The Tradeoff Between Consumption and Military Expenditures for the Soviet Union During the 1980s

Mark M. Hopkins, Michael Kennedy
with the assistance of Marilee Lawrence
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PREFACE

This study makes use of a relatively new methodology for the modeling of the Soviet economy--optimal control theory. It estimates tradeoff curves between Soviet consumption and defense spending for the 1980s. How these curves are affected by various parameters and uncertainties is examined in detail.

The study was sponsored by the Director of Net Assessment, Office of the Secretary of Defense. Because of its methodology, it should be of particular interest to economic modelers. It should also be of interest to students of the Soviet economy, as well as to those concerned with predicting Soviet defense expenditures.
SUMMARY

This study applies a relatively new methodology, optimal control theory, to the construction of a model of the Soviet Union. The resulting Hopkins-Kennedy (HK) model is then used to estimate the Soviet tradeoff curve between consumption and defense spending in the 1980s. The study investigates the implications for this tradeoff of the Rosefield-Lee versus CIA debate on the nature of the Soviet economy; expected demographic change; expected total factor productivity growth; weather; increasing energy costs; foreign trade; and various composite scenarios. These are all issues, trends, and variables which are considered by Western experts on the Soviet Union to be of critical importance to the Soviet economy in the next decade—a time when, due to a combination of deleterious factors, Soviet capabilities will be under a heavy strain to meet all of the demands placed upon them.

The more traditional approach to the modeling of centrally planned economies, such as the Soviet Union, is econometric. It is particularly useful for short-run predictions in cases where it is reasonable to assume that things will continue much as they have in the past. Optimal control theory, in contrast, estimates the set of all technically feasible efficient options for Soviet planners. It tells not what is most likely to happen in the future, but rather what can happen. The tradeoff curve that is of interest to this study is an example of such a set. This approach is particularly useful when one is concerned with tradeoff curves such as in this study, or when dealing with long-run scenarios which involve major departures from historic trends.
Our base case projection for 1980-90 produces a tradeoff curve which measures the average rate of growth of defense spending versus the rate for consumption for the decade. This curve reveals that the Soviets will not be able to maintain the same rate of growth in consumption and defense as they did during the 1970s. They will, however, be able to maintain at least enough to "muddle through." The minimum required rate of growth to "muddle through" we define somewhat arbitrarily to be the expected rate of growth of defense spending and a 1 percent per capita rate of growth of consumption (1.9 percent in terms of total consumption). The loose justification of the latter is that it has been argued that this is the minimum required, given the nature of Soviet politics and distribution systems, for there to be a positive consumption growth rate for every major group in society, and without a positive consumption growth rate political difficulties would arise.

The Hopkins-Kennedy model is historically verified by making a similar projection for the 1960-75 period. The point which represents the rate of growth of consumption and defense spending in this period is found to correspond almost exactly to a point on the tradeoff curve. The HK model is also used to predict the gross national product (GNP) for the period 1960 to 1975. This also comes very close to what actually occurred.

Most of the analysis in this study employs as data CIA estimates of historical parameters and their implications for the future. The remaining analysis employs the sharply differing Rosefilede-Lee estimates of historical parameters and their implications for the future. Rosefilede and Lee argue that the current size of the Soviet
GNP is substantially larger than what the CIA maintains. They also argue that the rate of growth of the Soviet economy and the level of Soviet defense spending have been substantially higher than those estimated by the CIA.

The Rosefield-Lee data, rather than the CIA data, are used with the model to repeat the historical verification analyses, with good results. Future projections are then made with assumptions concerning such matters as the size of the economy in 1980 and the future rate of technical progress, which are implied by the Rosefield-Lee view of Soviet economic history. The resulting difference when compared with our base case projection is dramatic. The question of whether the CIA or the Rosefield-Lee view is correct dominates the importance of all other issues examined in this study.

To measure the impact on the economy of the rapid shift in the ethnic composition of the Soviet labor force from Slavs to non-Slavs, the relative efficiency of these two groups of workers was measured and found to be 1.27 in favor of the Slavs, with an uncertainty range from 1.0 to 1.6.

For the base case, the rate of total factor productivity growth is assumed to be the same rate as was observed during the first half of the 1970s, 0.32 percent. This variable was parameterized. A low value was defined to be the same as during the last half of the 1970s, -0.75 percent, and a high value was defined to be about the same as implied in the new Soviet five-year plan for the first half of the 1980s, 0.94 percent. Where, within the uncertainty span bounded by the low and high values, the actual value will be has more effect on our results than any of the other issues investigated in this study save for the CIA versus
Rosefield-Lee debate. If the high or low value is correct, the Soviets will not "muddle through"; they will either do quite well (better than the 1970s) or be in a severe crisis.

A poor weather scenario was run in which the average weather for the decade was moderately worse than the average for the last half of the 1970s, a period noted for poor weather. It was found that the difference in the tradeoff curves between poor weather and the base case was not large compared with most of the effects in this study.

Expected increases in the cost of energy for the Soviets were modeled in part by using the equivalent of a negative rate of technological progress for the energy sectors to represent resource exhaustion. Substitution of other goods for energy in both production and consumption was allowed for in the model. Four energy scenarios were addressed: (1) elimination of Soviet subsidies from the price they charge for oil exported to Eastern Europe; (2) an increase in the supply of oil equivalent to the amount currently being produced by Iran; (3) the same for the equivalent of the total output of Iran, Kuwait, Iraq, and the Neutral Zone; and (4) a lower or higher rate of growth of the world price of oil than that specified in the base case. The first and last scenarios resulted in a small economic impact. For the two supply-of-energy scenarios, the effect was substantial.

Special attention in this study is directed to foreign trade. There is an extensive literature that debates the question of whether imported capital is more efficient than Soviet domestically built capital, and if so by how much. In the literature the range in the ratio of efficiencies for these two types of capital is between 1 and 10. The difference in our estimates when these values are used is substantial.
A second foreign trade issue concerns the importance of credit to the Soviets. This is of particular interest to policymakers because it is a variable which the United States can and has influenced. The resulting impact when we alternate between a credit level (defined as Soviet imports less exports) of zero and a level which is twice as high as the Soviets have obtained in recent years is relatively small. The impact is of course larger for higher values of the capital efficiency ratio than the 1.5 which we have assumed for our base case.

Two scenarios labeled "best" case and "worst" case are defined so that they combine our high and low scenarios, respectively, for the rate of increase of total factor productivity; the rate of increase of the world oil price; the ratio of the efficiency of foreign to domestic capital; and the amount of credit obtained by the Soviets. If the "best" case occurs, the Soviet economy will perform remarkably well, not only better than during the 1970s but almost as well as during the 1960s. If the "worst" case occurs, the Soviet economy will face a severe crisis. Not only will the Soviets be unable to "muddle through," they will not be able to obtain the level of consumption growth required to "muddle through," even by reducing defense growth to zero.

