A MODEL FOR PREDICTING AVERAGE FIRE COMPANY TRAVEL TIMES

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PREFACE

This report was written as part of the research program on the deployment of fire-fighting resources undertaken by The New York City-Rand Institute for the Fire Department of the City of New York. It documents a mathematical model relating average travel times to the number of fire companies stationed in a region. The model has been applied in New York and other cities.

SUMMARY

In this Report we derive and test a fundamental model relating the average fire company travel time in a region, ET, to the area of the region covered, A, the number of fire companies stationed there, n, the rate at which alarms are received, λ, the expected total service time per alarm, ES, and some speed parameters. The model is one of several developed by The New York City-Rand Institute to analyze fire company deployment problems in New York City and has been applied there extensively. The model is

$$ET = \alpha + \beta \left(\frac{A}{n - \lambda ES}\right)^{\gamma}.$$

For New York City, where the model has been validated, estimates the values of parameters are: $\alpha = 0$, $\beta = 2.2$, and $\gamma = 0.3$. The model has been used in analyzing fire company deployment problems, and recent changes in the number and location of fire companies in New York City were based partially on this analysis.

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I. INTRODUCTION

In this Report we propose and test a simple model for predicting expected fire company travel times (the elapsed time between when a company leaves its house and when it arrives at the scene of a fire). The model states that, in a given region of a city, the expected travel time of the closest responding fire company, ET, is given by

$$ET = \alpha + \beta \left(\frac{A}{n - \lambda ES}\right)^{\gamma}, \qquad (1)$$

where A is the physical area of the region (square miles), n is the number of fire companies stationed there, λ is the expected number of alarms received per hour, and ES is the expected total service time (in hours) of all the fire companies that respond to and work at an alarm. α, β and λ are parameters having values that depend on the physical characteristics of the region. Field measurements and simulation experiments have validated the model for New York City and have shown that, in most regions of the City, the model is approximately

$$ET = 2.2 \left(\frac{A}{n - 0.5\lambda}\right)^{0.3} \qquad (2)$$

The model has been used as a rule of thumb in estimating how average travel times change with increases or reductions in fire company allocations, alarm rate, etc. It has been employed in studies that have led to fire company deployment changes in New York City and, consequently, to improved fire-fighting effectiveness [6], [9].

Equation (1) is based upon two earlier models--one relating expected response distances to the square root of the area served per available company [7], and the other relating expected travel time to response distance [8]. Both models are motivated by simple physical reasoning, are quite robust, and have been tested empirically under a wide variety of conditions. (By "robust" we mean that the basic form and parameter values of the model did not change significantly when we tested the model in different regions, with different dispatching rules, and for different types of apparatus.) Although most of our testing has been in New York City, the robustness of the model and fragmentary data from other cities makes it appear likely that the model is valid in many other places.

Several caveats are in order with regard to applications of the model to real problems.

- First, it is not necessary to "believe" the model. It is an approximation motivated by other approximations, which are then combined into another approximation. What we do believe--on the basis of examination of simulation data--is that the approximation is quite accurate in its predictions of averages for moderately large regions.
- Second, while the model is certainly useful in narrowing the range of alternatives considered and suggesting possibly desirable policies, its results should be double checked and corroborated by more detailed analyses before any concrete actions are taken. The model provides an estimate of average travel times in an average region. When dealing with a particular problem, more accurate estimates can be obtained by direct computation of distances between alarm points and fire company locations. Such computations might be carried out to get a better estimate of the consequences of policies that appear good according to the simple power law proposed here. How this can be done when selecting fire house sites is detailed in [3].
- Third, average travel time is only one of many possible criteria of fire-fighting effectiveness. Maximum travel times, the probability of running short of fire companies, and fire company workloads are among other measures to be considered.

II. BACKGROUND: THE RESPONSE DISTANCE AND TRAVEL TIME MODELS

In [7] it is hypothesized that ED, the expected distance traveled by the closest responding fire company to a typical alarm, is given by

$$E = k\sqrt{\frac{A}{n - \lambda ES}} \, . \tag{3}$$

Since λES is approximately the expected number of busy fire companies, the denominator n - λES is approximately the expected number of available companies and (3) states that expected travel distance decreases with the square root of the number of available companies. This equation was motivated by several mathematical models and was tested extensively, but the main motivation is essentially a dimensional argument: Area = $(\text{length})^2$, and area served per company decreases linearly with the number of companies added.

