Estimating Aircraft Recoverable Spares Requirements with Cannibalization of Designated Items

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PREFACE

This report presents an approach to estimating aircraft recoverable spares requirements using data that reflect the relative ease with which aircraft components can be cannibalized. As a matter of policy, in determining recoverable spares requirements, the Air Force has traditionally assumed no cannibalization. The approach discussed in this report departs from that traditional policy and assumes designated cannibalization, i.e., some parts would be designated in the requirements database as readily cannibalized and other parts as difficult to cannibalize. The Air Force already uses this approach when computing war readiness spares. The report discusses the cost-effectiveness of this approach, presents a mathematical model to guide its implementation, and evaluates an arbitrarily specified case.

The research described here is part of a larger body of work carried out in the Logistics Requirements Project and described in the following companion reports:


The first of these reports describes the principal thrusts of the main body of work, discusses the effects of uncertainty on the performance of the spares asset position anticipated to evolve from the requirements computation and subsequent procurements, and evaluates system performance as it is affected by management adaptations such as cannibalization, lateral supply, and expedited repair, transportation, and handling. The second report describes improved methods for
forecasting the demand for aircraft recoverable spares and specifying the variance of the probability distribution describing the number of assets of a given type in resupply. The third report discusses data and data-processing issues related to estimating aircraft recoverable spares and repair requirements. The fourth describes Dyna-METRIC Version 6, an advanced capability assessment model used to evaluate the stockage postures that were anticipated to eventuate from purchases of particular mixes of recoverable spares. The last report summarizes the entire body of work.

The research described here was done in the Resource Management and Systems Acquisition Program of Project AIR FORCE, RAND's federally funded research and development center supported by the U.S. Air Force. This report should interest logisticians throughout the defense establishment and in the Air Force, especially those concerned with the determination of requirements for aircraft recoverable spares.
SUMMARY

To estimate requirements for primary operating stocks (POS) of aircraft recoverable spare parts, the Air Force has not, as a matter of policy, assumed that parts shortages can be consolidated among aircraft (cannibalized). In its computational model, parts shortages are assumed to be randomly distributed among aircraft. Until recently, however, it has assumed perfect consolidation of shortages in its computation of war readiness spares (WRS) requirements. The Air Force changed its policy for WRS requirements estimation to incorporate the assumption of designated cannibalization, under which certain parts are designated as relatively easy to cannibalize and others as more difficult to cannibalize. Although this dichotomy does not accurately represent the degree of difficulty, cost, or risk of cannibalization, it is an important first step toward a more realistic model of what happens in practice. Even in peacetime, cannibalization is done quite routinely.

Designating some parts as cannibalizable and others not implies that the total estimated investment level required to achieve specified aircraft availability goals is reduced, the total stockage of aircraft recoverable spare parts is reduced, the problem of long supply is mitigated somewhat, and the mix of spares procured is more cost-effective. The research described in this report shows that a policy of designated cannibalization in estimating POS requirements is cost-effective, although because of lack of data on the suitability of specific parts for cannibalization, we are unable to quantify precisely the cost savings and performance improvement. Nevertheless, this research strongly suggests that a designated cannibalization policy for POS can reduce safety stock requirements while maintaining traditional levels of system performance.

In this report we present a computational model that incorporates designated cannibalization. With it we estimate the cost-effectiveness of the policy. It is only one approach to the problem; there are probably better ones, and we hope that this work serves to stimulate additional thinking about the topic. The model discussed here determines the probability of meeting a specified aircraft availability goal with a specified confidence. Although this departs from the usual objective function of expected availability, it will yield a spares mix that approximates the mix from an expected availability computation and is a meaningful criterion in its own right.
We recommend that the Air Force adopt a designated cannibalization policy for POS requirements estimation. This model should be helpful in moving toward that goal.
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1. INTRODUCTION

The larger body of research of which this work is a part estimates the effects of uncertainty and management adaptations on the determination of requirements for aircraft recoverable spare parts. By management adaptations we mean such practices as cannibalization (described below), mutual base support (e.g., lateral supply and lateral repair), priority repair, and expedited handling, processing, and transportation. The acquisition of aircraft recoverable spares in the Air Force is important, both in its costs and implications for logistics system performance. During the mid to late 1980s, the Air Force spent roughly $5 billion annually on procurement and depot-level repair of these assets (roughly $3 billion for spares procurement and $2 billion for repair). Thus the issue is a major one.

This report focuses on the consolidation of parts shortages into the least number of next-higher assemblies, a practice known as cannibalization. Cannibalization is a very powerful management adaptation that helps the logistics system cope with uncertainties in parts demands.

When estimating spares and repair requirements, planners must cope with uncertain futures that evolve over long planning horizons. These uncertainties are reflected in part in the changes in the requirements database that we observe from year to year. We refer to the sum total of all such changes as churn. Churn results in poorer performance than is anticipated in the requirements computation; however, our evaluations suggest that it is more than offset by management adaptations such as cannibalization, lateral supply, priority repair, and expedited transportation, processing, and handling of assets in the repair pipeline. On balance, then, the performance of the logistics system exceeds the goals currently specified in the requirements computation.²

In the discussion that follows, we illustrate the effects of uncertainties and management adaptations on system performance and point

¹Recoverable parts are typically high-cost items and are subject to repair upon failure.
²Note, however, that when an aircraft availability goal of, say, 83 percent is specified repeatedly to the requirements computation in the face of system performance of, say, 94 percent, the decisionmaker’s real goal is 94 percent, for which the number 83 is only a numerical artifact of the computational methodology.
out the need for an alternative way to compute spares requirements that explicitly accounts for cannibalization. First, though, we define some fundamental ideas that underlie the discussions throughout the report.