The key conclusions to this study are: An optimal control model of the Soviet Union can produce interesting insights into the nature of the Soviet economy which would otherwise be difficult to obtain; the most likely course for the Soviet economy in the next decade is to "muddle through" at a performance level which is somewhat lower than during the 1970s; the source of the greatest uncertainty in our estimates (assuming that the CIA view of the Soviet economy is correct) is the uncertainty
as to what the rate of growth of total factor productivity will be; and
the issue which dominates all other concerns is whether the CIA view or
that of Rosefielde and Lee is correct.
ACKNOWLEDGMENTS

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We would like to thank Steven Salant, Edmund Brunner, and David Epstein for comments on earlier drafts which resulted in a number of important improvements. A special thank you goes to Daniel Bond, who graciously made available to us Wharton Econometric Forecasting Associates data on the Soviet economy.
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I. INTRODUCTION

This study develops a relatively new approach to the modeling of the Soviet economy that uses optimal control theory. The resulting Hopkins-Kennedy optimal control model is used to address the primary research question of the study: What will be the tradeoff between Soviet consumption and defense spending during the 1980s?

This analysis is one of the first uses of optimal control theory to study the Soviet economy, or for that matter, any centrally planned economy. It thus provides a test case for the new methodology. Optimal control theory allows us to calculate the tradeoff curves between various pairs of variables, such as consumption and defense. Such a tradeoff curve gives the set of all possible technically feasible efficient outputs of consumption and defense. Thus, it maps out the options available to Soviet decisionmakers. Knowledge of what the Soviets can do tells us something about what they are most likely to do by reducing the number of Soviet options that we need to consider. Perhaps more importantly, such knowledge provides information about the numerous scenarios which are feasible for the Soviet Union but which are not the most likely. Examples of such cases include various major shifts in priorities which could come about when the current leadership is replaced.

Section II is devoted to the model. Its strength is indicated by a number of historical scenarios in Sec. III in which the model makes predictions which can be checked against what actually occurred.
In most of this study we assume that Central Intelligence Agency (CIA) estimates concerning the Soviet economy are correct. Section IV examines the implications of an alternative view of Soviet economic history, that put forth by Rosefielde and Lee. They maintain that the Soviet gross national product (GNP) and defense spending share is currently larger and has been growing faster than the CIA believes.

We studied the implications for the tradeoff curve of a number of issues and uncertainties in the estimates of key parameters thought by experts on the USSR to be important for the Soviet economy during the next decade. One of these issues (examined in Sec. V) concerns demographic change, in terms of both growth of the labor force and its ethnic composition. Total labor force growth is expected to be dramatically lower than during the 1970s, which will result in lower GNP growth. What little growth in the labor force does occur will be concentrated in the less efficient (as compared with Slav) non-Slav portion of the labor force.

One way to counter low labor force growth is to increase the rate of technical change and thus increase the effectiveness of each worker. The impact of differing rates of productivity growth is studied in Sec. VI.

Poor weather has harmed the Soviet economy in recent years. A scenario in which poor weather continues into the next decade is investigated in Sec. VI.

As is true with many economies, the Soviets are expected to have additional difficulties during the 1980s because of the increasing cost of energy. Section VII describes how this is incorporated into the model and investigates a number of energy scenarios.
Foreign trade, which is likely to play a central role in the Soviet economy in the next decade, is examined in detail in Sec. VIII. Foreign trade interacts with technology (due to imports of high technology goods), with weather (due to agricultural imports), and with energy (due to oil exports).

Section IX compares a best case and a worst case scenario.

There are four key conclusions to this study. Optimal control models can produce reasonable and insightful forecasts of the Soviet economy; the Soviets will probably be able to "muddle through" the decade without a severe economic crisis, although they probably will not be able to do as well as they did during the 1970s; the rate of total factor productivity is the key uncertainty--low values leading the economy into a severe crisis and high values resulting in an excellent economic performance; and all other issues are dominated by the question of whether the CIA or Rosefielde-Lee view of the economy is correct.
II. THE HOPKINS-KENNEDY MODEL

The study of centrally planned economies (CPEs) such as that of the Soviet Union is relatively new to economic model builders. Most existing models deal with Western economies and are of a demand side type. They first determine demand for the goods produced by the various sectors and then outputs, employment, and prices. Such an approach makes sense when major policy questions concern the business cycle, inflation, and unemployment.

In CPEs, the key issues are different from those of Western economies. In these economies there is no unemployment, no business cycle, and prices are established by fiat. The key issues are the effects of various planning decisions on the economy, such as the allocation of labor and investment goods, and the implications for the economy of various events and uncertainties, such as changes in the world oil price and uncertainty in the future rate of technological change. To analyze these types of issues, model builders have found it appropriate to build supply side models—models which assume the economy is working at full capacity (full employment, etc.). The usefulness of supply side models is not, of course, restricted entirely to CPEs; they can provide insight into Western economies on certain types of issues. They have recently enjoyed a period of greatly heightened popularity because of their contribution to Reaganes, where they are used to address such issues as how to change the tax structure to increase the rate of growth and thus the capacity of the economy.
The Hopkins-Kennedy model employed in this study is a supply side optimal control model. Most other models of centrally planned economies are supply side econometric models. The use of an optimal control model instead of an econometric model for the modeling of centrally planned economies is relatively new.[1]

The Hopkins-Kennedy model was designed to measure the long-run (over the next 10 years) impact of major changes in the Soviet economy. Such changes could occur as a result of major policy shifts on the part of Soviet decisionmakers (e.g., a decision to markedly change defense spending); hypothetical changes in the key economic parameters (such as the rate of technical change); and exogenous economic shocks (such as the Soviets gaining control of the Iranian oil fields).

In general, the model can be used to analyze time intervals of any finite length regardless of whether these intervals occurred in the past, are projections into the future, or a combination of both the past and the future. The model can be applied to virtually any centrally planned economy--not just that of the Soviet Union--with remarkably few changes other than the obvious need to use different input data.

The model employed in this study has 21 sectors, each of which produces a certain good, such as chemicals, construction materials, and

[1] The most developed and famous econometric model of the Soviet Union is SOVMOD. For a discussion, see Donald Green and Christopher Higgins, 1977. Insight gained by one of the authors from the construction of an earlier simpler optimal control model of the Soviet Union, which was used to investigate the potential for Soviet economic recovery after a nuclear war, was of substantial help in the development of the Hopkins-Kennedy model. See Michael Kennedy and Dennis Smallwood, January 1978. There exists only one other significant optimal control model of the Soviet Union. This is the DYNEVAL model which is being developed in rough parallel to our own efforts. See "The DYNEVAL Model: A Generalized Overview," May 1981.
processed foods. The number of sectors and the definition of the good which is represented by a particular sector can be easily altered to analyze questions other than those examined in this study. For instance, our particular concern with energy led to the inclusion of four energy sectors (oil, gas, electric power, and coal and peat). If the model were used in a study where there was less concern for energy, the number of energy sectors could be decreased and the sectors increased in some other area of the economy which was of more concern to that study.