In [8], it is hypothesized that expected travel time, ET, obeys a simple function of the distance traveled, D. A derivation follows: Suppose that for short runs the fire company never reaches a full cruising velocity, but rather increases its speed for the first half of the trip as it accelerates, gets onto main thoroughfares, etc., and then decelerates for the last half of the trip as the process reverses. For longer runs, we hypothesize that there is a similar initial "acceleration" phase, but then the company runs at, or near, its cruising speed for some distance, finally "decelerating" as it nears the fire scene.

Let a = acceleration,

d_c = distance required to achieve cruising velocity,

v_c = cruising velocity.

Then, using basic physical relations, and assuming a constant acceleration and deceleration, a, during the initial and final phase, and a constant cruising velocity, v_c, during the middle phase, one obtains

$$E[T|D] = \begin{cases} 2\sqrt{D/a} & \text{if } D \leq 2d_c \\ v_c/a + D/v_c & \text{if } D > 2d_c. \end{cases} \tag{4}$$

This model is to be sure, an oversimplification. We do not believe that fire companies actually travel this way. Yet, it appears that the model gives accurate predictions. It has been extensively tested and validated for New York City.

III. COMBINING THE TRAVEL TIME AND RESPONSE DISTANCE MODELS

Now we combine the travel time model with the response distance model and, by so doing, we relate expected travel time in a region to a few easily measured parameters. To motivate the approximation and the supporting data analysis that follows, we begin with some mathematical formalities.

Consider a particular region of the city. Let $T(x)$ denote the conditional expected travel time when the response distance is x, and let $F_D(x)$ denote the cumulative probability distribution of response distance. Then ET, the unconditional expected travel time (to an alarm an arbitrary distance away), is

$$ET = \qquad (x).$$

Assuming the validity of the travel time model, this becomes

$$ET = \int_0^{2d_c} 2\sqrt{\frac{x}{a}}\, dF_D(x) + \int_{2d_c}^{\infty} \left(\frac{v_c}{a} + \frac{x}{v_c}\right) dF_D(x). \tag{5}$$

Recognizing that both segments of the $T(x)$ function are concave, by Jensen's inequality we have:

$$ET \leq \begin{cases} 2\sqrt{\dfrac{ED}{a}}, & \text{if } ED \leq 2d_c \\ \dfrac{v_c}{a} + \dfrac{ED}{v_c}, & \text{if } ED > 2d_c. \end{cases} \tag{6}$$

However, by ignoring the inequality, we use (6) to approximate ET. In addition, we replace ED by (3), obtaining the approximation

$$ET \simeq \begin{cases} c_1 \left(\dfrac{A}{n - \lambda ES}\right)^{1/4}, & \text{if ED is "small"} \quad (7a) \\ c_2 + c_3 \left(\dfrac{A}{n - \lambda ES}\right)^{1/2}, & \text{if ED is "large."} \quad (7b) \end{cases}$$

From (7a) and (7b), we are led to hypothesize the general form for expected travel time given by equation (1). (We note that approximations (1), (7a) and (7b) were derived from two inequalities, one $\geq$ and one $\leq$.)

IV. VALIDATING THE MODEL

In order to test the validity of these models, we ran a series of simulations similar to those discussed in [7]. We used the same simulation model of fire-fighting operations in the Bronx that is documented in [1] and [2]. Simulations were done at different alarm rates and with different numbers of companies assigned to one region of the City, the Borough of the Bronx. Each simulation was run for about 3,000 alarms. Data were collected separately for engine companies ("engines") and ladder companies ("ladders") and for two parts of the Borough--north Bronx and south Bronx.* In this series of simulations, the travel times to alarms were generated as follows:

- First, the response distance was calculated using an empirically validated function (see Reference 1, pages 35-38), the inputs to which are the coordinates of the house of the responding engine, the coordinates of the alarm, and the orientation of the street grid.
- Second, the travel time was calculated by applying the piecewise square-root-linear travel time vs. response distance function (4) with empirically determined parameter values for the Bronx. We must point out here that our simulation model incorporates one of the component models used in the heuristic derivation of our hypothesis.

The simulation outputs used here are average travel times, denoted by $\overline{T}$, and average number of available units, denoted by $\overline{N}$. These data were used to fit the following models by linear or nonlinear regression:

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*An engine company is a group of firemen who man a pumper and have the responsibility for delivering water onto a fire. A ladder company is a group of firemen who man a truck equipped with an aerial ladder and are responsible for forcible entry, rescue, ventilation, and salvage work.

