OF UTILITY OR NATURAL GAS DEMAND OBTAINED IN RELATED STUDIES

Study and Data Sample	Dependent Variable[a]	Regression Coefficients[b] for Prices[c] of	
		Natural Gas	Competing Fuel
Balestra [7] 47 states, 1951-62	$QG_t - QG_{t-1}$	-0.0311 (0.0104)	-0.0202 (0.0192)
same 47 states, 1957-62	$QG_t - QG_{t-1}$	-0.0566[d] (0.0221)	excluded
same 47 states, 1957-62	$QG_t - QG_{t-1}$	-6.526[e,f] (2.762)	f
McAvoy-Pindyck [9] 30 districts, 1962-69	$QG_t - 0.9QG_{t-1}$	-1.762×10^8 (0.480×10^8)[g]	1.121×10^8 (0.226×10^8)[g]
Anderson [8] 50 states, 1969	GPC_{69}	-0.97 (0.08)[h]	-.23 (0.11)[h]
same	QG_{69}	-2.31 (0.16)[h]	0.08 (0.21)[h]

[a] The dependent variables are defined in Table 13. Subscripts denote time period.

[b] All coefficients are estimated from linear specifications except those of Anderson, which are estimated from a log-linear specification. Standard errors are given in parentheses.

[c] All prices are average prices except as noted. The competing fuel price is an index of coal and oil prices in Balestra, the average wellhead price of crude oil in McAvoy-Pindyck, and the price of electricity in Anderson.

[d] The own-price elasticity calculated at the sample mean is -0.680.

[e] The own-price elasticity calculated at the sample mean is -0.675.

[f] The price variable used in this case was the ratio of natural gas to competing-fuel price.

[g] Calculated from reported t-ratios. For the own- and competing-price coefficients they are -3.67 and 4.95.

[h] Standard errors are not reported in [8].

than the cross elasticity. Indeed, in some of the gas equations, implausible *negative* cross elasticities are obtained.[1]

These results imply that large equal percentage increases in the prices of all types of energy sources would lead either to notable increases in energy conservation or to marked shifts from equipment using utility gas or electricity to the competing sources of energy whose prices were not included in the estimated equations, since the estimated cross-price effect (either positive and small or negative) would not offset the relatively large (and negative) own-price effect. The possibility of a major shift is difficult to accept, given that price increases would apply to all types of energy. But the alternative interpretation, that there is a quantitatively significant link between energy price and the degree of energy conservation, is not well supported by casual observation. In the absence of corroborating data and given the methodological weaknesses described above, neither interpretation is particularly appealing. Since the solution to this problem bears both upon the methodology of energy demand estimation and upon the evaluation of policies affecting energy price, this report seeks to provide some helpful clues. A second and equally important goal is to provide a more comprehensive statistical picture of residential energy demand behavior than has been available in the past.

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[1] A negative cross-price effect indicates complementarity; a positive one, substitutability. Examples of factors suggesting complementarity can be found: Electric motors are used to drive fans in forced-air heating systems that burn fossil fuels. Electricity may be used to run the hi-fi while fossil fuel is used to "produce" evening mealtime pleasure. Nevertheless, it is likely that substitution effects predominate over such complementarities. With the minor exceptions of electric motors or pilot lights in certain nonelectric devices, the selection of energy-using equipment involves choosing one type of energy source to the exclusion of others; and as shown in Section VI, the major portion of the effect of price appears to be upon the choice of equipment according to type of energy used.

II. THEORETICAL FRAMEWORK

The equations estimated for this report fall into two classes: those for predicting stocks of energy-using equipment and those for predicting energy consumption. The equations for equipment stocks provide information about the relative importance of fuel choice in the overall responsiveness of residential energy consumption to price. This section presents the theoretical framework upon which the estimates for both types of equations rest.

AGGREGATION

Since the data used for estimation consist mainly of annual averages for individual states,[1] a model of aggregate rather than individual or household demand behavior must be formulated. Nevertheless it is proper to begin discussion at the level of the individual decisionmaking unit.

In accordance with classical demand theory, one may assume that the quantity of any given commodity that a consumer or household would desire to consume in equilibrium can be written as a function of price, income, and a vector of taste parameters. For household i one has,

$$q_i = F(p_{i1},\ldots,p_{iJ};\ y_i;\ \tau_{i1},\ldots,\tau_{iK}) \ , \qquad (1)$$

where q_i is the amount demanded of some specified commodity, the p_i are the prices to household i of the J commodities that are available in the market, y_i is household income, and the parameters τ_{ik} describe the household's tastes. Equation (1) is an exact relationship. Since observations on the τ_{ik} are generally not available and since inclusion of all the p_{ij} is not compatible with the usual methods for estimating demand relationships, practical estimation of such relationships typically requires that a stochastic equation of the following sort be assumed:

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[1] See Section III and Appendix A for details.

$$q_i = G(p_{i1},\ldots,p_{iJ'};\ y_i;\ h_{i1},\ldots,h_{iL};\ u_i) \ , \qquad (2)$$

where J' denotes some small subset of the J prices, the h_{i1} are certain observable household characteristics, and u_i is a random error term. The h_{i1} are assumed to be functions of the omitted price and taste variables. Any effects of the omitted variables that are not reflected in the h_{i1} are assumed to form the random error component. When observations on individual households are not available, but averages for groups of households are, it is necessary to make the further assumption that the average amount demanded per household in group I can be estimated by an approximating function whose arguments are group averages of the arguments of the function $G(\cdot)$ in Eq. (2); that is, for group I,

$$q_I = H(p_{I1},\ldots,p_{IJ'};\ y_I;\ h_{I1},\ldots,h_{IL};\ u_I) \ . \qquad (3)$$

Under certain circumstances, the characteristics of the function $H(\cdot)$ will approximate those of the individual household function, $G(\cdot)$.

SUPPLY-PRICE RELATIONSHIPS

Utility gas and electricity are typically purchased according to declining block rate schedules. The correct price variable to appear in Eq. (3) for these commodities is, therefore, not the average unit price but the price of a marginal unit of consumption, provided that consumers are well-informed. It is possible, of course, that residential customers as a rule do not consider the marginal price. The likely existence of real-world imperfections suggests that average price or the size of a typical bill may be as good a measure of the rate that influences consumer behavior as marginal price, if indeed one can assume that consumers consider a single rate variable when making choices about energy consumption.[1] Declining block-rate

[1] A discussion of the choice-of-variable issue is contained in [3]. The problem is resolved here by using an approach that allows for more than one interpretation of the results, depending upon one's view of consumer behavior.

schedules raise a second issue: If either marginal or average price is supposed to be the proper price variable to include in the demand function, then one has a simultaneous demand-supply system, since average and, to some extent, marginal prices decline as the quantity consumed increases.

One way of dealing with both problems is to derive and estimate a reduced form consumption equation that incorporates the effect of the rate schedule into the estimated parameters of the equation. Specifically, if it is assumed that the marginal or average price (without necessarily specifying which) can be expressed by a function of a "standardized price"--for example, an average price or bill for some specified level of consumption--and the actual quantity consumed per household, then one can write for group I,

$$p_I = J(p_I^\circ, q_I, v_I) , \tag{4}$$

where p_I° is the standardized price and v_I is a random error term. If the functions $H(\cdot)$ and $J(\cdot)$ are of suitable form for each of the energy sources, they can be solved to eliminate the p_I for gas and electricity, which are replaced by the corresponding exogenous p_I°.[1] One can then write the reduced-form consumption equation as

$$q_I = R(p_{I1},\ldots,p_{Ig}^\circ, p_{Ie}^\circ,\ldots,p_{IJ}; y_I; h_{I1},\ldots,h_{IL}; w_I), \tag{5}$$

---

[1]The p_I° are not of necessity exogenous. If the marginal cost of production is dependent upon the total level of production, then the p_I° might be endogenous in certain circumstances. In the case at hand, however, it seems unlikely that state-to-state variations in the level of production would, of themselves, influence costs. Differences in long-run costs due to differences in production levels would presumably reflect economies of scale, yet in every state the total market volume greatly exceeds the output of any single optimally scaled plant. In addition, since the dependent variable in the demand equation is defined in per-household terms, the link between the p_I° and the dependent variable is considerably weaker than if total residential demand or, worse, total demand by all customers were being estimated.

where w_I is the error term and the subscripts g and e denote gas and electricity. The above reduced-form approach has the desirable property of yielding unbiased estimates of the total effect on consumption of changes in the explanatory variables, including the feedback effect due to declining block rates. On the other hand, reduced-form estimates do not permit one to estimate the effects of changes in the character of the rate schedule (as reflected in changes in the structure of an explicit supply-price relationship).

Estimates of Eq. (5) may be given a second interpretation. If consumers, in fact, tend to measure the level of electric or gas utility rates by some notion of a typical price, if the standardized price included in the estimated equation is adequately representative of this price, and, finally, if consumers' concepts of this price are not seriously affected by their own level of consumption, then Eq. (5) can be interpreted as a pure demand equation. The fact that utilities' declining block rate schedules generally tend to flatten out considerably after consumption has reached a certain level lends support to the pure demand interpretation. Table 3 presents composite rates for all U.S. electric utilities as of January 1, 1969. At that time, average annual consumption of electricity was about 6570 kWh per household, well into the region of consumption where utility rate schedules on the average offered no significant decreases in marginal price. If consumers do tend to make marginal evaluations--for example, as to how much the bill would change if a particular unit were installed or removed--then for most customers such evaluations would be unaffected by their current level of consumption.

Comparable evidence for utility gas rates is not available. The American Gas Association indicates that declining block rates are typically designed to approximate a curve of the form

$$p = a + \frac{b}{q} ,$$

where a and b are constants, p is the average price paid by the customer, and q is the quantity consumed by the customer. This curve is

Table 3

TYPICAL ANNUAL ELECTRIC BILLS AND PRICES
U.S. Averages, January 1, 1969

Electricity Consumption (kWh)	Annual Bill ($)	Average Price (¢/kWh)	Incremental Consumption (kWh)	Incremental Bill ($)	Marginal Price (¢/kWh)	Marginal Price[a] (¢/kWh)
1,200	49	4.08	1,200	49	4.08	
3,000	89	2.97	1,800	40	2.22	1.56
6,000	124	2.07	3,000	35	1.17	
9,000	168	1.67	3,000	44	1.46	
12,000	216	1.80	3,000	48	1.60	1.30
15,000	246	1.80	3,000	30	1.00	
20,000	311	1.56	5,000	65	1.30	--
25,000	375	1.50	5,000	64	1.28	--
30,000	439	1.46	5,000	64	1.28	--

[a]For alternative intervals of consumption, as indicated.

Sources: Federal Power Commission, *Typical Electric Bills* and *All Electric Homes*, 1969 issues.

such that *the marginal price faced by the consumer is constant at all levels of q:*

$$\frac{d(pq)}{dq} = a$$

In a sample rate schedule provided on p. 271 of [14], the marginal price declines only once after a monthly consumption level of 20 therms is reached: from 8¢/therm to 7¢/therm at 100 therms/mo. (Average consumption per customer in 1970 was approximately 108 therms/month.) If this sample rate schedule is representative of most such schedules, it is reasonable to expect that consumers' perceptions of marginal price might be little affected by their own level of consumption once it had exceeded some small amount.

STOCK EQUATIONS

A salient characteristic of household demand for stocks of energy-using devices is the interdependency among energy forms chosen to perform a given function. Simply put, if one form of energy—say, electricity--is used for central heating, water heating, cooking, air conditioning, and so on, alternative forms of energy—fossil fuels, for example--are not likely to be used in any significant quantity, if at all, since most households invest in a single major unit to perform each of the various tasks indicated above. Furthermore, the data on stocks of equipment that are used in this study indicate only "the fuel used most" for a given task. Each household thus fits into one and only one category for each energy-using task.[1] Of necessity, the sum of households in each category (including the "none" category) is equal to the total, and it is therefore desirable that the estimated relationships reflect this characteristic of the data.

Let N_{it} be the number of households using fuel type i for a given household function in period t, and let r_i be the fraction of households that retained existing equipment between period t-n and period t. Then installations at time t can be described by the following relationship:

$$N_{it} = r_i \cdot N_{it-n} + s_i \cdot (N_t - \sum_{i=1}^{m} r_i \cdot N_{it-n}) , \qquad (9)$$

where s_i is the fraction of new installations and replacements made from time t-n to t that use fuel type i for the specified function,[2] N_t is the total number of households at time t, and m is the total number of alternative fuel types (including none). Observe that

$\sum_{i=1}^{m} s_i = 1$ by necessity. Assume now that r_i is given by

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[1] See Table 5, especially note b, for a listing of these categories.

[2] One can interpret s_i as a market sales share based upon numbers of installations instead of the usual dollar sales volumes.

$$r_i = \begin{cases} \rho_i \cdot (N_{it}/N_{it-n}), & N_{it} \leq N_{it-n} \\ \rho_i, & N_{it} > N_{it-n}, \end{cases} \tag{10}$$

where ρ_i is a constant "retention" ratio. Eq. (10) implies that a given fraction of the "old" stock[1] existing in period t represents replacements made between periods t-n and t. If ρ_i is given, one can compute r_i, and then the market sales share, s_i, using observations on N_{it} and N_{it-n}. The problem at hand can then be conveniently formulated as one of estimating functions, $f_i(\cdot)$, that will explain variations in s_i across states for the time interval 1960-1970. If these functions are assumed to be of the following form,

$$f_i(\cdot) \equiv \frac{g_i(\cdot)}{\sum_{j=1}^{m} g_j(\cdot)}, \tag{11}$$

they necessarily sum to unity, so as to accord with observations on the s_i. It remains to specify the forms of the functions, $g_i(\cdot)$.

The estimates presented in Section IV assume that these functions are of the following form:

$$g_i(\cdot) = a_i p_i^{b_i} v_i^{c_i} y^{d_i} z_i, \tag{12}$$

where p_i is the price (standardized, where appropriate) of energy type i, v_i is a representative purchase price for the class of household devices or installations using energy type i to perform a given task, y is household income, and z_i is a remainder term including the effects of all the household-characteristic variables (which for simplicity are omitted here).[2]

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[1] That is, the total stock in period t minus new installations and conversions made between periods t and t-n.

[2] Household characteristic variables enter as h^a or e^{ah} terms. See Eqs. (24) and (25).

Equations (11) and (12) indicate that each $f_i(\cdot)$ is a nonlinear function not directly amenable to standard linear estimation techniques. But the form chosen for Eq. (12) permits the problem to be overcome in the following way: Let

$$s_i = \frac{g_i(\cdot)}{\sum_{j=1}^{m} g_j(\cdot)} \cdot u_i, \qquad i=1,\ldots,m \ , \tag{13}$$

where u_i is an error term. Then

$$\frac{s_i}{s_m} = \frac{g_i(\cdot)}{g_m(\cdot)} \cdot \frac{u_i}{u_m} = \frac{a_i}{a_m} \cdot p_i^{b_i - b_m} p_m^{c_i - c_m} v_i^{d_i - d_m} \cdot \frac{z_i}{z_m} \cdot \frac{u_i}{u_m} \ ,$$

$$i=1,\ldots,m-1 \ . \tag{14}$$

Taking the logarithms of both sides of Eqs. (14) one obtains a set of (m-1) linear equations that are appropriately estimated by Zellner's method for sets of seemingly unrelated equations.[1] In addition, the estimates of b_m and c_m must be constrained to be identical in each of the (m-1) estimated equations.[2]

The approach outlined above has the advantage not only of incorporating certain desirable constraints but also (as shown below) of yielding price and income elasticities that have suitably realistic properties. Nevertheless, the benefits of this approach are not attained without costs. First, the coefficient estimates obtained by the procedure are not unique but vary with the choice of denominator (s_m) in Eqs. (14). (Section IV presents alternative sets of estimates having different denominators.) Second, it is not possible to deduce from the estimates of Eqs. (14) the values of the coefficients of individual

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[1] See [11].

[2] This procedure is adapted from one designed for estimating complete systems of demand equations. See [12].

expanatory variables other than the price variables. For example, as shown in Eq. (14) the coefficients actually estimated for y are the (m-1) coefficients, $(d_i - d_m)$, not the individual d_i themselves. Nevertheless, it is possible to estimate the partial elasticity of a change in any given explanatory variable upon each of the s_i. In the case of income changes, one has

$$\frac{\partial s_i}{\partial y} \cdot \frac{y}{s_i} = (d_i - d_m) - \sum_{j=1}^{m-1} s_j (d_j - d_m). \tag{15}$$

The partial elasticities with respect to own and competing energy prices are, respectively,

$$\frac{\partial s_i}{\partial p_i} \cdot \frac{p_i}{s_i} = (1 - s_i) b_i \, , \tag{16}$$

and

$$\frac{\partial s_i}{\partial p_m} \cdot \frac{p_m}{s_i} = -s_m \cdot b_m \, . \tag{17}$$

Note that these elasticities are not constant but vary systematically with the s_i in ways that are intuitively satisfying. Cross elasticities, for example, are proportional to the market penetration of competing equipment.