In this study the model is used primarily to calculate the tradeoff between the level of consumption and defense for the economy under various circumstances. The model can also analyze the tradeoff between any two or more goods and combinations of such goods. For example, we could look at the tradeoff between the two sectors of weapons production and military services (military pay, operations, and maintenance). In addition, the model can be employed to make economic predictions, such as the growth rate in GNP or consumption, and used to analyze how these growth rates are altered by such changes as a decrease in the availability of foreign credit.

OPTIMAL CONTROL MODELS

The general nature of optimal control models is first illustrated by a simplified optimal control model[2] of an economy which consists of two goods (guns and butter) and one time period. The optimizing technique employed with optimal control models allows the calculation of the set of all net outputs which are technically feasible for this economy. The noncoordinate axis boundary of this set of net outputs is

[2] This model is a simplified optimal control model because it contains only one time period.
called the production possibility frontier (see Fig. 1). This is also the set of all guns and butter net outputs which are economically efficient in the sense that for any point on the production possibility frontier there does not exist a feasible set of net outputs which can produce more of one of the two goods without producing less of the other. The tradeoff curve between guns and butter for this model is the production possibility frontier. In the more complicated Hopkins-Kennedy optimal control model a similar production possibility frontier is the tradeoff curve between consumption and defense.

Fig. 1—A simplified optimal control model
THE RELATIVE ADVANTAGES OF ECONOMETRIC AND OPTIMAL CONTROL MODELS

In this project we employ an optimal control model of the Soviet economy. The application of this type of model to the study of the policy options open to the Soviet Union is relatively recent. The more traditional approach is to use econometric models.

Econometric models represent an economy with a system of simultaneous equations. By estimating the parameters of the equations, the regularities that exist in historical data can be determined. These models can then be used with relatively high accuracy for short-run forecasting scenarios which assume that historic trends will continue. In contrast, optimal control models employ optimizing techniques. They are relatively useful for long-run scenarios which allow for major departures from historic trends. The study of the tradeoff between consumption and defense during the 1980s falls largely in the latter category. In this context, optimal control models can help uncover the range of options open to Soviet decisionmakers.

The two methods are complementary. Optimal control models can reveal insights that cannot be obtained by econometric models, and vice versa. The use of both types of models can often greatly enrich our understanding of an issue.

THE CORE OF THE HOPKINS-KENNEDY OPTIMAL CONTROL MODEL

We will describe the core of the Hopkins-Kennedy (HK) model separately from a number of the model's special features which were added in order to investigate questions of special concern to this study. Three major groups of special features were added which will be described in more detail below: (1) a division of the labor supply
between Slavs and non-Slavs with a differential labor efficiency assigned to each of the two groups; (2) special parameters and equations to incorporate the effects of increasing energy costs--costs which in the core of the model are assumed constant; and (3) a foreign trade sector. The core of the model is described in this section, and the three groups of special features are dealt with in Secs. V, VII, and VIII, respectively. The base case presented in Sec. III uses the core of the model as modified by these special features. Scenario-specific alterations to the model are also discussed below along with the relevant scenarios. Examples include alterations which enable an analyst to study the effects of poor weather.

The model has 21 sectors--each with its own estimated translog production function.[3] Labor and two types of capital (structures and machinery) are the inputs to the production functions. In addition to the amounts of labor and capital specified by the translog production function, each sector also requires for each unit of output a certain amount of inputs of other goods. For example, a unit of output of the ferrous metallurgy sector requires as an input 0.023 units of the output of the chemicals sector as well as inputs from other sectors. In general, the amount of input for each of the sectors required to obtain a unit of output from any particular sector is given by an input-output matrix. The model allows each of its time periods (taken in this study

[3] The translog production function has an elasticity of substitution between any two inputs which is variable and in general different for each pair of inputs. In contrast, the Constant Elasticity of Substitution (CES) production function has constant elasticities which have the same value for every pair of inputs. The Cobb Douglas production function is a special case of the CES production function where the elasticity of substitution between every pair of inputs is not only equal to the same constant value, but this value is specified as 1.0.
to be one year) to have a different input-output matrix. This feature is particularly useful when using the model to track history for model verification purposes (see Sec. IV, for example). The tradeoff curve we are particularly interested in is between consumption and defense. Each is defined as aggregates of the output of the 21 sectors. For consumption, the weights given to certain components are allowed to change with time in accordance with historical time trends which were determined by econometrically estimating a group of Engel-like curves. At the beginning of the first time period, each sector has a specified amount of each of the two types of capital.

The first group of decisions that policymakers must make in the model during the first period is determination of the share of the supply of labor to be allocated to each of the sectors. Every set of labor shares corresponds to at least one point in the set of technically feasible net outputs. The model uses an optimizing procedure to determine which of these sets of labor shares correspond to the net output points which make up the production possibility frontier or tradeoff curve between consumption and defense.

Once the labor shares are determined, the model uses the exogenously specified labor supply and the estimated production functions to determine outputs. Some of these outputs come from the investment goods sectors. The second group of decisions that policymakers must make in the first period is determination of the share of these investment goods to be allocated to each of the sectors. This is done by an optimization procedure analogous to that employed in determining the labor shares.
The determination of the investment goods allocation yields the amount of capital associated with each sector for the second period. The process is then repeated for the remaining periods.

The model's result will be a production possibility curve—the tradeoff curve—between consumption and defense.

A detailed technical description of the Hopkins-Kennedy model is given in App. A. The appendix first describes the core of the model in more detail and then describes in detail the various special features of the model which are discussed in a less technical fashion in the text.

THE DATA FOR THE CORE OF THE MODEL

For most of our calculations the base year is taken to be the beginning of the 1980s and projections are run up to 1990. A key item of data is an input-output matrix for the economy. None is available for our base year; we thus use one calculated by Treml et al. for 1972[4] and assume that it is valid for our base year.

The net outputs of the economy are also required for the base year for each of the 21 sectors. Available CIA estimates[5] for the base year exist only in a fairly aggregated form. They were disaggregated by weights calculated by Greenslade.[6]

Given the net outputs and the input-output matrix (which, among other information, gives the amount of the two types of capital required to produce a given unit of output for each sector), the base year level of each type of capital associated with each sector can be calculated.

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The production function parameters are calculated primarily from the input-output data as discussed in detail in App. B.

The data inputs needed for the years after the base year in the projection are few. These are mainly the labor supply[7] and the rate of technological progress.[8]

The first 18 of the 21 sectors of the HK model are concerned with producing the economic or material product portion of the economy. The 18-sector level of aggregation was chosen rather than 10 or 30 because these 18 sectors are those for which data are readily available. The 21 sectors are:

1. Ferrous metallurgy
2. Nonferrous metallurgy
3. Machine building metal working
4. Forest products
5. Soft goods
6. Processed foods
7. Construction materials
8. Coal and peat
9. Oil
10. Gas
11. Electric power
12. Chemicals
13. Paper and pulp

[7] The labor supply data are discussed in Sec. V.
[8] Data on the rate of technological progress are discussed in Sec. VI.
14. Construction
15. Agriculture and forestry
16. Transportation and communication
17. Trade and distribution
18. Industry not elsewhere classified (NEC) and other branches
19. Weapons production
20. Military services
21. Other

The energy portion of the economy was disaggregated somewhat more than usual because of its particular importance in this study. Sectors 19 and 20 cover defense. The last sector covers the remainder of the economy, which is primarily services.