Model I: $\bar{T} = \alpha + \beta \; (A/\bar{N})^{\gamma}$

Model II: $\bar{T} = \alpha + \beta \; (A/\bar{N})^{0.50}$

Model III: $\bar{T} = \alpha + \beta \; (A/\bar{N})^{0.25}$

Model IV: $\bar{T} = \beta \; (A/\bar{N})^{\gamma}$

Model V: $\bar{T} = \beta \; (A/\bar{N})^{0.50}$

Model VI: $\bar{T} = \beta \; (A/\bar{N})^{0.25}$

Regression analyses were performed, with $\bar{T}$ being the average travel times of closest ladders and $\bar{N}$ the average number of ladders available, for the following sets of data.

Set 1: The South Bronx. This set consists of 12 (pairs of) observations ($\bar{T}$, $\bar{N}$) from the South Bronx region, where each observation is the average over one entire simulation.

Set 2: The North Bronx. This set consists of 12 pairs of observations as above except that the observations are from the North Bronx region.

Set 3: The Entire Bronx. This set consists of 12 pairs of observations as above except that the observations are from the entire Bronx. Since the data all come from the same simulations, Set 3 is not independent of Sets 1 and 2.

Set 4: A concatenation of Sets 1 and 2. This set consists of the 24 pairs of observations comprising Sets 1 and 2. Note that it is different from Set 3.

No engine data are included in the above, nor were any engine data used in model fitting. Instead, engine data were held in reserve and used subsequently to test the models that had been fitted to the ladder data.

Table 1 summarizes the results of the regression. It contains the parameter estimates and proportion of original variance explained, r^2, for each model using each of the four sets of data. Inspection of the tabulated results shows that:

Table 1

RESULTS OF REGRESSIONS OF SIMULATED TRAVEL TIMES VS. AVAILABILITY FOR LADDERS

		Sets of Data															
		Set 1: South Bronx				Set 2: North Bronx				Set 3: All Bronx				Set 4: North & South Bronx (Set 1 ∪ Set 2)			
	Models	α	β	γ	r^2	α	β	γ	r^2	α	β	γ	r^2	α	β	γ	r^2
I.	$\overline{T} = \alpha + \beta(A/\overline{N})^{\gamma}$	0.33	1.85	0.36	0.96	-1.91	4.00	0.19	0.87	1.30	0.69	0.69	0.96	-1.29	3.45	0.20	0.94
II.	$\overline{T} = \alpha + \beta(A/\overline{N})^{0.50}$	0.95	1.22	0.50	0.96	1.37	0.93	0.50	0.85	0.81	1.15	0.50	0.96	1.18	1.03	0.50	0.92
III.	$\overline{T} = \alpha + \beta(A/\overline{N})^{0.25}$	-0.67	2.85	0.25	0.96	-0.52	2.68	0.25	0.87	-1.06	2.96	0.25	0.96	-0.50	2.68	0.25	0.94
IV.	$\overline{T} = \beta(A/\overline{N})^{\gamma}$	--	2.17	0.32	0.96	--	2.19	0.29	0.89	--	1.92	0.35	0.95	--	2.18	0.29	0.94
V.	$\overline{T} = \beta(A/\overline{N})^{0.50}$	--	1.92	0.50	0.62	--	1.60	0.50	0.37	--	1.63	0.50	0.78	--	1.69	0.50	0.49
VI.	$\overline{T} = \beta(A/\overline{N})^{0.25}$	--	2.25	0.25	0.92	--	2.30	0.25	0.85	--	2.12	0.25	0.87	--	2.28	0.25	0.92

- Model V, $\overline{T} = \beta(A/\overline{N})^{0.50}$, is distinctly inferior to the others as measured by the proportion of the original variance explained. This holds for each data set.
- The other models are essentially equivalent in the proportion of variance explained, but notice that Models I, II, and III--all of which have constant terms--have only slightly higher r^2 values. Notice also that the estimated parameter values for Models I, II, and III vary quite a bit across the different data sets, even though the fits are good. On the basis of this fact, we eliminate them from further consideration.
- Models IV and VI are the remaining contenders. Letting γ be a free parameter results in values near, but different from, the hypothesized value of 0.25. The differences are statistically, but perhaps not operationally, significant. (See Table 2, which gives approximately 95 percent confidence intervals on the parameters of Model IV.)

Figures 1, 2, and 3 show Models IV and VI fitted to Set 1, Set 2 and Set 3. The fits of each are good and it is interesting to note that the resulting curves of $\overline{T}$ vs. $\overline{N}$, although different, are quite close. Figure 4 shows Models IV and VI fitted to Set 4. (The curves appear reversed from the convex form of Figures 1 through 3 because, since two sets of data with different areas were combined, we used $A/\overline{N}$ as the abscissa instead of $\overline{N}$.) Again, the difference between the curves is very slight. Finally, Figure 5 shows the curves of Models IV and VI and the travel time data for engines corresponding to Set 3 for ladders (the whole Bronx). These data points were not used to fit the functions, yet the correspondence is quite good.

