Resource Allocation Under the COWPS Price Guideline: The Case of Fixed Proportions

Frank Camm, Charles E. Phelps, P. J. E. Stan
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Gasoline shortages returned to the United States for the second time in a decade during the spring of 1979. While many persons believed that U.S. Department of Energy (DOE) refinery controls had caused those shortages, significant evidence emerged to refute this explanation. Shortages also prevailed in products not controlled by DOE, and DOE-administered gasoline price controls did not constrain most refiners' prices throughout the period of the shortages. Thus the authors of the present report turned to the price guidelines administered by the Council on Wage and Price Stability (COWPS) as an alternative explanation for the shortages.

This report, sponsored by the Department of Energy with supplemental funding from The Rand Corporation, develops the theory of a profit-maximizing firm constrained by COWPS price guidelines. The nature of COWPS guidelines for refiners changed several times during the period of the shortages. The authors provide here a general formulation of such controls which encompasses all of the actual variants employed by COWPS. It is primarily an exposition of economic theory. Empirical evidence surrounding these events is discussed in a companion Rand report by Charles E. Phelps, Frank Camm, and P.J.E. Stan, Petroleum Product Shortages, 1979-1980, R-2766-RC (forthcoming).

These results should be of interest to those agencies and individuals concerned with U.S. energy policy and the causes of the 1979-80 petroleum shortages. The results should also interest those concerned with theories of the regulation of business, those studying prices and price controls, and those analyzing aggregate data on prices through time periods in U.S. economic history that include similar price controls.
SUMMARY

Refiners faced "voluntary" price and wage guidelines covering all products administered by the Council on Wage and Price Stability (COWPS) from October 1978 through December 1980. They also faced controls for gasoline prices from the Department of Energy (DOE), but there were no DOE controls on other products for which shortages emerged at the refinery level during 1979 and 1980. Further, the DOE gasoline price controls were not a binding constraint on the pricing of most U.S. refiners during the period of shortages. This forces the question: If the DOE controls were not binding, what caused the shortage?

The COWPS guidelines allowed refiners to choose one of two standards for calculating allowable revenues: The "gross margin" standard allowed all increased costs of crude oil to be passed through with higher revenues by refiners ("dollar-for-dollar passthrough"), but placed constraints on which other cost increases could be passed through. An alternative "profit margin" standard was similar, except that more input costs were allowed to be passed through dollar for dollar. Both standards can be characterized by a formula specifying

\[
\text{allowable revenues} = \left( \frac{\text{actual allowable costs}}{\text{allowable costs}} \right) + \left( \frac{\text{historic other costs}}{\text{other costs}} \right) + \left( \frac{\text{adjustment factor}}{\text{factor}} \right)
\]

where the adjustment factor may or may not depend upon the level of output of the firm. In general, this constraint appears in the general form

\[
\sum p_i y_i = w_a a + \alpha + \beta \sum y_i,
\]

where \( p_i \) are selling prices of product \( i \), \( y_i \) is the amount of product \( i \) sold, \( a \) is "allowed" inputs (crude oil, or a broader set of inputs), and \( w_a \) is the price of input \( a \). The parameters \( \alpha \) and \( \beta \) depend upon historical data for each firm regulated as well as public policy choices.
From these guidelines, one can derive an implied constraint on the profits of the complying firm and a nonlinear constraint on product prices. But the firm also faces the constraint of market prices for its products and the constraints imposed by its productive technology. Taken together, these constraints modify the behavior of a complying firm significantly from that of a simple profit-maximizing firm.

If and only if the productive technology of the firm embodies certain types of fixed proportions (as appears to be the case for refiners), then complying with COWPS guidelines can lead to a shortage of refined product from any given refiner. If the COWPS constraint becomes so binding that the profit-maximizing refiner cannot charge market-clearing prices for its products at any level of output, then it will select the output that leads to highest COWPS-allowed profits and will charge less than market prices. This, of necessity, leads to rationing of the product by the refiner among its customers and is a prima facie condition of shortage. Importantly, this theory shows how rationing of products can occur for products that were totally uncontrolled by the DOE, such as diesel fuel, heating oil, jet fuel, and other products. And indeed, there was nonprice rationing for each of those products by some refineries during the shortages, rationing that cannot logically be attributed to the DOE price controls on refineries.

These types of controls need not cause shortages, even if firms are complying and even if the COWPS constraint is binding. As the controls were written during the COWPS regime, a refiner could possibly find some level of output that provided maximum possible COWPS profits while still charging market-clearing prices for its products. If so, the refiner would not have to ration its product, and there would be no shortage.

Forces that increase the likelihood of shortage (for a given set of COWPS regulations) include increases in market demand for refined products, or increases in the cost of "nonallowed" factors of production. In either case, the profit-maximizing refiner will respond by reducing output (in the single-product case) and by shifting the product mix toward products with lower marginal costs of production in a
multiproduct case. (The weighted sum of product outputs will also likely fall in the multiproduct case.) If many refiners in the market are responding similarly, the reduced market supply could cause market prices to increase still further, thus exacerbating the original condition leading to reduced supply. As market price continues to rise, some refiners may eventually be forced into a position of shortage, whereby their output becomes unresponsive to market prices.

Other economic conditions can lead to the eventual elimination of the shortages. In the case of the refiner, for example, higher crude oil prices eventually can eliminate shortages facing refiners and (paradoxically) at the same time increase the output of the COWPS-constrained refiner. This arises through an interplay between forces in an unconstrained market and the regulations. In a free market, if a factor price (such as that of crude oil) increases, there will typically be less than 100 percent passthrough of that increased cost of productive factors, so long as there are any specialized factors of production in the industry. (Refinery capital would, of course, be a highly specialized factor of production.) In such a case, the reduced demand for the final product (arising from higher product price) would reduce demand for those specialized factors, thus causing the value of the special factors to fall. These special factors thus "absorb" some of the increase in costs of the other factor (crude oil, in the refining sector). This implies that market prices would not rise as much as crude oil costs have increased: The market does not typically allow 100 percent passthrough of increased factor costs. On the other hand, the COWPS rules do allow a 100 percent passthrough of increased crude oil costs in computing allowable revenues. Thus, as prices of crude oil increase, there is an inherent tendency toward "decontrol" of a COWPS-bound firm, because the cost-passthrough provisions of COWPS are relatively more generous than the market in their treatment of increasing crude oil costs. In such a way, increasing crude oil costs can remove a firm from a shortage position into one where market prices are being charged.

Another way to reduce the chance of shortages is to relax the COWPS guidelines. This, in fact, occurred in January 1980 for
refiners using the "gross margin" standard of COWPS. At that time, COWPS for the first time allowed refinery fuel costs to be included as an "allowed" factor of production, rather than those costs being embedded in the refiners' gross margins. Since refinery fuel costs are primarily petroleum products and their derivatives, the same forces causing crude oil costs to rise (the Iran oil shutdown, among others) would have led to increased refinery fuel costs. By allowing them to be included in the "allowed" factor costs beginning in 1980, COWPS thereby reduced the constraint on some refiners considerably. This, too, would tend to eliminate shortage conditions and to increase output of the constrained firms.

Some casual empirical evidence supports the belief that the COWPS controls were in fact the determinate cause of refinery-level shortages in 1978-1980. The variability in refinery prices for specific products increased just after COWPS was introduced and the price variability across firms seemed to diminish markedly upon the partial decontrol of January 1980. Evidence from retail gasoline markets further supports this notion—upward pressures on retail gasoline price "margins" appeared coincident with the COWPS controls and began to diminish shortly after the partial "decontrol" of January 1980. This evidence is consistent with a shortage at the refinery level induced by COWPS.

While much of the theory of refiner behavior under a COWPS constraint is the same, whether or not the firm is treated as a multiproduct or a single-product firm, several important differences in fact emerge in a multiproduct analysis. The first, briefly considered previously, is that the effects of market and regulatory forces on output of individual products become ambiguous, whereas firm-specific predictions can be made in the single-product firm. A second important difference arises in product pricing. Under a binding COWPS constraint, the refiner may literally choose any set of product prices he wishes so long as the overall COWPS revenue constraint is not violated, and so long as he does not attempt to charge prices higher

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than the market constraints will allow. This implies that the relative prices of products among firms will become more variable under COWPS than in a free market (where the relative prices should be very similar across firms). Casual evidence shows that refiner relative pricing decisions did indeed become more random after COWPS, further supporting the belief that COWPS had induced significant changes in refiner behavior.
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I. INTRODUCTION

During 1979 and 1980, many American petroleum refining companies engaged in voluntary, nonprice rationing of a variety of petroleum products, including gasoline, kerojet and diesel fuel, and no. 2 fuel oil. Such rationing is prima facie evidence of a shortage. Department of Energy (DOE) price regulations might have induced some of these shortages, but (a) by 1979 the regulations applied only to gasoline, and (b) there was strong evidence that they were not effective. The refining companies themselves claimed that their rationing behavior resulted from their intentions to comply with the voluntary guidelines of the Council on Wage and Price Stability (COWPS). Setting aside the important issue of why the oil companies would forgo profits, simply to comply with voluntary price guidelines, this report analyzes how these guidelines, if followed, would affect a profit-maximizing, competitive firm's decision on general resource allocation.

The model developed is general enough to apply to industries other than oil refining. That is, while COWPS provides special guidelines for petroleum refining firms, a generalized form of those guidelines applies to a wide variety of activities. Basically, firms who "cannot" meet COWPS general goals for reducing the rate of output price increase have the option of holding the rate of increase in some COWPS-prescribed

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1Companies who voluntarily rationed their gasoline simultaneously built up their "banked costs" under the DOE regulations. Banked costs are price increases that the regulations allow a company at a particular time but that the company elects not to take at that time. Under such circumstances, the DOE regulations do not bind prices; they are not effective. For details, see Phelps and Smith (1977).

2Forgone profits were substantial. A lower bound estimate is provided by the increase in gasoline retailer margins occurring in 1979, from about 7c/gal to over 14c/gal (Lundberg, 1980). For annual gasoline consumption of 120 billion gallons, this represents forgone profits of over $8 billion annually, ignoring dollar equivalences of queuing and forgone profits on other products.