A more detailed discussion of the data used by the core of the model is given in the first part of App. B. Data of a general nature concerning subjects other than the core of the model such as historical verification, special model features, and various scenarios are given in the remainder of the text. Additional data of a more detailed nature concerning these noncore of the model subjects are given in the second part of App. B.
There is some controversy over what the history of the Soviet economy has been since World War II. For our base case we employ CIA estimates of recent economic data and their implications. These data reflect the dominant view of Soviet economic history held by Western experts on the Soviet Union. An alternative view expressed by Rosefielde and Lee will be briefly described in Sec. IV and its implications for the future discussed. We plan to undertake a much more detailed comparison of these alternative views in a follow-on project.

According to CIA data, the rate of growth of the Soviet GNP has been falling since the 1950s. It was 6 percent during the 1950s, 5 percent in the 1960s, and 3 percent in the 1970s. This decline is due in part to a fall in the rate of growth of capital from 10 percent in the 1950s to 8 percent in the 1960s and 7 percent in the 1970s. As can be seen from these figures, the capital/GNP ratio has been rising. This indicates a fall in the rate of return on capital and thus helps to explain the fall in the rate of growth of capital.

Another major factor contributing to the long-run decline in Soviet GNP growth rates is the fall in the rate of growth of total factor productivity. This was 2 percent in the 1950s, 1 percent in the 1960s, and -0.2 percent in the 1970s. This fall, and the fact that the rate is as low as it is, are remarkable, given the enormous commitment to research and development in the Soviet Union compared with other economies and the fact that Soviet technology is relatively backward.
In general, technically backward nations have an advantage in obtaining high rates of growth in total factor productivity because they can copy the technology of advanced nations rather than having to undertake the more difficult task of developing it themselves. Exactly why the Soviets have done so poorly in technical improvements is not known. This is not surprising since economists have a poor understanding of why Western industrial democracies have differing rates of total factor productivity growth—economies for which far more data are available than for the Soviet Union. There is, however, a consensus that the highly centralized, bureaucratic, change-resistant nature of the Soviet economy makes innovation and technical diffusion more difficult than in Western economies and that this is an important part of the explanation.

The decade of the 1970s is a useful benchmark to measure the predictions of the Hopkins-Kennedy model for the 1980s. The 1970s, as we have already seen, was a period of relatively poor performance compared with earlier periods, particularly for the last half of the decade, which in many respects was one of the worst economic periods in modern Russian history. It has led a number of Western observers to begin to speak of a crisis in the Soviet economy. Total factor productivity growth fell from 0.3 percent to -0.75 percent. Total factor productivity in industry grew at 1.1 percent per year during 1970-75, but then declined by 0.6 percent per year in 1976-80.

During the 1980s there are certain factors that will cause increased economic difficulties for the Soviets in general and for their tradeoff between consumption and defense in particular. The growth rate
of the labor force will fall from 1.8 percent in the 1970s to 0.4 percent in the 1980s because of the second echo of the demographic effects of World War II. Further, what little increase in the labor force does occur will be concentrated in the relatively low-producing non-Slav sector of the labor force. This group will grow at a rate of 2.1 percent, while the Slavs will actually be declining at a rate of 0.1 percent. This demographic change will lower GNP growth and will shift the tradeoff between consumption and defense unfavorably from what it otherwise would be. A second problem is resource exhaustion and the increasing need for expenditures to protect the environment. Resource exhaustion will push the Soviet energy and mineral industries increasingly into the higher production cost areas in the northern and eastern part of the country. This problem is particularly acute for the energy sectors.

BASE CASE PROJECTION: 1980 TO 1990

The base case employs the core of the HK model, as described in the preceding section, modified with regard to the labor supply, energy sector, and foreign trade, as mentioned in that section and described in more detail in Secs. V, VII, and VIII, respectively. The model takes the base case and makes a projection from 1980 to 1990.

Figure 2 shows the result. The horizontal and vertical axes are the average annual rates of growth for consumption and defense from 1980 to 1990, respectively. The tradeoff curve between consumption and defense is the boundary of the technically feasible rates of growth. This curve was calculated as follows. A given annual rate of growth of defense spending was selected. The model was then used to calculate for this rate of defense growth a maximum discounted sum of consumption for
the period 1980 to 1990. This sum of consumption was then converted into an equivalent average annual rate of growth of consumption. The process was repeated for different rates of growth of defense spending until enough points were calculated to graph the tradeoff curve at the desired level of accuracy. Investment was determined endogenously in the model. The rate of discount was 10 percent, the same rate as that calculated for the 1960-75 period by a method to be discussed in the next section.

Both defense and consumption as used here are intertemporal goods. For instance, consumption is composed of the discounted sum of consumption for each year. Of course, even within a given year, defense and consumption are composite goods consisting of aggregates of the outputs of the 21 sectors.
In general, the model can use any set of intertemporal weights in its definition of a composite good, such as defense or consumption. For questions different from those addressed in this study, other sets of intertemporal weights may be more appropriate.

It is instructive to compare the tradeoff curve in Fig. 2 with the 1970s experience. If the base case is correct, the Soviets will not be able to do as well as they did during the 1970s, mainly because of lower effective labor force growth.

In Fig. 2 there is a point identified as the minimum required economic performance to "muddle through." This point is arbitrary. The intention is to define a single point for illustrative purposes which can be used to give a rough lower bound on the obviously loosely defined concept of "muddling through." We have taken "muddling through" to mean the ability to maintain a rate of growth of defense spending of 4.5 percent over the period and a rate of growth in consumption spending sufficient to prevent political discontent. The 4.5 percent figure is in the vicinity of what the CIA believes will be the rate of growth of defense spending.[1] It has been argued by some Western experts on the Soviet Union that, due to various distortions, political considerations, etc., a 1 percent rate of growth of per capita consumption is required to ensure a positive rate of growth of per capita income for all major groups in Soviet society and that this is roughly what is required to prevent political discontent. Given the expectation of a 0.9 percent rate of increase in the population in the 1980s, this translates into a 1.9 percent rate of growth of total consumption. It is worth

[1] See, for example, Central Intelligence Agency, January 1981.
emphasizing that such concepts as "muddling through" and political
discontent are qualitatively uncertain terms. In defining a point to be
(for the purposes of this study only) the minimum requirement for
"muddling through," we are not arguing that this is the correct
definition, but only that it is a plausible one for our purposes.

The tradeoff curve goes between our minimum required point and the
point of the 1970s experience in Fig. 2. We conclude that, if the base
case is correct, the Soviets will be able to "muddle through." They
will be able to obtain the minimum required economic performance, but
not enough to duplicate the 1970s experience.