**SOME BASIC CONCEPTS**

We begin with a brief, somewhat simplified overview of the recoverable spares management system that supports the combat force. At bases with aircraft, problems sometimes arise with the hardware components of the aircraft. Sometimes those problems are diagnosed and repaired on the aircraft with an adjustment, minor repair, or replacement of consumable parts. On other occasions, repair involves the replacement of a recoverable component, typically described as a line-replaceable unit (LRU). When an LRU believed to be defective is removed from an aircraft, a base repairable generation is said to occur. In the Air Force's traditional maintenance structure, the repairable component is sometimes diagnosed and repaired at an intermediate-level maintenance activity. If the repair is beyond the capability of the intermediate-level repair activity, it is declared to be not-repairable-this-station (NRTS) and is usually returned to the depot-level repair activity for repair and return to serviceable stock. The LRU is often repaired at intermediate level by removing and replacing a shop-replaceable unit (SRU), typically also a recoverable item.

**Some Common Terms**

Unserviceable assets en route from the base to the depot are said to be in retrograde. After repair and return to serviceable condition, assets are available for issue to a base. Bases submit requisitions to the depot for replacement assets any time they NRTS an asset back to a depot repair facility. Assets on order by the base are called due-ins. Assets that have been requested from base supply by base maintenance but not provided by supply are called due-outs or base backorders. The base's generation of replenishment requisitions is governed in part by a reorder policy, which in the case of recoverable

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3 This discussion draws heavily, often verbatim, from the introductory discussion in Abell et al. (1993).

4 Unlike recoverables, consumables are discarded upon failure or consumed in use.

5 The term repairable (accent on the second syllable) is defined to mean defective, i.e., unserviceable, in contrast to repairable (accent on the first syllable), an awkward synonym for recoverable popularized by the other services.
items is described as a continuous review reorder policy with an order quantity of one. This implies that whenever the base's stock on hand plus due-ins minus due-outs falls below the stock level, a replenishment requisition is sent to the depot by the base. If the depot fills the requisition upon receipt, the transaction is referred to as an issue. If it does not, a depot backorder, or depot due-out, is created. The allocation of a serviceable asset to a backorder is called a backorder release.

Note that depot backorders do not necessarily imply that an aircraft is short a component; it simply means that the depot owes the base an asset. On the other hand, a base backorder does imply that a component is missing from an aircraft. These shortages are sometimes referred to as holes in aircraft. The consolidation of such holes, or parts shortages, into a smaller number of aircraft (typically the smallest possible number), i.e., cannibalization, is the practice of fundamental interest here. In most models of the logistics system, cannibalization is not modeled accurately, being assumed to be done either for all parts or for no parts. The important extension to this logic, and the principal focus of this report, is the idea of designated cannibalization—the designation of parts in the requirements database as cannibalizable or easy to cannibalize, or noncannibalizable or difficult to cannibalize.

Clearly, the ease or difficulty of cannibalization depends on the type of part. The cannibalization of a fuse, for example, is a trivial task, something that technicians do almost invariably rather than taking the trouble to draw one out of stock. The cannibalization of a wing spar, on the other hand, is very costly, very hazardous, and virtually unheard of in the real world of aircraft maintenance. Cannibalization of parts that are relatively easy to cannibalize provides serviceable parts when there is no spare in stock. Again depending on the type of part, it is often quick and easy to remove a serviceable component from an aircraft that is unavailable for other reasons and gain a serviceable component for an aircraft that may need only that part to make it available for use.

On the other hand, cannibalization is sometimes imprudent. It may be too costly in labor or time, or too risky in the sense that the component may be damaged during removal, or the item may require fitting to an aircraft as is often the case with canopies, sheet metal parts, and the like. Thus emerges the logic of the idea explored in this report, that of designating certain items in the requirements database as cannibalizable and others as noncannibalizable. Despite the traditional resistance in the Air Force to the idea of assuming any
cannibalization in its computation of requirements for aircraft spare parts, the austere budgetary environment the Air Force currently faces weakens that position considerably, and the idea that it can designate which parts are “easy” to cannibalize and which are not weakens it even further. Note that the Air Force already uses such a scheme in computing requirements for war readiness spares. Thus it is not a new idea, although its computational method is apparently undocumented. The computational method currently used for war readiness spares may be the same as the method described here, or it may be an attractive alternative. Our motivation for publishing this report is simply to show that the computational problem involved in implementing designated cannibalization does not pose an insurmountable barrier and to provide a self-contained exposition of an important policy issue.

The Concept of Item Pipelines and Resupply

The models of the inventory system used in computing spares requirements are built around the concept of an item’s pipeline. The notion of a pipeline in this context is different from the generic usage of the term. Suppose that one component of an aircraft, say an F-16A, is a radar transmitter. Suppose, too, that each F-16A aircraft has one radar transmitter. When a transmitter fails, or is suspected of failure, it is generally removed from the aircraft, in the manner we have already described, for repair in the intermediate-level repair activity (typically located on the same base). It is either repaired there or sent back to the depot (or a contractor facility) for repair. Eventually it emerges from repair and is typically allocated to a base as a replacement asset. If there were no serviceable spare transmitters in the supply system, the base at which the failure and removal occurred would have to wait for the repair of the component removed from the aircraft. To avoid such delays, we provision the system with spare transmitters. Thus, some transmitters may be in resupply, i.e., in intermediate-level repair, in retrograde to the depot, undergoing depot repair, in shipment from the depot to bases, or waiting for replacement after condemnation (when an asset is beyond economical repair). The expected number of items of a particular type in resupply is commonly referred to as the pipeline, or item pipeline, thus the word pipeline takes on meaning according to the context of its use; it may mean the logistics pipeline in the sense of a conduit, or the expected number of assets in the pipeline. In the modeling context, it is simply a mathematical expectation of the number of assets, transmitters say, that would be missing from aircraft if there were no spares
in the supply system (assuming the flying hour program is actually flown).