3Phelps, Camm, and Stan (forthcoming) present an empirical analysis of the response of petroleum refining firms to the COWPS guideline. The present report, while providing background for that study, reports results of importance not just to petroleum refining, but to a wide variety of activities subject to the COWPS price guideline.
accounting margin below a level defined by a specific rate of price increase and the rate of output change. For some firms, that margin is effectively defined as returns to capital; for others it includes all revenues except those used to pay for materials. In each case, however, the guideline displays two features. First, it allows firms who increase output to increase their margins and thereby increase profits. Second, it allows firms to include the costs of some inputs but not others in calculating allowable revenues (or, what is the same thing, excludes some input costs but not others from the controlled margin). This report examines firm and market response to a price control with these two general features.

In this report, we concentrate on the effects of COWPS guidelines on a firm with fixed-production coefficients between certain subsets of inputs. The fixed-proportion technology is a good basis for analysis of the petroleum refining industry and other material-intensive industries like metal wire, sheet metal, and cement. These industries, where the product is a transform of one input, display very limited opportunities to substitute between materials and other inputs. In the particular case of refining, as a refinery attempts to substitute toward crude oil, holding product mix constant, it eventually reaches a point where further substitution is impossible.

Section II reviews the COWPS guidelines and states the general results on resource allocation. Section III examines the case of firms with one output and the market behavior that the guideline induces when only single-output firms are present in an industry. Section IV extends the results to the multiproduct case. Section V concludes by providing some casual evidence that the phenomena implied by the model developed here can be observed in the petroleum refining industry.

\[4\] In subsequent work, we plan to analyze the consequences of COWPS guideline compliance with completely variable proportions.

\[5\] We base this understanding on Griffin (1971) and on conversations with refining operators.
II. PROFIT MAXIMIZATION UNDER THE COWPS PRICE GUIDELINE

The COWPS price guideline, though relatively simple in concept, has proven quite complex in practice. Though it was in place for 2-1/4 fiscal years, it was changed numerous times. And, at any given time, COWPS allowed some leeway in its precise interpretation. In this section we use COWPS initial definition of the gross margin guideline relevant to petroleum refining to develop a simplified analytic version of the guideline. Properly interpreted, this analytic representation spans the changes that the guideline has undergone and in fact can represent many of COWPS guidelines for other industries and the price controls of previous administrations (e.g., the Nixon era controls).

Having derived a simple form of the price guideline, we then integrate it into the price theory of a multiproduct firm with a special kind of fixed proportions in production. Again, this is the situation relevant to petroleum refining; it could also represent many other manufacturing industries. In this section we develop a basic profit function that serves as a basis for the analysis in the remainder of the report.

A SIMPLE ANALYTICAL SPECIFICATION OF THE GUIDELINE

The price guideline under COWPS had two distinctive features. First, it concentrated on controlling not price but an accounting margin defined as the difference between total revenues and "allowable costs":

$$\sum_{i=1}^{m} p_i^s y_i - w_a$$

(2.1)

where the firm produces m outputs, the ith sold at a price of $p_i^s$ and a level of $y_i$, and uses an aggregate allowable input with an exogenously determined price, $w_a$, at a level of a. For example, "for petroleum refinery operations, the gross margin is equal to net sales
(gross sales adjusted for discounts, rebates, and other allowances) less the cost of petroleum, petroleum products, natural gas, natural gas liquids, and natural gas liquid products used in refinery operations." (COWPS, 1979, pp. 2-7.) The alternative profit margin constraint was similar, except that a broader set of input factors could be subtracted from net revenues. The structural similarity to utility rate-of-return (AJ) regulation (Averch and Johnson, 1962) is apparent, but the consequences of such regulation in a competitive industry are, as we shall demonstrate, considerably different from those in the usual AJ case.

Second, the guideline limited the percentage growth in this margin to an amount based on the percentage change in the firm's output level:

\[
\frac{\sum_{i=1}^{m} p_i y_i - w_a}{\sum_{i=1}^{m} p_i y_i} - 1 \leq (1.065) \left[ \frac{\sum_{i=1}^{m} y_i}{\sum_{i=1}^{m} y_i} - 1 \right] \quad (2.2)
\]

where the null superscript indicates the base period value of a variable. During the first year of application, FY79, reductions in output were not counted against the margin; the limit on the right side became a constant of 0.065. Increases in output were treated as shown in Eq. (2.2), so that a rise in output increased the size of the allowable gross margin. During the second year, FY80, output increases and decreases were treated symmetrically as in Eq. (2.2). The second year also incorporated several changes in the indices used and expanded the scope of allowable costs by treating petroleum and petroleum products consumed as fuel in refining as an allowable expense. But, properly interpreted, Eq. (2.2) serves as a reasonable representation of the COWPS gross-margin and profit-margin guidelines during its entire application.

Rearranging Eq. (2.2) provides a useful formulation of the guidelines:
\[
\sum_{i=1}^{m} p_{1}^{S} y_{1} - w_{a} a \leq (1.065) \left[ \frac{\sum_{i=1}^{m} p_{i}^{y} y_{1}^{o} - w_{a}^{o} a}{\sum_{i=1}^{m} y_{1}^{o}} \right] \sum_{i=1}^{m} y_{1}^{o}
\]

By substituting constants \( \alpha = (1.065) \left[ \sum_{i=1}^{m} p_{i}^{y} y_{1}^{o} - w_{a}^{o} a \right] \) and \( \beta = 0 \) for output reductions in the first year and \( \alpha = 0 \) and \( \beta = (1.065) \left[ \sum_{i=1}^{m} p_{1}^{y} y_{1}^{o} - w_{a}^{o} a \right] / \sum_{i=1}^{m} y_{1}^{o} \) for output increases in the first year and all output adjustments in the second year of COWPS, we can rewrite Eq. (2.3) as

\[
\sum_{i=1}^{m} p_{1}^{S} y_{1} - w_{a} a \leq \alpha + \beta \sum_{i=1}^{m} y_{1}^{o}
\]

Here one important difference between the COWPS regulation and AJ regulation becomes apparent; the analog to Eq. (2.4) in AJ regulation would contain no constant term (\( \alpha \)) and there would be a capital-related scale term rather than an output term. The various forms that the COWPS guideline has taken can be represented as special cases of a two-parameter policy instrument.¹ Even within a given policy regime, however, note that the presence of firm-specific, historical data in \( \alpha \) and \( \beta \) suggests that the levels of these important parameters can differ across firms.

**PROFIT MAXIMIZING UNDER THE GUIDELINE**

A normal, competitive firm maximizes profit,

\[
\pi = \sum_{i=1}^{m} p_{1}^{S} y_{1}^{S} - w_{a} a - w_{n} n
\]

¹During FY79, firms could also select a profit-margin constraint for compliance, rather than the "gross-margin" constraint, and approximately half the U.S. refining industry chose this standard. It is easily shown that such a formulation also characterizes, for example, the price controls of Nixon's Phase II era, and even British price controls of 1972-74 (Hunter, 1975).
subject to a production function

\[ f(y_1, \ldots, y_m; a, n) \leq 0 \quad (2.6) \]

and a market price constraint

\[ p_s^a \leq p_s^m \quad (2.7) \]

where \( n \) is the level of the aggregate "nonallowed" factors implicitly included in the gross margin above, \( w_n \) is its exogenously determined price, and \( p_s^m \) is the market price for the \( i \)th good, again exogenously determined.

For this analysis, Eq. (2.6) is assumed to display a special form of fixed proportions. Essentially, one and only one efficient method can be used to produce any given vector of outputs \( \vec{y}_1, \ldots, \vec{y}_m \). But the production function allows that vector to change. Hence, we wish to preserve substitutability among outputs but specify that one and only one level of \( a \) and \( n \) is efficient to use in the production of any given vector of output the firm chooses to produce, no matter what the relative factor prices, \( w_a \) and \( w_n \). If we assume that the firm pursues production efficiently,\(^2\) we can embody Eq. (2.6) in factor demand functions for \( a \) and \( n \) dependent only on output levels:

\(^2\)Ronald Braeutigam has pointed out to us that this assumption may be unwarranted in some circumstances. Using Kuhn-Tucker conditions, it can be shown that the firm is indifferent between efficient and inefficient use of allowed factors \( (a) \), in terms of its profit constraint. That is, while it may waste factor \( a \), it gains no profits from doing so. The decision to do this will be made on other criteria, e.g., if there are cleanup costs from excess use of factor \( a \)—oil spills, for example—then it is unprofitable. But if there are large costs to managing queues induced by the controls, allowable price may be increased through waste of factor \( a \) sufficiently to reduce or eliminate queues, in which case it is a desirable activity from the profit-maximizing perspective. But for simplicity, we have chosen to pursue the simpler formulation here, under the assumption that waste will not be the most desirable tactic.
\[ a = g(y_1, \ldots, y_m) \]
\[ n = h(y_1, \ldots, y_m). \]

Price theory does not normally consider the market price constraint in Eq. (2.7) explicitly. It is important here because, under the COWPS guideline, Eq. (2.7) need not be binding. (This is also a departure from the usual AJ case.) For a normal, competitive firm in a free market, however, we can assume that Eq. (2.7) holds as an equality. Substituting this equality and Eq. (2.8) into Eq. (2.4) yields

\[ \pi_m = \sum_{i=1}^{m} p_i y_i - w_a g(y_1, \ldots, y_m) - w_n h(y_1, \ldots, y_m). \]

A normal, competitive firm in a free market chooses its output mix to maximize this function. We call it the market profit function \( \pi_m \).