Table 2

CONFIDENCE INTERVALS FOR PARAMETERS OF MODEL IV:
95% CONFIDENCE INTERVALS ON β AND γ FOR MODEL $\overline{T} = \beta(A/\overline{N})^{\gamma}$

Data Set	Interval on β			Interval on γ		
	L_β	$\hat{\beta}$	U_β	L_γ	$\hat{\gamma}$	U_γ
1	2.127	2.173	2.219	0.2908	0.3158	0.3393
2	2.112	2.187	2.262	0.2639	0.2874	0.3090
3	1.871	1.918	1.966	0.3290	0.3512	0.3728
4	2.142	2.184	2.225	0.2746	0.2923	0.3093

Note: The L_β and U_β columns contain observations on random variables L_β and U_β having the property that

$$P\{(L_\beta \leq \beta)\cap(U_\beta > \beta)\} = 0.95.$$

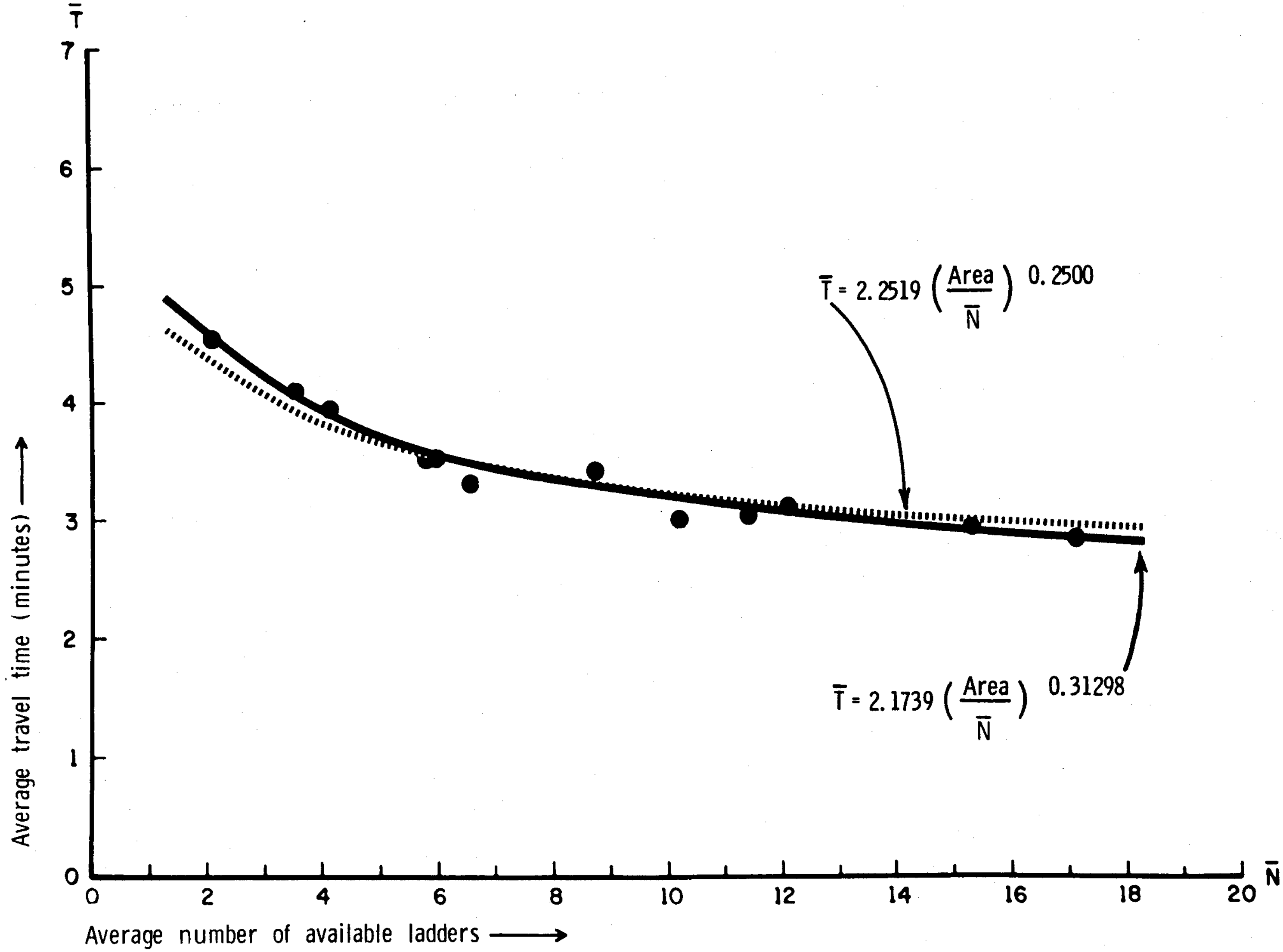

Figure 1. Average ladder travel time vs. average number of ladders available--South Bronx Region, Data Set 1.