DYNAMIC VERSUS STATIC FORMULATIONS

The estimation procedure outlined above reflects a dynamic view of the capital stock adjustment mechanism: Eqs. (13) predict installation shares (new installations + conversions + replacements) between two periods. This approach is theoretically preferable to one that predicts capital stocks at a given point in time, as long as households are not generally at a position of long-run static equilibrium. Nevertheless, there are several reasons why, in the case at hand, a static approach may be desirable.

The stock data used in this study are available only for the rather widely spaced decennial population census intervals. But since household investment goes on continuously, the most desirable formulation of a capital stock adjustment mechanism would be a very short-interval model, for example for predicting monthly or quarterly installation shares. Use of wide intervals weakens the validity of the assumption that consumers acted as if they responded to some fixed set of values for the explanatory variables during the entire interval. The seriousness of this problem is somewhat mitigated by the fact that most of the explanatory variables either changed smoothly and monotonically between 1960 and 1970 or did not vary greatly during the interval. But the lack of abrupt changes also suggests that a dynamic equilibrium model might be applicable. Since static and dynamic models have similar implications under conditions of dynamic equilibrium, a static approach may provide a suitable approximation.[1]

The relative attractiveness of a static formulation is enhanced in another way by the length of the intercensal period: The average economic lifetime for most household energy-using devices, with the exception of central heating, is probably well under twenty years. Thus, the retention fraction, ρ_i, is likely to be fairly low for an interval as long as ten years. Inspection of Eqs. (10) and (11) shows that as $\rho_i \to 0$, $s_i \to N_{it}/N_t$. That is, s_i approaches a static, capital-stock "saturation" ratio as ρ_i becomes appropriately small. Moreover, direct observations on the ρ_i are not available nor are they easily estimated from the data using regression techniques, given the structure implied by Eqs. (9) and (11). Thus, when the dynamic approach is taken, the estimates obtained are conditional on the values assumed for the ρ_i. Given the potential inaccuracies involved in fixing these parameters, one may introduce less error by resorting to the simpler static approach.[2]

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[1] See [2], pp. 13-15.

[2] Results obtained by Halvorsen [3] and Anderson [2] for electricity suggest that the static approach is probably as good as the dynamic.

ENERGY EQUATIONS

Estimates of energy-consumption or demand equations are presented in Section V for utility gas and electricity. These account for almost 70 percent of total residential energy consumption (excluding transportation). Fuel oil accounts for the bulk of the remainder (25 percent). Data for household consumption of fuel oil by states are not available.[1]

As in the case of the stock equations, both dynamic and static specifications of the estimated equations are tried. The two dynamic specifications used are based upon an assumed underlying capital stock adjustment mechanism of the sort described above:

$$\left.\begin{array}{l} \dfrac{Q_{it} - \bar{\rho}_i Q_{it-n}}{C_{it} - \bar{\rho}_i C_{it-n}} = \\ \\ \dfrac{Q_{it} - \bar{\rho}_i Q_{it-n}}{C_{it}} = \end{array}\right\} g_i(\cdot)u_i \quad \cdot \qquad \begin{array}{l} (18a) \\ \\ (18b) \end{array}$$

Q_{it} is total residential consumption of energy type i in period t; C_{it} is the average number of type i customers in period t; $\bar{\rho}_i$ can be described roughly as a fixed mean retention ratio for stocks of all types of energy-using equipment between periods t-n and t; and u_i is an error term. Version (a) assumes that there are $\bar{\rho}_i \cdot C_{t-n}$ "locked-in" customers who have a zero elasticity of consumption with respect to all explanatory variables, that these customers are a representative cross-section of period t-n customers, and that, as such, they consume $\bar{\rho}_i \cdot \bar{Q}_{it-n}$ of energy i in period t. Version (b) assumes that $\bar{\rho}_i \cdot \bar{Q}_{it-n}$ of period t-n demand has zero elasticity of consumption but does not attribute the fixed component of demand to any specific portion of period t-n customers. Variations of Eqs. (18) are also estimated using numbers of households rather than customers in the denominator of the dependent variable. The assumption of a zero elasticity for locked-in

[1]The available data do not distinguish sales to residential customers from those to commercial customers.

demand and customers in version (a) is, of course, a caricature of reality since even customers who choose to make no alteration in their stocks of energy-using devices can influence their consumption by changes in operating conditions or habits. Thermostats can be adjusted, windows closed or opened, lights turned off, and so on. It is unlikely, however, that these responses would be very sensitive to the variations observed in the explanatory variables. Version (a) assumes that they are small enough to ignore. Version (b) is also a caricature of sorts. It allows for short-run responsiveness in demand, but it makes no distinction between customers who vary stocks and those who vary only the conditions of use.

The specification of the $g_i(\cdot)$ is the same for both gas and electricity:

$$g_i(\cdot) = a_i\left(\prod_{j=1}^{m'} p_j^{b_{ij}}\right) y^{c_i} z_i, \quad i=1, 2 . \tag{19}$$

The p_j are own- and competing-energy prices (standardized for gas and electricity); z_i is a remainder consisting of household characteristic terms, which are not shown; and m' is the number of energy prices included. Since no suitable indices for the purchase prices of energy-using equipment as a whole by energy type could be found or constructed, such prices are not included in the energy equations.

Like the stock equations, Eqs. (19) form a pair of seemingly unrelated equations. Since both are identically specified, no loss of estimation efficiency would occur if they were estimated separately by ordinary least squares, but simultaneous estimation permits the cross elasticities with respect to gas and electricity prices to be constrained in accordance with classical consumer demand theory. According to that theory, the cross elasticity of demand for good i with respect to price j can be written as follows:

$$\eta_{ij} = -\omega_j \cdot \eta_{iy} + S_{ij} \frac{p_j}{q_i} , \tag{20}$$

where ω_j is the share of income expended upon good j, η_{iy} is the income elasticity of demand for good i, and S_{ij} is the pure substitution effect. If $-\omega_j\eta_{iy}$ and $-\omega_i\eta_{jy}$ are sufficiently small relative to the substitution term,[1] we can write:

$$\frac{\eta_{ij}}{\eta_{ji}} \simeq \frac{p_j q_j}{p_i q_i}, \tag{21}$$

since $S_{ij} \equiv S_{ji}$. It seems reasonable, therefore, to estimate η_{ji} subject to Eq. (21), where the right hand side variables are set equal to their sample mean values.[2] In Section V, both constrained and unconstrained estimates are presented.

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[1]The shares of disposable income taken by gas and electricity purchases are 0.0076 and 0.0137 for 1970. Assuming that the income elasticities do not markedly exceed unity (see Tables 11 and 12), the terms $\omega_j\eta_{iy}$ and $\omega_i\eta_{jy}$ can safely be ignored.

[2]But see p. 45 for additional qualifications.

III. DATA

The *U.S. Census of Housing: 1960* and the corresponding publication for 1970 provide data on stocks of energy-using devices. These data are based upon a 5 percent sample survey of total population and indicate by state the number of occupied housing units (households) using various fuels for heating, water heating, cooking, and clothes drying. In addition, they indicate the number of households having a single room air conditioner, more than one room air conditioner, central air conditioning, a wringer clothes washer, an automatic clothes washer, a dishwasher (1970 only), a food freezer, one television, and two or more television sets.[1] The above items account for all major household energy-using devices except refrigerators, which had achieved nearly 100 percent saturation throughout the United States by 1960.[2] Little would be gained by trying to explain state-to-state variations in refrigerator installations. Additional demographic data obtained from the censuses of housing include figures by state for population in housing units, numbers of occupied housing units, numbers of year-round housing units, numbers of housing units in urban areas, and numbers of single family detached housing units.

Total residential consumption, average number of residential customers, and total value of residential sales for 1960 and 1970 are contained in the annual statistical summaries of the American Gas Association (*Gas Facts*) and the Edison Electric Institute (*E.E.I. Statistical Yearbook*).[3] The 1960 electricity figures exclude a small fraction of residential data contained in a separate rural category.[4]

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[1] In 1970, air conditioning installations were measured for year-round housing units from a 15 percent sample of the population.

[2] In 1960, 93.5 percent of all U.S. households had a refrigerator; in 1970 the figure was 98.6. Electric refrigerators accounted for 98-99 percent of total installations.

[3] See Appendix A for estimates made for 1960 Alaska.

[4] This omission does not appear to have serious consequences. See [3], pp. 297-300, for a discussion of the problem.

Typical monthly electrical bills for 1000 kWh for the years 1965 and 1970, published by the Federal Power Commission in *Typical Electric Bills*, are used as the standardized electricity prices for 1960 and 1970. 1965 data are used for 1960 since 1000 kWh bills are not reported for earlier years. According to data for smaller bills, electricity rates hardly changed between 1960 and 1965.[1] Typical monthly gas bills appear not to be available for states. To obtain standardized gas prices, Bureau of Labor Statistics data for January 1, 1969 on typical gas bills in selected Standard Metropolitan Statistical Areas (SMSAs) were used to estimate a supply-price function given by

$$\left(\frac{p_g q_g}{p_g^\circ q_g^\circ}\right) = \left(\frac{q_g}{q_g^\circ}\right)^{b_g} \cdot u_g \,. \tag{22}$$

Values of $p_g q_g$ corresponding to monthly q_g of 10, 25, and 40 therms were available for 23 SMSAs and to a monthly q_g of 100 therms for 20 of the 23 SMSAs. The value selected for q_g° was 40 therms, since it was nearest the midpoint of the range of q_g values considered. The estimate obtained for b_g was -0.6173. This coefficient has a standard error of 0.0151 with 65 degrees of freedom and is highly significant statistically. The coefficient of determination of the regression is 0.9544. Using this result, one can estimate a "typical" 40-therm monthly bill from annual consumption data:

$$\text{PGAS} = \left(\frac{\text{VGPC}}{12}\right)\left(\frac{\text{GPC}}{12 \cdot 40}\right)^{0.6173} \,, \tag{23}$$

where PGAS is the standardized price of gas (equal to the typical 40-therm monthly bill), VGPC is the average annual residential gas bill per customer, and GPC is average annual consumption of gas per customer. Eq. (23) is used for both 1960 and 1970 on the assumption that the average structure of quantity discounts remained about the same during

[1] The average U.S. bill for 500 kWh/month declined from $10.62 to $10.41 during this interval.

the decade. The values obtained for PGAS are presented in Appendix A.[1]

The prices of fuel oil, kerosene, and bottled gas were obtained from *Agricultural Prices* for 1960 and 1970.[2] These data reflect prices paid by farmers. It is possible that farmers in a given state would pay prices somewhat different from those paid by residential consumers, even though both groups are likely to consist mainly of small users purchasing from retail dealers; the latter may have better access to larger, more competitive urban markets where prices may tend to be lower.

Data on the price of coal to residential users by state could not be found. The average price paid by manufacturing firms for coal is available for most states for 1958 and 1962 from the *Census of Manufactures*. These data can be used to construct a reasonably accurate 50-state series for 1962, although prices paid by manufacturers are likely to be somewhat lower than those paid by small residential users and the types of coal purchased may also differ. It is assumed here that the average retail price of coal is roughly proportional to the average price paid by manufacturers. The latter can then be used as a proxy for the retail price of coal in the regression equation. The fact that 1962 data are used for both 1960 and 1970 is not necessarily a matter of serious consequence where cross-sectional rather than time series estimation is involved. If relative coal prices across states remained fairly constant throughout the period, the estimated coefficients for the price of coal would be unaffected. Table 4 shows that domestic coal prices declined somewhat during the early 1960s and underwent a substantial increase in the later years, especially between 1969 and 1970. A constant absolute price assumption thus cannot be maintained, unless one postulates a certain inertia or lag to the

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[1] All data series for which substantial modifications of published information were required are shown in Appendix A. Minor emendations of other data series are also noted.

[2] See [16] and [17].

Table 4

U.S. AVERAGE PRICE OF COAL, DOMESTIC SIZES, SELECTED YEARS 1960-1970 ($/short ton, f.o.b. mine)

Year	Price of Coal
1960	7.69
1965	6.93
1967	6.80
1969	7.49
1970	9.65

Source: *Survey of Current Business*, various issues.

formulation of consumer price expectations. The assumption of fairly uniform rates of change across regions is probably tenable, however.[1]

Representative prices by states for ranges (both gas and electric), wringer and automatic clothes washers, electric clothes dryers, food freezers, and room air conditioners were obtained from the 1970 edition of *Agricultural Prices*. As noted above, it is necessary to assume that prices paid by farmers are representative of prices paid by households in general. Price indicators for equipment not listed above could not be located.

Personal income by states for 1960 and 1970 was obtained from the *Survey of Current Business*, August 1971. A cost-of-living index for normalizing prices and incomes across states was not directly available. The 1970 BLS data for SMSAs on the relative living costs of a family of four having an "intermediate" budget permitted construction of an index for state metropolitan areas. Indices for state non-metropolitan areas were set at 90/103 of the metropolitan indices, based upon the U.S. averages for these two types of areas. The overall index for a state is a population-weighted average of the two indices. The indices for 1960 assume the same relative price levels across states as in

[1] Moreover, the proxy coal-price variable does well in the estimated equations. See Tables 6 and 7.

1970 for metropolitan and non-metropolitan areas. The absolute level of prices in 1960 is assumed to be 88.7/116.3 that of 1970, based upon the U.S. consumer price index. The 1960 population weights are used to compute the overall state indices for that year. Appendix A shows the 1970 metropolitan index used as the basis for the constructed series.

Climatic variations are measured by mean January and mean July temperatures.[1] Estimates for states are obtained from data for cities covering the period 1931-1960 as reported in the *1971 World Almanac*. See Appendix A.

Table 5 defines the variables used in the stock equations and lists their units of measurement. Estimation procedures and results are discussed in the next two sections.

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[1] A heating degree days variable was tried in place of WTEMP, but with less satisfactory results. This is perhaps because of the uniform weighting of temperatures effects in the degree day variable; that is, one day at 20° below 65° is counted the same as 10 days at 2° below 65°, but the effects upon heating demand may be different.

Table 5

IDENTIFICATION OF VARIABLES

Variable[a]	Definition	Units of Measurement
S_iH	heating, type i[b]	fraction of total installations[c]
S_iWH	water heating, type i	
S_iC	cooking, type i	
S_iCW	clothes washing, type i	
S_iCD	clothes drying, type i	
S_iAC	air conditioning, type i	
SFF	food freezers	
SDW	dishwashers	
S_iTV	air conditioning, type i	
EPC	annual electricity consumption per customer	kWh/customer-year
EPH	annual electricity consumption per household	kWh/household-year
GPC	annual utility gas consumption per customer	therms/customer-year
GPH	annual utility gas consumption per household	therms/household-year
PGAS	price of gas[d]	$/40 therms
POIL	price of oil	mills/gallon
PCOAL	price of coal	$/short-ton
PELEC	price of electricity	$/1000 kWh
PBGAS	price of bottled gas	mills/gallon
PKER	price of kerosene	mills/gallon
PGR	price of gas range	$/unit
PER	price of electric range	$/unit
PWCW	price of wringer clothes washer	$/unit
PACW	price of automatic clothes washer	$/unit
PRAC	price of room air conditioner	$/unit
PEFF	price of electric food freezer	$/unit
PTV	price of television	$/unit
YPH	annual income per household	$/household-year
HS	household size	persons/household
SHU	single detached housing units	fraction of total housing
NUHU	nonurban housing units	fraction of total housing
WTEMP	mean December temperature	°F
STEMP	mean July temperature	°F
CL	cost-of-living	index number

[a]Where necessary, variables pertaining to 1960 are identified by a trailing "6"--income per household in 1960 is YPH6. Variables that represent averages of 1960 and 1970 variables are identified by a leading A--average income per household during the period 1960-70 is AYPH.

[b]Heating types include gas, oil, coal, electricity, bottled gas, wood, other, and none. Water heating types are gas, oil, electricity, bottled gas, other, and none. Cooking types consist of gas, electricity, bottled gas, other, and none. Washer types are wringer, automatic, and none. Dryer types include gas, electric, and none. Air conditioning types are room, central, and none. Television "types" consist of one, more than one, and none. The following symbols are used to designate unit types: G = utility gas, O = oil, C = coal (heating and water heating), E = electricity, BG = bottled gas, W = wood, OTH = other fuels, N = none, W = wringer, A = automatic, R = room, C = central (air conditioning), M = multiple installations (air conditioning and television), S = single installation (television).

[c]See Eq. (9) and the accompanying discussion for further clarification.

[d]All prices and income per household are divided by (CL/100) or (CLG/100) to normalize for cross-state variations in the price level.