In practice, the number of assets of each type in resupply is a random variable. Its probability distribution is the basis for the computation of spares requirements. If the number of transmitters removed from aircraft every day never varied, and their repair and return to the base always took exactly the same amount of time, the pipeline would have little or no variability, and the system could operate effectively simply by buying enough spares to fill the pipeline.

In actuality, however, there is great uncertainty in every segment of the pipeline, and in all of the factors used to estimate its numerical value. Demand rates, NRTS rates, repair times, shipment times, and other measures of system performance change over time. Depot repair times and the times that elapse between an asset's arrival at the depot and its induction into the depot repair shop also vary greatly among shops and items and over time. Thus, although the mathematical expression for the expected number of assets of a given type in resupply is simple and straightforward, it is very difficult to "get it right" in a practical sense; that is, the world often eventuates in ways that we cannot predict. When resupply times are longer than forecast, spares shortages may develop. When repair activities respond more quickly than planned, more serviceables may be available than anticipated. The vagaries of the processes underlying the richness or paucity of serviceable spares in the supply system are substantial. Our uncertainties about factors that affect the numbers of assets in resupply greatly complicate the problem of deciding on the appropriate levels of investment in aircraft recoverable spares and on the best mix of spares to buy to achieve our aircraft availability goals at least cost.

Because of our uncertainties about the actual number of assets in resupply, we invest in safety stock as well as in the assets required to cover the pipeline. Safety stock protects the inventory system against variability in the number of items in resupply; it also helps protect the system against errors in estimating item pipelines.

The problem, though simple enough to state, is difficult to solve: determine the level of investment in aircraft recoverable spares and the best mix of spares to buy to achieve specified aircraft availability goals at least cost. This is the fundamental problem addressed by the larger body of research of which this work is a part. It is made difficult by its sheer dimensions, the quality of data used to support requirements computations, and the complexity of the logistics system whose behavior the requirements system tries to model, but most of
all, perhaps, by the formidable uncertainties that underlie the factors
that shape the pipelines of aircraft recoverable spares and our esti-
mates of those factors.

The approach the Air Force Materiel Command (AFMC) uses to es-
imate procurement and repair requirements for these assets has long
absorbed the attention of many persons in the logistics research com-
unity. Progress has been made on several issues, most notably,
perhaps, on the logic underlying the computation of safety stock for a
large and important subset of items in the recoverable inventory sys-
tem. Despite this progress, the Air Force has been heavily and per-
sistently criticized, especially in recent years, owing to a situation
typically described as the long supply problem that people often per-
ceive as symptomatic of overstated requirements. A designated can-
nibalization policy in spares requirements estimation can help miti-
gate the effects of the long supply problem.

UNCERTAINTY AND MANAGEMENT ADAPTATIONS

In an attempt to quantify the effects of uncertainty and the effective-
ness of management adaptations in overcoming it, we replicated the
Air Force’s spares requirements computation using the March 1986
database. Spares procurement actions initiated as a result of that
computation would have been delivered to the Air Force, on average,
13 quarters later, i.e., in June 1989. By accounting for expected con-
demnations between the buy point in 1986 and an average procure-
ment lead time beyond the buy point (1989), we are able to estimate
the anticipated stockage posture in 1989 using a replica of the Air
Force’s central stock leveling system (D028). Using Dyna-METRIC
Version 6, the latest version of RAND’s capability assessment model,
we are able to estimate the system performance delivered by the an-
ticipated stockage posture in terms of the percentage of aircraft that
would be unavailable owing to lack of recoverable parts. Although we
replicated the requirements computation for the entire database, we
use the F-16 weapon system (all series) as a case study here.

We evaluated the anticipated stockage posture using two different
sets of assumptions: (1) the item characteristics (demand rates,
NRTS rates, repair times, etc.) in 1989 are identical to those in the
1986 requirements database, and (2) the item characteristics in 1989
are as observed in the March 1990 database, which contains eight
quarters of past demands, NRTS actions, average repair times, etc.,
for the period 1 April 1988 through 31 March 1990. As pointed out
above, an average lead time beyond the buy point occurs in June
1989, roughly the midpoint of the observations in the 1990 database.
Figure 1.1 illustrates the results of these evaluations under a variety of assumptions about management adaptations. The darker bars represent the results of evaluating the anticipated stockage posture under the first assumption (item characteristics in 1989 are the same as expected in 1986), and the lighter bars reflect the results of the second assumption (the observed item characteristics). The top pair of bars simply represents the 83 percent aircraft availability goals (i.e., not more than 17 percent unavailable) specified to the requirements computation that produced the anticipated stockage posture underlying the results associated with both assumptions. The second pair of bars shows the estimated system performance given the assumptions about management adaptations that underlie the requirements system's computations: no cannibalization, no lateral supply, and no expedited handling, processing, or transportation. The remaining cases reflect the effects of management adaptations in various combinations, as shown in the text on the left of the chart. The term “quick response” means that average depot repair turnaround times are reduced on average from about 89 days to about 52 days.