The Hopkins-Kennedy model is in an early stage of development.
This is the first study in which it has been employed. Further
improvements in the model and in the data it uses can be expected to
change the quantitative results to some extent. However, the authors
believe that it is improbable that the major qualitative results of this
study, such as the one given in the previous paragraph, will be altered.

HISTORICAL VERIFICATION

One way to test a model is to use it to predict history and then
compare the predictions with what actually occurred. In Fig. 3 the
analysis in Fig. 2 was repeated, except this time instead of projecting
from 1980 to 1990, we projected from 1960 to 1975. The input data are,
of course, different. In particular, a different base year is used.

The resulting tradeoff curve is depicted in Fig. 3, which gives the
average rate of growth of consumption over the 15-year period versus the
average rate of growth of defense. A point representing the actual
experience according to data is also shown. As can be seen from the
figure, this actual experience point is directly on the tradeoff curve.
Fig. 3—Base case projection: 1960-1975

Another way of testing how well the model does at predicting history is to use it to forecast GNP for each year in the period 1960 through 1975 and then compare these forecasts with the CIA estimates of GNP. Recall that the model gives the tradeoff between consumption and defense, with investment calculated endogenously. Thus, by taking defense as being equal to the CIA's estimate, we obtain for our forecast not only defense but consumption and investment as well. Adding these three together gives our GNP forecast. The results are shown in Fig. 4, where both the forecasted GNP and the CIA estimates are graphed over time. It is seen that the forecasted GNP closely traces actual GNP.

In the preceding calculations, optimal control theory was used to calculate for given sets of defense expenditures the maximum discounted
Fig. 4—Forecasted and actual GNP for the base case: 1960-1975

The sum of consumption over the relevant period. The discount rate used for these calculations was 10 percent. In a series of calculations similar to those depicted in Fig. 4, with different discount rates, it was found that 10 percent gives the best fit. This 10 percent was then used not only for our historical analysis but also for all of our projections into the future based on historical data. This amounts to assuming that the effective discount rate used by the Soviets in the recent past would also be used in the 1980s.

The forecasted and actual GNP numbers in Fig. 4 differ for a number of reasons. The Soviet economy does not operate, as is well known, with optimal efficiency. However, the Hopkins-Kennedy model is able to track history as well as it does because its optimization process is carried
out under the assumption that inefficiencies in the economy in the base year of a projection continue into the future. This eliminates most but not all of the reasons to expect that the optimization process per se might lead to a divergence between forecasted and actual GNP. There remains the possibility of mistakes by decisionmakers in the decisions that they make after the base year.

Mistakes can be defined as the extent to which actual decisions differ from what the model forecasts as being correct. In turn, mistakes can be subdivided into two categories, human error and fictitious mistakes. Human error occurs when decisionmakers make mistakes even though they have all the information required to make a correct decision. We also define human error to include mistakes which are made because real-life decisionmakers must guess some of the information which is known to the model, and sometimes guess wrong. For example, one of the model's inputs is the rate of total factor productivity growth. For the historical projection this is set at the value which actually occurred.[2] Real-life decisionmakers would not know what this will be in the future for which they have to make decisions, and as a result will make mistakes.

Fictitious mistakes occur when decisionmakers make decisions which are correct as far as their individual objective functions and constraints are concerned, but which are mistakes with respect to what the model forecasts. In the Hopkins-Kennedy model it is assumed that all decisionmakers in the Soviet Union behave as if they were maximizing a discounted sum of consumption for their nation as a whole. In

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[2] More precisely, total factor productivity growth for the CIA historical projections is set at a value which is consistent with the rate of growth of labor, capital, and output that the CIA estimates to have occurred.
actuality the objective function for the Soviet Union is far more complex. In fact there is not a well-defined nationwide objective function at all. Rather, each decisionmaker maximizes a personal objective function where the interests of the nation as a whole are not perfectly taken into account. The national objective function depends, loosely speaking, on an aggregation of these individual functions with weights that depend on the importance of the various decisionmakers. Further, unlike what is assumed in the model, the national objective function changes over time due to shifts in the relative importance of individual decisionmakers and other factors. The constraints which affect the options available to the Soviet Union are also more complex than we have assumed. They depend in part on the constraints faced by individual decisionmakers. For example, a decisionmaker may allocate more labor to a sector than is optimal because his options are constrained by political considerations. Much but not all of this is corrected by the assumption that base year inefficiencies continue into the future. For instance, political constraints which were not important in the base year may become important later. It is also possible to add constraints to the model to eliminate otherwise possible decisions which because of inefficiencies in the Soviet system or for some other reason it is reasonable to assume will not occur—even if these inefficiencies, etc. are applicable after the base year. None such were made for the historical projections, but one was made for the 1980 to 1990 projections. This was that investment growth in any one year will not be negative and for the decade as a whole will not be less than about one-third of what it was in the 1970s.
Other factors that contribute to the divergence of the forecasted and actual GNP which we have ignored until now include errors in the input data and errors caused by the fact that the model is necessarily built from a set of simplifying assumptions. Examples include the assumption that the production functions are translog instead of more complicated; the assumption that the input-output coefficients are, with some exceptions, fixed; the assumption that capital once invested in a given sector cannot be moved to another sector; and the assumption that the only relevant decision that decisionmakers make is how to allocate labor and investment across the sectors in each year.

The fact that the model tracks history as well as it does demonstrates that the cumulative effect of all of these problems is small.
IV. THE "CIA WORLD" VERSUS THE "ROSEFIELD-Lee WORLD"

In recent years, a debate has arisen over the actual nature of Soviet economic history. Steven Rosefielde and William Lee have argued that Soviet historical GNP growth rates, levels, and shares of defense spending have been higher than those estimated by the CIA. The Hopkins-Kennedy model can be employed to examine the future implications of this alternative view, as well as other alternative views concerning such issues as defense spending or historical economic growth. We will employ the model here in an exploratory and illustrative fashion, considering only the Rosefielde-Lee alternative and its relationship to CIA estimates. Since both Rosefielde and Lee arrive at about the same conclusion, we will use the data provided by Lee. Henceforth, we shall refer to the CIA view of economic history and its implications for the future as the "CIA world." Similarly, we shall refer to the Rosefielde-Lee view of history and its implications for the future as the "Rosefielde-Lee world."

For the period 1960 to 1975, the CIA estimates a GNP growth rate of 4.8 percent and a rate of growth in defense spending of 4.6 percent,[1] whereas Lee estimates 7.7 percent and 10.9 percent, respectively.[2]

[1] The Wharton Econometric Forecasting Associates, Inc. data bank for SOVMOD. Access to these data was generously provided by Dr. Daniel Bond.
HISTORICAL VERIFICATION ("LEE WORLD")

In Fig. 5 we repeat for the "Lee world" what was done in Fig. 4 for the "CIA world." We used the Hopkins-Kennedy model to forecast GNP changes over time in the "Lee world" and compared this with the actual values of GNP according to the Lee estimates.