IV. STOCK EQUATIONS

Space heating is undoubtedly the single largest component of residential energy demand. It seems appropriate, therefore, to begin the presentation of stock equation estimates with those for space heating. For this household function, both static and dynamic specifications are tried. In the static case, only data pertinent to 1970 are used. The retention ratio, ρ_i, is set equal to zero. In the dynamic case, the explanatory variables are averages of the corresponding 1960 and 1970 variables. Weighted geometric averages are used for the price and income variables as well as for HS.[1] Simple arithmetic averages are used for SHU and NUHU. No averaging is required for WTEMP. The retention ratio is set equal to 0.75 for all types of installations including the "none" category.[2]

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[1] When 1963 data are used in place of 1960 data, they are given a weight of 2/3 (1/3 for the 1970 data) owing to their greater proximity to the midpoint of the period. (See Appendix A.) APELEC is not an average (since data are not available for the early 1960s), but a 1965 figure deflated by the geometric mean of CL and CL6, ACL. APCOAL undeflated is the same as PCOAL. But the latter is deflated by CL and the former by ACL.

[2] It might seem curious at first to think of a retention ratio for the "none" category, but the idea is not so bizarre if one considers that many housing units without heat might be structurally unsuited for central heating. Retention is no doubt closely tied to housing age and structure, for all types of heating installations. Alternatively, the retention ratio in the "none" case can be interpreted as an estimate of the fraction of households in that category who did not consider purchasing a heating unit during the 1960-70 period, and who were thus "out of the market." The retention ratio of 0.75 corresponds to the fraction of installations replaced every 10 years out of stock of constant size and uniform age distribution if each unit lasts 40 years. After the stock equations had been estimated conditional upon the 0.75 retention ratio, additional information came to light that permits one to calculate national average heating retention ratios for several energy types. Data from [18] and [20] yield the following estimates: 0.588 (oil), 0.629 (coal), 0.578 (gas), 0.634 (electricity). These are somewhat below the 0.75 assumed in the dynamic formulation. It was decided, however, not to recompute the estimates, since the results below show that they are quite insensitive to variations in the retention ratio between 0 and 0.75. Recomputation would have yielded estimates more similar to those of the static formulation. The same is

The form of the equations actually estimated is as follows:

$$\ell n \left(\frac{S_i H}{S_j H}\right) = a^{\circ}_{ij} + a_i \cdot \ell n P_i + a_j \cdot \ell n P_j + a^{1}_{ij} \cdot \ell n YPH + a^{2}_{ij} \cdot \ell n HS$$

$$+ a^{3}_{ij} \cdot SHU + a^{4}_{ij} \cdot NUHU + a^{5}_{ij} \cdot WTEMP + u_{ij} ,$$

$$i=1,\ldots,8, \text{ except } i \neq j . \qquad (24)$$

For a given j, seven equations are estimated simultaneously by generalized least squares, subject to the constraint that the estimates of a_j be the same in all seven. Where i corresponds to the wood, other fuels, or none categories, no corresponding P_i variable appears on the right hand side of the estimated equation.

The signs expected for the coefficients of Eqs. (24) can be easily determined in some cases but not in others. From classical demand theory one must expect the a_i to be negative and the a_j to the positive. The coefficient of ℓnYPH should be positive or negative as heating type i is *relatively* superior or inferior (in a consumption, not a technological, sense) to heating type j, for a given household size. The coefficient of household size should be positive or negative as heating type i is *relatively* inferior or superior to heating type j, for a given household income. The coefficient of SHU should be positive or negative as heating type i tends to be chosen more frequently than type j for single unit dwellings. Choices would be related to such factors as relative economies of scale for single heating plants in multiple-unit dwellings and differences in the structural and spatial characteristics of multiple- and single-unit dwellings. The variable NUHU is included as a proxy measure of the degree of availability of utility gas in a given state, since utility gas service tends to be

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true for the water-heating case. As indicated in the text below, a value of 0.5 is assumed for the water-heating retention ratio. Calculations based upon data from the Census of Housing and [21] yield the following estimates: 0.399 (utility and bottled gas), 0.456 (electricity). Recomputation of the econometric estimates was also judged to be unnecessary.

restricted to urban areas. For equations in which the utility gas installation variable is in the numerator of (S_iH/S_jH), the coefficient of NUHU should be negative. The reverse should hold true when it is in the denominator. When the gas installation variable is neither in the numerator nor in the denominator, the expected sign of the coefficient is indeterminable, depending upon the tastes of customers who would have chosen utility gas had it been available. The sign of WTEMP should be positive or negative as type i installations have higher or lower operating costs than type j installations, since a higher WTEMP implies a lower rate of operation, on the average, and thus improves the competitive position of high-operating-cost installations.

Table 6 displays the results for the static case and Table 7 those for the dynamic. Only five of the possible eight S_jH are used as denominators. These five correspond to the S_iH for which P_j are available. The equations for wood, other fuels, and none are not shown, owing to their lesser importance.[1] In both tables the equations are presented in pairwise fashion. Unconstrained OLS estimates for each pair would be identical (except for a reversal of signs). The observed divergence reflects the effect of the constrained simultaneous estimation procedure. Owing to the non-uniqueness of the estimates, the coefficients directly estimated by regressing $\ell n(S_iH/S_jH)$ on the explanatory variables are not necessarily the same as those derived from the regressions of $\ell n(S_iH/S_kH)$ and $\ell n(S_jH/S_kH)$, or their inverses, on the explanatory variables.

Table 6 shows that the coefficients for energy prices are generally quite significant statistically, with the exception of those for POIL, which are nevertheless of proper sign and plausible magnitude.[2] The coefficients of YPH are sizable in some cases but are all insignificantly different from zero at probability levels higher than 0.2.

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[1] Those for the "none" case are shown in Appendix C.

[2] At 40 degrees of freedom the relationship between statistical significance and the t-ratio is as follows:

T-ratio	1.69	2.03	2.71	3.56	4.33	5.06	5.77
Significance	0.10	0.05	10^{-2}	10^{-3}	10^{-4}	10^{-5}	10^{-6}

Table 6

RESIDENTIAL HEATING INSTALLATIONS,
CONSTRAINED ESTIMATES, 50 STATES, 1970

Dependent Variable	Explanatory Variables[a]											Unconstrained R^2	Degrees of Freedom
	Constant	PGAS	POIL	PCOAL	PELEC	PBGAS	YPH	HS	SHU	NUHU	WTEMP		
SOH/SGH	-10.33	4.01	-1.31				1.80	-3.38	-8.92	6.56	-0.0207		
	(-0.39)	(4.47)	(-0.89)				(0.68)	(-0.90)	(-4.03)	(3.31)	(-1.37)		
SGH/SOH	-0.90	-6.95	2.13				-0.38	4.01	5.55	-4.40	0.0261	.89	42
	(-0.05)	(-13.15)	(1.77)				(-0.19)	(1.42)	(3.51)	(-3.03)	(2.32)		
SCH/SGH	-15.57	4.01		-3.22			0.04	10.11	-4.23	7.21	-0.0136		
	(-0.52)	(3.63)		(-4.59)			(0.01)	(2.35)	(-1.61)	(3.20)	(-0.78)		
SGH/SCH	14.40	-4.70		3.30			0.02	-10.13	3.50	-6.69	0.0149	.65	42
	(0.49)	(-11.46)		(4.51)			(0.01)	(-2.37)	(1.46)	(-3.13)	(0.85)		
SEH/SGH	9.78	4.01			-2.65		0.98	-1.92	-1.04	3.49	0.0560		
	(0.06)	(7.14)			(-7.29)		(0.59)	(-0.85)	(-0.75)	(2.96)	(6.29)		
SGH/SEH	2.08	-3.59			2.21		-1.08	2.11	1.43	-3.72	-0.0574	.77	42
	(0.13)	(-9.88)			(3.42)		(-0.65)	(0.92)	(1.09)	(-3.25)	(-6.42)		
SBGH/SGH	1.27	4.01				-1.71	-0.33	-3.48	4.53	2.86	0.0238		
	(0.06)	(5.40)				(-2.32)	(-0.16)	(-1.23)	(2.61)	(1.92)	(2.07)		
SGH/SBGH	-3.61	-2.73				2.23	-0.06	3.70	-2.65	-4.35	-0.0303	.53	42
	(-0.18)	(-6.87)				(2.71)	(-0.03)	(1.41)	(-1.67)	(-3.15)	(-2.75)		
SCH/SOH	-10.70		2.13	-3.39			-1.65	14.12	4.52	0.44	0.0057		
	(-0.31)		(1.02)	(-4.99)			(-0.48)	(2.87)	(1.75)	(0.18)	(0.28)		
SOH/SCH	15.10		-2.95	3.30			1.70	-14.67	-4.51	-0.01	-0.0024	.51	42
	(0.46)		(-2.20)	(4.18)			(0.50)	(-3.09)	(-1.75)	(-0.01)	(-0.12)		
SEH/SOH	1.98		2.13		-1.71		-1.03	1.61	7.79	-3.70	0.0745		
	(0.07)		(1.16)		(-3.97)		(-0.34)	(0.37)	(3.55)	(-1.75)	(4.35)		
SOH/SEH	-6.10		-1.84		2.21		0.91	-1.63	-7.91	3.43	-0.0752	.66	42
	(-0.20)		(-1.22)		(1.88)		(0.30)	(-0.38)	(-3.48)	(1.62)	(-4.44)		
SBGH/SOH	31.19		2.13			-4.18	-3.16	-0.72	11.24	-2.14	0.0557		
	(0.94)		(1.02)			(-3.74)	(-0.97)	(-0.15)	(4.29)	(-0.91)	(2.95)		
SOH/SBGH	-9.37		-2.59			2.23	2.34	-0.54	-12.90	3.85	-0.0433	.76	42
	(-0.25)		(-1.54)			(1.43)	(0.66)	(-0.11)	(-4.32)	(1.48)	(-2.08)		
SEH/SCH	9.04			3.30	-1.05		0.53	-12.79	3.51	-4.26	0.0697		
	(0.28)			(4.21)	(-1.95)		(0.16)	(-2.77)	(1.38)	(-1.86)	(3.70)		
SCH/SEH	-14.27			-3.19	2.21		-0.86	12.22	-3.24	3.82	-0.0701	.63	42
	(-0.42)			(-4.82)	(1.63)		(-0.25)	(2.52)	(-1.22)	(1.60)	(-3.63)		
SBGH/SCH	34.91			3.30		-3.51	-1.17	-14.49	7.25	-3.03	0.0468		
	(0.96)			(3.82)		(-3.62)	(-0.32)	(-2.85)	(2.50)	(-1.17)	(2.21)		
SCH/SBGH	-21.35			-3.07		2.23	0.49	13.79	-8.21	3.82	-0.0415	.60	42
	(-0.54)			(-4.74)		(1.37)	(0.13)	(2.59)	(-2.57)	(1.35)	(-1.83)		
SEH/SBGH	-2.71				-3.20	2.23	1.65	2.06	-5.20	0.39	0.0290		
	(-0.16)				(-8.34)	(3.17)	(1.00)	(0.90)	(-3.77)	(0.32)	(3.01)		
SBGH/SEH	0.67				2.21	-1.53	-1.14	-1.27	5.66	-0.65	-0.0334	.75	42
	(0.03)				(2.98)	(-2.12)	(-0.61)	(-0.49)	(3.69)	(-0.47)	(-3.15)		

[a]Figures in parentheses are t-ratios.

Table 7

RESIDENTIAL HEATING INSTALLATIONS,
CONSTRAINED ESTIMATES, 50 STATES, 1960-70

Dependent Variable[a]	Explanatory Variables[b]											Unconstrained R^2	Degrees of Freedom
	Constant	APGAS	APOIL	APCOAL	APELEC	APBGAS	AYPH	AHS	ASHU	ANUHU	WTEMP		
SOH/SGH	9.82	3.54	-2.72				0.39	-3.46	-8.22	7.46	-0.0140	.87	42
	(0.40)	(4,17)	(-1.60)				(0.16)	(-0.92)	(-3.45)	(3.13)	(-0.81)		
SGH/SOH	-19.09	-5.75	3.61				0.68	3.84	4.86	-5.31	0.0207		
	(-0.98)	(-11.17)	(2.50)				(0.36)	(1.30)	(2.72)	(-2.93)	(1.54)		
SCH/SGH	5.72	3.54		-2.97			-2.24	10.32	-3.77	4.99	-0.0301	.65	42
	(0.22)	(3.58)		(-4.40)			(-0.82)	(2.59)	(-1.46)	(2.00)	(-1.64)		
SGH/SCH	-6.69	-3.97		2.99			2.46	-10.34	3.18	-4.51	0.0319		
	(-0.26)	(-9.93)		(4.34)			(0.92)	(-2.61)	(1.36)	(-1.92)	(1.75)		
SEH/SGH	2.39	3.54			-1.41		0.17	-3.79	-0.16	2.72	0.0522	.75	42
	(0.18)	(7.38)			(-4.13)		(0.12)	(-1.85)	(-0.12)	(2.12)	(5.69)		
SGH/SEH	-0.78	-3.16			1.29		-0.03	3.85	0.32	-3.13	-0.0540		
	(-0.06)	(-9.27)			(2.54)		(-0.25)	(1.90)	(0.26)	(-2.58)	(-6.01)		
SBGH/SGH	8.21	3.54				-2.08	-0.88	-3.11	4.21	4.48	0.0260	.62	42
	(0.43)	(4.77)				(-3.00)	(-0.45)	(-1.11)	(2.28)	(2.52)	(1.99)		
SGH/SBGH	-9.85	-2.68				2.55	0.51	3.39	-2.49	-5.91	-0.0329		
	(-0.54)	(-6.79)				(3.61)	(0.28)	(1.25)	(-1.45)	(-3.54)	(-2.60)		
SCH/SOH	-8.43		3.61	-2.97			-2.69	14.28	4.14	-2.83	-0.0189	.50	42
	(-0.27)		(1.55)	(-4.59)			(-0.87)	(3.01)	(1.53)	(-1.01)	(-0.86)		
SOH/SCH	10.09		-3.95	2.99			2.70	-14.49	-4.02	2.96	0.0199		
	(0.34)		(-2.54)	(3.82)			(0.88)	(-3.13)	(-1.51)	(1.08)	(0.93)		
SEH/SOH	-14.26		3.61		-1.03		-0.32	0.04	8.20	-5.19	0.0639	.65	42
	(-0.50)		(1.73)		(-2.73)		(-0.11)	(0.01)	(3.40)	(-2.11)	(3.34)		
SOH/SEH	12.01		-3.50		1.29		0.29	-0.08	-8.15	5.09	-0.0635		
	(0.42)		(-1.98)		(1.25)		(0.10)	(-0.02)	(-3.40)	(2.08)	(-3.30)		
SBGH/SOH	17.19		3.61			-5.12	-1.80	-0.60	8.62	-0.35	0.0580	.82	42
	(0.64)		(1.78)			(-6.59)	(-0.68)	(-0.15)	(3.43)	(-0.14)	(3.12)		
SOH/SBGH	7.43		-4.63			2.55	1.47	-1.21	-11.23	3.30	-0.0370		
	(0.24)		(-2.47)			(2.20)	(0.48)	(-0.25)	(-3.72)	(1.16)	(-1.70)		
SEH/SCH	-8.67			2.99	-0.60		2.32	-14.40	4.21	-2.46	0.0833	.66	42
	(-0.32)			(4.10)	(-1.17)		(0.81)	(-3.39)	(1.72)	(-0.99)	(4.28)		
SCH/SEH	4.11			-2.93	1.29		-2.41	14.15	-3.94	2.26	-0.0828		
	(0.14)			(-4.66)	(1.15)		(-0.80)	(3.15)	(-1.53)	(0.87)	(-4.09)		
SBGH/SCH	11.09			2.99		-3.21	1.19	-13.98	6.63	0.59	0.0637	.57	42
	(0.36)			(3.63)		(-4.69)	(0.37)	(-2.93)	(2.35)	(0.21)	(2.87)		
SCH/SBGH	-5.99			-2.68		2.55	-1.37	13.67	-7.21	-0.20	-0.0616		
	(-0.19)			(-4.30)		(2.10)	(-0.42)	(2.81)	(-2.36)	(-0.07)	(-2.67)		
SEH/SBGH	-7.59				-1.68	2.55	1.14	-0.35	-3.60	-2.16	0.0227	.69	42
	(-0.45)				(-4.91)	(4.10)	(0.69)	(-0.14)	(-2.30)	(-1.42)	(1.96)		
SBGH/SEH	6.85				1.29	-2.11	-1.04	0.69	3.97	1.82	-0.0261		
	(0.38)				(1.92)	(-3.40)	(-0.60)	(0.27)	(2.40)	(1.14)	(-2.19)		

[a] $\rho = 0.75$.

[b] Figures in parentheses are t-ratios.

They nevertheless do provide weak evidence of a preference ordering. Pairwise comparisons of the coefficients imply that high income levels promote the use of electricity relative to any fossil fuel and that oil dominates the other three fossil fuels. Coal dominates both utility and bottled gas, which cannot be ranked relative to one another. The failure of (1) utility gas to dominate oil and (2) coal to be dominated by all the others is surprising.[1] Both anomalies may have resulted from errors inherent in the static approach, since the results for the dynamic case put coal at the bottom and do not permit a ranking of utility gas, oil, and electricity.