Figure 1.1—The Effects of Churn and Management Adaptations on System Performance
Note the dramatic effect of cannibalization on system performance. Cannibalization alone reduces the estimated (86–89) percentage unavailable from 48 percent to 14 percent; however, this result overestimates the effect of cannibalization because it naively assumes that all recoverable components of the aircraft are cannibalizable. In this report we explicitly treat cannibalization as applicable only to subsets of components of aircraft, rather than to all components.

The requirements database used by the Air Force in estimating its requirements for aircraft recoverable spares and depot repair does not indicate which items are cannibalizable; therefore, to evaluate the cost-effectiveness of assuming designated cannibalization in the spares requirements computation, we rather arbitrarily designated, by federal stock class, certain items as cannibalizable and others not using only our own judgment. For example, we declared airframe and engine components noncannibalizable and fire control system components cannibalizable. As a result of this arbitrary exercise, we designated about 58 percent of the items in the database cannibalizable. We do not know how closely this percentage approximates the actual percentage of items that would be designated cannibalizable by maintenance experts. Although our designation was arbitrary, it helped us gain some intuition about the approach of computing spares requirements with an assumption of designated cannibalization.

In the case of war readiness spares, the Air Force designated items as cannibalizable or not in the database; as a result, roughly 70 percent of those items were designated cannibalizable. Note, however, that war readiness spares comprise mostly high-demand items that tend to be fairly readily removable as a matter of necessity. One would expect that the percentage of all of the recoverable, cannibalizable items in the requirements database would be somewhat smaller than for war readiness spares. Although 58 percent is just an arbitrary number, it might not be far off.

Figure 1.2 contrasts the effects of computing spares requirements with the current system and with using the designated cannibalization approach we describe in Section 2. The usual 83 percent aircraft availability goal was specified in both computations; our example is again the F-16 weapon system. The darker bars in Figure 1.2 represent system performance assuming historical values of item pipelines. In this case, the computational approach that assumes designated cannibalization yields roughly the same aircraft availability as the current approach, but reduces the required investment by $247 million owing to the leaner mix of spares purchased under the designated cannibalization assumption. The lighter bars in Figure
1.2 tell essentially the same story for the case where item pipelines are decreased through quick response, as previously described. Again, we achieve about the same level of performance as the current system. These evaluations were done with DyNA-METRIC Version 4 (Isaacson et al., 1988) using 1989 item characteristics to evaluate stockage postures computed with the 1986 database.\textsuperscript{6}

We conclude that AFMC should incorporate designated cannibalization when estimating POS requirements; that is, items should be designated in the database as cannibalizable or not, and a requirements computational model should be used to model this explicitly. In the remainder of this report, we present the mathematical underpinnings of such an approach, describe an implementation of it that we used to compute the stockage posture underlying the results in Figure 1.2 portrayed by the bottom pair of bars, and suggest that the approach be refined, evaluated with more realistic data, and implemented in the spares and repair requirements estimation system.

\begin{figure}[h]
\centering
\includegraphics[width=0.5\textwidth]{Figure12.png}
\caption{Designated Cannibalization Pays Off}
\end{figure}

\textsuperscript{6}The variance-to-mean ratios observed in 1989 were capped at 5.0 in the evaluations to avoid the extreme values that seemed to be associated with data errors. About 5 percent of the items were affected.
The first step in the process is to designate items that are especially difficult, costly, or risky to cannibalize. It would then be possible to replicate the analysis discussed here and then evaluate the effects of the policy on cost and system performance. The evidence presented here strongly supports such a policy.
2. MODEL FORMULATION

Our approach examines aircraft availability as a function of stock levels allocated to a single base. Its underlying logic extends readily to the Air Force's Aircraft Availability Model (AAM), a multi-echelon computation of safety stock for aircraft recoverable spares. The model discussed here can be thought of as the model for an "average base," a concept that is implemented in the AAM. We assume that the distribution of backorders is known; thus we assume that the depot stock level has been specified. Lateral supply and any form of prioritization scheme are ignored, although prioritization may influence the numerical values of repair or transportation times observed historically and used in the computations. We focus on solving the problem of allocating stock levels of both cannibalizable and noncannibalizable items to a base.

In the discussion that follows, we use bold-face characters to denote random variables and underlining to denote vectors; underlined bold-face characters denote random vectors. Upper-case characters are used to name random variables, lower-case to denote realizations of random variables.

Suppose there are a aircraft on a base. Each aircraft has a complement of \( q_i \) LRUs of type \( i \) (\( i = 1, 2, \ldots, N \)); \( q_i \) is the quantity of type-\( i \) LRUs per aircraft (QPA), and is an integer that is frequently one, but may be larger. We assume for simplicity here that all aircraft on the base are configured similarly; therefore, there are a total of \( o q_i \) installations of type-\( i \) LRUs at the base. We call these \textit{slots} for short or \textit{i-slots} if we want to distinguish types. Because of redundancy, an aircraft may require only \( r_i < q_i \) of type-\( i \) LRUs to be mission-capable, but we assume here that all \( q_i \) of type-\( i \) LRUs are essential. We denote the stock level of item \( i \) by \( s_i \). The inventory system operates with an \((s, s - 1)\) reorder policy, i.e., a continuous review policy with an order quantity of one. The objective will be to maximize a quantitative measure of aircraft availability by choice of \( s_i \) allowing for budget constraints. The stock level, \( s_i \), comprises the item pipeline (the expected number of assets of type \( i \) in resupply) and safety stock (additional assets to cover the variability in the actual number of assets in resupply).