The same model can be used to make accurate estimates for both the "CIA world" and the very different "Lee world" because of altered input data. In particular, after the model calculates the tradeoff curve between consumption and defense (investment being endogenous), instead of specifying for each year a level of defense spending equal to the "CIA world" value and then adding consumption, defense, and investment to obtain GNP, we specify defense spending equal to the "Lee world"
value and then add. We also employ the higher rate of total factor productivity growth of the "Lee world" as compared to the "CIA world" and recalculate the discount rate according to the procedure previously discussed, obtaining 12 percent instead of 10. The 12 percent is then used for all of the "Lee world" calculations.

Since GNP in the "Lee world" grows at roughly a constant rate without such things as cyclical fluctuations, the change in the total factor productivity data inputs assures that the value of GNP predicted by the model will be in the right ballpark if the model also roughly predicts the rate of growth of capital. This is accomplished by the adjustment in the discount rate. All of the reasons previously discussed, as to why there is a divergence between forecasted and CIA-estimated GNP in Fig. 4 also apply to the divergence between forecasted and Lee-estimated GNP in Fig. 5. The close fit in Fig. 5 is a further indication that the cumulative impact of these effects is small.

In addition to making GNP estimates for the "Lee world" similar to those made for the "CIA world," a "Lee world" projection was also calculated from 1960 to 1975, and compared in Fig. 6 with the previously presented "CIA world" projection for the same period. Also shown in Fig. 6 are points representing the actual experience for this period according to the CIA and Lee. Note that the Lee actual experience point is close to the Lee tradeoff curve, as was the case for the CIA actual experience point and the CIA tradeoff curve. Figure 6 also provides another important insight--the tremendous difference between the rates of growth in defense and consumption between the "CIA world" and the "Lee world."
"LEE WORLD" VERSUS "CIA WORLD" PROJECTION: 1980 TO 1990

Recall that the base case projection assumes that the "CIA world" is correct. This projection is shown for a second time in Fig. 7. Accompanying it, for comparison purposes, is a "Lee world" projection over the same period. Lee gives data on defense and consumption growth only up to 1975. Thus, it is not possible to compare a Lee actual experience point for the 1970s decade. Instead, a Lee experience point is given for the 1971-75 period. For the "CIA world," an experience point for 1971-75 is given in addition to the experience point for the 1970s. As mentioned earlier, assuming that the "CIA world" is correct
and that the future corresponds to the base case, the Soviets will not do as well in the 1980s as they did during the 1970s. The same is true in the "CIA world" if the 1980s are compared with the 1971-75 experience. A similar result is obtained for the "Lee world." The Lee tradeoff curve indicates that if the "Lee world" and the "Lee world" base case are correct, then the Soviets will not be able to do as well as they did during the 1971-75 period.

Recall from Fig. 7 the large difference in the growth rates of consumption and defense between the "CIA world" and the "Lee world." This difference is in many ways more important than any other single factor studied in this project. The actual difference in one sense is even greater than depicted in Fig. 7 because the levels of consumption
and defense in 1980 for the "CIA world" and the "Lee world" are substantially different. In the "Lee world" they are in the ballpark of one-fifth and two-thirds larger than in the "CIA world."

Suppose we ask a somewhat different question than the one answered in Fig. 7. Consider the perspective of someone in the CIA who is studying the economic potential of the Soviet Union during the next decade. Now assume that this person's boss announces that the CIA has discovered that Lee was right all along, and then asks the analyst how this changes his perspective. To answer the question the analyst would make a new Lee projection like that in Fig. 7, except that this time instead of using Lee world data for the base year, CIA data would be used. The result would be to move the Lee tradeoff curve to the northeast of its current position, resulting in a substantially greater difference between the two worlds than is depicted in Fig. 7.

There are serious implications associated with an assumption that the Lee, or more generally, the "Rosefielde-Lee world" is a better representation of reality than the "CIA world." The Soviet GNP in 1980 would then be on the order of one-quarter larger than the current CIA estimates. Further, Soviet GNP and total factor productivity growth during the next decade would be expected to be substantially higher than for the United States. The resulting image is one of a strong Soviet Union with a dynamic and rapidly expanding economy—an economy which can afford rapid increases in an already high level of defense spending without causing serious problems for the consumption sector. Indeed, there is enough growth in GNP to allow both the defense and consumption sectors to grow at a rapid rate. In comparison, the U.S. economy would be one of stagnation. The existing notion that communism is an
inefficient economic system would have to be revised. Finally, the long-range possibility that the faster growing Soviet economy would eventually "bury" the United States would have to be faced.
V. THE LABOR SUPPLY AND THE RELATIVE EFFICIENCY OF SLAVS AND NON-SLAVS

The labor supply for the Soviet economy is expected to increase at an average rate of 0.4 percent per year[1] during the decade of the 1980s. This is much lower than the 1.8 percent average rate of increase experienced during the 1970s, a fact which is a substantial factor in explaining why our base case predicts that the Soviet economy will not perform in the 1980s as well as it did in the 1970s.

Changes in the efficiency of a labor force, as well as changes in its size, are important for an economy. One source of efficiency change is improved technology, which is addressed in the next section. This section concentrates on another source of efficiency change--changes in ethnic demographics. In particular, we examine changes in the mix of Slav and non-Slav workers and how this affects the economy due to the labor efficiency discrepancy between the two groups.

Changes in the ethnic composition of the Soviet Union have been going on for some time and are almost certain to continue well into the 1980s. The predominantly Slav, or northern, populations have not been increasing at as rapid a rate as the predominantly non-Slav, or southern, populations. During the 1980s the Slav labor force is expected to actually decrease at a rate of 0.1 percent per year.[2] In contrast, the non-Slav labor force is expected to grow at 2.1 percent per year.[3] Currently non-Slavs comprise about 19 percent of the total

[1] Feshbach, 1980. His data is subject to revision.
[2] Ibid.
[3] Ibid.
labor force. Historic data indicate a lower efficiency for non-Slav labor compared with Slav labor. A continuation of past demographic trends into the 1980s implies a slowing effect on the Soviet economy. This negative impact on the Soviet economy could be lessened to the extent that the efficiency of the non-Slav labor force is increased to match that of the Slav labor force.