The coefficients of HS are statistically significant at the .05 level only in the equations in which SCH appears. Moreover, only in these equations do the coefficients take on relatively large absolute values (ranging from 10.11 to 14.67). This pattern presumably reflects a historical tendency for coal space-heating units to have been retained relatively more heavily in states with large average household sizes. It is not clear whether this tendency is merely a statistical artifact or reflects a functional relationship. The latter interpretation is certainly possible: If coal heating can be classified as an inferior good, given the alternatives available in the market, then a larger household size, holding household income constant, implies a lower household income per capita and increases household demand for the inferior good. As with income, the relationship of fuel choice to household size can be illuminated by examining the ranking of choices implied by the coefficients. In this case, however, the rankings reflect not only the ordering of preferences (as indicated by the income coefficients) but also the effect of household size upon the average size or operating level of installations and hence upon the importance of variable costs relative to fixed costs in the evaluation of alternative heating systems. The coefficients of Table 6 imply the following ordering:

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[1] Utility gas would seem to have the edge on oil because of its cleanliness and because it does not need a storage tank. On the other hand, it is possible that oil units can be retrofitted into older coal-burning systems at less cost. The indicated "preference" for oil might thus be due to excluded purchase and installation price variables.

1. bottled gas/oil
2. electricity
3. utility gas
4. coal.

The relative positions of bottled gas and oil are ambiguous. Small households are more likely to choose an energy source near the top of the list, other things equal, than are large households.

Without exception, the equations in which the coefficients of SHU are significant at the .05 level or better are those containing either SOH or SBGH. In all cases, the results show that (1) oil is preferred most often (other things equal) in areas having high density housing, perhaps reflecting desirable characteristics as a boiler fuel for central heating systems in apartment buildings; and (2) bottled gas is preferred most often in areas of low density housing. As expected, the only equations in which the coefficients of NUHU are statistically significant are those where SGH appears on the left hand side of the equation. In every case, these estimates show that urbanity has a positive effect upon the level of gas heating installations. Looking finally at the coefficients of WTEMP, one sees that with the exception of the gas-coal and oil-coal choices, WTEMP has a generally significant effect upon heating fuel choices. Moreover, the estimated coefficients suggest the following temperature-related ranking of heating-system choices: electricity, bottled gas, utility gas, coal, oil. Energy types at the beginning of the list are most likely to be found in warm climates, those at the end in cold climates, other things equal.

The estimates obtained with the dynamic specification (Table 7) are generally quite similar to those obtained with the static. With respect to energy prices, the most notable changes are (1) an increase in the absolute size and statistical significance of the coefficients for the price of oil and (2) a decrease in these factors for the coefficients of the price of electricity. The income coefficients continue to be statistically insignificant but, as noted above, imply a slightly different preference ordering. Household size exhibits the same

relationship with respect to coal heating as in the static case. The ranking of energy choices with respect to household size differs a bit, in that the position of bottled gas is not clearly determined. If the ambiguous relationship between oil and bottled gas is ignored, then the latter falls between electricity and utility gas on the list presented for the static case; the ordering is otherwise unchanged. The results obtained for the single-detached-housing and urbanity variables are similar, and the results for WTEMP imply the same ranking as in the static case, except that the positions of oil and coal are reversed. The lack of marked contrasts between the results of the dynamic and static specifications suggests that the latter may be as good an approximation of reality as the former.

Water heating is probably the second most important source of household energy demand. The approach taken is identical to that for space heating, except that (1) PCOAL is used as a proxy for the price of "other" water heating fuels since the bulk of the "other" category consists of coal-fired units; and (2) in the dynamic case ρ_i is assumed to be equal to 0.5 for all installations, because water heaters have shorter average lifetimes than space heating systems. Eqs. (24) thus continue to apply. However, the motivation for including WTEMP is somewhat different than in the case of heating. It seems plausible to think that the same fuel may be used for both space and water heating because of cost interrelationships.[1] To the extent that climate affects the choice of space heating unit it will also influence the choice of water heating unit. Tables 8 and 9 present the results for the static and dynamic specifications.

In the static case, the energy-price coefficients are highly significant statistically, except those for POIL, which are generally significant at the .05-.10 levels. None of the coefficients of YPH or HS is significant at the .05 level and only one at the .10 level,

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[1] Two factors may be noted: (1) Installation of a service inlet or tank for heating provides a ready source of fuel or electricity for water heating as well. No additional major hook-up charges are likely to be levied. (2) Use of a given source of energy for heating usually ensures that the user will be able to obtain additional energy for water heating at a relatively low marginal price.

Table 8

RESIDENTIAL WATER HEATING INSTALLATIONS,
CONSTRAINED ESTIMATES, 50 STATES, 1970

Dependent Variable	Explanatory Variables[a]										Unconstrained R^2	Degrees of Freedom
	Constant	PGAS	POIL	PELEC	PBGAS	YPH	HS	SHU	NUHU	WTEMP		
SOWH/SGWH	13.44	3.99	-2.47			0.34	-4.94	-15.23	6.77	-0.0184	0.91	42
	(0.50)	(4.30)	(-1.74)			(0.12)	(-1.27)	(-6.66)	(3.32)	(-1.19)		
SGWH/SOWH	-19.87	-6.45	2.65			0.84	5.11	12.45	-4.74	0.0247		
	(-0.94)	(-11.05)	(2.06)			(0.40)	(1.69)	(7.33)	(-3.05)	(2.04)		
SEWH/SGWH	8.67	3.99		-2.80		0.26	-0.32	0.40	4.34	0.0143	0.83	42
	(0.63)	(8.17)		(-6.34)		(0.18)	(-0.16)	(0.33)	(4.21)	(1.84)		
SGWH/SEWH	-8.44	-4.32		2.60		-0.06	0.42	-0.80	-4.00	-0.0134		
	(-0.61)	(-11.07)		(4.68)		(-0.04)	(0.21)	(-0.70)	(-3.98)	(-1.75)		
SBGWH/SGWH	12.05	3.99			-2.07	-1.02	-3.10	0.98	3.25	0.0138	0.53	42
	(0.67)	(6.23)			(-3.04)	(-0.57)	(-1.29)	(0.66)	(2.55)	(1.40)		
SGWH/SBGWH	-17.45	-3.35			2.76	1.01	3.42	0.33	-4.33	-0.0194		
	(-0.98)	(-8.14)			(3.66)	(0.58)	(1.44)	(0.23)	(-3.45)	(-1.95)		
SEWH/SOWH	-8.22		2.65	-2.30		-0.19	4.51	15.70	-2.65	0.0324	0.81	42
	(-0.33)		(1.78)	(-4.59)		(-0.08)	(1.29)	(8.67)	(-1.55)	(2.35)		
SOWH/SEWH	7.26		-2.74	2.60		0.12	-4.71	-15.64	2.60	-0.0319		
	(-0.29)		(-2.03)	(2.70)		(0.05)	(-1.33)	(-8.42)	(1.49)	(-2.29)		
SBGWH/SOWH	19.58		2.65		-4.30	-2.30	0.88	14.25	-1.86	0.0442	0.83	42
	(0.68)		(1.47)		(-4.45)	(-0.82)	(0.22)	(6.29)	(-0.92)	(2.71)		
SOWH/SBGWH	-11.76		-3.20		2.76	1.65	-2.01	-15.55	3.30	-0.0337		
	(-0.04)		(-1.97)		(2.00)	(0.53)	(-0.46)	(-5.94)	(1.45)	(1.85)		
SBGWH/SEWH	9.06			2.60	-2.55	-1.44	-2.93	0.13	-0.67	0.0021	0.67	42
	(0.50)			(3.76)	(3.76)	(-0.83)	(-1.22)	(0.09)	(-0.52)	(0.21)		
SEWH/SBGW	-8.42			-3.16	2.76	1.66	3.28	-0.04	0.66	-0.0036		
	(-0.49)			(-6.43)	(3.96)	(1.02)	(1.46)	(-0.03)	(0.55)	(-0.38)		

[a] Figures in parentheses are t-ratios.

Table 9

RESIDENTIAL WATER HEATING INSTALLATIONS,
CONSTRAINED ESTIMATES, 50 STATES, 1960-70

Dependent Variable	Explanatory Variables[a]										Unconstrained R^2	Degrees of Freedom
	Constant	APGAS	APOIL	APELEC	APBGAS	AYPH	AHS	ASHU	ANUHU	WTEMP		
SOWH/SGWH	27.48	3.98	-4.29			-0.19	-5.72	-13.39	7.07	-0.0156	.91	42
	(1.16)	(4.81)	(-3.02)			(-0.08)	(-1.58)	(-5.79)	(3.08)	(-0.94)		
SGWH/SOWH	-34.87	-6.18	4.80			1.29	5.88	10.17	-4.77	0.0235		
	(-1.86)	(-12.75)	(3.44)			(0.69)	(2.07)	(5.91)	(-2.74)	(1.81)		
SEWH/SGWH	7.70	3.98		-1.86		-0.22	-1.91	0.69	3.95	0.0186	.89	42
	(0.75)	(10.79)		(-6.02)		(-0.21)	(-1.22)	(0.67)	(4.01)	(2.63)		
SGWH/SEWH	-8.19	-4.29		1.83		0.39	1.91	-1.12	-3.59	-0.0173		
	(-0.81)	(-14.22)		(4.73)		(0.37)	(1.24)	(-1.16)	(-3.82)	(-2.52)		
SBGWH/SGWH	11.56	3.98			-2.15	-0.98	-3.08	1.08	3.35	0.0179	.53	42
	(0.71)	(6.24)			(-3.62)	(-0.59)	(-1.27)	(0.68)	(2.19)	(1.60)		
SGWH/SBGWH	-15.51	-3.45			2.83	0.81	3.44	0.42	-4.61	-0.0248		
	(-0.96)	(-10.00)			(4.52)	(0.50)	(1.42)	(0.28)	(-3.10)	(-2.21)		
SEWH/SOWH	-23.53		4.80	-1.66		-0.08	4.03	13.96	-3.37	0.0328	.84	42
	(-1.11)		(3.07)	(-4.67)		(-0.04)	(1.26)	(7.74)	(-1.83)	(2.30)		
SOWH/SEWH	21.43		-4.58	1.83		0.05	-3.96	-13.98	3.25	-0.0333		
	(0.99)		(-3.34)	(2.35)		(0.03)	(-1.24)	(-7.75)	(1.77)	(-2.35)		
SBGWH/SOWH	4.23		4.80		-5.13	-1.28	1.51	10.86	-1.00	0.0524	.87	42
	(0.18)		(2.72)		(-7.03)	(-0.56)	(0.43)	(4.92)	(-0.48)	(3.24)		
SOWH/SBGWH	15.06		-5.16		2.83	0.95	-2.81	-13.39	3.41	-0.0354		
	(0.55)		(-3.33)		(2.79)	(2.79)	(0.35)	(-0.69)	(1.37)	(-1.86)		
SBGWH/SEWH	8.44			1.83	-2.73	-0.84	-1.43	-0.27	-0.03	0.0033	.67	42
	(0.52)			(3.03)	(-4.96)	(-0.54)	(-0.61)	(-0.19)	(-0.02)	(0.30)		
SEWH/SBGWH	-7.43			-2.09	2.83	0.88	1.58	0.30	-0.01	-0.0043		
	(-0.49)			(-5.68)	(5.07)	(0.59)	(0.71)	(0.21)	(-0.01)	(-0.42)		

[a] Figures in parentheses are t-ratios.

although in some cases they are of notable absolute size. The income coefficients indicate no clear preference ordering except for bottled gas, which is dominated by the other three energy sources. The household size coefficients imply the following ordering of choices: oil, bottled gas, electricity, and utility gas, with small households tending to choose more toward the top of the list than large households. The coefficients of SHU are significant only in the cases where SOWH appears on the left hand side of the equation. As in the case of space heating, it appears that oil has special advantages in multiple-unit structures.[1] Also, as in the case of space heating, the coefficients of NUHU are significant only in the equations containing SGWH and indicate a strongly positive relationship between utility gas availability (as represented by urbanity) and installations using utility gas. WTEMP has a number of marginally significant coefficients. The pattern of climate-related preference rankings parallels that for heating. The apparent preference for electricity over bottled gas in warm climates is insignificant, however.

In the dynamic case, all energy price coefficients are significant at the .01 level or better, except one coefficient of APELEC. As in the case of heating, the coefficients for electricity prices are lower and those for oil prices larger than in the static formulation. All other estimates are similar for both cases.

Because of the lack of important differences between the results obtained with the static and dynamic specifications for space and water heating, the equations reported below for other types of equipment pertain to the static case only. The equations for cooking units are specified as in the heating case, with the following changes: WTEMP is dropped, and the prices of electric and gas ranges are entered in the same fashion as the corresponding energy prices. Table 10 displays the results. All energy price coefficients except the constrained PELEC coefficient are significant at the .02 level or better. Those for the price of gas ranges are significant at the .05 level in the utility-gas-electricity and bottled-gas-electricity equations. Those

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[1]The use of combined oil-fired space and water heating systems in apartment buildings is an example.

Table 10

RESIDENTIAL COOKING UNITS,
CONSTRAINED ESTIMATES,
50 STATES, 1970

Dependent Variable	Explanatory Variables[a]										Unconstrained R^2	Degrees of Freedom
	Constant	PGAS	PELEC	PBGAS	PGR	PER	YPH	HS	SHU	NUHU		
SEC/SGC	2.83	3.06	-1.34		2.78	-1.98	-0.54	0.89	1.76	1.83	.68	41
	(0.19)	(5.53)	(-3.25)		(2.36)	(-1.56)	(-0.34)	(0.43)	(1.33)	(1.89)		
SGC/SEC	-0.51	-3.07	1.06		-3.17	2.07	0.66	-0.83	-1.61	-1.73		
	(-0.03)	(-6.44)	(1.47)		(-2.17)	(0.94)	(0.41)	(-0.38)	(-1.15)	(-1.75)		
SBGC/SGC	6.52	3.06		-1.35	0.48		-1.07	-0.75	0.55	4.07	.73	42
	(0.49)	(6.34)		(-3.17)	(0.58)		(-0.85)	(-0.43)	(0.51)	(5.13)		
SGC/SBGC	-6.76	-3.07		1.40	-0.58		1.13	0.74	-0.47	-4.08		
	(-0.47)	(-6.37)		(2.45)	(-0.69)		(0.83)	(0.42)	(-0.43)	(-5.10)		
SBGC/SEC	6.00		1.06	-1.40	-2.49	2.07	-0.48	-1.55	-1.22	2.35	.64	41
	(0.43)		(1.81)	(-3.21)	(-2.17)	(1.20)	(-0.37)	(-0.89)	(-1.03)	(3.03)		
SEC/SBGC	-3.81		-1.42	1.40	2.37	-2.05	0.58	1.68	1.22	-2.23		
	(-0.28)		(-3.24)	(2.81)	(2.34)	(-1.56)	(0.48)	(1.04)	(1.19)	(-3.05)		

[a]Figures in parentheses are t-ratios.

for PER are not. All price coefficients have the proper signs. (The expected signs for the coefficients of PGR are indeterminable in the gas-bottled-gas equations.) None of the coefficients of YPH, HS, or SHU is significant at even the 0.1 level. The coefficients of NUHU are highly significant in the equations where SBGC is on the left hand side of the equation and suggest that the lack of availability of utility gas in rural areas has raised the demand for bottled gas for cooking in such areas. The coefficients of NUHU in the utility-gas-electricity equations weakly indicate a similar effect upon the demand for electri city for cooking.

The equations for clothes washing and drying installations are specified in nearly the same fashion as the preceding ones. In the clothes washer equations no gas prices appear and washer prices replace range prices. The coefficient of NUHU in the washer case cannot be presumed to reflect the availability of utility gas. Rather it is assumed to reflect the relative proximities of laundry and dry cleaning establishments (including self-service types). In the clothes dryer equations, both PGAS and PELEC are included as is the price of electric dryers. State data on the prices of gas dryers could not be found. WTEMP is included on the grounds that households in warmer climates are more likely to rely upon outdoor clothesline drying than those in colder climates. Table 11 presents the estimates for washing and drying installations.

In the case of washers, PELEC appears to have influenced the relative attractiveness of automatic and wringer washers, but its effect upon the decision whether to purchase a washer is not made clear by the results. The coefficients of the equipment purchase prices are generally significant statistically and possess the signs one would expect. The coefficients of YPH provide weak evidence of a positive income effect on the attractiveness of automatic washers. Household size plays no important role. Areas with high proportions of single detached housing units tend to have a higher saturation of washers of either type. The coefficients of NUHU suggest that urbanity reduces the demand for washers, other things equal.