Assume that items labeled \( i = 1, 2, \ldots, n, \ 0 < n \leq N \), are not cannibalizable. If such an item fails on an aircraft and a spare is not available at the base, then it \textit{cannot} be replaced by one from another
aircraft (i.e., by cannibalization) but must be replaced from serviceable stock, depot replenishment, lateral supply, withdrawal from war readiness spares stocks, or another source. On the other hand, items designated \( i = n + 1, n + 2, \ldots, N \) are cannibalizable. If one of this ilk fails on an aircraft and there is no spare available at the base, then a serviceable component of this type can be removed from another aircraft, often one with a noncannibalizable failure, and used to replace the failed item. Clearly it is advantageous to consolidate shortages within as few aircraft as possible to minimize the number of unavailable aircraft and maximize the number available. We will always try to cannibalize from an aircraft that has a hole for a noncannibalizable part until all such opportunities are exhausted. In general, we will simply cannibalize from as few aircraft as possible.

Our first objective is to characterize the number of available aircraft as a function of backorders, or holes in aircraft. It is straightforward to compute the expected or mean number but more difficult to describe the probability distribution. However, we need the probability distribution and we propose an approach to approximating it in the remainder of this section.

**AIRCRAFT AVAILABILITY WITHOUT CANNIBALIZATION**

In the paragraphs that follow, we develop a model of the relationship between shortages of noncannibalizable items and aircraft availability. As a first step, we calculate the probability that an aircraft selected at random is unavailable as a function of stockage posture.

At any time \( t \) of system evaluation, let \( B_i(t) \) be the number of backorders or holes for items of type \( i \). These holes are presumably scattered randomly among different individual aircraft. If the particular item is type \( i \), where \( 1 \leq i \leq n \), then the holes cannot be filled from another aircraft; the particular holes cannot be rearranged or concentrated because item \( i \) is noncannibalizable.

Given \( B_i(t), i = 1, 2, \ldots, n \), it is interesting to ask how many distinct aircraft are now unavailable because of noncannibalizable shortages. Even when \( B_i(t) \) is known, the number of unavailable aircraft is still a random variable because the shortages are scattered among aircraft, so we assume for simplicity that all \( i \)-slots are equally likely to yield type-\( i \) holes. This apparently plausible assumption may be wrong, for there may be a systematic tendency for such holes to concentrate on some particular aircraft (tail numbers) rather than others, but such an effect would tend to produce more available aircraft, other things being equal, than would a uniformly random distribution. So an as-
umption of equal likelihood of holes across all slots is conserva\textit{tive}, in
that it tends to underestimate availability.

Given the backorder status for noncannibalizable items at time \( t \), i.e.,
the numbers of noncannibalizable items of each type backordered, we
have for the number of \textit{unavailable} aircraft resulting from noncannib-
alizalble failures alone

\begin{equation}
D_{nc}(t) = I_{1}(t) + I_{2}(t) + \ldots + I_{a}(t),
\end{equation}

where

\begin{equation}
I_{j}(t) = \begin{cases} 
1 & \text{if there is at least one noncannibalizable failure} \\
0 & \text{on aircraft } j \text{ at time } t, \; j = 1, 2, \ldots, a ; \\
& \text{otherwise.}
\end{cases}
\end{equation}

The expected number of unavailable aircraft given the backorder sta-
tus, \( \mathbf{B}(t) \), is given by

\begin{equation}
E[D_{nc}(t) | \mathbf{B}(t)] = E\left[ \sum_{j=1}^{a} I_{j}(t) | \mathbf{B}(t) \right] = \sum_{j=1}^{a} E[I_{j}(t) | \mathbf{B}(t)] .
\end{equation}

If all aircraft are assumed equally likely to have shortages, then
(suppressing \( t \))

\begin{equation}
E[D_{nc} | \mathbf{B}] = a (1 - P[I_{j} = 0 | \mathbf{B}] )
\end{equation}

\begin{equation}
= a P[D_{nc} | \mathbf{B}] ,
\end{equation}

where \( P[D_{nc} | \mathbf{B}] \) denotes the conditional probability that an aircraft
is down (unavailable) given the backorder position denoted by
\( \mathbf{B} = \{ B_{1}, B_{2}, \ldots, B_{n} \} \). Assuming independence of failures across LRU
types (not necessarily justified, but consistent with the AAM currently
used\textsuperscript{1}), the probability that aircraft \( j \) is available with respect to
noncannibalizable parts is given by:

\textsuperscript{1}O'Malley (1983).
\[ P\{ I_j = 0 | \mathcal{B} \} = \prod_{i=1}^{n} P\{ \text{no type-} i \text{ holes in aircraft } j | \mathcal{B}_i \} \]
\[ = \prod_{i=1}^{n} \left[ \frac{q_i (a - 1)}{q_i (a_i)} \right] = \prod_{i=1}^{n} P(A_{nc} | \mathcal{B}_i) \]
\[ = 1 - P(D_{nc} | \mathcal{B}) \]  

(5)

where we define \( P(A_{nc} | \mathcal{B}) \) as the probability that any (the \( j \)th) aircraft has no noncannibalizable shortage.