For reasons of simplicity and data availability, the term non-Slav has been defined to be the peoples residing in the Transcaucasus and Central Asian republics (Armenia, Georgia, Azerbaijan, Uzbekistan, Kazakhstan, Kirgizstan, Turkmenistan, and Tadzhikstan), and the term Slav used to describe the people living in the rest of the Soviet Union (RSFSR, Ukraine, Belorussia, Lithuania, Moldavia, Latvia, and Estonia). This regional dichotomy approximately captures the demographic distinctions which are of concern to this study. As of 1970, 70 percent of the population living in the Transcaucasus and Central Asian republics were ethnically non-Slav, and 95 percent of all ethnic non-Slavs lived in this region. [4]

Data from 1970 indicate an income per capita ratio of 1.37 for Slavs compared with non-Slavs. [5] The corresponding ratio for income per worker is 1.01. [6] The difference between these ratios is primarily due to a higher labor force participation rate by the Slavs. Official Soviet data for 1978 indicate 46 percent of the Slav population and 31 percent of the non-Slav population was employed. [7] The labor force

[4] For our purposes the people living in the Baltic republics of the USSR are considered to be Slavs.
participation rates of the two groups are determined largely by cultural and environmental factors, which are significantly different in the two regions. The southern, or non-Slav region, has a substantially higher rate of population growth than the northern, or Slav, region. In general, the populace of the non-Slav region tends to be Moslem, a faith which encourages the practice of raising large families, and of semi-nomadic background, a tradition which discourages employment of women outside of the home. Whereas the participation of women in the labor force has increased from 38.7 percent in 1960 to 42.6 percent in 1970 in the non-Slav regions, it is still less than the respective rates of 48.1 percent and 52.3 percent for the Slav regions.[8]

The ratio of GNP per worker for Slavs relative to non-Slavs is 1.15.[9] The discrepancy between the almost equal distribution of income per worker (1.01) and the GNP per worker ratio is due in part to the effects of taxation and transfer payments which appear to benefit the non-Slav republics. Turnover taxes are levied most heavily against the capital-producing sectors of the economy, which are predominantly in the northern, Slav regions. In addition to having different tax bases as a result of the economic mix of the republic, the portion of tax a republic may keep of the total tax collected is determined by the USSR Ministry of Finance. For the period 1961 to 1979, the southern, non-Slav republics were authorized to retain 91 percent of the turnover tax collected, compared with 50 percent for the northern, Slav republics.[10] The ratio of fixed capital per worker for Slav relative

to non-Slav republics is 0.96 (as of 1966), suggesting the extent of
GOSPLAN efforts to deepen capitalization in the Transcaucasus and
Central Asian republics.[11] Although extensive capital investment has
taken place in these areas, the labor force has serious deficiencies.
In both education and skill levels, the Central Asian republics are at a
disadvantage compared with the northern republics. There is some
evidence that workers from the north are "imported" or enticed with
special wages and privileges into the Central Asian republics to fill
jobs requiring either education or skills which are in short supply in
the southern, non-Slav republics.

It is possible to calculate an index of relative total factor
productivity for the two regions. We define this mathematically in the
usual manner as

\[
\text{Relative TFP} = \frac{X_S}{L_S^{\alpha} K_S^{\beta}} / \frac{X_{NS}}{L_{NS}^{\alpha} K_{NS}^{\beta}}
\]

where

- \( K \) = capital
- \( L \) = labor
- \( NS \) = non-Slav
- \( S \) = Slav
- TFP = total factor productivity
- \( X \) = output
- \( \alpha \) = labor's share of output
- \( \beta \) = capital's share of output

For a discussion of relative total factor productivity indexes in general and this index in particular (which is technically a geometric total factor productivity index), see Fogel and Engerman, 1974.

Using the ratio of GNP per worker as a measure of output per worker, and the ratio of fixed capital per worker as a measure of capital per worker, the relative total factor productivity index can be calculated. The index is the ratio of the output per unit of input indices for the two regions, with the inputs geometrically aggregated, using labor and capital shares as weights. The estimated value of the index is 1.17. Assuming that for a given amount of effective labor, capital is equally productive in the two regions, the relative efficiency of Slav to non-Slav labor is estimated to be 1.27. Boundaries to the uncertainty of this estimate (resulting from errors in the data and questions on the precise variables to be used as estimates of the ratios of output and capital per worker) were judgmentally determined to be 1.0 and 1.6.

Given the existence of a differential in the relative efficiencies of Slav and non-Slav labor, it would be illuminating to estimate the effect on the tradeoff curve between consumption and defense of eliminating this differential by 1990 via a constant rate of reduction throughout the next decade. Three tradeoff curves are depicted in Fig. 8. The innermost is the base case. The middle curve shows the improvement in the economy that would occur if the efficiency of non-Slav labor caught up with that of Slav labor in the manner postulated. The middle curve assumes that the current efficiency differential is in fact equal to our best estimate of 1.27. If the current differential is
in fact equal to the upper bound of our estimate, 1.6, then the relevant curve would be the outer dotted curve instead of the middle one.

In our comparisons of the different curves in Fig. 8, we are effectively using a two-dimensional figure of merit. For some purposes it is useful to summarize the difference between two tradeoff curves by a one-dimensional figure of merit—that is, a single number. Attempting to describe the difference between two of our typical tradeoff curves by a single number, however, leads to ambiguity as to what this number should be. We have chosen to measure the difference between a pair of curves in terms of the difference in the rate of growth of defense spending at a 2 percent rate of growth of consumption. It is actually a little more complicated, since we are not looking just at the difference
in the rate of growth of defense between a pair of curves, but rather at the cumulative effects that this difference in defense spending has over the decade. To be precise, we measure the difference in defense spending in 1990 which would occur if in both cases consumption grew at 2 percent per year over the decade.

Thus in Fig. 8, if the relative efficiency differential is equal to 1.27, then the difference between the corresponding curve and the curve which represents the base case is such that if in both cases consumption grew at 2 percent per year, then in 1990 defense spending would be approximately 13 percent higher.[12] If the current efficiency differential is instead 1.6, then the value of our figure of merit becomes 23 percent.

[12] All figure of merit estimates in this study are approximations in the sense that they were defined by graphically measuring the relative distance between the two curves that were being compared instead of being estimated in a slightly more precise and significantly more expensive way by computer.
VI. TECHNOLOGY AND WEATHER

The Soviets hope to accelerate their rate of growth of total factor productivity during the 1980s. If successful, they can use this growth to partially offset the slowdown in the rate of growth of the labor force and its shift toward a less efficient ethnic composition. This will occur because total factor productivity growth (loosely speaking) increases the efficiency of workers, making each one worth the equivalent of more than one before the period of growth.

Weather has become an item of interest to Soviet specialists. In the last half of the 1970s weather was particularly poor compared with historical averages, raising the question, what effect will continuing adverse weather have on the Soviet economy?

TOTAL FACTOR PRODUCTIVITY GROWTH

The rate of future total factor productivity is uncertain. The Soviet five-year plan for the first part of the decade calls for an average rate of about 0.94 percent, and there is no reason to believe that the Soviets will do better in the second half of the decade than in the first. Given the history of failure to meet total factor productivity goals in previous five-year plans, it is reasonable to take this as an upper bound on what the actual rate will be.

The base case and lower bound rates of growth used in the model were chosen arbitrarily. They are the same as the rates for the first and second half of the 1970s decade--0.3 percent and -0.75 percent, respectively--these assumptions imply that the -0.75 percent rate for the second half of the decade is abnormally low. Given the long-run
secular decline in Soviet total factor productivity growth since World War II, this may not be entirely true. But if the normal future course for the Soviets is one of negative total factor productivity growth, then the Soviet economic structure will eventually collapse.

The results for the upper and lower bound cases are shown in Fig. 9. As is evident in the figure, the difference between the two tradeoff curves is enormous. This difference is the most important of all those investigated within the context of the "CIA world." (Only the CIA versus Rosefielde-Lee controversy is more important.) It completely spans the range which we have defined as "muddling through." If the upper bound case is correct, the Soviets will be doing better than the 1970s experience—hence better than "muddling through." If the lower bound case is correct, they will be doing worse than the minimum
required performance of 4.5 percent growth for defense and 1.9 percent for consumption—hence worse than "muddling through."