A firm subject to the COWPS price guideline faces an additional constraint characterized by Eq. (2.4). In its presence, the firm can face one of three profit functions. First, if the COWPS guideline is ineffective, the firm continues to use \( \pi_m \) in Eq. (2.9) as though COWPS did not exist. Second, if the COWPS guideline binds but the market price constraint does not, we can substitute Eq. (2.4) into Eq. (2.5) to yield

\[ \pi = \alpha + \beta \sum_{i=1}^{m} y_i - w_n. \]

This is the profit available to a firm not bound by the requirement that \( p_s^i \leq p^m_i \). In practice, this means that the COWPS guideline is so binding that \( p_s^i < p^m_i \) for at least one output. Market price exceeds selling price, inducing a need for nonprice rationing devices like queues to allocate the firm's output of this good to consumers. It implies a COWPS-induced shortage. Recalling that Eq. (2.8) defines the efficient
firm's technology in fixed proportions, the firm this tightly con-
strained chooses its output mix to maximize what we call a COWPS profit
function ($\pi_c$)

$$\pi_c = \alpha + \beta \sum_{i=1}^{m} y_i - w_n(y_1, \ldots, y_m). \quad (2.11)$$

Third, suppose the COWPS guideline is effective but not so binding as
to force the firm's selling prices below market prices. Then, Eqs.
(2.4) and (2.7) bind simultaneously, and the firm charges prevailing
market prices for all its outputs. To represent the profit function
of a firm in this situation, we must consider a Lagrangian constrained
maximization. In particular, assume that the COWPS constraint in Eq.
(2.4) is binding and maximize profit subject to Eq. (2.7). A zero
value for the Lagrange multiplier then indicates the presence of a
shortage.

Saying that both Eqs. (2.4) and (2.7) bind is equivalent to say-
ing that the firm maximizes $\pi_c$ subject to the constraint that $\pi_c = \pi_m$
(cf. Bailey, 1973). The Lagrangian that expresses this relationship is

$$L = \pi_c - \lambda(\pi_c - \pi_m)$$

$$= (1 - \lambda)\pi_c + \lambda\pi_m$$

$$= (1 - \lambda)[\alpha + \beta \sum_{i=1}^{m} y_i - w_n(y_1, \ldots, y_m)]$$

$$+ \lambda[\sum_{i=1}^{m} p_i y_i - w_a g(y_1, \ldots, y_m) - w_n(y_1, \ldots, y_m)]. \quad (2.12)$$

The normal, competitive firm chooses an output mix (and $\lambda$) to maximize
a convex combination of its market profit function and COWPS profit
function. The market profit function is a polar case of Eq. (2.12) when \( \lambda = 1 \) and COWPS is ineffective. The COWPS profit function emerges at the other end for \( \lambda = 0 \), when a shortage occurs. For \( 0 < \lambda < 1 \), both constraints bind and the firm alters its behavior in response to the guideline but faces no shortages. Equation (2.12) sums up the firm's general profit function in the presence of COWPS and \( \lambda \) defines the relative bindingness of the market price and COWPS constraints. This is the basic model we will use in this report to predict the behavior of a profit-maximizing firm subject to the COWPS guideline.

To maximize profits, the firm will set the first derivatives of \( L \) in Eq. (2.12) with respect to \( y_i \) and \( \lambda \) equal to zero.

\[
\frac{\partial L}{\partial y_i} = (1 - \lambda)(\beta - w h_i) + \lambda(p_i^m - w a g_i - w h_i) = 0 \quad \text{for all } i \quad (2.13a)
\]

\[
\frac{\partial L}{\partial \lambda} = \sum p_i^m y_i - wa g - \beta \sum y_i = 0 \quad \text{or } \lambda = 0 \quad (2.13b)
\]

where \( g_i = \partial g / \partial y_i \), \( h_i = \partial h / \partial y_i \). Equation (2.13a) simply states that the firm will choose each output so that a marginal change in \( y_i \) sufficient to increase profit by a given amount under the COWPS profit function alone reduces the profit available under the market profit function alone by the same amount for every output:

\[
\frac{d\pi_m}{d\pi_c} = \frac{\pi_i c_i}{\pi_i m_i} = -\frac{\beta - w h_i}{p_i^m - w a g_i - w h_i} = \frac{\lambda}{1 - \lambda} \quad \text{for all } i \quad (2.14)
\]

for \( \pi_{c_i} \equiv \partial \pi_c / \partial y_i \). The firm is literally balancing between the two disparate goals represented in its polar market and COWPS profit

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3The basic symmetry of Eq. (2.12) suggests that maximizing \( \pi_c \) subject to \( \pi_c = \pi_m \) is equivalent to maximizing \( \pi_m \) subject to \( \pi_c = \pi_m \), an outcome that can easily be shown.

4"We use \( x \) to represent \( c \) or \( m \) when a condition holds for both COWPS and market price functions. In this case, for example, \( \pi'_{x} \equiv \partial \pi_{x} / \partial y \) implies that \( \pi'_{c} \equiv \partial \pi_{c} / \partial y \) and \( \pi'_{m} \equiv \partial \pi_{m} / \partial y \).
functions. Equation (2.13b) simply states that, so long as COWPS is binding, either \( \pi_c = \pi_m \) and the firm charges market prices or \( \lambda = 0 \) and the firm faces some shortages.

These simple results provide the basis for all that is to come. We can use them to derive the firm's supply functions and thence to predict how COWPS affects market behavior. We can also use them to derive how changes in \( \alpha \) and \( \beta \) will affect firm behavior. We examine first the case of a firm with one output to develop the intuition behind these results and those that follow from them. Following that, we return to the m-good case for further development.
III. COWPS AND THE SINGLE-OUTPUT FIRM

Aside from being an important case in itself, the case of a firm with only one output parallels the typical price theoretic development of the theory of the firm. Its simple geometric analogs also provide intuitions that make the leap into hyperspace simpler when we move to the general case of m-outputs.

This section first explains what output level a single-output firm under fixed proportions chooses to maximize its profits under the COWPS price guideline. It then examines how this output choice changes in response to variations in key prices and policy parameters. Finally, it carries the results of the analysis of individual firm behavior to the market in an industry under the COWPS price guideline.

PROFIT MAXIMIZATION UNDER THE COWPS PRICE GUIDELINE, SINGLE PRODUCT

When $m = 1$, Eqs. (2.12) and (2.14) become, respectively,

$$L(y) = (1 - \lambda)[\alpha + \beta y - w_n h(y)] + \lambda[py - w_a g(y) - w_n h(y)] \quad (3.1)$$

$$\frac{d\pi_m}{d\pi_c} = -\frac{\pi'_c}{\pi'_m} = -\frac{\beta - w_n h'}{p - w_a g' - w_n h'} = \frac{\lambda}{1 - \lambda} \quad (3.2)$$

for $\pi'_x \equiv \partial\pi_x / \partial y$, $g' \equiv dg/dy$, and $h' \equiv dh/dy$. These relationships offer the possibility of four distinct geometric representations. In the first, only Eq. (2.4) binds in the single-output case ($\lambda = 0$) and shortages ensue. In the second and third, both Eqs. (2.4) and (2.7) bind in the single-output case ($0 < \lambda < 1$) and the COWPS guideline is effective without inducing shortages. In the fourth, only Eq. (2.7) binds ($\lambda = 1$) and the COWPS guideline is ineffective. This subsection develops the first three cases geometrically and discusses the degenerate geometric nature of the fourth.
Firm Behavior When $\lambda = 0$

Figure 1 represents the first case. Figure 1a displays examples of the profit functions contained in Eq. (3.1) as functions of $y$. Figure 1b displays the restrictions that these profit functions implicitly impose on price and a set of revenue functions that derive from these price restrictions. Both figures illustrate the effect of the COWPS guideline from two different vantage points—profits and prices—when $\lambda = 0$.

The two profit functions in Fig. 1a are the polar cases developed in the last section. $\pi_m$ is the one-output representation of the market profit function in Eq. (2.9); $\pi_c$ is that of the COWPS profit function in Eq. (2.11). Both take the form of the profit "hills" in Fig. 1a because $\pi''_m \equiv \partial \pi'_m / \partial y < 0$ for both functions. That is equivalent to saying that marginal cost (MC) rises more quickly with $y$ than marginal revenue (MR) in both cases. The relative position of the hills can be explained by looking at these underlying MR and MC functions. Figure 1b presents these.

Consider the functions underlying $\pi_m$ first. Here we have the "unconstrained" firm familiar from traditional price theory. $\text{MR}_m$ equals average revenue ($\text{AR}_m$) or the exogenously determined market price, $p_m$, in this case. $\pi_m$ in Fig. 1a shows the maximum profit a firm can achieve at any output level; its analog in Fig. 1b, $\text{AR}_m$, shows the maximum price the firm can charge at any output level. MC, determined strictly by the firm's fixed proportions technology, rises in response to decreasing returns to scale. It intersects $\text{MR}_m$ at $y_m$, the output level at which $\pi_m$ peaks. The efficient firm picks this output level and charges a price of $p^*_m$, if unconstrained by COWPS.

$\pi_c$ and $\text{AR}_c$ play similar roles for the COWPS profit function. $\pi_c$ shows the maximum profit a firm can earn at any output level when the firm can charge any price consistent with Eq. (2.4). $\text{AR}_c$ shows what maximum prices are consistent with Eq. (2.4) at various outputs. In particular,

$$\text{AR}_c = p^s \frac{a}{y} + \beta + w \frac{a}{y}.$$
When $\pi_c$ constrains $\pi_m$ in Fig. 1a ($\pi_c < \pi_m$), then $AR_c$ constrains $AR_m$ in Fig. 1b ($AR_c < p_m$). This is true in Fig. 1 over the range of $y$ to $\bar{y}$. When the firm charges the maximum price available at any output level, the marginal revenue function, $MR_c$, associated with this price follows directly

$$MR_c = \frac{d(p^S_y)}{dy} = \beta + w_a g' > 0.$$ (3.4)

We can sign Eq. (3.4) by noting that, with fixed proportions, the COWPS guideline does not affect the firm's factor demand functions. Increases in output continue to elicit increased demand for $a$. The absence of any effect on factor demand also assures that the firm faces the same MC function under both profit functions. Note that while $MR_c$ is upward sloping, it does not rise with $y$ as fast as MC does if the firm's technology displays decreasing marginal returns to scale in $n$. We use this assumption throughout this report. Hence, the firm can maximize profits where $MC = MR_c$, at $\bar{y}_c$. $\pi_c$ peaks at this output level.