It is obvious that energy prices have notably influenced the decision to purchase a gas, electric, or no dryer. Evidence for the

Table 11

RESIDENTIAL WASHING AND DRYING UNITS,
CONSTRAINED ESTIMATES,
50 STATES, 1970

Dependent Variable	Explanatory Variables[a]								Unconstrained R^2	Degrees of Freedom
	Constant	PELEC	PWCW	PACW	YPH	HS	SHU	NUHU		
SACW/SWCW	-18.47	-0.93	3.30	-1.25	1.96	0.20	-1.26	-2.02	.54	42
	(-1.52)	(-2.08)	(3.41)	(-2.04)	(1.69)	(0.13)	(-1.38)	(-2.97)		
SWCW/SACW	18.33	0.93	-3.29	1.24	-1.95	-0.19	1.26	2.02		
	(1.50)	(2.06)	(-3.03)	(0.97)	(-1.58)	(-0.12)	(1.24)	(-2.79)		
SNCW/SWCW	-14.83	-0.69	3.30		0.67	0.27	-2.57	-2.83	.52	43
	(-1.05)	(-1.33)	(3.04)		(0.51)	(0.15)	(-2.48)	(-3.64)		
SNCW/SACW	3.65	0.24		1.24	-1.29	0.07	-1.30	-0.81	.21	
	(0.60)	(0.97)		(2.06)	(-1.99)	(0.08)	(-2.37)	(-2.05)		
	Constant	PGAS	PELEC	PECD	YPH	HS	SHU	NUHU	WTEMP	
SECD/SGCD	19.01	2.17	-1.73	-0.04	-1.02	-0.46	0.09	3.39	0.0205	.64	41
	(1.17)	(3.68)	(-4.28)	(-0.04)	(-0.59)	(-0.19)	(0.06)	(2.71)	(2.16)		
SGCH/SECD	-18.81	-2.19	1.58	0.20	1.03	0.59	-0.20	-3.37	-0.0209		
	(-1.08)	(-3.45)	(2.23)	(0.09)	(0.57)	(0.23)	(-0.12)	(-2.43)	(-1.98)		
SNCD/SGCD	14.26	2.17			-1.80	-0.81	-1.23	3.18	0.0459	.56	43
	(0.86)	(3.63)			(-1.02)	(-0.33)	(-0.84)	(2.54)	(4.82)		
SNCD/SECD	-4.51		1.58	0.20	-0.78	-0.22	-1.42	-0.20	0.0250	.64	42
	(-0.54)		(4.59)	(0.21)	(-0.92)	(-0.17)	(-1.81)	(-0.35)	(5.14)		

[a] Figures in parentheses are t-ratios.

influence of PECD is nil, though the estimated coefficients do possess the proper signs. Omission of the price of gas dryers may have had something to do with this poor result. The coefficients of YPH (in the equations where SNCD appears on the left hand side) weakly suggest that there is a positive income elasticity of demand for clothes dryers. As in the case of washers, household size is of minimal importance. SHU appears not to exert much influence upon the relative attractiveness of gas and electricity, but perhaps has some positive effect upon the saturation of dryers, as would be expected. The availability of gas (represented by NUHU) has clearly affected both the choice of a gas rather than an electric dryer and that of a gas dryer or no dryer. As postulated, the coefficients of WTEMP (in the equations with SNCD on the left hand side) show a clear downward effect of climate warmth upon the decision to purchase a dryer. In addition, warm climates appear to bias choices in favor of electricity, as might be expected, given the preceding results on space and water heating.

The 1970 housing census data distinguish households having no air conditioning, a single room air conditioner, more than one room air conditioner, and central air conditioning. That these installation categories do not correspond to differences in the type of energy used has two notable effects upon the specification of the air conditioning equations:

(1) The same set of energy prices appears in each $g_i(\cdot)$ of Eqs. (13). In practice, this set can be reduced to the price of electricity, since alternative types of air conditioning play a negligible role. In 1970, for example, gas-fueled units accounted for little more than one percent of total air conditioning installations.[1] Since the same price appears in each $g_i(\cdot)$, it is not appropriate to constrain the coefficients of PELEC in any way.

(2) The equipment price that should apply to the $g_i(\cdot)$ corresponding to room units is different from that corresponding to central

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[1] According to data in the *Census of Housing* and [15], gas installations accounted for 1.1 percent of all U.S. air conditioning installations and 3.7 percent of all U.S. central air conditioning installations (assuming all gas units to be central).

installations. Since a representative price series for the latter could not be found, the price of room air conditioners was tried in all equations, on the assumption that it could serve as a proxy for the prices of larger units. The results obtained with this price variable were unsatisfactory in all cases, the relevant coefficients being insignificant even at the 0.3 level and having the wrong sign. The equations reported in Table 12 exclude this variable. The identical specification of the variables on the right hand side in these equations and the lack of constraints across equations permits one to estimate them individually by OLS, although they form a set of "seemingly unrelated" regressions.

Although gas air conditioning is an insignificant component of total air conditioning installations, NUHU is retained in the regression equation. In this case, it serves not as proxy for gas availability, but rather as an indicator of the effect of possible lifestyle differences--for example, greater reliance upon tree shade or screened porches in rural areas. STEMP replaces WTEMP for obvious reasons.

The results show air conditioning installations to be highly responsive to household income and household size but not to the price of electricity. The coefficients of PELEC have the proper sign, though, and do increase in absolute value and statistical significance as one proceeds from smaller to larger installations. SHU appears to have no systematic effect; nor does NUHU. STEMP, as expected, is shown to have had a substantial influence upon installations.

Since nearly all home food freezers are electric, the only energy price variable appearing in the food freezer equation is PELEC. An index of food freezer prices (PEFF) also appears. Despite the absence of utility-gas-using installations, NUHU is retained in the regression equation as an indicator of lifestyle. For example, greater home-to-market distances or the benefits of storing home-produced foodstuffs may promote the use of food freezers in rural areas. According to the 1970 housing census, 21.7 percent of urban households had food freezers, and 47.9 percent of rural households had them. Inclusion of WTEMP is based upon the hypothesis that cold winters raise the attractiveness of storing food because shopping trips are burdensome.

Table 12

AIR CONDITIONING, FOOD FREEZING, DISHWASHING AND TELEVISION, CONSTRAINED ESTIMATES, 50 STATES, 1970

Dependent Variable	Explanatory Variables[a]								Adjusted R^2	Degrees of Freedom
	Constant	PELEC	YPH	HS	SHU	NUHU	STEMP			
SRAC/SNAC	-37.91	-0.14	3.54	-6.18	1.74	-0.21	0.1317		.70	43
	(-3.07)	(-0.26)	(2.74)	(-3.59)	(1.68)	(-0.25)	(7.56)			
SMAC/SNAC	-50.64	-0.76	4.69	-6.28	-1.35	0.85	0.2262		.70	43
	(-2.81)	(-0.96)	(2.49)	(-2.50)	(-0.89)	(0.68)	(8.90)			
SCAC/SNAC	-50.93	-1.09	5.21	-6.83	3.57	-0.24	0.1740		.65	43
	(-2.70)	(-1.32)	(2.64)	(-2.60)	(2.25)	(-0.18)	(6.54)			
	Constant	PELEC	PEFF	YPH	HS	SHU	NUHU	WTEMP		
SFF/SNFF	-8.02	-0.80	1.19	0.33	1.79	2.86	0.35	-0.0208	.66	42
	(-1.03)	(-2.81)	(1.35)	(0.46)	(1.80)	(5.29)	(0.60)	(-3.34)		
SFF/SNFF	-0.04	-0.62	-1.14	0.70	1.19	2.61	1.71	—	.58	43
	(-0.01)	(-1.98)	(-1.87)	(0.89)	(1.09)	(4.38)	(3.70)			
	Constant	PELEC	YPH	HS	SHU	NUHU				
SDW/SNDW	-6.98	-0.91	1.42	-0.82	0.05	-0.70			.45	44
	(-1.13)	(-3.64)	(2.24)	(-0.95)	(0.10)	(-1.90)				
	Constant	$PELEC_2^4$	PTV_t	YPH	HS	NUHU				
SMTV/SSTV	-10.24	-0.07	-0.38	1.29	0.09	-1.20			.66	44
	(-2.23)	(-0.41)	(-1.30)	(3.02)	(0.14)	(-4.60)				
SNTV/SSTV	4.78	-0.52	-0.11	-0.64	2.62	0.22			.10	44
	(0.58)	(-1.63)	(-0.20)	(-0.84)	(2.38)	(0.47)				

[a] Figures in parentheses are t-ratios.

Table 12 shows two alternative estimates for the food-freezer equation, one that includes WTEMP and one that does not. The results in these two cases differ notably in several respects: When WTEMP appears, its coefficient is statistically significant and bears a negative sign, thus confirming the hypothesis just mentioned. But the coefficient for PEFF, though not very significant, is incorrectly signed. When WTEMP is dropped, the coefficient reverses sign and its statistical significance improves somewhat. Moreover, the coefficient of NUHU increases dramatically in magnitude and significance. Collinearity appears to be the source of the problem.[1]

The dishwasher equation differs in specification from the food-freezer equation only in that PGAS and PEFF are dropped. The estimated equation, displayed in Table 12, shows PELEC and YPH to have influenced dishwasher installations. Evidence for the effects of HS and SHU is lacking. That for NUHU is weak.

The data for television stocks are broken into three categories in the housing census: households with a single unit, those with two or more units, and those with none. The list of explanatory variables is the same as for the preceding equation, except that PTV, an index of television prices, is included. The results are presented in Table 12. Of the two equations shown, only the first yields much in the way of positive results. The dependent variable in this case is the ratio of multiple- to single-unit installations. The coefficient for PELEC is

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[1] Regression of each of the independent variables against the others yields the following interesting results (omitting all but the three variables of interest here):

	PEFF	NUHU	WTEMP	F(6,43)
PEFF	0.726	4.640	8.385	18.98
NUHU	0.578	0.730	-6.439	19.34
WTEMP	0.788	-0.701	0.730	19.36

The diagonal elements of the 3 x 3 matrix are the multiple correlation coefficients of the regressions. The corresponding F-statistics are given in the fourth column. Below the diagonal are the partial correlation coefficients, and above it are the associated t-ratios of the estimated regression coefficients. The results show that both PEFF and NUHU are closely related to WTEMP (statistically).

nearly zero and that for PTV is statistically insignificant, but both income and urbanity appear to raise the relative number of multiple-unit installations.

V. ENERGY CONSUMPTION EQUATIONS

Table 13 presents an additional list of variables pertinent to the estimates presented in this section. As noted in Section II, gas and electricity equations can be estimated separately by OLS with no loss of efficiency, though at the cost of omitting any constraint upon the cross-price coefficients. The unconstrained estimates are thus presented separately for gas and electricity. The constrained estimates follow. All explanatory variables appearing in the stock equations are included except appliance prices, for which a satisfactory 50-state index is not available. The form assumed for the estimated equations is as follows:

$$\begin{aligned} \ell nX = a_0 &+ a_1 \ell nPELEC + a_2 \ell nPGAS + a_3 \ell nPOIL + a_4 \ell nPCOAL \\ &+ a_5 \ell nPBGAS + a_6 \ell nYPH + a_7 \ell nHS + a_8 SHU + a_9 NUHU \\ &+ a_{10} WTEMP + a_{11} STEMP + u \ , \end{aligned} \tag{25}$$

where X is the (unspecified) dependent variable.

ELECTRICITY

In the static formulation, $\bar{\rho}_e$ is assumed equal to zero,[1] and the dependent variable is therefore either EPC or EPH. Since the number of electricity customers is close to the number of households in every state, one would expect the differences in the estimated coefficients to be small. Indeed, the results obtained for both 1960 and 1970 cross-sections show this to be the case. (See Table 14.) The results for the two time periods suggest the possibility of structural changes or of differences in the extent to which the two periods approximated long-run equilibrium positions. Noteworthy are the changes in magnitude

[1] See p. 15 for definition of $\bar{\rho}_e$.

Table 13

ADDITIONAL VARIABLE DEFINITIONS
AND UNITS OF MEASUREMENT

Variable	Definition	Units of Measurement
QE	Total annual residential electricity consumption	kWh/year
CE	Average number of residential electricity customers	customers
QG	Total annual residential utility gas consumption	therms/year
CG	Average number of utility gas customers	customers
NH	Number of households	households
EPC	QE/CE	kWh/customer-year
EPH	QE/NH	kWh/household-year
GPC	QG/CG	therms/customer-year
GPH	QG/NH	therms/household
DEPH[a]	$(QE_{70}-\bar{\rho}_e \cdot QG_{60})/NH_{70}$	kWh/household-year
DGPH[a]	$(QG_{70}-\bar{\rho}_g \cdot QG_{60})/NH_{70}$	therms/household-year
REPH[a]	$(QE_{70}-\bar{\rho}_e \cdot QE_{60})/(NH_{70}-\bar{\rho}_e \cdot NH_{60})$	kWh/household-year
RGPH[a]	$(QG_{70}-\bar{\rho}_g \cdot QG_{60})/(NH_{70}-\bar{\rho}_g \cdot NH_{60})$	therms/household-year
REG	QE/QG	kWh/therm
RPEPG	PELEC/PGAS	therm/kWh
GEPH	12.5 · GPH + EPH	equivalent kWh/household-year[b]
PEG	1.53 · PGAS + PELEC	$/1000 kWh-mo. + $/80 therms-mo.

[a]For definitions of $\bar{\rho}_e$ and $\bar{\rho}_g$, see p. 15. The subscripts 70 and 60 refer to the two census years 1970 and 1960.

[b]Based upon typical gas and electric appliance operating efficiencies.

Table 14

RESIDENTIAL ELECTRICITY CONSUMPTION
50 STATES, 1960, 1970 AND 1960-70

Dependent Variables	Explanatory Variables[a]												Adjusted R^2	Degrees of Freedom
	Constant	PELEC	PGAS	POIL	PCOAL	PBGAS	YPH	HS	SHU	NUHU	WTEMP	STEMP		
EPC	2.16	-1.07	0.33	0.73	-0.05	-0.09	1.06	0.04	0.69	0.42	0.0014	0.0001	.70	38
(1960)	(0.57)	(-6.70)	(2.27)	(2.52)	(-0.43)	(-0.62)	(3.05)	(0.08)	(1.71)	(1.03)	(0.39)	(0.02)		
EPH	-0.78	-0.99	0.36	0.87	0.05	0.15	1.01	0.07	1.03	-0.03	-0.0040	0.0057	.70	38
(1960)	(-0.21)	(-6.20)	(2.40)	(2.98)	(0.50)	(0.98)	(2.90)	(0.13)	(2.54)	(-0.07)		(0.70)		
EPC	10.48	-1.28	0.27	0.32	0.06	-0.27	0.67	-0.56	0.23	0.97	0.0054	0.0106	.63	38
(1970)	(2.24)	(-6.80)	(1.62)	(1.14)	(0.59)	(-1.21)	(1.52)	(-0.94)	(0.60)	(2.93)	(1.60)	(1.39)		
EPH	6.83	-1.12	0.30	0.27	0.12	0.00	0.80	-0.94	0.76	0.88	0.0030	0.0113	.66	38
(1970)	(1.47)	(-6.00)	(1.84)	(0.97)	(1.13)	(0.01)	(1.84)	(-1.60)	(2.03)	(2.68)	(0.38)	(1.49)		
DEPH	7.25	-0.99	0.35	0.01	0.26	0.06	0.76	-1.89	0.58	1.18	0.0047	0.0202	.70	38
($\bar{\rho}_e$ = 0.5)	(1.66)	(-6.17)	(2.08)	(0.05)	(2.44)	(0.32)	(1.87)	(-3.26)	(1.46)	(3.09)	(1.31)	(2.55)		
REPH	-3.90	-0.95	0.28	-0.05	0.25	0.10	0.51	-1.88	0.61	1.19	0.0019	0.0221	.76	38
($\bar{\rho}_e$ = 0.5)	(-1.05)	(-6.91)	(1.98)	(-0.19)	(2.79)	(0.63)	(1.48)	(-3.79)	(1.81)	(3.67)	(0.62)	(3.26)		
DEPH	8.60	-1.03	0.38	-0.15	0.32	0.03	0.73	-2.29	0.43	1.40	0.0067	0.0239	.65	38
($\bar{\rho}_e$ = 0.75)	(1.67)	(-5.39)	(1.94)	(-0.41)	(2.52)	(0.12)	(1.52)	(-3.33)	(0.92)	(3.09)	(1.58)	(2.54)		
REPH	0.42	-0.91	0.24	-0.25	0.29	0.10	0.18	-2.30	0.49	1.45	0.0004	0.0278	.75	38
($\bar{\rho}_e$ = 0.75)	(0.10)	(-6.20)	(1.58)	(-0.89)	(3.05)	(0.60)	(0.48)	(-4.34)	(1.33)	(4.17)	(0.12)	(3.82)		

[a]Figures in parentheses are t-ratios.

or statistical significance of the coefficients for PGAS, POIL, YPH, and NUHU. The signs of the coefficients generally agree with expectations, however. In the few instances where such is not the case, the coefficients involved are not significantly different from zero even at the 0.1 level.

The estimates obtained for alternative dynamic formulations are also shown in Table 14. Two values for $\bar{\rho}_e$ are tried, 0.5 and 0.75. The latter is perhaps a bit high for a ten-year interval but is included to provide a fuller test of the sensitivity of the estimates to the value chosen for $\bar{\rho}_e$. Two types of dependent variables are used, DEPH and REPH, corresponding to the two versions of the dynamic adjustment mechanism discussed in Section II.