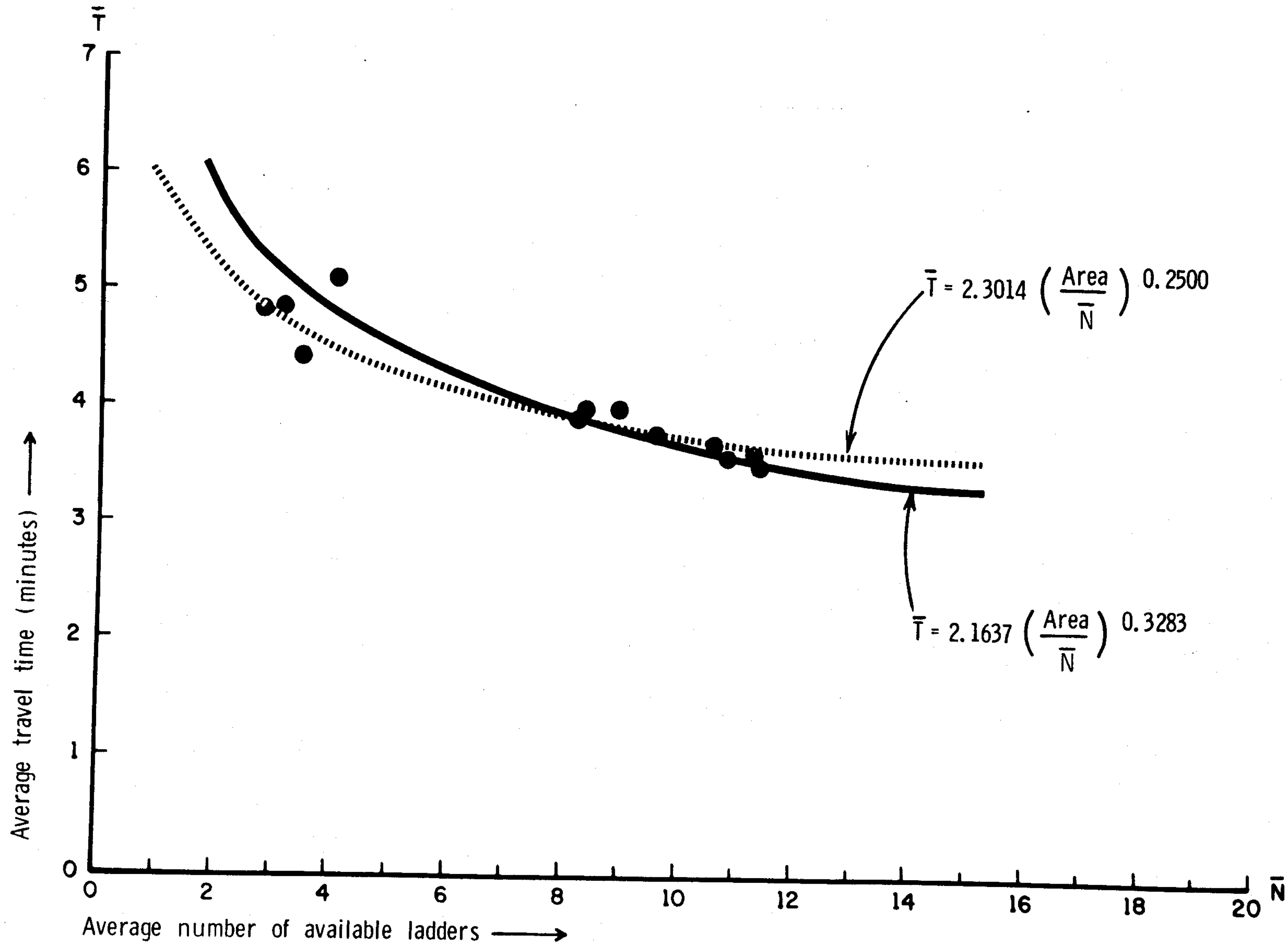

Figure 2. Average ladder travel time vs. average number of ladders available--North Bronx Region, Data Set 2.