A competitive firm attempting to maximize profits in the presence of COWPS must accept the lower profit allowed under these two profit functions at any output level:

$$\max \pi^*(y) = \min [\pi_c(y), \pi_m(y)] \quad (3.5)$$

A restriction on selling price in Fig. 1b parallels this choice set in Fig. 1a:

$$p^S(y) \leq \min [AR_c(y), p^m] \quad (3.6)$$

Bold loci in Figs. 1a and 1b define these allowable sets. $\pi^*$ follows $\pi_m$ for $y < \bar{y}$ or $y > \bar{y}$ and $\pi_c$ for $\bar{y} < y < \bar{y}$; $p^S$ follows $p^m$ for $y < \bar{y}$ or $y > \bar{y}$ and $AR_c$ for $\bar{y} < y < \bar{y}$. (The inequalities here display the notion that $\pi_c = \pi_m$ and $p_s = p_m$ at $y = \bar{y}$ and $y = \bar{y}$.)

\[ \frac{dMC}{dy} = w_a g'' + w_n h'', \text{ while } \frac{dMR_c}{dy} = w_a g''. \text{ Thus } \frac{dMC}{dy} > \frac{dMR_c}{dy} \text{ iff } h'' > 0. \]


Where \( \lambda = 0 \), a firm attempting to maximize profits at the level of output it would choose in the absence of COWPS, \( y_m \), finds its selling price constrained below the market price and hence its profits constrained below their free market level. By increasing its output, the firm can increase allowable profits until it reaches \( y_c \). Its price remains constrained below the market level but any further output expansion induces growing costs from diseconomies in the nonallowed factor (n) that outweigh the increased gross margin allowed to cover the cost of that factor.\(^2\)

This output choice has two distinct features. First, it occurs at a point where selling price lies below market price. Some nonprice device will be required to manage the shortage that must result from this underpricing. Second, the maximum on \( \pi_c \) lies below \( \pi_m \). The market considerations embodied in \( \pi_m \) are irrelevant to a firm maximizing profits at \( y_c \) because \( \pi_m \) does not bind these. From one point of view, this feature is no different from the first; it simply says in another way that \( p^s < p^m \). But saying that \( \pi_m \) is irrelevant to the firm's decisions is equivalent to saying that \( \lambda = 0 \) in Eqs. (3.1) and (3.2). In this case, the one-output case of \( \pi_c \) in Eq. (2.11) is the firm's effective profit function. Hence, the firm maximizes profits where \( \beta = w_n h' \), as implied in Eq. (3.2) when \( \lambda = 0 \). The output choice is independent of both market price and selling price. In fact, the firm chooses selling price only after it has determined its desired output level to satisfy Eq. (3.3).\(^3\) These two features will be important both in our consideration of the single-output firm's supply behavior and the market behavior that it implies and in our discussion of a multiproduct firm's choice of selling prices.

Before we leave this case, note that \( \lambda = 0 \) does not necessarily imply that the COWPS guideline increases desired output. \( y_c > y_m \) only

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\(^2\)Formally, \( \pi'_c = \beta - w_n h' \). \( \beta \) is the policy parameter in Eq. (2.4) that expands the allowed gross margin in response to increases in output. \( w_n h = w_n h \) is the cost that the firm must cover with the revenue in the gross margin. The firm continues to expand output until \( \beta = w_n h' \).

\(^3\)As implied by \( p^s \) in Fig. 1b any output level determines a unique maximum selling price.
because of the way we have drawn our example. A simple condition determines whether \( y_c > y_m \). Recall that the firm attempting to produce at \( y_m \) found that it could increase profits by increasing output because \( \pi_c \) was upward sloping at \( y_m \), since \( \beta > w_n h' \). If the firm had found that it could increase profits by decreasing output, the maximum profit under \( \pi_c \) would have lain to the left of \( y_m \) and \( \beta < w_n h' \). That is, for \( \lambda = 0 \), we can say\(^4\) that at \( y_m \)

\[
y_c > y_m \iff \beta > w_n h'.
\] (3.7)

It seems likely that \( \beta < w_n h' \), so that \( y_c < y_m \) because COWPS allows only 6.5 percent annual increases in \( n \)'s average cost, whereas general inflation has substantially exceeded this rate since inception of the COWPS guideline. Also, COWPS allows only \( w_n \) times average use of \( n \), whereas maximizing \( \pi_c \) considers marginal use of \( n \).

Given the nature of the COWPS price guideline, this result could have been expected. It restricts neither profit nor price but a specified margin. If the change in allowed margin resulting from an increase in output, \( \beta \), exceeds the marginal nonallowed costs, \( w_n h' \), at the preferred output level in a free market, we should expect the firm to expand output beyond that level. If it falls short we should expect the firm to reduce output. In effect, the COWPS price guideline makes the allowable factor a free good and either subsidizes or taxes the nonallowable factor. The firm with fixed proportions, constrained only by the COWPS guideline, reacts to such incentives as we would expect it to.

**Firm Behavior When \( 0 < \lambda < 1 \)**

Two cases of firm response to the COWPS guideline are important when \( 0 < \lambda < 1 \). They are differentiated by the relative positions of

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\(^4\)The result is conditional on the well behavedness—in particular, on the single-peakedness—of \( \pi_c \). This is assured by the rather weak requirements for decreasing returns to scale cited above. Note that we can derive the same result from Fig. 1b by noting that \( y_c > y_m \) iff \( MR_c > MC \) at \( y_m \). By definition, \( MR_c > MC \) iff \( \beta + w_ag' > w_ag' + w_n h' \).
y_c and y_m. Figure 2 illustrates the case when y_c > y_m. All the functions shown are defined in the same way as are their counterparts in Fig. 1. The example in Fig. 2 differs from that in Fig. 1 in that (a) \( \pi_c \) has moved to the right relative to \( \pi_m \) and (b) AR_c and hence MR_c have moved to the left relative to MC.  \(^5\) The change moves the maximum point on \( \pi_c \) to an output level greater than that at which \( \pi_c = \pi_m \); it moves the maximum point on \( \pi_c \) "outside" \( \pi_m \). What is the same thing, it moves the intersection of MC and MR_c to an output level higher than that at which AR_c = AR_m. Such a move is important because it means that the maximum points on \( \pi_c \) and \( \pi_m \) now both lie above the bold locus in Fig. 2a that denotes \( \pi^* \). \( \pi_m \) now effectively binds \( \pi_c \) in the same way that \( \pi_c \) bound \( \pi_m \) in Fig. 1 and continues to bind it here. \( \pi_c \) and \( \pi_m \) mutually constrain one another, whence the symmetric form of Eq. (3.1) when \( 0 < \lambda < 1 \).

The symmetry of this constraint is reflected in Fig. 2b. At \( y_m \), \( p_m \) lies above the price allowed by \( p^{S*} \), again denoted by the bold locus. At \( y_c \), also, AR_c lies above \( p^{S*} \). Neither price is achievable at the preferred locations. Under these circumstances, the firm maximizes profits by choosing an output level that sets MC equal to MR* implied by the revenue attainable under \( p^{S*} \). MR* equals \( p_m \) for \( y < \bar{y} \) or \( y > \bar{y} \) and MR_c for \( \bar{y} < y < \bar{y} \). It is discontinuous at \( \bar{y} \) and \( \bar{y}_c \). And in this case, MC must pass through the discontinuity at \( \bar{y}_c \). The firm chooses \( \bar{y}_c \).

As Fig. 2a confirms, \( \bar{y}_c \) maximizes profit under such a double constraint. In fact, the relevant profit function, \( \pi^* \), is kinked at \( \bar{y}_c \) because MR* is discontinuous at \( \bar{y}_c \). Note that the slope of \( \pi^* \) to the left of \( \bar{y}_c \) is \( \pi'_c = \beta - w_nh' = MR_c - MC \). This is simply the vertical difference between MR_c and MC at \( \bar{y}_c \) in Fig. 2b. The slope of \( \pi^* \) to the right of \( \bar{y}_c \) is \( \pi'_m = p_m - w_ag' - w_nh' = p_m - MC \), the inverse of the vertical distance between MC and \( p_m \) at \( \bar{y}_c \). Combining these features of Fig. 2 with Eq. (3.2) gives us two ways to interpret \( \lambda \) geometrically.

\(^5\) The changes in Fig. 1a obviously imply those in Fig. 1b and vice versa.

\(^6\) This was true in Fig. 1 as well. But because MC = MR_c at an output level between \( \bar{y} \) and \( \bar{y}_c \), the discontinuities were irrelevant.
Fig. 2 — Profit maximization under COWPS when $0 < \lambda < 1$ (i)
From Fig. 2a, \( \lambda/(1 - \lambda) \) equals the ratio of the absolute values of \( \pi'_C \) and \( \pi'_m \). As \( |\pi'_C| \) rises relative to \( |\pi'_m| \) at \( \bar{y} \), \( \lambda \) moves from 0 toward 1 as the COWPS constraint becomes relatively less binding. From Fig. 2b, \( \lambda \) is a measure of MC’s relative position between MR\(_C\) and MR\(_m\) (= p\(^m\)) at \( \bar{y} \). As MC moves from MR\(_C\) to p\(^m\), \( \lambda \) moves from 0 toward 1 and, again, the COWPS constraint becomes relatively less binding. The two parts of the figure are, of course, simply different reflections of the same underlying phenomenon.

As in the case of \( \lambda = 0 \) in Fig. 2, the case of \( 0 < \lambda < 1 \) does not necessarily imply that \( y'_C > y'_m \). Once again, the relative levels of \( y'_C \) and \( y'_m \) depend on the size of \( \beta \) relative to \( w\,h' \). Using an argument completely analogous to that used for \( \lambda = 0 \), suppose a firm attempts to produce at \( y'_m \). If it finds that profit rises as output increases, \( \pi'^* = \pi'_C > 0 \) and \( \beta > w\,h' \). If profit falls as output increases, \( \pi'^* = \pi'_C < 0 \) and \( \beta < w\,h' \). Although the firm will not proceed to the point where \( \beta = w\,h' \) in this case, the slope of \( \pi'_C \) at \( y'_m \) points it in the right direction.\(^7\) Equation (3.7) holds as well for \( 0 < \lambda < 1 \) as for \( \lambda = 0.8 \) (Recall also the discussion near Eq. (3.7) suggesting that \( \beta < w\,h' \) appears likely.)