The results suggest that there is no great sensitivity to the type of dependent variable or level of $\bar{\rho}_e$ chosen. Among the four examples shown in the table, some coefficients appear to have been affected a bit more by the former factor (PGAS, YPH, WTEMP) and others by the latter (HS, NUHU, STEMP). The only coefficients to display a sign contrary to expectations are those for POIL, and they are all statistically insignificant. Notable differences between the dynamic results and those for the corresponding 1970 static case (EPH) include substantial increases in the magnitude and statistical significance of the coefficients for HS and STEMP. Similar, less striking changes occur in the coefficients for PCOAL and NUHU.

GAS

In contrast to the electricity case, it makes a good deal of difference whether the number of customers or the number of households is used in the denominator of the dependent variable for gas. The first four equations of Table 15 illustrate this point for the static formulation. Most striking are the changes in the estimated coefficients of PGAS and WTEMP, because these two variables substantially influence the proportion of households that become gas customers. Likewise, the marked increase in the absolute size of the coefficient for NUHU probably reflects the effect of gas availability on the number of gas customers, though in this case the large standard errors of the coefficients weaken the evidence.

Table 15

RESIDENTIAL GAS CONSUMPTION
50 STATES, 1960, 1970, AND 1960-70

Dependent Variable	Explanatory Variables[a]												Adjusted R^2	Degrees of Freedom
	Constant	PELEC	PGAS	POIL	PCOAL	PBGAS	YPH	HS	SHU	NUHU	WTEMP	STEMP		
GPC	15.97	-0.98	-1.30	-0.41	-0.27	-0.37	0.49	1.47	0.55	0.70	-0.0137	-0.0049	.75	38
(1960)	(2.61)	(-3.79)	(-5.45)	(-0.87)	(-1.56)	(-1.54)	(0.88)	(1.68)	(0.83)	(1.07)	(-2.35)	(-0.37)		
GPH	13.34	-0.84	-2.92	-0.46	-0.08	-0.29	0.59	2.27	-0.43	-1.24	-0.0296	0.0407	.76	38
(1960)	(1.21)	(-1.80)	(-6.80)	(-0.54)	(-0.24)	(-0.67)	(0.58)	(1.44)	(-0.36)	(-1.05)	(-2.83)	(1.71)		
GPC	6.23	-0.62	-0.98	-0.30	-0.36	0.16	0.81	0.80	0.79	0.23	-0.0178	0.0034	.75	38
(1970)	(1.15)	(-2.84)	(-5.10)	(-0.52)	(-3.02)	(0.62)	(1.60)	(1.17)	(1.81)	(0.60)	(-4.58)	(0.38)		
GPH	-0.63	-0.67	-2.75	-0.25	-0.41	0.47	1.42	1.24	1.10	-1.70	-0.0372	0.0424	.71	38
(1970)	(-0.05)	(-1.32)	(-6.08)	(-0.32)	(-1.45)	(0.75)	(1.19)	(0.77)	(1.07)	(-1.89)	(-4.05)	(2.03)		
DGPH	5.37	-1.08	-2.55	-0.69	-0.21	0.21	1.53	0.57	0.55	-0.87	-0.0330	0.0413	.68	38
($\bar{\rho}_g$ = 0.5)	(0.44)	(-2.40)	(-5.46)	(-0.81)	(-0.71)	(0.40)	(1.35)	(0.35)	(0.50)	(-0.82)	(-3.31)	(1.86)		
RGPH	-3.47	-1.04	-2.62	-0.76	-0.22	0.25	1.29	0.58	0.59	-0.85	-0.0358	0.0431	.69	38
($\bar{\rho}_g$ = 0.5)	(-0.28)	(-2.30)	(-5.57)	(-0.88)	(-0.73)	(0.48)	(1.13)	(0.36)	(0.53)	(-0.80)	(-3.57)	(1.93)		
DGPH	3.56	-1.35	-2.66	-0.87	-0.22	0.25	1.98	0.19	0.56	-0.65	-0.0335	0.0455	.59	38
($\bar{\rho}_g$ = 0.75)	(0.24)	(-2.51)	(-4.42)	(-0.85)	(-0.61)	(0.40)	(1.46)	(0.10)	(0.42)	(-0.51)	(-2.82)	(1.72)		
RGPH	-2.32	-1.24	-2.61	-0.97	-0.24	0.33	1.43	0.17	0.62	-0.55	-0.0397	0.0494	.62	38
($\bar{\rho}_g$ = 0.75)	(-0.16)	(-2.32)	(-4.71)	(-0.96)	(-0.67)	(0.53)	(1.06)	(0.09)	(0.47)	(-0.47)	(-3.37)	(1.88)		

[a] Figures in parentheses are t-ratios.

The results for several dynamic formulations are shown--also in Table 15--where the household variable appears in the denominator. This case is of greater importance for the present study since it can be related directly to the results obtained for the stock equations. (See Section VI.) The four variants displayed in the table correspond to those used for electricity. The lack of contrasts among the equations indicates little sensitivity to the alternative formulations.

A troublesome aspect of both the dynamic and static results is the large number of negative signs obtained for the cross-price coefficients, particularly those for PELEC, which in most cases are significantly different from zero at better than the .05 level. It seems likely that these are erroneous estimates perhaps in part because of aggregation but even more because of specification errors inherent in the estimated equations. In particular, if the stock equations are good approximations of reality, then price elasticities are not constant as implied by Eq. (25) but vary according to the relative importance of each type of energy in residential use.

GAS AND ELECTRICITY

It is pointed out above that the cross-price coefficients in the gas and electricity consumption equations are theoretically interdependent. In the case of energy this interdependence is really more complicated than is indicated by the simple constraint discussed in Section II. Energy consumption is influenced by purchase, installation, and maintenance costs for equipment as well as by energy costs. The constraints upon the cross-price coefficients ought to be formulated for a larger set of price coefficients. Moreover, even a constraint restricted only to the two energy cross-price coefficients should, in principle, vary from observation to observation according to relative expenditure shares. But the practical requirements of estimation force one to base the constraint upon a single representative pair of expenditures shares--for example, sample mean values. Finally, since a different constraint applies to each individual customer, the validity of a constraint based upon aggregated data must be further questioned.

The first two equations of Table 16 are constrained estimates for the static formulation, with EPH and GPH appearing as the dependent variables. The most noteworthy contrasts between the constrained and unconstrained estimates are, of course, the changes in the cross-price coefficients. The shift in the sign of the coefficient for PELEC from negative to positive is gratifying, but the decline in statistical significance of the coefficient of PGAS somewhat diminishes the attractiveness of the constrained results.

The second pair of equations in Table 16 tests the hypothesis that the own- and cross-price effects are such that the relative amounts of gas and electricity consumed depend only upon the relative prices of gas and electricity, not upon their absolute levels. The first equation includes only the relative price variable, RPEPG; the second adds PGAS. If the relative amounts of utility gas and electricity consumed were unaffected by the absolute level of prices, the coefficient of PGAS in the second equation would be zero. In fact, it is significantly different from zero at the .01 level. Moreover, the change in variance explained between the two equations is significantly different from zero at the .005 level, suggesting that one ought to reject the relative-price-only hypothesis.

The last equation in the table aggregates gas and electricity into a single utility-energy commodity. A fixed weight reflecting the average efficiencies of gas and electric equipment is used (1 therm = 12.5 kWh).[1] The utility-energy price index, PEG, is the sum of bills for equivalent amounts of energy (1000 kWh/mo. and 80 therms/mo.). The results of this aggregate model appear to be consistent with the coefficients obtained in the 1970 regressions for EPH and GPH. Noteworthy are the improved levels of statistical significance for YPH and SHU, the perversely signed but significant coefficient for PCOAL, and the decline in significance of NUHU apparently because of offsetting effects. A similar offsetting phenomenon may be taking place with respect to the influence of HS. The large standard errors for HS in all equations do not allow for much certainty, however.

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[1] See Tables 17 and 18 below for the data used to compute this weight.

VI. OWN- AND CROSS-PRICE ELASTICITIES

The stock equations reported in Section IV can be used to provide indirect estimates of the magnitudes of the own- and cross-price elasticities of energy consumption. Average per household consumption of energy type i, q_i, may be expressed as follows:

$$q_i = \sum_{k=1}^{n} e_{ik} \cdot s_{ik} \tag{26}$$

where e_{ik} is the average amount of energy type i used for household function k and s_{ik} is the fraction of households using energy type i for function k. The s_{ik} are, of course, the dependent variables of the stock equations (static formulation). The e_{ik} reflect the effects of installation size, efficiency, and operating conditions upon average consumption per installation. Assuming the e_{ik} to be given constants, one has

$$\frac{\partial q_i}{\partial p_j} \frac{p_j}{q_i} = \sum_{k=1}^{n} \left(\frac{e_{ik} s_{ik}}{\sum_k e_{ik} s_{ik}} \cdot \frac{\partial s_{ik}}{\partial p_j} \cdot \frac{p_j}{s_{ik}} \right) . \tag{27}$$

That is to say, the elasticity of consumption of energy type i with respect to the price of energy type j is a weighted sum of the elasticities of installation of units using energy type i with respect to the price of energy type j. If estimates of the e_{ik} are available, Eq. (27) can be used to calculate elasticities of consumption at the sample mean values of the s_{ik}. These can then be compared with the elasticities estimated directly in the energy consumption equations.

Generally, one would expect the indirectly computed elasticities to be biased toward zero, since they assume the effects of price upon the e_{ik} to be zero. Such bias should, however, be quite small in the case of cross-price effects, since it is difficult to imagine why the

price of, say, gas or oil should measurably affect the size, features, or average operating levels of electrical equipment.[1] By contrast, one might expect the bias in the case of the own-price effect to be of notable magnitude, since some, if not all, the e_{ik} are likely to be affected by the price of the fuels actually used.

If the directly estimated elasticities are assumed to be unbiased estimators of the true sample mean elasticities, then the indirectly estimated own-price elasticities should be of lower absolute magnitude than the former. In addition, the directly and indirectly estimated cross elasticities should all be positive (assuming substitutes), with the indirect estimates being slightly smaller than the direct.

Tables 17-19 present the data used to calculate price elasticities by means of Eq. (27). The typical annual consumption figures listed for the major household functions are based mainly upon industry estimates and not upon census or other large-scale sampling results. The data for fuel oil and bottled gas usage levels are adapted from the figures for utility gas. Only the relative magnitudes of the typical consumption estimates are important from the standpoint of error in these two cases, since only they affect the shares of average household consumption accounted for by a given function. Data for coal are not shown. It is assumed that 100 percent of household consumption of coal is used for space heating.

Table 20 presents the indirectly estimated average (long-run) price elasticities for utility gas, electricity, fuel oil, bottled gas, and coal. Also shown are the directly estimated price elasticities for 1970 obtained with EPH and GPH as dependent variables (in both the constrained and unconstrained versions). The calculated own-price elasticities for gas and electricity are substantially lower in absolute value than their counterparts obtained from the energy consumption equations. If both estimates are "accurate," then the difference reflects the influence of price upon the average rate of consumption

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[1] The effects of price upon the installation levels of equipment not included in the calculations can be assumed to be quite small owing to low energy consumption per installation, to low saturation levels, or, as in the case of electric lighting and refrigeration, to the lack of competitive substitutes.

Table 17

ESTIMATED PATTERN OF GAS CONSUMPTION
BY HOUSEHOLD FUNCTION, 1970

Function	Typical Annual Consumption (therms/yr.)	Average U.S. Saturation	Contribution to Average Household Consumption (therms/yr.)	Share of Average Household Consumption
Heating	1000	0.552	552	0.711
Water heating	270	0.551	149	0.192
Cooking	95	0.492	47	0.061
Clothes drying	85	0.124	11	0.014
Other	--	--	17[a]	0.022
Total	--	--	776[b]	1.000

[a]Residual.

[b]Actual, not estimated.

Sources: [13], [14], and [15].

Table 18

ESTIMATED PATTERN OF ELECTRICITY CONSUMPTION ACCORDING TO HOUSEHOLD FUNCTION, 1970

Function	Typical Annual Consumption (kWh/yr.)	Average U.S. Saturation	Contribution to Average Household Consumption (kWh/yr.)	Share of Average Household Consumption
Refrigerator	1,200	1.000	1,200	0.170
Heating	14,500	0.077	1,117	0.158
Water heating	4,500	0.250	1,125	0.159
Television	400	1.242	497	0.070
Cooking	1,175	0.406	477	0.068
Room air conditioning	1,350	0.178	240	0.034
Multi-room air conditioning	2,700	0.072	194	0.027
Central air conditioning	4,000	0.107	428	0.061
Food freezing	1,400	0.282	395	0.056
Clothes drying	990	0.294	291	0.041
Dishwashing	360	0.189	68	0.010
Wringer clothes washing	75	0.112	8	0.001
Automatic clothes washing	100	0.559	56	0.008
Other[a]	--	--	962[b]	0.136
Total	--	--	7,058[c]	1.000

[a]Includes lighting, pumping, small appliances, etc.

[b]Residual.

[c]Actual, not estimated.

Sources: Edison Electric Institute, [18] and [19].

Table 19

ESTIMATED PATTERN OF FUEL OIL AND BOTTLED GAS CONSUMPTION BY HOUSEHOLD FUNCTION, 1970

Function	Typical Annual Consumption[a]	Average U.S. Saturation	Contribution to Average Household Consumption[a]	Share of Average Household Consumption
Fuel oil:				
Heating	856	0.260	223	0.86
Water heating	231	0.098	23	0.09
Other[b]	--	--	12	0.05
Total	--	--	258	1.00
Bottled gas:				
Heating	1,000	0.060	60	0.70
Water heating	270	0.050	14	0.16
Cooking	95	0.084	8	0.09
Other[b]	--	--	4	0.05
Total	--	--	86	1.00

[a]Units of measurement are gal/yr. for oil and therms/yr. for gas.

[b]Assumed to be 5 percent of average household consumption.

Sources: [14] and author's estimates.

Table 20

ESTIMATED MEAN PRICE ELASTICITIES
OF CONSUMPTION, 1970

Energy Types	Energy Prices PGAS	PELEC	POIL	PBGAS	PCOAL
Indirect Estimates[a]					
Utility gas	-1.73	0.28	0.43	0.13	0.07
Electricity	0.81	-0.84	0.10	0.05	0.02
Fuel oil	2.10	0.21	-1.58	0.13	0.08
Bottled gas	2.04	0.26	0.43	-2.04	0.01
Coal	2.21	0.17	0.55	0.13	-3.29
Direct Estimates					
Utility gas	-2.75	-0.67[b]	-0.25[b]	0.47[b]	-0.41[b]
Electricity	0.30[b]	-1.12	0.27[b]	0.00[b]	0.12[b]
Utility gas[c]	-2.68	0.20[b]	-0.26[b]	0.57[b]	-0.48[b]
Electricity[c]	0.11[b]	-1.19	0.20[b]	0.15[b]	0.11[b]

[a]The indirect estimates are based upon the following values taken from Tables 6, 8, and 10-12 for the price coefficients--the b_i of Eq. (12):

Function	Energy Type Utility Gas	Electricity	Bottled Gas	Oil	Coal
Heating	-4.01	-2.21	-2.23	-2.13	-3.39
Water heating	-3.99	-2.60	-2.76	-2.65	--
Cooking	-3.06	-1.06	-1.40	--	--
Clothes drying	-2.17	-1.58	--	--	--

The coefficients for food freezing, multiple room air conditioning, central air conditioning and dishwashing apply only to electricity. They are -0.80, -0.76, -1.09, and -0.90.

[b]Not significant at .05 level.

[c]Constrained estimates. See Table 16.

per installation. An alternative hypothesis, that price has no effect upon the rate of consumption per installation and that only one of the two coefficients is correctly estimated, can be rejected on the basis of evidence presented in Appendix C.

One interpretation of the cross-elasticity results is that the indirectly computed values are close to the true values, while the direct estimates have a large error component primarily because of inappropriate specification of the energy consumption equations. Energy equations require aggregation of demands across individual household energy-using functions and cannot be readily adapted to account for the dependency of price effects upon the varying extents of installation of competing types of equipment. Moreover, the rather large standard errors and negative signs of many of the directly estimated cross-elasticities do not easily lend themselves to other interpretation.

VII. CONCLUSIONS

The conclusions of this report are offered as conjectures rather than as matters of fact, since they rest upon a few interesting clues and not upon decisive proofs. With respect to estimation techniques: Aggregation across different household energy-using functions leads to good estimates of own-price elasticities and erroneous estimates of cross-price elasticities. Cross-price elasticities can be estimated fairly accurately with the indirect method described in Section VI, which disaggregates according to function. Interdependencies among energy demands are important and should be modeled explicitly. Energy demand equations should account for the effects of installation levels upon price elasticities.

As regards household energy-consumption behavior: The difference between the directly and indirectly estimated own-price elasticities of demand is a measure of the responsiveness of "conservation" to price. According to the estimates shown in Table 20 this conservation effect is about 30-35 percent of the total mean elasticity of demand. The remaining 65-70 percent reflects inter-fuel substitution effects. The mean values of the indirectly estimated cross elasticities range from 3 to 16 percent of the directly estimated mean own-price elasticity for utility gas and from 2 to 72 percent of that for electricity, depending upon the energy source considered.