Note that the above calculation is for the mean or expected proportion of aircraft that suffer from shortages of noncannibalizable parts, given the backorder state. The unconditional version must also be computed.

In the course of a period of time, e.g., a year, or quarter thereof, backorders will fluctuate randomly. Suppose \( X_i \) is the random number of type-\( i \) items in resupply; then

\[ \mathcal{B}_i = \begin{cases} X_i - s_i & \text{if } X_i > s_i, \\ 0 & \text{if } X_i \leq s_i. \end{cases} \]  

(6)

It is customary to assign a distribution to \( X_i \), and furthermore to assume \( \{X_i, i = 1, 2, \ldots, n\} \) independent, so for the unconditional probability that aircraft \( j \) is unavailable at a randomly chosen point in time, given the spares allocation \( \mathcal{S} \),

\[ P(D_{nc} | \mathcal{S}) = E[P(D_{nc} | \mathcal{B}) | \mathcal{S}] = 1 - \prod_{i=1}^{n} E\left[ \frac{q_i (a - 1)}{q_i (a_i)} \right] | \mathcal{S} \].  

(7)

This expectation over all backorders must be computed numerically, given the joint distribution of \( \{X_i\} \), the number of assets of each type in resupply.

**AIRCRAFT AVAILABILITY WITH CANNIBALIZATION**

Suppose failed items of type \( i, i = n + 1, n + 2, \ldots, N \), can be replaced by serviceable items cannibalized from other aircraft at the base under discussion in the event that there are no other serviceable spares available. We discuss in the paragraphs that follow the number of aircraft unavailable owing to shortages of type-\( i \) parts.
Because shortages can be consolidated across aircraft, not more than \( k \) aircraft are down for holes of type \( i \) if

\[
B_i \leq kq_i. \tag{8}
\]

Therefore, not more than \( k \) aircraft are down for holes of all cannibalizable types if \( B_i \leq kq_i \) is simultaneously true for \( i = n + 1, n + 2, \ldots, N \), namely

\[
\bigcap_{i=n+1}^{N} (B_i \leq kq_i).
\]

If \( D_c \) is the number of aircraft down for cannibalizable items,

\[
P\left[ D_c \leq k \mid \mathbb{S} \right] = P\left[ \bigcap_{i=n+1}^{N} (B_i \leq kq_i) \mid \mathbb{S} \right] = P\left[ \bigcap_{i=n+1}^{N} (X_i \leq kq_i + s_i) \right]. \tag{9}
\]

As noted above, it has been customary to model the numbers of assets in pipelines, \( X_i \) for type \( i \), as independent, so

\[
P\{ D_c \leq k \} = \prod_{i=n+1}^{N} P\{ X_i \leq kq_i + s_i \}, \quad k = 0, 1, 2, \ldots, a. \tag{10}
\]

Equation (10) provides the probability distribution of the number of aircraft down for shortages of cannibalizable items.

**MODELING AIRCRAFT AVAILABILITY WITH DESIGNATED CANNIBALIZATION**

Let \( D_{nc} \) be the number of aircraft down for noncannibalizable parts and \( D_c \) the number of aircraft down for cannibalizable ones. Let \( D^* \) be the number of aircraft down for parts of either kind. Then

\[
D^* = D_{nc} \quad \text{if} \quad D_{nc} \geq D_c,
\]

since all cannibalizable shortages can be moved to the aircraft on which noncannibalizable holes occur. Otherwise

\[
D^* = D_c \quad \text{if} \quad D_c \geq D_{nc}, \tag{12}
\]
because all cannibalizable shortages possible will be moved to the aircraft with noncannibalizable holes, consolidating the surplus on as few aircraft without noncannibalizable holes as possible.

This just means that

$$D^* = \max \{ D_{nc}, D_c \}$$

and, hence,

$$P\{ D^* \leq \ell | \mathcal{G} \} = P\{ D_{nc} \leq \ell | \mathcal{G}, D_c \leq \ell | \mathcal{G} \};$$

assuming independence between backorders for cannibalizable and noncannibalizable parts,

$$P\{ D^* \leq \ell | \mathcal{G} \} = P\{ D_{nc} \leq \ell | \mathcal{G} \} \cdot P\{ D_c \leq \ell | \mathcal{G} \}. \quad (15)$$

Then the expected number of aircraft down in terms of spares allocations is

$$E[ D^* | \mathcal{G} ] = \sum_{\ell=0}^{a} P\{ D^* > \ell | \mathcal{G} \} = a - \sum_{\ell=0}^{a} P\{ D_{nc} \leq \ell | \mathcal{G} \}P\{ D_c \leq \ell | \mathcal{G} \}, \quad (16)$$

based on the well known identity

$$E(X) = \sum_{x=1}^{\infty} [1 - F(x)], \quad 0 \leq x < \infty.$$

In terms of the availability, $A$,

$$E[ A | \mathcal{G} ] = \sum_{\ell=0}^{a} P\{ D_{nc} > \ell | \mathcal{G} \}P\{ D_c > \ell | \mathcal{G} \}. \quad (17)$$

There seems to be no way of avoiding direct numerical evaluation of the sums in Equations (16) and (17).