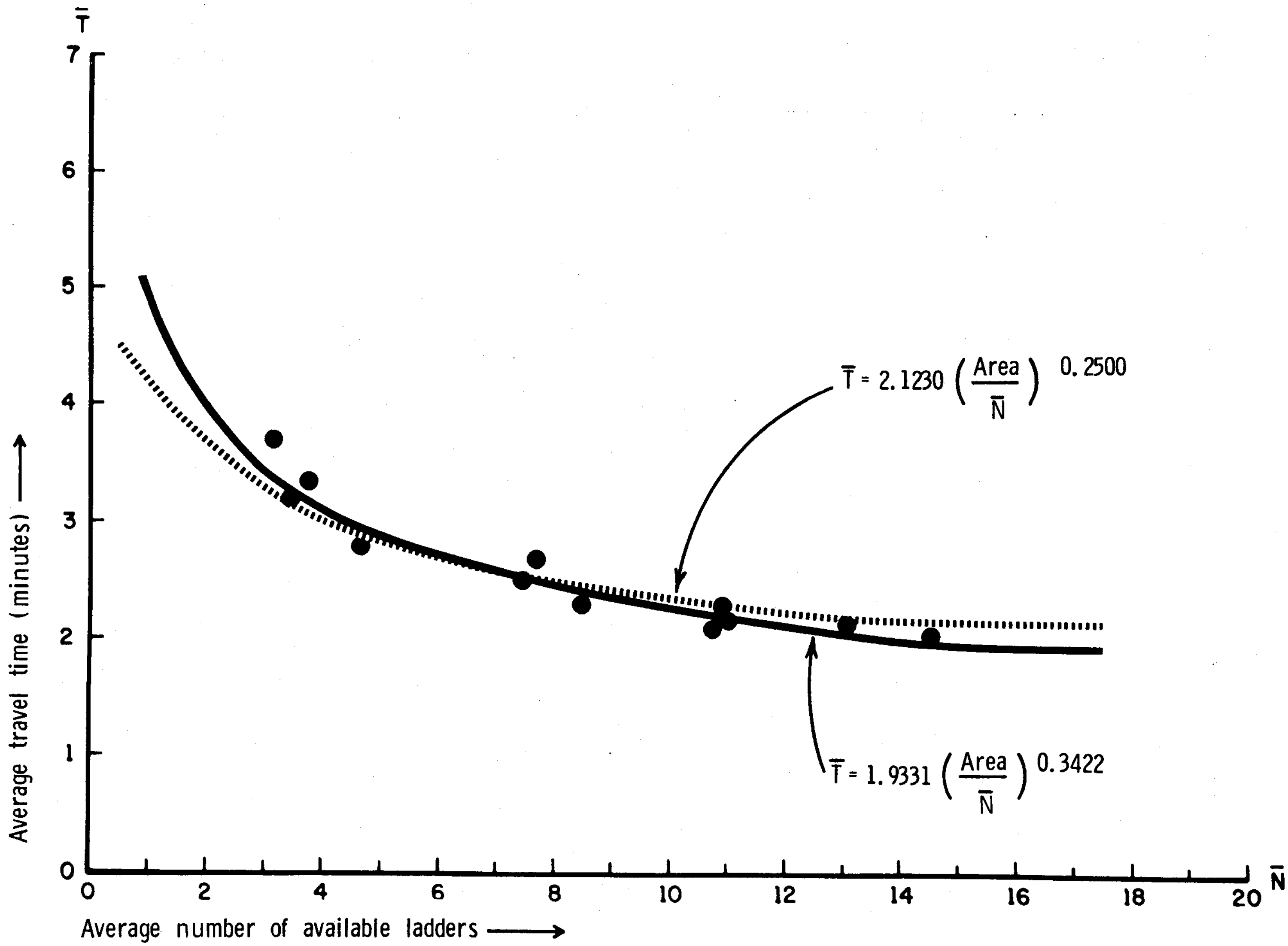

Figure 3. Average ladder travel time vs. average number of ladders available--the entire Bronx, Data Set 3.