Appendix A
CONSTRUCTED AND AMENDED DATA

This appendix presents certain details relating to the data used for estimation. Minor amendments or modifications to published series are noted, and in cases where a substantial amount of "construction" was required, the entire data series are presented.

MINOR AMENDMENTS TO PUBLISHED DATA SERIES

Data series for POIL, POIL6, PKER, and PKER6 are provided in [16] and [17]. June 15 values are used except in the case of POIL for Hawaii, which uses a September 15 value. Observations for Alaska are lacking for both 1960 and 1970, as are those for Hawaii in 1960. As a result, the Alaska figures for POIL and PKER are set equal to those for Hawaii on the grounds that such fuel costs would tend to be similar in both extra-continental states. For POIL6 and PKER6, Alaska and Hawaii are assumed to have values equal to the average of the three Pacific Coast states.

Values for QG6, NGC6, and VG6 are not available for Alaska. These are estimated to be 9×10^6 therms, 5×10^3 customers, and 1.6×10^6 \$, based on 1965-70 data in [13].

Data on appliance prices are also contained in [17]. All such data used for estimation are March 15 values, except as noted. Range prices (PER, PGR) refer to types having four-unit tops plus oven. The Hawaii observation refers to December 15. Idaho (lacking) is set equal to the unweighted average of Utah, Wyoming, and Colorado values. Air conditioner prices (PRAC) refer to window units of 7,000-10,000 Btu capacity. The television price index (PTV) is a weighted average of the unit prices of 18" black and white portables and 23" color consoles, the weights being 3/4 and 1/4, respectively. Clothes washer prices (PWCW, PACW) and clothes dryer prices (PECD) refer to 12-16 lb. capacity units. Food freezer prices (PFF) are simple averages of those for 16 ft^3 chest and upright types. Values for Alaska for all the above-mentioned prices are set equal to those for Hawaii on the grounds that appliances prices would be similarly high in both areas.

CONSTRUCTED DATA SERIES

Table A-1 displays the data series for which it was necessary to construct many or all observations from related series. A general description of the sources and methods is presented in the main body of the report and need not be repeated here.

Table A-1

SELECTED DATA SERIES

1960 and 1970

State	MCL[a] (index)	PGAS ($/40 therms)	PGAS6 ($/40 therms)	PBGAS (mills/gallon)	PBGAS6[b] (mills/gallon)	PCOAL[c] ($/short ton)	WTEMP (°F)	STEMP (°F)
Alabama	91	6.00	5.57	180	161	6.29	49	82
Alaska	136[d]	9.78	11.74	185	181	12.89	7	58
Arizona	101	5.49	4.61	140	130	5.53	50	90
Arkansas	95	4.58	3.80	147	128	6.67	41	82
California	101	5.01	4.55	178	174	8.40	52	69
Colorado	97	4.48	4.14	130	108	5.57	28	74
Connecticut	109	9.74	8.27	215	235	10.40	30	72
Delaware	102	8.22	7.68	148	175	8.12	33	76
Florida	89	7.24	7.84	185	205	7.35	61	82
Georgia	89	5.64	5.32	188	176	7.53	47	80
Hawaii	120	12.43	11.41	185	181	12.89	73	79
Idaho	101	7.48	6.69	177	160	9.06	29	75
Illinois	102	6.56	6.10	151	135	6.44	27	77
Indiana	102	6.65	5.25	158	147	6.64	29	75
Iowa	100	6.19	5.73	137	123	6.87	20	76
Kansas	100	4.43	3.59	111	94	5.82	31	80
Kentucky	94	5.27	4.87	183	151	5.25	35	78
Louisiana	91	3.91	3.24	145	158	11.65	55	82
Maine	102	10.58	9.96	200	237	12.04	19	67
Maryland/D.C.	102	7.84	7.38	163	215	7.39	36	77
Massachusetts	113	9.41	8.59	235	173	11.13	30	74
Michigan	99	6.75	5.80	175	166	8.38	23	70
Minnesota	102	7.11	6.78	150	150	9.55	11	70
Mississippi	91	4.74	4.64	168	141	14.87	49	82
Missouri	99	6.00	4.87	145	113	5.75	32	80
Montana	101	5.54	3.96	167	163	6.85	19	68
Nebraska	100	5.57	5.39	124	110	6.23	21	77
Nevada	101	7.27	7.43	174	180	9.27	30	68
New Hampshire	102	9.11	8.18	210	222	10.92	21	69
New Jersey	108	8.34	7.65	190	260	9.21	32	76
New Mexico	94	4.89	4.38	123	115	5.57	35	79
New York	112	7.04	6.34	210	217	8.96	29	74
North Carolina	96	7.12	6.79	180	200	7.72	40	75
North Dakota	100	6.82	4.74	156	153	3.43	10	72
Ohio	103	6.07	5.10	160	143	6.26	32	76
Oklahoma	90	4.67	3.84	118	95	8.87	37	83
Oregon	103	7.98	7.54	206	245	9.13	38	67
Pennsylvania	99	7.20	6.07	185	166	6.81	31	74
Rhode Island	109	8.67	7.64	200	197	8.74	29	72
South Carolina	93	6.85	6.57	195	195	8.09	50	81
South Dakota	100	6.47	5.67	146	145	8.88	18	74
Tennessee	91	5.17	5.29	175	163	5.84	41	80
Texas	90	4.81	3.75	120	104	9.30	50	84
Utah	99	4.93	4.37	170	150	5.53	27	77
Vermont	102	8.02	9.71	220	202	12.11	16	69
Virginia	104	7.81	7.64	215	205	7.70	40	79
Washington	103	7.50	7.27	206	235	7.61	35	67
West Virginia	95	5.52	5.05	196	204	4.80	35	76
Wisconsin	104	7.38	6.52	162	142	9.31	21	69
Wyoming	101	4.30	3.84	155	145	4.81	19	71

[a] MCL = metropolitan area cost-of-living index

[b] Data actually pertain to 1963

[c] 1962 Data

[d] Anchorage index. There are no SMSAs in Alaska. The non-metropolitan index in this case is also 136.

Appendix B

THE EFFECTS OF AGGREGATION: A SIMPLE EXAMPLE

Let D_i, the ith individual's demand for some commodity, be given by

$$D_i = a_i p_i^{b_i}, \quad i = 1, \ldots, I,$$

where a_i and b_i are constants and p_i is the price paid by individual i. Then total and average demand (D and $\bar{D}$) are given by

$$D = \sum_{i=1}^{I} D_i \qquad \text{and} \qquad \bar{D} = D/I.$$

If the average market price ($\bar{p}$) is defined as follows:

$$\bar{p} = \sum_i \frac{p_i D_i}{D},$$

then $\bar{b}$, the elasticity of $\bar{D}$ with respect to $\bar{p}$, is given by

$$\bar{b} = \left(\sum_i b_i \cdot w_i \cdot \frac{dp_i}{p_i} \right) \Bigg/ \left(\sum_i [v_i + b_i(v_i - w_i)] \cdot \frac{dp_i}{p_i} \right),$$

where w_i is D_i/D and v_i is $p_i D_i / \bar{p}\, D$.

If variations in price are proportional across individuals--i.e., if

$$\frac{dp_i}{p_i} = \alpha$$

for all i--then

$$\bar{b} = \sum_i w_i b_i / [1 + \sum_i (v_i - w_i) b_i] \ .$$

If, in addition,

$$\sum_i (v_i - w_i) b_i = 0 \ ,$$

then $\bar{b}$ is an exact quantity-weighted average of the individual price elasticities. The above conditions are, of course, met if all individuals pay the same price. If all b_i are the same, the aggregate elasticity of demand is identical to that for each individual.

Appendix C
REGRESSION RESULTS FOR THE "NONE" CASE

This appendix presents constrained regression results for heating, water heating, and cooking installations where the variable corresponding to the "none" category appears on the left hand side. Table C-1 shows the results for heating using both the static and dynamic formulations; Table C-2, those for water heating; Table C-3, those for cooking (static only). Discussion of the price coefficients is omitted, since they are constrained to be identical to those reported in Tables 4-8 in the text. (Of the 24 price coefficients reported, 16 are significant at the .05 level or better, and all but one are larger than their standard errors.)

HEATING

The income coefficients are somewhat smaller than their standard errors and the pattern of signs is mixed, tending (correctly) toward negative in the static formulation and (incorrectly) toward positive in the dynamic. More useful information about income effects can be obtained from the coefficients of HS and AHS. They indicate that, with the exception of coal, an increase in household size tends to raise the number of households without heating, other things equal, this almost assuredly reflecting the implied lowering of income per capita. The opposite sign obtained for coal is consistent with the role of an inferior good, as hypothesized in the text. The results obtained for the housing characteristic and urbanity variables are generally consistent with those reported in Tables 4 and 5, while those for WTEMP indicate that a warm climate has a depressive effect upon the level of heating installations.

WATER HEATING

Both the dynamic and the static equations demonstrate a responsiveness to household size similar to that for heating. Further, according to the estimated coefficients, average household income has a uniformly

Table C-1

HOUSEHOLDS WITHOUT HEATING UNITS,
CONSTRAINED ESTIMATES,
50 STATES, 1970 AND 1960-70

Dependent Variables	Explanatory Variables[a]											Unconstrained R^2	Degrees of Freedom
	Constant	PGAS	POIL	PCOAL	PELEC	PBGAS	YPH	HS	SHU	NUHU	WTEMP		
SNH/SGH	-18.60	4.01	--	---	---	---	-0.54	8.19	-4.29	0.29	0.1060	0.73	43
	(-0.70)	(4.21)					(-0.19)	(2.13)	(-1.83)	(0.14)	(6.98)		
SNH/SOH	-13.00	--	2.13	---	---	---	-2.34	12.14	4.57	-6.63	0.1240	0.69	43
	(-0.33)		(0.90)				(-0.61)	(2.18)	(1.58)	(-2.44)	(5.62)		
SNH/SCH	-2.66	--	--	3.30	---	---	-0.63	-1.95	-0.01	-7.00	0.1189	0.64	43
	(-0.06)			(3.16)			(-0.14)	(-0.32)	(-0.00)	(-2.30)	(4.73)		
SNH/SEH	-17.44	--	--	---	2.21	---	-1.42	10.31	-3.32	-3.08	0.0498	0.66	43
	(-0.80)				(2.54)		(-0.63)	(3.31)	(-1.97)	(-2.07)	(4.14)		
SNH/SBGH	-25.03	--	--	---	---	2.23	0.01	11.92	-8.37	-2.98	0.0792	0.69	43
	(-0.80)					(1.74)	(0.00)	(2.82)	(-3.30)	(-1.34)	(4.46)		
	Constant	APGAS	APOIL	APCOAL	APELEC	APBGAS	AYPH	AHS	ASHU	ANUHU	WTEMP		
SNH/SGH	-26.48	3.54	--	---	---	---	0.37	6.53	-2.24	-0.65	0.0994	0.68	43
	(-1.03)	(3.80)					(0.14)	(1.64)	(-0.87)	(-0.26)	(5.55)		
SNH/SOH	-40.64	--	3.61	---	---	---	-0.08	10.50	5.68	-8.47	0.1106	0.67	43
	(-1.11)		(1.33)				(-0.00)	(1.89)	(1.81)	(-2.65)	(4.45)		
SNH/SCH	-32.20	--	--	2.98	---	---	2.61	-3.79	1.54	-5.65	0.1294	0.64	43
	(-0.85)			(2.93)			(0.65)	(-0.64)	(0.45)	(-1.63)	(4.76)		
SNH/SEH	-28.11	--	--	---	1.29	---	0.22	10.36	-2.44	-3.34	0.0470	0.61	43
	(-1.28)				(1.56)		(0.01)	(3.11)	(-1.30)	(-1.78)	(3.24)		
SNH/SBGH	-38.24	--	--	---	---	2.55	1.32	9.87	-5.91	-5.58	0.0702	0.69	43
	(-1.31)					(2.34)	(0.45)	(2.24)	(-2.15)	(-2.08)	(3.42)		

[a]Figures in parentheses are t-ratios.

Table C-2

HOUSEHOLDS WITHOUT WATER HEATING UNITS,
CONSTRAINED ESTIMATES,
50 STATES, 1970 AND 1960-70

Dependent Variables	Explanatory Variables[a]										Unconstrained R^2	Degrees of Freedom
	Constant	PGAS	POIL	PELEC	PBGAS	YPH	HS	SHU	NUHU	WTEMP		
SNWH/SGWH	19.00	3.99	--	---	---	-3.88	3.48	1.03	5.00	0.0331	0.84	43
	(1.21)	(7.09)				(-2.33)	(1.53)	(0.75)	(4.25)	(3.69)		
SNWH/SOWH	4.57	--	2.65	---	---	-4.22	8.54	16.25	-1.84	0.0509	0.74	43
	(0.13)		(1.22)	---	---	(-1.20)	(1.67)	(6.14)	(-0.74)	(2.52)		
SNWH/SEWH	11.30	--	--	2.60	---	-4.10	3.89	0.60	0.72	0.0187	0.51	43
	(0.58)			(3.32)		(-2.04)	(1.39)	(0.40)	(0.54)	(1.73)		
SNWH/SBGWH	0.16	--	--	---	2.76	-2.57	6.92	0.65	1.22	0.0154	0.53	43
	(0.01)				(3.29)	(-1.28)	(2.50)	(0.39)	(0.84)	(1.32)		
	Constant	APGAS	APOIL	APELEC	APBGAS	AYPH	AHS	ASHU	ANUHU	WTEMP		
SNWH/SGWH	20.19	3.98	---	--	--	-4.09	3.30	1.77	3.66	0.0224	0.86	43
	(1.46)	(7.92)				(-2.78)	(1.53)	(1.27)	(2.73)	(2.32)		
SNWH/SOWH	-9.77	--	4.80	--	--	-3.93	9.30	14.97	-3.61	0.0364	0.74	43
	(-0.30)		(1.97)			(-1.22)	(1.87)	(5.32)	(-1.26)	(1.63)		
SNWH/SEWH		--	---	1.83	--	-3.87	5.22	1.07	-0.28	0.0038	0.47	43
				(2.73)		(-2.14)	(1.94)	(0.71)	(-0.18)	(0.32)		
SNWH/SBGWH	3.50	--	---	--	2.83	-3.01	6.70	1.46	-0.35	-0.0002	0.51	43
	(0.19)				(4.17)	(-1.63)	(2.44)	(0.85)	(-0.21)	(-0.02)		

[a] Figures in parentheses are t-ratios.

Table C-3

HOUSEHOLDS WITHOUT COOKING UNITS,
CONSTRAINED ESTIMATES,
50 STATES, 1970

Dependent Variables	Explanatory Variables[a]										Unconstrained R^2	Degrees of Freedom
	Constant	PGAS	PELEC	PBGAS	PGR	PER	YPH	HS	SHU	NUHU		
SNC/SGC	-17.24	3.06	---	---	2.78	---	-0.77	-0.92	-0.28	1.08	0.51	43
	(-0.95)	(4.63)			(2.31)		(-0.40)	(-0.37)	(-0.18)	(0.94)		
SNC/SEC	-19.02	---	1.06	---	---	2.07	-0.18	-1.68	-2.15	-0.67	0.29	43
	(-1.64)		(2.23)			(1.94)	(-0.15)	(-1.04)	(-2.07)	(-0.98)		
SNC/SBGC	-24.53	---	---	1.40	2.37	---	0.32	-0.15	-0.85	-3.02	0.41	43
	(-1.34)			(2.06)	(2.20)		(0.18)	(-0.07)	(-0.60)	(-3.00)		

[a]Figures in parentheses are t-ratios.

negative effect upon the number of households lacking water heating, although only in the cases of electricity and gas are the coefficients significant at the .05 level. The effect of housing structure is largely restricted to water heating by oil. As suggested in the text, oil is used relatively more heavily in areas of high-density housing. The role of NUHU/ANUHU is clearly that of a proxy for utility gas availability, the coefficients of the urbanity variables being positive and reasonably large in relation to their standard errors only in the utility gas equations. Finally, warm climates may tend to depress the level of water heating installations, particularly those using gas or oil, according to the results obtained for WTEMP.

COOKING

Of the 16 coefficients obtained for YPH, HS, SHU, and NUHU, only two are significant at the .05 level or better. No generalizations about the effects of these variables can be made with confidence.