**BUYING SPARES USING A SEPARABLE AVAILABILITY MEASURE**

The AAM that computes requirements for aircraft recoverable spare parts assumes that none of the parts is cannibalizable (O'Malley,
1983). The AAM exploits the fact that expected aircraft availability depends in approximate product form on individual item stock levels, provided backorder distributions for those individual items are (approximately) independent. Such an objective function is conveniently separable. The overall aircraft availability is a concave function of the stock levels; a marginal analysis algorithm guides an expedientious computation of the number of spares of each type that maximizes expected aircraft availability for any specified total investment level, \( C_{nc} \). The result for each value of \( C_{nc} \) is a suite of spares, \( s(C_{nc}) = s_1(C_{nc}), s_2(C_{nc}), \ldots, s_n(C_{nc}) \), that maximizes the expected aircraft availability with respect to noncannibalizable items. Given this optimal suite of spares, the probability that an individual aircraft chosen at random is available can be expressed as

\[ 1 - P(D_{nc} \mid \mathcal{E}) \]

by evaluating Equation (7).

Now suppose that one wishes to procure spares according to a policy of designated cannibalization. If this were to be done using the expected availability criterion, then expression (17) shows that a non-separable expression for availability results, so the previous analysis cannot be used. To easily adapt the existing software to incorporate designated cannibalization, we must change the objective function. Instead of optimizing expected availability, we will optimize a specified probability, \( \beta \), of achieving at least a specified aircraft availability goal, \( \alpha \), \( 0 < \alpha \leq 1 \). For example, if the availability goal \( \alpha = 0.85 \) and the probability \( \beta = 0.5 \), then spares are optimally selected so that the probability that at least 85 percent of the \( \alpha \) aircraft are available is one-half. That is, \( P(A \geq 0.85\alpha) = 0.5 \). Such an objective function has been successfully used before, in both the Dyna-METRIC series of models and in DRIVE (Distribution and Repair in Variable Environments).\(^2\)

Such a spares determination roughly resembles the optimal allocation for achieving the more traditional expected 85 percent availability, but the probability objective function is a meaningful criterion in its own right. Separability of the above criterion under the assumption of designated cannibalization follows, since if \( D^a \) is the number of aircraft down for both noncannibalizable and cannibalizable parts, \( D_{nc} \) being the number down for noncannibalizable parts, and \( D_c \) the number down for cannibalizable parts, then under the independence assumptions made previously,
\[ P\{A \geq a | \alpha \} = P\{D^\circ \leq (1 - \alpha)a\} \]
\[ = P\{D_{nc} \leq (1 - \alpha)a\} P\{D_e \leq (1 - \alpha)a\}. \] (18)

This function has the property of separability, but is to be distinguished from the partwise separability of the expected availability objective function incorporated in the AAM mentioned previously.

Assume an investment of \(C_e\) in cannibalizable spares of type \(i\), \(i = n + 1, n + 2, \ldots, N\). The method of determining \(\{s_{n+1}, s_{n+2}, \ldots, s_N\}\) that will maximize the probability
\[ \beta_e(C_e) = P\{D_e \leq (1 - \alpha)a\} \] (19)
since any specified investment constraint is well known.\(^3\)

Similarly, suppose an investment of \(C_{nc}\) is made in noncannibalizable items. The AAM will determine \(\{s_1, s_2, \ldots, s_n\}\) that will maximize \(P(A_{nc} | C_{nc})\), which we define as
\[ P\{A_{nc} | C_{nc}\} = \max_{s_1, s_2, \ldots, s_n} \left[ 1 - P\{D_{nc} | s\} \right], \] (20)
where \(P\{D_{nc} | s\}\) is given by Equation (7).

To approximate the probability that the number of aircraft unavailable for noncannibalizable items does not exceed the goal, use the binomial expression
\[ P\{D_{nc} \leq (1 - \alpha)a\} \]
\[ = \sum_{j=0}^{(1-\alpha)a} \binom{a}{j} [1 - P(A_{nc} | C_{nc})]^j [P(A_{nc} | C_{nc})]^{a-j}. \] (21)

The above approximation is discussed in the appendix. Since the above probability is a monotonically increasing function of \(P(A_{nc} | C_{nc})\), the spares allocation that maximizes \(P(A_{nc} | C_{nc})\) clearly also maximizes the above probability, giving
\[ \beta_{nc}(C_{nc}) = \sum_{j=0}^{(1-\alpha)a} \binom{a}{j} [1 - P(A_{nc} | C_{nc})]^j [P(A_{nc} | C_{nc})]^{a-j}. \] (22)

\(^3\) Isaacson et al. (1988).
Now we can define the optimal overall probability function of
\((C = C_{nc} + C_c)\) as
\[
\max_{c = C_{nc} + c_c} \beta_{nc}(C_{nc}) \cdot \beta_c(C_c) = \max_{0 \leq C_c \leq C} \beta_{nc}(C - C_c) \cdot \beta_c(C_c) \equiv \beta(C) \tag{23}
\]
for any specified value of \(C\). By construction, this is an increasing function of \(C\), as portrayed in Figure 2.1.

Consequently, if we want to specify that, say, 85 percent of aircraft are available with probability 0.5, then it is only necessary to read off the total cost value, \(C(0.5)\), and we can recover the optimal overall suite of spares. Here optimal means the least-cost way of achieving
\[
P\{A \geq \alpha \cdot \alpha\} = \beta ,
\]
where \(\alpha = 0.85\) and \(\beta = 0.5\) may be specific values of interest. Of course, the above procedure can be made to work for quite different values of both \(\alpha\) and \(\beta\). For example, if a decisionmaker wishes high assurance of at least \(\alpha\) (perhaps 0.85) aircraft availability he can set \(\beta = 0.90\); the algorithm based on the above logic will provide the spares list.