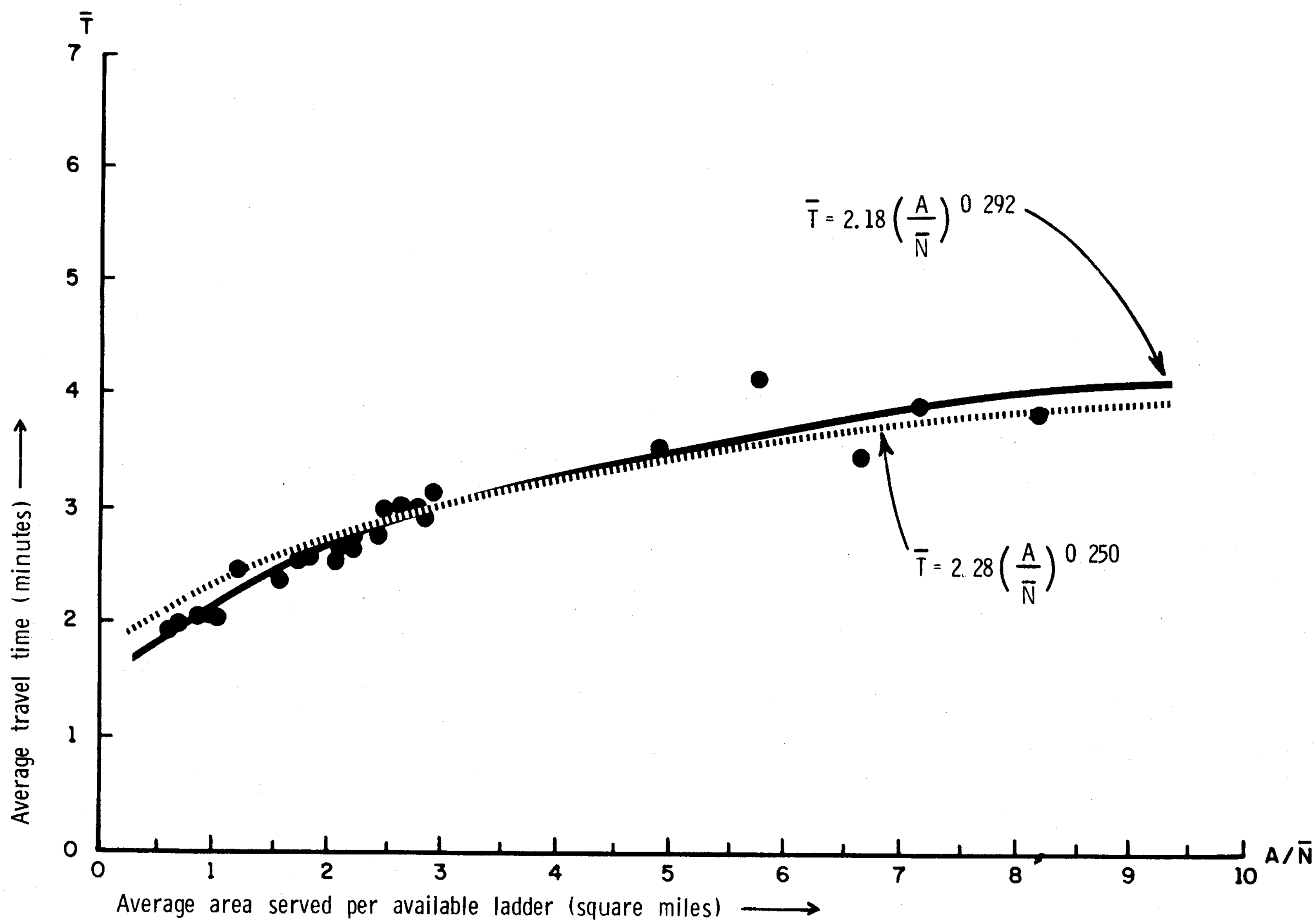

Figure 4. Average ladder travel time vs. average area served per available ladder--North Bronx and South Bronx, Data Set 4.