Figure 3 displays the case for \( \beta < w\,h' \) at \( y'_m \) that parallels that for \( \beta > w\,h' \) in Fig. 2. The functions shown are all defined as are their counterparts in Fig. 2. AR\(_C\) and MR\(_C\) have moved to the right relative to MC, forcing \( \pi'_C \) to the left relative to \( \pi'_m \). The firm maximizes profits at the point where MC passes through the lower discontinuity in MR\(_C\), at \( \bar{y} \); the discontinuity points to a kinked maximum in \( \pi'^* \), now at an output level below \( y'_m \). Just as in Fig. 2, \( \lambda \) can be defined either by the relative slopes of \( \pi'^* \) to be left and right of \( \bar{y} \) or by MC’s relative position between \( p\(^m\) \) and MR\(_C\) at \( \bar{y} \).

**Firm Behavior When \( \lambda = 1 \)**

The final possibility is the polar case of \( \lambda = 1 \). Here, only the market price constraint, \( p^S \leq p^m \) binds; the COWPS guideline is

\(^7\) Again, the result requires \( \pi'_C \) to have only one local maximum.

\(^8\) The proof relating to Fig. 2b also continues to hold.
Fig. 3 — Profit maximization under COWPS when $0 < \lambda < 1$ (ii)
ineffective. We could present this case graphically but, as a "degenerate" case, it is not particularly important. In Fig. 3a, $\pi_m$ would lie fully within $\pi_c$. In accord with Eq. (3.5), only $\pi_m$ would bind, leading of course to $\lambda = 1$ in Eq. (3.1). Similarly, in Fig. 3b, $AR_c$ would lie wholly above $p^m$ and, in accord with Eq. (3.6), $p^m$ would wholly determine allowable price. The traditional results of the theory of the firm would hold.

**COMPARATIVE STATICS UNDER THE COWPS PRICE GUIDELINE**

So far, we have examined a firm's response to COWPS under the presumption that input and prices and policy parameters were fixed. In this subsection we use the model we have developed to predict how profit-maximizing firms respond to changes to such parameters. Understanding the comparative statics of both prices and policy instruments will allow us to exploit differences in these parameters across time and across firms in our empirical work, and help set the stage for analysis of market behavior (cf. McNicol, 1973).

**Firm Response to Changes in $p^m$**

$p^m$ appears only once in Eq. (3.1), in the $\pi_m$ component. We can predict its effect on output by perturbing $\pi_m$ in Figs. 1a, 2a, and 3a. Raising $p^m$ raises $\pi_m$ slightly and shifts its maximum to the right. Figures 1b, 2b, and 3b can confirm our results.

The effect depends very much on the value of $\lambda$ and $\beta - w_n h'$ at $y_m$. When $\lambda = 0$ (see Fig. 1), recall that the firm is insensitive to market price because the firm uses $\pi_c$ to make decisions. Hence, changes in $p^m$ do not affect output choice.

When $0 < \lambda < 1$, the effect depends on the value of $\beta - w_n h'$ at $y_m$. For $\beta > w_n h'$ (see Fig. 2), increasing $p^m$ raises $\pi_m$ and tends to make it less binding relative to $\pi_c$. The firm moves up along $\pi_c$ in Fig. 2a.

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9 That work will be reported in Phelps, Camm, and Stan (forthcoming), which will also present a more detailed discussion of empirical differences in these variables across firms and time.

10 Raising $p^m$ in Figs. 1b, 2b, and 3b leads to intersection with MC at a higher output, as we would expect. Equation (2.9) shows that it also raises profit in proportion to the level of output at each level of output.
increasing output. In Fig. 2b, raising $p^m$ moves the discontinuity in $MR_c$ to the right. This tends to reduce $\lambda$, thus confirming the impression that $\pi_m$ becomes relatively less binding. As increases in $p^m$ push $\bar{y}$ to $y_c$, $\pi_m$ rises so that it lies completely above $\pi_c$. (We will refer to this phenomenon hereafter as $\pi_m$ "enveloping" $\pi_c"). Any rise in $p^m$ that pushes $\bar{y}$ beyond $y_c$ pushes $\lambda$ to zero and forces the firm to become unresponsive. Price reductions, on the other hand, push $\bar{y}$ to the left, raise $\lambda$, make $\pi_m$ relatively more binding, and in the limit make the COWPS price guideline ineffective. That occurs when $\bar{y}$ moves anywhere to the left of $y_m$ and $\pi_c$ "envelops" $\pi_m$.

By summarizing the firm's response to market price, we can trace out its supply curve (see Fig. 4). The firm's supply curve is $S_0$ in the absence of COWPS; it is simply its marginal cost curve. The supply curve when $\beta > w_n h'$ at $y_m$ is $S_1$. It follows $S_0$ up to the price where the COWPS guideline just becomes effective. This is $p_1$ in Fig. 4, identical to $AR_c$ where it intersects $MC$ in Fig. 2. When the guideline becomes effective, $S_1$ jumps to the right. It follows $AR_c$ in Fig. 2 up to the firm's preferred COWPS output level, $y_c$. At this point, it becomes vertical. COWPS allows no further rise in selling price and the firm becomes insensitive to market price.

When $\beta < w_n h'$ at $y_m$ (Fig. 3), increases in $p^m$ still tend to loosen the relative bindingness of $\pi_m$. But because $y_c < y_m$ in this case, increases in $p^m$ tend to reduce desired output. We can now trace out the full supply curve of the firm. Again, at low $p^m$, the COWPS guideline is unbinding and the firm follows its marginal cost curve, $S_0$. At $p_2$ in Fig. 4, which is $AR_c$ where it intersects $MC$ in Fig. 3, the guideline becomes effective.\textsuperscript{12} Further price increases push the firm along $AR_c$ in Fig. 3. This leads to a downward sloping supply function. When $y$ reaches $y_c$, $\pi_m$ "envelops" $\pi_c$, the firm can raise its selling price no more, and it becomes unresponsive to output price.

\textsuperscript{11} It does so by reducing the gap between $MR_c$ and $MC$.

\textsuperscript{12} The $AR_c$ curve will typically differ for $\beta > w_n h'$ and $\beta < w_n h'$ at $y_m$. That is why $S_1$ and $S_2$ do not trace out one full $AR_c$ function.
Fig. 1 — Profit maximization under COWPS when $\lambda = 0$
Fig. 4 — Firm supply functions under the COWPS price guideline

Firm Response to Changes in $w_a$

In this subsection, we consider the consequences of a change in $w_a$, holding constant both $w_n$ and product price $p^m$. This could occur, for example, with a domestic tax on crude oil in a market with infinitely elastic product import supply (see Phelps and Smith, 1977).

Like $p^m$, $w_a$ appears only in the $\pi_m$ component of the Lagrangian shown in Eq. (3.1). Raising $w_a$ slightly lowers $\pi_m$ slightly and shifts its maximum to the left. Raising $w_a$, then, has the same qualitative effect as lowering $p^m$. When the COWPS guideline is unbinding, it has

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13 Raising $w_a$ raises MC in Figs. 1b, 2b, and 3b and pushes the intersection of $p^m$ and MC to the left (i.e., the competitive output declines). Equation (2.9) shows that it also lowers profit in proportion to the level of $a$ at each level of output.
the normal effect of raising marginal cost and reducing the desired level of output, since \( p^m \) is assumed unchanged.

When the guideline is effective but not so binding as to induce a shortage, an increase in \( w_a \) reduces desired output when \( \beta > w_n h' \) at \( y_m \) and increases it when \( \beta < w_n h' \) at \( y_m \). This odd result is entirely parallel to that for \( p^m \). Raising \( w_a \) tends to make \( \pi_m \) relatively more binding and hence moves desired output away from \( y_c \).

When COWPS binds tightly enough to induce shortages, the firm makes output decisions without reference to \( w_a \) because allowable factors are effectively free goods to the firm. The demand curve for factor \( a \) becomes vertical.

**Firm Response to Changes in \( w_n \)**

Since \( w_n \) appears in both \( \pi_c \) and \( \pi_m \), perturbations in \( w_n \) will perturb both, making comparative static analysis in Figs. 1a, 2a, and 3a difficult. Fortunately, \( w_n \) appears in Figs. 1b, 2b, and 3b only in MC. We can trace the effects of varying \( w_n \) by noting that increases in \( w_n \) raise MC.

The firm responds quite differently to changes in \( w_n \) than to changes in \( p^m \) or \( w_a \) when the COWPS guideline is effective. Consider the effects of \( w_n \) as the guideline becomes progressively tighter. In the extreme, when \( \lambda = 1 \) and the guideline is ineffective, the firm responds to a rise in \( w_n' \), as we would expect, by reducing desired output. As COWPS begins to bind but fails to cause shortages (\( 0 < \lambda < 1 \)), the firm finds itself at the discontinuity in \( MC \). Within this discontinuity, changes in \( w_n \) have no effect on desired output. If \( \beta > w_n h' \) at \( y_m' \), increases in \( w_n \) move MC from \( p^m \) toward \( MR_c \), thereby increasing the relative bindingness of \( \pi_c \). If \( \beta < w_n h' \) at \( y_m \), however, increases in \( w_n \) have the opposite effect. As MC rises, it moves from \( MR_c \) toward \( p^m \) and \( \pi_m \) becomes relatively more binding.\(^{14}\) As soon as the guideline binds

\(^{14}\) This asymmetric result is consistent with the relationship in Figs. 2 and 3 between rightward movements of \( AR_c \) and \( MR_c \) relative to MC and leftward movements of \( \pi_c \) relative to \( \pi_m \). These too make \( \pi_c \) relatively more or less binding as \( \beta > w_n h' \) at \( y_m \). Note, however, that movements in \( w_n \) do not precipitate the same continuous movement in desired output that these movements did for \( 0 < \lambda < 1 \).
tightly enough to induce a shortage, increases in \( w_n \) reduce the desired level of output in the shortage. In general, increases in \( w_n \) tend to push a firm under a given COWPS guideline from a free market solution to a shortage if \( \beta > w_n h' \) at \( y_m \) and from a shortage to a free market solution if \( \beta < w_n h' \) at \( y_m \).