Figure 2.1—Total Probability of Achieving (100 - \(\alpha\)) Percent
Availability vs Cost
We emphasize that no claim is made that the above procedure, which has been programmed in the form of accessible and transportable software at RAND, optimizes expected availability when $\beta = 0.5$, except coincidentally. However, trial calculations have indicated that if spares are acquired as prescribed above, and the resulting expected availability evaluated using Dyna-METRIC Version 4, an analytic capability assessment model developed at RAND, the expected availability is close to the target (e.g., 85 percent) and this is achieved with a substantial reduction in spares cost over that required when an exclusive, traditional, no-cannibalization policy is followed.\textsuperscript{4}

If a higher degree of certainty is desired than the median, i.e., 0.5, that $100 \cdot \alpha$ percent aircraft availability is achieved, then replace 0.5 in the above by $x(0 \leq x < 1)$ and revise the calculations accordingly. To repeat, do not expect that the procedure suggested will give a spares allocation that precisely agrees with an allocation that maximizes the expected aircraft availability, but the two may well approximately agree. The procedure suggested, which has been implemented at RAND, can be used as a practical initial approach to the mean-optimal allocation; a search in the neighborhood of the present median solution for an allocation that maximizes expected value could prove efficient.

\textsuperscript{4}Isaacson et al. (1988).
Appendix

APPROXIMATING THE DISTRIBUTION OF THE NUMBER OF UNAVAILABLE AIRCRAFT WITHOUT CANNIBALIZATION

In this appendix we propose and examine approximations to the distribution of the number of aircraft down only for noncannibalizable items, i.e., of the random variable \( D(t) \) defined above. The approximations are based on the assumption that the number of holes (unfilled slots) at any time is likely to be small:

\[
h_i << a q_i, \quad i = 1, 2, \ldots, n.
\]

The approach taken is that of Poissonification (Gaver, 1976; Aldous, 1989). Suppose that there are \( h_i \) holes of type \( i, \ i = 1, 2, \ldots, n \), scattered at random over \( a \) aircraft. Then it is a somewhat complicated combinatorial exercise to write down a formula for the probability that exactly \( j \) aircraft have at least one hole of some type, whereas the others are hole-free, such that \( D(t) = j \). Such a calculation resembles the derivation of the classical hypergeometric distribution (see Feller, 1967). But suppose we allow \( n \) independent Poisson processes to generate holes in \( a \sum_{i=1}^{n} q_i \) slots in \( a \) aircraft for a unit time, the type-\( i \) rate being \( h_i \). By properties of homogeneous Poisson processes, the rate of arrival of type-\( i \) Poisson events per aircraft is \( h_i / a \). Therefore, the probability that a given aircraft experiences no (zero) events (holes) from any type of noncannibalizable failure is

\[
P_A(h) = \prod_{i=1}^{n} \exp\left(-\frac{h_i}{a}\right) = \exp\left[-\left(\sum_{i=1}^{n} h_i\right)/a\right] = e^{-h/a}, \quad (A.1)
\]

where \( h \) is the total number of noncannibalizable shortages. Then Equation (A.1) is an approximation to Equation (5) in the text; others have derived it by Stirling's approximation and Taylor series methods, but the present approach is more direct. Furthermore, because of the properties of the Poisson process, each individual aircraft receives holes independently and with the same probabilities, so if \( h = \{h_1, h_2, \ldots, h_n\} \), and the subscript \( p \) signifies the approximation by Poisson, then

\[
P\{A_p(t) = j | h\} = \binom{a}{j} \left(P_A(h)\right)^j [1 - P_A(h)]^{a-j}, \quad (A.2)
\]
the binomial. This tends to justify the binomial as a first approximation.

It is likely that spares will be bought in such a way that \( h = h_1 + h_2 + \ldots + h_n \) is of smaller order than \( a \). In this case, for large \( a \),

\[
E[A_p(t) | h] = aP_A(h) = ae^{-h/a} \equiv a, \quad (A.3)
\]

but if \( D_p(t) \) is the number of aircraft down, equal to \( a - A_p(t) \), then

\[
E[D_p(t) | h] = a \left( 1 - e^{-h/a} \right) \equiv h. \quad (A.4)
\]

As \( a \to \infty \) with \( h \) fixed, the distribution of unavailable aircraft approaches the Poisson:

\[
P\{D_p(t) = k | h\} = e^{-h} \frac{h^k}{k!}, \quad k = 0, 1, 2, \ldots \quad (A.5)
\]

Note that the results in Equations (A.2) and (A.5) are both conditional on the numbers of holes, which is equivalent to the number of backorders. Now the calculation of Equation (21), the optimal probability that the number of aircraft down for noncannibalizable items does not exceed \( (1 - \alpha)a \), should be made by first using Equation (A.2):

\[
P\{D_{nc} \leq (1 - \alpha)a | h\} = P\{A_p \geq \alpha a | h\}
= \sum_{j=\alpha}^{a} \binom{a}{j} \left[ P_A(h) \right]^j \left[ 1 - P_A(h) \right]^{a-j}, \quad (A.6)
\]

and then taking the expectation thereof to remove the condition on \( B = h \) for a given suite of spares, \( s \). Finally, the above should be optimized under a cost constraint such that the sum of the spares cost is \( C_{nc} \). In fact, the optimization proposed here reverses the order of expectation and optimization and is therefore an approximation. The justification for the approximation is the relative ease with which the software developed for the AAM can be adapted to the present purpose. The quality of the approximation deserves further investigation.
BIBLIOGRAPHY