Appendix D
ANALYTICAL FRAMEWORK FOR SECTION VI

This appendix presents the analytical framework that underlies the estimates presented in Section VI and introduces some evidence concerning the influence of price upon the rate of consumption per installation. Following the notation of that section, let

$$q = \sum_{k=1}^{n} e_k(p) \cdot s_k(p) \ . \tag{D.1}$$

For simplicity, the subscript i, denoting energy type, is dropped; also e_k and s_k are written as functions of the own-price of energy alone. Let

$$\hat{q} = \sum_{k} \hat{e}_k \cdot s_k(p) \ , \tag{D.2}$$

where $\hat{e}_k$ is an estimate of $\bar{e}_k$. (A bar above a variable denotes a mean value; a circumflex denotes an estimated quantity.) From (D.1),

$$\varepsilon_{q,p} = \varepsilon^{s}_{q,p} + \varepsilon^{e}_{q,p} \tag{D.3}$$

That is, the elasticity of q with respect to p is the sum of the elasticity of q with respect to p holding the s_k constant and the elasticity of q with respect to p holding the e_k constant. Also from (D.1),

$$\bar{\varepsilon}^{e}_{q,p} = \sum_{k} \bar{\varepsilon}_{s_{k,p}} \cdot \bar{w}_k \tag{D.4}$$

$$\bar{\varepsilon}^{s}_{q,p} = \sum_{k} \bar{\varepsilon}_{e_{k,p}} \cdot \bar{w}_k \tag{D.5}$$

where $\bar{w}_k \equiv \bar{e}_k \bar{s}_k / \bar{q}$. From (D.2),

$$\bar{\varepsilon}_{\hat{q},p} = \sum_k \bar{\varepsilon}_{s_{k,p}} \cdot \hat{\bar{w}}_k \ , \tag{D.6}$$

and from (D.1) and (D.2) one has

$$\bar{\varepsilon}_{\underset{\hat{q}}{q},p} = \sum_k \bar{\varepsilon}_{e_{k,p}} \cdot \bar{w}_k + \sum_k (\bar{w}_k - \hat{\bar{w}}_k)\bar{\varepsilon}_{s_{k,p}} \ , \tag{D.7}$$

where $\hat{\bar{w}}_k = \hat{e}_k \cdot \bar{s}_k/\hat{\bar{q}}$. If $\hat{e} = \bar{e}_k$, then $\hat{\bar{w}}_k = \bar{w}_k$ and

$$\bar{\varepsilon}^{e}_{q,p} = \bar{\varepsilon}_{\hat{q},p} \quad \text{and} \tag{D.8}$$

$$\bar{\varepsilon}^{s}_{q,p} = \bar{\varepsilon}_{\underset{\hat{q}}{q},p} \ . \tag{D.9}$$

Note that the above equalities apply only to elasticities evaluated at the mean. Together, (D.8) and (D.9) indicate that the mean elasticity components of (D.3) can be estimated indirectly from (D.2) and from an equation of the form

$$\frac{q}{\hat{q}} = f(p,u) \ , \tag{D.10}$$

where u is an error term, provided that $\frac{f'(\bar{p})\bar{p}}{f(\bar{p})}$ is a good estimator of $\bar{\varepsilon}_{\underset{\hat{q}}{q},p}$.

Alternatively, if a better estimate of the relationship between q and $\hat{q}$ can be obtained from the following approximation:

$$q - \hat{q} = g(p,u) \ . \tag{D.11}$$

then $g'(\bar{p})\frac{\bar{p}}{\bar{q}}$ is an estimator of $\frac{d(\bar{q}-\hat{q})}{dp} \cdot \frac{\bar{p}}{\bar{q}}$, which in turn is equal to $\bar{\varepsilon}^{s}_{q,p}$--as can be verified from (D.1) and (D.2).

Suppose now that the "true" $\hat{q}$ (whose $\hat{e}_k$ are equal to the $\bar{e}_k$) is not observed but rather some $\hat{\hat{q}}$ whose $\hat{\hat{e}}_k$ are not equal to the $\bar{e}_k$. Suppose also that

$$\bar{\hat{\hat{q}}} = \bar{\hat{q}} \quad ,$$

that is, $\hat{\hat{q}}$ is an unbiased estimator of $\hat{q}$. The effect of such systematic error can be illustrated most simply if only two energy-using functions are assumed, one for which saturation is 1.0 ($s_0 \equiv 1$) and one having variable saturation ($0 \leq s_1 \leq 1$). Then

$$\hat{q} = \hat{e}_0 + \hat{e}_1 s_1 \tag{D.12}$$

and

$$\hat{\hat{q}} = \hat{\hat{e}}_0 + \hat{\hat{e}}_1 s_1 \quad . \tag{D.13}$$

Also,

$$\hat{q} = \alpha + \beta\hat{\hat{q}} \tag{D.14}$$

where

$$\alpha = \frac{\hat{e}_0\hat{\hat{e}}_1 - \hat{e}_1\hat{\hat{e}}_0}{\hat{\hat{e}}_1} \text{ and } \beta = \frac{\hat{e}_1}{\hat{\hat{e}}_1} \quad .$$

Since it is assumed that

$$\hat{e}_0 + \hat{e}_1\bar{s}_1 = \hat{\hat{e}}_0 + \hat{\hat{e}}_1\bar{s}_1 \quad ,$$

it follows that $\beta > 1 \rightarrow \alpha < 0$ and $\beta < 1 \rightarrow \alpha > 0$. The systematic error of $\hat{q}$ in relation to $\hat{q}$ must therefore be one of the two types shown in Figure D-1. If Eq. (D.11) holds, then from (D.14) one has

$$q - \hat{\hat{q}} = g(p,u) + \alpha + (\beta - 1)\hat{\hat{q}} \quad ; \tag{D.15}$$

i.e., $\hat{\hat{q}}$ must appear on the right hand side of the estimated equation, if $\hat{\hat{q}}$ rather than $\hat{q}$ appears on the left. Note also that the coefficient of $\hat{\hat{q}}$ must be greater than -1.

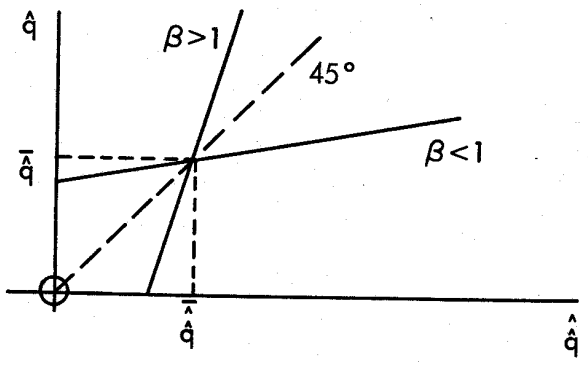

Fig. D-1 — Systematic Error in $\hat{\hat{q}}$, I

If the simplifying assumptions are dropped, and one writes

$$\hat{q} = \sum_k \hat{e}_k s_k \quad , \tag{D.16}$$

and

$$\hat{\hat{q}} = \sum_k \hat{\hat{e}}_k s_k \quad , \tag{D.17}$$

it is no longer possible to derive a simple relationship between $\hat{\hat{q}}$ and $\hat{q}$. Nonetheless it is clear that systematic error could exist. For example, suppose that there is a group of e_k whose saturations vary relatively much across observations (analogous to $\hat{e}_1$ above), then it is possible that a systematic relationship between the observed $\hat{\hat{q}}$ and the true (but unobserved) $\hat{q}$ would exist. In the general case, however, it is not clear that systematic error of the type portrayed in Figure D-1 would be observed.

In this case, a purely empirical approach must be taken. That is, a particular functional form is assumed to be appropriate to describe the relationship between $\hat{\hat{q}}$ and $\hat{q}$ and the *ex post* results compared with other results. If the estimate obtained for $\bar{\varepsilon}_{e_{k,p}}$ appears to be more nearly consistent[1] with the estimates obtained for $\bar{\varepsilon}_{s_{k,p}}$ and $\bar{\varepsilon}_{q,p}$, then one has limited evidence in favor of the systematic error hypothesis.

[1] Than the results of the formulation that assumes $\hat{\hat{q}} = \hat{q}$.

For several of the estimates presented below, it is assumed that

$$\hat{q} = \alpha \hat{\hat{q}}^{\beta} \quad \text{with } \alpha, \beta > 0 \ . \tag{D.18}$$

Figure D-2 displays the kinds of systematic errors that such a formulation may depict. Qualitatively, the errors shown in Figure D-2 differ little from those depicted in Figure D-1, although the properties of (D.17) in the vicinity of the origin are probably superior to those of (D.14) in the general case where the relationship between $\hat{q}$ and $\hat{\hat{q}}$ is given by (D.16) and (D.17).

Together (D.16) and (D.18) imply that

$$\frac{q}{\hat{\hat{q}}} = f(p,u)\alpha \ \hat{\hat{q}}^{(\beta-1)} \ . \tag{D.19}$$

As in the linear case, given by (D.15), $\hat{\hat{q}}$ appears on the right hand side with a coefficient that is algebraically greater than -1.

Table D-1 presents a number of alternative estimates of $\overline{\varepsilon}^{s}_{q,p}$, the own-price elasticity of electricity consumption due to variations in usage levels. Depending upon the assumed error function, that is, linear given by (D.14) or log-linear given by (D.18), either the

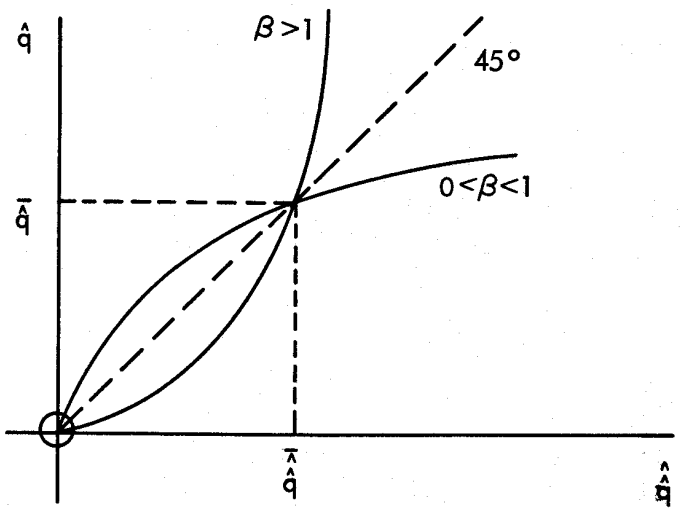

Fig. D-2 — Systematic Error in $\hat{q}$, II

Table D-1

MEAN ELASTICITY ESTIMATES FOR ELECTRICITY
FROM 1970 STATIC MODELS

Elasticity Type	Approach or Source of Estimate		Elasticity[a]
Total ($\bar{\varepsilon}_{q,p}$)	unconstrained electricity eq.		-1.12 (-6.00)
	constrained electricity eq.		-1.19 (-5.74)
Saturation ($\bar{\varepsilon}^{e}_{q,p}$)	indirect, from stock eqs.		-0.84[b]
Usage level ($\bar{\varepsilon}^{s}_{q,p}$)	with systematic error term	linear	-0.53 (-3.92)
		log-linear	-0.37 (-3.11)
	no systematic error term	linear	-0.32 (-2.51)
		log-linear	-0.19 (-1.78)
	($\bar{\varepsilon}_{q,p} - \bar{\varepsilon}^{e}_{q,p}$)		-0.28/-0.35[b]

[a]The t-ratio in parentheses refers to the estimated price coefficient, not to the elasticity estimate shown, in cases where the two differ.

[b]T-ratio not available.

no-systematic-error or the systematic-error model yields an estimate within the range of values given by the difference ($\bar{\varepsilon}_{q,p} = \bar{\varepsilon}^{e}_{q,p}$) as evaluated in the table. (It should be noted that if a systematic error model is assumed to apply, then the estimate of $\bar{\varepsilon}^{e}_{q,p}$ obtained indirectly from the stock equations may be inaccurate. The presence of systematic error leads to the presumption that $\hat{\bar{w}}_k \neq \bar{w}_k$, since the $\hat{\bar{w}}_k$ must then be based upon incorrect $\hat{e}_k$ rather than the theoretically accurate $\hat{e}_k$). Empirically, then, it is difficult to say whether the systematic error hypothesis should be accepted or rejected in the case of electricity. The estimates presented in Table D-2 below include only those where EEPH is on the right hand size because (1) the t-ratios of the estimated coefficients are lower than those in the alternative formulation, and (2) the coefficient of EEPH is of plausible magnitude and statistically significant in all examples. The coefficients of PELEC are also statistically significant in all examples and yield reasonable estimates of $\bar{\varepsilon}^{s}_{q,p}$. Note that competing energy prices are not included in the estimated equations. This procedure is correct if, as suggested in the text, the size, features, and operating rates of equipment are not affected by prices of the energy types not used by that equipment.

In the case of gas, the systematic-error problem appears less likely to exist. When EGPH is included on the right hand side in various specifications, both linear and nonlinear, its coefficient is generally quite insignificant statistically and the t-ratios of the statistically significant variables decline. One exception is the log-linear case. Although the t-ratios of the explanatory variables decline and the coefficient of PELEC takes on an incorrect positive sign when EGPH appears on the right hand side, nevertheless the coefficient of the latter variable is of plausible magnitude (0.34) and highly significant (t-ratio = 3.92). The equations reported in Table D-3 do not include EGPH on the right hand side. The table shows that when GPH-EGPH is the dependent variable, the coefficients of PGAS are quite insignificant and are improperly signed. When GPH/EGPH is the dependent variable, more satisfactory estimates of the coefficient of PGAS and accordingly of $\bar{\varepsilon}^{s}_{q,p}$ are obtained.

Table D-2

ACTUAL VS. ESTIMATED ELECTRICITY CONSUMPTION,
50 STATES, 1970

Dependent Variables	Constant	Explanatory Variables							Adjusted R^2	Degrees of Freedom	Estimated $\bar{\varepsilon}_{q,p}$
		PELEC	YPH	HS	SHU	NUHU	WTEMP	STEMP	EEPH			
EPH-EEPH	-167.4	-2.126	0.1351	-608.9	1625	3640	-2.673	48.72	-0.1950	.33	41	-0.53
(linear)	(-0.05)	(-3.92)	(0.95)	(-0.77)	(1.12)	(2.82)	(-0.20)	(1.59)	(-3.05)			
EPH-EEPH	8440	-4118	2059	-1550	1385	3955	-3.196	59.75	-0.2251	.37	41	-0.54
(semi-log)	(0.50)	(-4.37)	(1.16)	(-0.63)	(0.98)	(3.09)	(-0.25)	(1.98)	(-3.48)			
EPH/EEPH	0.9096	-1.771×10^{-4}	2.315×10^{-5}	-0.1463	0.2513	0.4520	3.301×10^{-4}	5.372×10^{-3}	-1.850×10^{-5}	.26	41	-0.34
(linear)	(1.94)	(-2.57)	(1.27)	(-1.45)	(1.36)	(2.76)	(0.20)	(1.38)	(-2.28)			
EPH/EEPH	0.4318	-0.3412	0.3171	-0.4295	0.2291	0.4774	2.735×10^{-4}	6.312×10^{-3}	-2.094×10^{-5}	.28	41	-0.34
(semi-log)	(0.20)	(-2.79)	(1.37)	(-1.34)	(1.25)	(2.88)	(0.16)	(1.61)	(-2.50)			
EPH/EEPH	0.6870	-0.3713	0.3727	-0.4359	0.2455	0.5262	-6.701×10^{-5}	6.987×10^{-3}	-0.1999	.31	41	-0.37
(log-linear)	(0.31)	(-3.11)	(1.64)	(-1.42)	(1.37)	(3.17)	(-0.04)	(1.83)	(-2.71)			

Table D-3

ACTUAL VS. ESTIMATED GAS CONSUMPTION,
50 STATES, 1970

Dependent Variables	Constant	Explanatory Variables							Adjusted R^2	Degrees of Freedom	Estimated $\bar{\epsilon}_{q,p}$
		PGAS	YPH	HS	SHU	NUHU	WTEMP	STEMP			
GPH-EGPH	-460.7	10.15	0.01271	116.5	659.5	-511.7	-10.22	-1.15	0.319	42	0.10
(linear)	(-0.63)	(0.56)	(0.44)	(0.75)	(2.13)	(-1.99)	(-4.13)	(-0.21)			
GPH-EGPH	-1741	91.15	134.8	368.1	672.9	-530.2	-10.23	-1.07	0.320	42	0.13
(semi-log)	(-0.50)	(0.71)	(0.37)	(0.74)	(2.18)	(-2.04)	(-4.18)	(-0.19)			
GPH/EGPH	0.9034	-4.236×10^{-2}	1.401×10^{-5}	0.1733	0.6386	-0.6777	-1.380×10^{-2}	-1.705×10^{-3}	0.509	42	-0.30
(linear)	(1.14)	(-2.16)	(0.45)	(1.02)	(1.90)	(-2.43)	(-5.14)	(-0.27)			
GPH/EGPH	-2.281×10^{-2}	-0.2326	0.1158	0.5520	0.6868	-0.7372	-1.433×10^{-2}	-6.891×10^{-3}	0.487	42	-0.23
(semi-log)	(-0.01)	(-0.63)	(0.28)	(0.99)	(2.00)	(-2.56)	(-5.28)	(-0.11)			
GPH/EGPH	-1.410	-0.3384	0.1764	0.6672	0.6537	-0.6794	-1.608×10^{-2}	-1.848×10^{-3}	0.441	42	-0.34
(log-linear)	(-0.29)	(-1.87)	(0.34)	(0.95)	(1.51)	(-1.867)	(-4.69)	(-0.23)			

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