**Firm Response to Changes in \( \alpha \)**

\( \alpha \) appears only in the \( \pi_c \) component of Eq. (3.1). An increase in \( \alpha \) shifts \( \pi_c \) up homothetically. Hence, if \( \lambda = 0 \), changes in \( \alpha \) have no effect on the output level at which \( \pi_c \) peaks; the desired output level is unresponsive to changes in \( \alpha \). When \( 0 < \lambda < 1 \), a rise in \( \alpha \) effectively loosens the relative bindingness of \( \pi_c \). If \( \beta > w_n h' \) at \( y_m \), a rise in \( \alpha \) induces a firm to move up along \( \pi_m \) and reduce desired output. If \( \beta < w_n h' \), moving up along \( \pi_m \) increases desired output. For \( \lambda = 1 \), \( \pi_m \) alone is operative and changes in \( \alpha \) have no effect on output choice. In general, sufficient increases in \( \alpha \) tend to move a firm from shortage to charging a market price under an effective COWPS guideline and from a shortage-free effective COWPS environment to a free market environment.

**Firm Response to Changes in \( \beta \)**

Like \( \alpha \), \( \beta \) appears only in the \( \pi_c \) component of Eq. (3.1). Increases in \( \beta \) shift \( \pi_c \) up and increase the level of output at which it peaks. Hence, for \( \lambda = 0 \), increases in \( \beta \) increase the desired level of output. For \( 0 < \lambda < 1 \), increases in \( \beta \) tend to loosen the bindingness of \( \pi_c \) relative to that of \( \pi_m \). Hence, they have the same

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15. This is verified by the fact that under \( \pi_n \) the firm chooses the output level at which \( \beta = w_n h' \). This is independent of \( \alpha \).

16. Formally, for \( 0 < \lambda < 1 \), Eq. (2.13b) requires that \( \pi_c = \pi_m \) or \( \alpha = (p - \beta)y - w_ag(y) \). \( dy/d\alpha = [(p - \beta) - w_ag'(y)]^{-1} = \pi_m - \pi_c^{-1} \). Equation (3.2) requires that \( sgn(\pi_m - \pi_c^{-1}) = \pi_c^{-1} \). As noted above, \( \pi_c^{-1} > 0 \) as \( \beta > w_n h' \). Hence \( dy/d\alpha > 0 \) as \( \beta > w_n h' \). We also show later, in the multiproduct case, that \( \partial \lambda/\partial \alpha > 0 \).

17. \( MR_c = \beta + w_ag' \) in Figs. 1b, 2b, and 3b. Because \( MC' > MR_c \), increases in \( MR_c \) always increase the desired level of output. From Eq. (2.11), increases in \( \beta \) increase \( \pi_c \) in proportion to the level of output at each level of output.
qualitative effects as have increases in $\alpha$.\footnote{Formally, as noted in footnote 16 (this section), when $0 < \lambda < 1$, $\alpha = (p - \beta)y - w_\text{ag}(y)$. Fully differentiating with respect to $\beta$ and $y$ yields $dy/d\beta = y[(p - \beta) - w_\text{ag}'(y)]^{-1} = y(dy/da)$.} Increases in $\beta$ increase or decrease the desired level of output as $\beta \lesssim w_n h'$. For $\lambda = 1$, $\pi_n$ alone is operative and changes in $\beta$ have no effect on desired level of output. In general, changes in $\beta$ affect the likelihood of shortage or effectiveness of the guideline much as do changes in $\alpha$.

**Summary**

Table 1 summarizes the results of this subsection. A number of observations are important for empirical testing. First, the case of $\lambda = 1$ in Table 1a represents the free market case and is a useful reference case or alternative hypothesis. Second, departures in behavior from the reference case in Table 1a depend heavily on whether or not the COWPS guideline induces shortages and whether it encourages increases or decreases in output. Third, to detect the effects of changes in one variable on output, any empirical analysis will have to control for changes in the others. This applies to crosssection or time series analysis. Fourth, Table 1b suggests that qualitative analysis to predict incidence of shortages and effectiveness will be relatively simpler. Directional effects do not depend on the value of $\lambda$ and in only one case on that of $\beta - w_n h'$. And variable perturbations always move the probabilities of effectiveness and shortage in the same direction.

**MARKET BEHAVIOR UNDER THE COWPS PRICE GUIDELINE**

The individual firm supply functions developed above allow us to construct an industry supply function and use it to analyze behavior in the output market for the firms competing in any particular industry.\footnote{We could also examine input markets in a similar way if the firm in question accounted for a large enough share of those markets to affect input prices with their aggregate behavior. We restrict our analysis here to the case where the relevant industry faces exogenously determined, constant factor prices. The analysis becomes considerably more difficult when industry behavior affects input and output prices simultaneously.} We will examine COWPS effect on industry output and price.
Table 1
SUMMARY OF COMPARATIVE STATICS

a. Desired Level of Output

<table>
<thead>
<tr>
<th>$\lambda$</th>
<th>$\beta - w^h_n$</th>
<th>$p^m$</th>
<th>$w$</th>
<th>$w_n$</th>
<th>$\alpha$</th>
<th>$\beta$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\lambda = 0$</td>
<td>+</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>+</td>
<td></td>
</tr>
<tr>
<td></td>
<td>-</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>+</td>
<td></td>
</tr>
<tr>
<td>$0 &lt; \lambda &lt; 1$</td>
<td>+</td>
<td>+</td>
<td>-</td>
<td>0</td>
<td>-</td>
<td></td>
</tr>
<tr>
<td></td>
<td>-</td>
<td>-</td>
<td>+</td>
<td>0</td>
<td>+</td>
<td></td>
</tr>
<tr>
<td>$\lambda = 1$</td>
<td>±</td>
<td>+</td>
<td>-</td>
<td>-</td>
<td>0</td>
<td>0</td>
</tr>
</tbody>
</table>

b. Probabilities of a Shortage and of Effectiveness

<table>
<thead>
<tr>
<th>Probability of</th>
<th>$\beta - w^h_n$</th>
<th>$p^m$</th>
<th>$w$</th>
<th>$w_n$</th>
<th>$\alpha$</th>
<th>$\beta$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Shortage</td>
<td>+</td>
<td>+</td>
<td>-</td>
<td>+</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>Effectiveness</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
</tr>
</tbody>
</table>

Summing individual firm supply functions horizontally yields the industry's supply function. Recall from Sec. II that $\alpha$ and $\beta$ depend on historical data on individual firms. Hence, even if firms have identical technologies, they can be expected to have different $\alpha$ and $\beta$ and hence different supply functions under the COWPS guideline. Differences in technology lead to further differences. Such differences are more important under the guideline than in a free market because they suggest that, within one industry, some firms can have upward sloping functions while others have downward sloping functions (at intermediate prices). Hence (see Fig. 4), while we can expect the industry supply function to be upward sloping at lower prices (when COWPS is ineffective for all firms) and inelastic at high prices (when COWPS induces shortages
in all firms), the midsection of the supply function can take any shape that assigns only one output level to each price. Such a supply function can be expected to induce unusual market behavior.

Figure 5, with market output on its abscissa and market price on its ordinate, compares two examples with the free market supply function, \( S_0 \); others are obviously possible. \( S_1 \) represents a case where most firms in the industry face \( \beta > w_n' \) at \( y_m' \); they are upward sloping and lie to the right and below \( S_0 \). In this case, the COWPS guideline holds price below its free market level by inducing firms to produce more at each price than they would in a free market. \( S_2 \) represents the function that can result if \( \beta < w_n' \) at \( y_m' \) for most firms. In this case, the COWPS guideline induces a price rise in the industry.

When the COWPS guideline does constrain prices, then, it does so indirectly. Direct restraint would drive price below a market clearing level, thereby immediately inducing shortages. COWPS, on the other hand, induces excess supply at prevailing prices by lowering firms' reservation sales prices. This drives prices down because too much supply is present at the market price. Therefore, COWPS can lower prices without inducing shortage. This is, of course, the same thing that utility rate of return (AJ) regulation does. And, as in that case, such supply expansion is not a free good. COWPS not only drives production beyond the socially desirable level, but also assures, except in the case of fixed proportions, that all output is produced inefficiently. Furthermore, as noted earlier, COWPS does not always lower prices. When COWPS raises prices, it imposes the same type of social costs associated with price reduction under COWPS, but in greater measure.

While aggregation of individual firm supply curves allows us to identify a market clearing price that each firm then takes as exogenous, individual firms need not charge this price. In fact, this price need

\[ \text{Note that while COWPS is meant to deal with the dynamic problem of price change— inflation—our analysis is static. } p_0^W \text{ is a hypothetical price that would prevail in COWPS's absence, not the market price that could be observed before the COWPS guideline was implemented or after it ends.} \]
not be charged by any firm in the market. When firm and market prices diverge under the COWPS guideline, we can expect to observe a variety of prices in the market that persists over time, and with this variety come shortages.

Whenever a firm produces in the vertical portion of its supply curve, it does so because it would collect excess revenues at that output level if it charged the market price. It should now be clear that any number of firms can find themselves in this circumstance at any given market clearing price. In the limit, all firms face this circumstance together, each firm produces at the output ($y_C$) at which
its supply curve is vertical, and the industry supply curve is vertical. This implies that the highest price in the market must lie below the market clearing price. Therefore, the price implied by the intersection of supply and demand curves is not observable in strict monetary terms. Each firm charges the price consistent with Eq. (3.3) at \( y_c \); that price is likely to differ from firm to firm. The market must clear using rationing, queuing, or some other nonprice mechanisms.

If even one firm is not on the vertical portion of its supply curve, however, such an industrywide shortage could be eliminated. If the firm responds positively to price, the shortage resolves itself in the standard fashion. Excess demand drives the price up, the firm responds by expanding output, and the market clears at a higher price, higher level of production, and a lower level of consumption. If the firm responds negatively to price, the market may or may not clear. Excess demand pushes price up and the firm responds by reducing output. If it reduces output more slowly than consumption falls with rising price, excess demand will finally be relieved at some price level above the initial price. If not, the firm finally reaches the point where price is so high that its market constraint no longer binds; it can raise price no higher, and we return to the condition where COWPS holds the price of every firm below the market clearing price.

It is important to note that the establishment of a "market clearing" price does not eliminate shortage in the market. A shortage will persist unless any customer can consume as much of the good as he likes at any price that prevails in the market. Under this definition, the COWPS guideline can lead to widespread shortage in an industry. Note that, given a market clearing price, the COWPS guideline can reduce firm-specific prices arbitrarily far below the market clearing level. Any time a firm's price lies below the market clearing price, total

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22 Note that the converse does not hold; a vertical industry supply curve at any price does not imply that all firm supply curves are vertical and hence that the market constraint is slack for all firms.