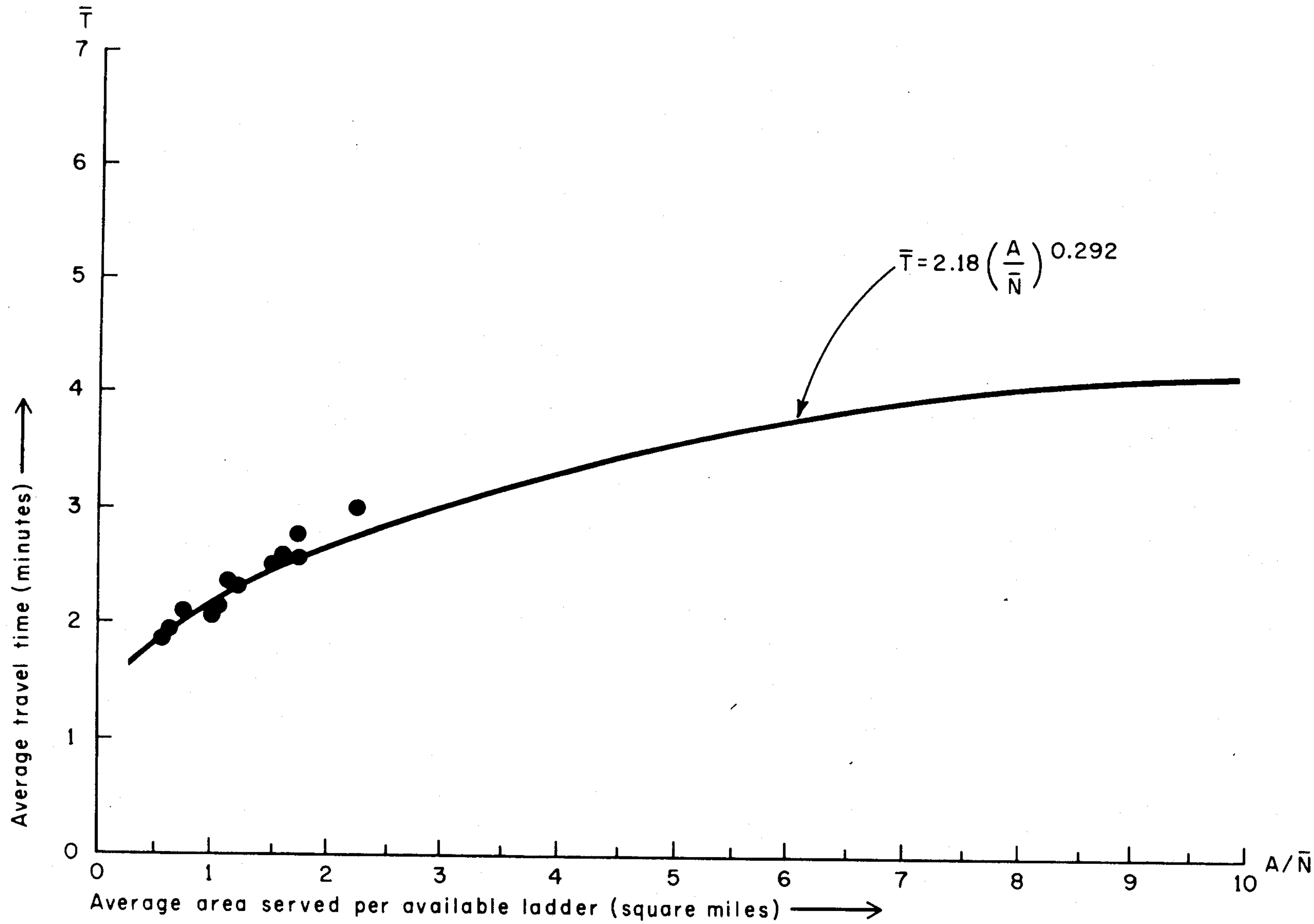

Figure 5. Average engine travel time vs. average area served per available engine.

V. CONCLUSIONS

The results presented in the previous section show that the postulated power models give good estimates of simulated average fire company travel times for the Bronx. In the Bronx, travel distance is short--about one-half mile on the average--and so, as suggested by equation (7a), we found that Model VI gave a good fit. Still better estimates were given by Model IV with a parameter value of β of about 0.3. This is not surprising since equation (7a) is a lower bound on expected travel time, and a better approximation will be obtained with coefficient greater than 0.25.

Because several of the components of the power model have been validated in other parts of New York City, we are confident that it gives good estimates of average travel times throughout the City and that Model IV applies in regions where, like the Bronx, average travel distances are short. Because of the robustness of the model and partial validation of its components in other cities (see [5], for example), we have little reason to doubt its wider applicability. Of course, in those regions where average travel distances are long, we expect that Model II would give the best estimates.

Finally, some remarks about estimating the parameters of the model. The results of Section IV show that there is little difference between Models IV and VI, so a precise estimate of the exponent γ is not crucial, particularly when using the model as a rule of thumb. However, 0.3 seems a good choice. The coefficient β can be derived from (3) and (4) and is equal to $\sqrt{k/a}$. Both k, the scale factor of the distance model, and a, the acceleration parameter of the time-distance model, can be estimated for any region of interest from simple field experiments as discussed in [4], [7], and [8]. Actually, in our experience, the values of k and a have been quite invariant across several regions and cities, and one cannot go far wrong using the value of β estimated in New York, namely $\beta \approx 2.2$.

In cities or regions where average response distances are longer (7b) can be expected to apply, that is

$$ET \approx c_2 + c_3 \left(\frac{A}{n - \lambda ES}\right)^{1/2} .$$

The values of c_2 and c_3 may be estimated by relating them to the parameters of the two component models. First, c_2 is v_c/a and c_3 is k/v_c. As mentioned above, our experience in several cities and in several regions within New York City indicates that the estimates derived in New York are quite robust. In the absence of better data, they can be used to give approximate results for other cities. The numerical values obtained in New York are $c_2 = 1.35$ minutes and $c_3 = 0.76$ minutes/mile.

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