23 This is true up to the point where the firm cannot recover variable costs in the short run or variable costs and capital amortization in the longer run.
demand at that price can be said to exceed market supply (because the demand curve is downward sloping). Hence, a shortage exists—excess demand is positive—whenever even one firm charges a price below the market clearing level.

In sum, differences across firms in technology and the firm-specific form of the COWPS guideline lead to what can only be called a bizarre industry supply function. Through that function, COWPS can hold the industry price down or force it to rise. It can sustain multiple equilibria, stable and unstable. It can sustain as many different monetary prices as there are firms for whom the market constraint is slack. If, in equilibrium, the market constraint binds for even one of these firms, this firm will charge a price that clears the market for the industry as a whole. Nonetheless, the existence of such a price does not eliminate excess demand. So long as even one firm charges a price lower than the market clearing price, nonprice devices must be used to eliminate excess demand for its product. This in itself is prima facie evidence of a COWPS-induced shortage.
IV. COWPS AND THE MULTIPLE-OUTPUT FIRM

Firms that produce several outputs react to the COWPS price guideline very much as single-output firms do. But their reaction differs in three important ways: (a) Within a multiple-output firm, it is possible for the COWPS guideline to produce increases in the preferred level of output for some products and reductions in that of others; (b) as the COWPS guideline becomes progressively more binding (λ falls), the firm's desired levels of each output need not move monotonically from the level preferred under a free market (λ = 1) to that preferred in a COWPS-induced shortage (λ = 0); (c) when the COWPS guideline induces a shortage (λ = 0), the constraints that COWPS imposes give a multiple-product firm no way to translate its allowed revenue into prices for individual outputs. All three divergences from responses in the single-output case occur because the presence of multiple outputs gives the firm more freedom to react than that of a single output. This section first uses a geometric illustration of the two-output case to develop the distinctive characteristics of multiple-output production in the firm and then reexamines market behavior when firms can have more than one output.

FIRM BEHAVIOR WITH MORE THAN ONE OUTPUT

As one would expect, the multiple-output case is more difficult to represent geometrically than the single-output case. Figure 6 illustrates the case of two outputs. Figure 6a shows three axes, one each for two levels of output, \( y_1 \) and \( y_2 \), and one for the profit, \( \pi \). The large profit hill, labeled \( \pi_m \), illustrates Eq. (2.9) for \( m = 2 \). The firm can produce output levels within the area defined by the hill's intersection with the \( y_1 - y_2 \) plane without violating the market price constraint. Its maximum occurs at \( \nu_m \) which, when projected, indicates that in the absence of the COWPS constraint, the firm would produce levels of output indicated by \( \eta_m \). The smaller profit hill, labeled \( \pi_c \), illustrates Eq. (2.11) for \( m = 2 \). The firm can produce output levels within the area defined by its intersection with the \( y_1 - y_2 \)
Fig. 6 — Profit maximization under COWPS with two outputs
plane without violating the COWPS price guideline. Its maximum occurs at $\kappa_c$; without the market constraint, the firm would choose levels of output consistent with $\eta_c$. Because the firm is subject to both constraints, it can retain profits equal to the height of the lower hill at whatever level of output the firm chooses. This profit will be maximized if the firm chooses output levels along a locus between $\eta_m$ and $\eta_c$. Projected on $\pi_m'$, this becomes the locus between $\mu_m$ and $\mu_c$.

In Fig. 6a, $\pi_c$ lies totally within $\pi_m$. Therefore, the market constraint is not binding; $\lambda = 0$ in Eq. (2.13a) and the firm chooses outputs at $\eta_c$. We show next what happens when $\alpha$ is relaxed. Equation (2.13a) tells us that, as in the single-output case, changes in $\alpha$ do not affect output choice. Raising $\alpha$, then, raises $\pi_c$, thereby raising $\kappa_c$ so that it continues to project onto $\eta_c$. The firm continues to produce at $\eta_c$ ($\lambda = 0$) until $\kappa_c$ just reaches $\mu_c$, which also projects onto $\eta_c$. At this point, the market constraint just becomes binding. Any further rise in $\alpha$ will move $\lambda$ from zero.

As $\lambda$ grows, the firm begins to adjust its output level toward that at $\eta_m$. Figure 6b shows one point for which $0 < \lambda < 1$. Now $\kappa_c$ lies above $\mu_c$ and the maximum profit consistent with both constraints lies along the locus from $\mu_c$ to $\mu_m$. In the case illustrated, it lies at a point $\mu_{\lambda}$, which projects into $\eta_{\lambda}$ in output space. As $\alpha$ continues to rise, $\lambda$ rises farther until $\mu_{\lambda}$ coincides with $\mu_m$. At this point, COWPS becomes unbinding and production stabilizes at $\eta_m$. Any further rise in $\alpha$ will leave production---and profits---unchanged.

This case differs from the one good case in three ways, as indicated earlier. We consider each in turn.

**COWPS Increases and Decreases Output Levels Simultaneously**

COWPS can induce increases in the preferred level of output for some goods and decreases in those of others. In Fig. 6, the firm

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Note that this is not the point at which $\pi_c$ first breaks the surface of $\pi_m$. That will occur, in our example, at some point on $\pi_m$ beyond $\mu_c$ from the origin (cf. Viner, 1952). The market constraint does not become binding at this first contact because the firm produces at the maximal point on $\pi_c$, $\kappa_c$. This point does not become revenue-constrained until some point after $\pi_m$. Or equivalently, it lies on a locus from $\kappa_c$ to $\kappa_m$ (not shown), the point at which $\mu_m$ projects onto $\pi_c$. 
prefers more of $y_1$ and less of $y_2$ under the guideline alone than under the free market. This result is fully generalizable to more than two outputs.

The Locus of Adjustments from $y^*_m$ to $y^*_c$ Need Not Be Well Behaved

The gradient of $\pi^*_c$ at $y^*_m$ no longer tells us whether the COWPS price guideline will tend to increase or decrease the preferred level of individual outputs. As in the one-output case, the gradient of $\pi^*_c$ at $y^*_m$ does tell us the direction in which the firm adjusts its output mix when the COWPS price guideline just becomes binding. If marginal COWPS profit for product $i$, $\beta - w_i h_i$, is positive at $y^*_m$, the firm produces marginally more of it. If $\beta - w_i h_i < 0$, the firm produces marginally less. As the guideline becomes increasingly binding ($\lambda$ falls), however, single peakedness of $\pi^*_c$ no longer guarantees that the firm will continue to adjust output levels in the directions suggested at $y^*_m$. With two outputs, this is equivalent to saying that the locus between $y^*_m$ and $y^*_c$ need not be monotonic in $y_1 - y_2$ space.

Figure 7 illustrates this in the two-output case. The concentric ellipses about output vectors $y^*_m$ and $y^*_c$ represent isoprofit contours of concave $\pi^*(y_1, y_2)$ and $\pi^*_c(y_1, y_2)$, respectively. Begin at a regulatory policy characterized by the pair $(\alpha^0, \beta^0)$ such that the market constraint is slack ($\lambda = 0$). In this case the firm produces at $y^*_c$. If $\alpha$ is increased monotonically, while $\beta$ is held at $\beta^0$, then $\pi^*_c$ shifts homothetically upward, and the firm adjusts its output vector along the indicated path until production occurs at $y^*_m(\lambda = 1)$. Note that, while $dy_1^\lambda > 0$ everywhere along this path, there exists an $\alpha^*$ such that $dy_2^\lambda < 0$ as $\alpha < \alpha^*$.

To see what lies behind this behavior, recall that in the one-output case $d\alpha = (d\pi^M/dy - d\pi^C/dy)dy$, and hence that $\text{sgn}(dy/da) = \text{sgn}(d\pi^M/dy - d\pi^C/dy)$. In the $m$-output case with $\lambda \in (0, 1)$, we note first that maximum profit must occur on the $(m - 1)$-dimensional surface defined by $\pi^m = \pi^c$ lying in $(m + 1)$-space. Explicitly, $\pi^m = \pi^c$ implies that

$$\alpha + \beta \sum_i y_i = \sum_i p_i^m y_i - w a_g(y_1, \ldots, y_m)$$  \hspace{1cm} (4.1)
Fig. 7 — Isoprofit contours that generate a non-monotonic output path for $0 < \lambda < 1$
Holding $\beta$ and prices constant and perturbing this condition, we obtain

$$d\alpha = \sum_{i} [(p_i^m - \beta) - w_a g_i] dy_i$$

$$= \sum_{i} (\pi_i^m - \pi_i^c) dy_i,$$

(4.2)

where $\pi_i^m = \partial \pi^m / \partial y_i$, and $\pi_i^c = \partial \pi^c / \partial y_i$. Now from Eq. (2.14) we have that

$$0 < \frac{1 - \lambda}{\lambda} = \frac{\pi_i^m}{-\pi_i^c},$$

and hence that $-\text{sgn}(\pi_i^c) = \text{sgn}[(p_i^m - \beta) - w_a g_i] = \text{sgn}(\pi_i^m)$. If the COWPS constraint becomes less binding ($d\lambda > 0$), then the firm moves locally in the direction of the gradient of $\pi^m$, denoted $\nabla \pi^m$. Thus, $\text{sgn}(dy_i) = \text{sgn}(\pi_i^m)$ and $(\pi_i^m - \pi_i^c) dy_i > 0$, all $i$. Hence, $d\alpha$ must be positive as the sum of positive terms. Likewise, if the COWPS constraint becomes more binding ($d\lambda < 0$), the firm moves locally in the direction of $\nabla \pi^c$, and $(\pi_i^m - \pi_i^c) dy_i < 0$, all $i$. Hence, $d\alpha < 0$. We thus conclude that $d\lambda / d\alpha > 0$ and that each position of the optimal output path connecting $y^c$ and $y^m$ is traversed exactly once as $\alpha$ increases monotonically from $\alpha^0$.

Note, in particular, that if $d\lambda$ and hence $d\alpha$, is positive, say, then literally any configuration of signs is possible for $(dy_1, \ldots, dy_m)$ in Eq. (4.2), since the sign of a $dy_i$ is determined solely by $\pi_i^m$. Hence, $dy_i / d\alpha$ is generally of indeterminate sign, and it is this indeterminacy that gives rise to the behavior illustrated in Fig. 7. From another perspective, suppose that $d\lambda > 0$, and hence that $\text{sgn}(dy_i) = -\text{sgn}(\pi_i^c = \beta - w_n h_i)$. Since $\beta$ and $w_n$ are fixed, we have that a decrease in $y_i$ for any $i$ must be induced by a fall in $h_i$, and that indeterminacy in output adjustment can be viewed as arising from the curvature properties of $h$.

When $\beta$, $w_a$, or any output price changes for a fixed level of $\alpha$, we might expect output adjustments to be no better behaved, since such
changes generally induce nonhomothetic shifts in $\pi^c$ and thus change its critical points. Differentiating Eq. (4.1) we obtain:

$$d\beta = \frac{1}{\Sigma y_i} \Sigma (\pi^m_i - \pi^c_i) dy_i$$

$$dw_a = \frac{1}{g(y_1, \ldots, y_m)} \Sigma (\pi^m_i - \pi^c_i) dy_i$$

$$-dp^m_j = \frac{1}{y_j} \Sigma (\pi^m_i - \pi^c_i) dy_i,$$

where the only exogenous variable permitted to vary in each equation is that shown on the left-hand side, and $a$ is held constant in each case. Since $\Sigma y_i$ and $g(y_1, \ldots, y_m)$ are positive, exactly the same arguments apply, and we conclude that $d\lambda/dw_a$ and $d\lambda/d\beta$ are positive, while $dy_i/dw_a$ and $dy_i/d\beta$ are of indeterminate sign for all $i$. Likewise, if $y_j > 0$, then $d\lambda/dp_j < 0$ and the sign of $dy_i/dp_j$ is indeterminate. If $y_j = 0$, on the other hand, then a small change in its output price is irrelevant to the firm, and $dy_i = 0$, all $i$.

A COWPS-Induced Shortage Eliminates Price Determinacy

Third, when the market price constraint fails to bind, pricing is freer with several outputs than with one. When $\lambda = 0$, the firm produces at $y_c$ in Fig. 6 or its equivalent in $m$-space. The output mix is well defined: The firm chooses output levels so that $\beta = w_n h_i$ for all $i$. As in the one-output case, the firm chooses output level without reference to market or selling price. The prices the COWPS guideline allows the firm to sell at are defined by a multiple-output version of Eq. (3.3):

$$\sum_{i=1}^{m} p^s_i y_i \leq \alpha + \beta \sum_{i=1}^{m} y_i + w_a g(y_1, \ldots, y_m).$$

(4.3)
Once the output mix is chosen, the expression on the right is fixed and the outputs \( y_i \)---coefficients on \( y_i^S \) on the left---are fixed. The COWPS guideline effectively allows the firm to choose any nonnegative prices for \( m-1 \) outputs and price the \( m \)th output to satisfy Eq. (4.3).\(^2\) So long as Eq. (4.3) holds as an equality, the firm's profit is invariant to the specific set of prices chosen. Hence, it has no basis for choosing among them.\(^3\) This is true even if the firm faces market-determined prices in some markets and is free to choose prices in other markets with shortages. In fact, when \( \lambda = 0 \), the firm can choose the markets in which it will experience shortages. This is not true in the one-output case because the firm has only one market in which to allow the shortage.

**MARKET BEHAVIOR WITH MULTIPRODUCT FIRMS**

As with single-product firms, the supply function for an industry with multiproduct firms is simply the horizontal summation of the individual firm supply functions. The nonmonotonicity of the output locus between \( y_m \) and \( y_c \) suggests that individual supply functions can look even more bizarre in this case than in the one-output case. But that changes nothing fundamental about the industry supply function. It will still tend, for normal goods, to be positively responsive to output price at prices low enough so that the COWPS guideline fails to bind, inelastic at prices high enough for the guideline to induce a shortage, and of indeterminant shape in between. Nothing changes qualitatively in the determination of industry output when we move to industries with multiproduct firms.

A subtle change in pricing, however, does occur. When \( \lambda = 0 \) for a firm, it can choose any price for an output up to the market clearing

\(^2\)Equation (4.1) defines a hyperplane in price \( m \)-space. The firm can choose any point on or below this hyperplane in the positive orthant of price \( m \)-space.

\(^3\)Our colleague, James Dertouzos, has pointed out that other price controls can give the firm a set of preferences among these prices. In Phelps, Camm, and Stan (forthcoming), we develop a rationale under which the Department of Energy price controls on gasoline encourage a petroleum refiner to choose price consistent with Eq. (4.3) so that the price of gasoline is artificially low, lower even than the price required by DOE.
price. Hence, it can determine the severity of the shortage it "causes" in any given output market. The lower it sets its selling price, the worse it makes the shortage. Because raising the price of one product forces the profit-maximizing firm to lower the price of another, the firm can balance the severity of the shortages it faces for different goods. Relative prices may tend to reflect the marginal costs of the nonprice rationing required for different outputs. Hence, to the extent that shortages in different markets affect different firms differently, we can expect to see relative prices of outputs vary across multiproduct firms in ways unrelated to relative production costs.

SUMMARY

The effects of the COWPS price guideline differ in the one-output and multiple-output cases in important ways. The guideline can increase and decrease preferred output levels simultaneously; with only one output, that is obviously impossible. Unfortunately, whether the guideline tends to increase or decrease the desired level of output can no longer be deduced for data available at \( y_m \). As the guideline becomes increasingly binding, it can first increase and then decrease the preferred level of any particular output or vice versa. All of these results make empirical analysis far more tenuous in the multiproduct case than in the single-product case. Finally, when a multiproduct firm faces a COWPS-induced shortage, its output prices become indeterminate. It can set prices to place shortages where they are least costly and will tend to reflect relative marginal rationing costs in its relative product prices. With one output, the firm has only one output in which to place a shortage and one profit-maximizing price to charge for it.

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4When \( \lambda = 0 \) for all firms in the industry, selling price is limited by the effective market price: the marginal willingness to pay for the last unit of output supplied to the market.

5Because profit is invariant to price choice and product costs are fixed once the output mix is chosen, these more subtle costs of rationing can give the firm its principal basis for choice. They may involve such intangibles as political support and customer good will. Recall from footnote 3 (this section) that other price regulation can also help the firm choose relative prices.
V. CONCLUSION AND EMPIRICAL PREVIEW

The COWPS price guideline does not behave the way an economist typically thinks a price control will behave, nor is it equivalent to AJ forms of regulation. Without cataloging all the results derived above, simply recall that the guideline can control prices without inducing shortages. It can also induce firms to produce more at every price and simultaneously induce widespread shortages. It can even simultaneously generate a market clearing price and widespread shortages facing individual firms. It can induce competitive firms to respond to product price increases by cutting output and to factor price increases by increasing output. It can sustain differences in prices across firms for long periods of time. And in the end, it need not control prices at all; it can even force them to rise.

We can respond to these bizarre implications by reassuring ourselves that the guideline is neither a price control nor a profit constraint, but a control on firms' gross margins. The analysis above emphasizes this. Hence, we should not be surprised that the guideline does not act like a price control. But it was being used explicitly to control prices. And similar forms of the same generic regulation have been used for many years to control prices. We would be surprised if they do not recur in some form in the future. So long as they persist, practical price control holds the potential for all the odd behavior discussed here.

To that suggestion, a detractor might retort that, while a control like the COWPS price guideline can precipitate unusual phenomena, it need not if properly applied. We agree. The proof that the phenomena hypothesized above occur under the guideline must await empirical verification. We are proceeding with that now in an examination of the petroleum refining industry. Casual empirical observation suggests that no other theory can account for the behavior of the petroleum refining industry, taken as a whole, over the last several years. Consider three phenomena:
1. **True shortages across entire markets.** Department of Energy rules specify the maximum prices at which gasoline can be sold by refiners but place no controls on prices of other products. Yet in mid-1979, serious shortages were reported for a wide variety of other petroleum products in addition to gasoline, including diesel fuel, heating oil, and jet fuel. (The jet fuel shortage, incidentally, occurred at a time when the nation's largest carrier, United Airlines, was not in service because of a labor strike, and later while the entire DC-10 fleet of the nation was grounded by the Federal Aviation Agency for safety checks—a time when aggregate demand for fuel was unusually low.) Since the refining industry seems to be well-characterized by the type of fixed production technology required in our model to produce shortages, the emergence of such shortages shortly after the COWPS regime was established is not surprising.

2. **Dispersion of prices across firms.** As we have noted, the COWPS mechanism sets firm-specific maximum prices for refined products (or, more accurately, maximum allowable revenues). Given a wide divergence in access to long-term contract oil at favorable prices, we would expect to see such a dispersion emerging in the refining market under COWPS. This dispersion carried through the entire COWPS regime with gasoline prices diverging by as much as 15¢ to 20¢ per gallon or more across firms. DOE data for July 1979 show that the dispersion was considerably larger: Maximum refiner prices for gasoline in that month were $1.09 per gallon, and minimum prices were 55¢ per gallon, a two-to-one ratio.

3. **"Random" relative prices among products displayed across firms.** Our model shows that firms, unless faced with other constraints such as minimizing costs of dealing with excess demand, will have little interest in the exact pattern of product prices. While we are certain that other factors do enter, they may well be quite different from firm to firm. Thus we expect to observe different relative prices among products in today's
market when looking across firms. Indeed, this is the case. For example, some firms set the prices of their regular and unleaded gasoline the same, while setting their premium gasoline prices much higher. Other firms have set the same prices for their premium and unleaded gasoline, while setting their regular gasoline prices much lower. Still other firms maintain a more traditional pattern of pricing their unleaded gasoline somewhere between the prices of regular and premium.

Similarly, we observe substantial variations in the relationship of gasoline prices to diesel fuel across firms, with some pricing no. 2 diesel at near their regular gasoline prices, while others set the price somewhat lower. (We have not found an instance where diesel prices rose above gasoline prices.)

Subsequent analysis will provide a more complete empirical analysis of refiner behavior in response to the COWPS guideline. But the price guideline affects the entire economy. Extension of the analysis to firms with variable proportions technology and market power and to the general equilibrium implications of such a pervasive regulation will be required before we can fully appreciate the extent of the COWPS price guideline's effect.
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