Higher total factor productivity growth affects the estimates produced by the model in essentially the same way as does higher labor force growth for a given rate of growth of population. The more efficient labor force directly contributes to increased output. There is also a second-order effect. The larger labor force (in terms of efficiency units) makes the rate of return on capital higher than it otherwise would be. This in turn induces a faster rate of capital growth, and hence an even larger increase in GNP growth than would occur in the absence of this second-order effect.

The difference in the tradeoff curves in Fig. 9 is due mainly to the difference in the available labor efficiency units. Some of the difference is explained by differing capital growth rates. This effect is mitigated by a third-order effect: The increase in the rate of growth of capital, while favorable to defense and consumption over the period as a whole, is not favorable in the first few years. The increased rate of capital growth requires a higher level of investment which in the first few years is found by decreasing the amount of consumption and defense which would otherwise be available.

POOR WEATHER

The Hopkins-Kennedy model has a weather adjustment factor which is applied only to the agricultural sector. It is used to increase or decrease the efficiency of the agricultural sector, depending upon the weather.
Our optimizing procedure is also affected when we take into account the variability of weather and do not simply assume (as we do in the base case) that the weather will be average for every year in the 1980s. The revised procedure is as follows. First we optimize the model as before, assuming normal weather every year of the decade. As a result of this optimization, we have a set of inputs for the agricultural sector in the first year. Next, the weather for the first year is taken into account by adjusting the output of the agricultural sector accordingly. The inputs for the first year are left unchanged. Having calculated the position of the economy after the first year, we reoptimize the model for the second and later years under the assumption of average weather for the second and later years. Next we take into account the weather from the second year and reiterate.

Note the theory behind this procedure. We are assuming that at the beginning of the first year the decisionmakers know nothing about what the future weather will be and hence base their decisions on the assumption that it will be average in every year. By the beginning of the second year the decisionmakers know what the weather was for the first year but found it out too late to affect the inputs to the agricultural sector. Armed with their new knowledge, they make a new set of decisions about what to do in the second and later years. The process is then reiterated.

To incorporate the effects of future weather in a scenario, we must predict what the future weather will be. For the period 1955-77 Green has calculated what the agricultural output would have been for each year if weather in every year was average.[1] We extrapolated Green's

work to cover the years 1978-80. The six-year period 1975 to 1980 was one of particularly poor weather for the Soviets. For our poor weather scenario, we took the five worst years from this six-year period and assumed that this is what would occur during the first half of the 1980s. For the second half of the 1980s, we assumed that this weather pattern would repeat itself.

The resulting difference between the poor weather and base case scenarios is shown in Fig. 10. The difference between the two curves is smaller than some might expect. As measured by our figure of merit, the difference in defense spending in 1990 is 12 percent, assuming a 2 percent rate of growth of consumption in both cases.

![Fig. 10—Weather](image-url)
The effects of poor weather feed through the model as follows. The poor weather causes agricultural output to be unexpectedly low, which increases agricultural imports. To pay for the imports, the Soviets must increase exports or decrease nonagricultural imports in some combination. In the HK model, most of this adjustment comes about by reducing the level of machine building metal working (MBMW) imports. This loss of MBMW imports has a negative effect on investment and hence GNP growth. The drop in the availability of agriculture and MBMW has a direct deleterious effect on the possibilities for consumption and defense.

Reductions in investment due to poor weather in a given year are partly made up in subsequent years, because the reduction in investment causes the stock of capital to be lower than it would otherwise be, and thus the rate of return on capital to be higher. In our optimizing procedure, this process causes investment to be higher in subsequent periods. Thus, if the time period we are concerned with was infinite instead of merely a decade, then the effect of the poor-weather-induced reduction in investment on the capital stock would eventually be entirely eliminated.

The reason our poor weather scenario is not much different from the base case can be better understood on an intuitive level by the following reasoning. Small changes in rate of growth of technology have a large effect because the impact for each year in the decade adds to the impact of the previous year. In the case of weather, this cumulative effect is minor (there is some second-order cumulative effect through the impact on investment). We can thus direct our attention to the
effect of poor weather in a single year. The central reason for a lack of much of an effect is that poor weather reduces total agricultural output by a small fraction of what total output would otherwise be and agriculture output is only a small fraction of GNP. Thus, the first-order effect consists of a small fraction of a small fraction.
VII. ENERGY

Our division of the Soviet economy into 21 sectors includes a particularly detailed representation of the energy sector. The model has coal and peat (sector 8), oil (9), gas (10), and electric power (11). Energy was chosen for special treatment for several reasons. It is vital to the Soviet domestic economy, Soviet exports, and foreign policy in such areas as Eastern Europe and the Middle East. Further, Soviet policies can have major impacts on the world energy market, which in turn can significantly affect the domestic economies of the Western powers. Finally, energy was chosen for special treatment because it is reasonable to assume that the real cost of energy for the Soviets (as for the world as a whole) will continue to rise at a rate which should not be ignored. This rising cost is due to the expectation of resource exhaustion occurring at a faster pace in the energy sector than technical change. This requires that the model incorporate a number of complicated effects, including the substitutability of other factors for energy in production and consumption, which for the other sectors can be ignored.

THE MODEL

Of the four energy sectors, special treatment is required for only those three which depend on an exhaustible resource and hence are subject to resource exhaustion: coal and peat, oil, and gas.

Resource exhaustion is handled as a form of negative technological change. The production function for the three relevant energy sectors is modeled to become less efficient at a rate of 2 percent per year.
Also, a maximum was placed on the amount that each of those sectors can produce in a given year (for details, see App. B).

The increasing cost of energy affects production in the real world by causing the partial substitution of other input factors for energy. The input-output coefficients in the model are adjusted as the cost of energy rises. Recall that in the HK model once capital is invested in a particular sector it cannot be moved at a later date to some other sector. The HK model assumes that a particular unit of capital has associated with it a fixed requirement for each of the three relevant types of energy. The energy per unit of capital requirement for the stock of capital that a sector has changes for two reasons. First, the capital created by investment in any given period has energy requirements associated with it which are optimal for capital produced in that period. Thus, the new capital which is added to the stock of capital from period to period has differing energy requirements. Second, depreciation reduces the fraction of the total amount of the capital stock which is composed of the older vintages of capital.

The optimal energy requirements for new capital for a given sector are determined as a function of the base-year requirements, the change in the prices of the different types of energy from their base-year values, and the extent to which an increase in price causes other goods to be substituted. The technical name for the last parameter is the price elasticity of substitution. Since no one has measured this for the Soviet Union, we have assumed that it is equal to the value which has been estimated for the world as a whole.

A unit of consumption in the model is a linear combination of the outputs of the various sectors where the weights in the linear
combination depend on a number of factors, including the prices of the three relevant types of energy. This dependency of the weights on the prices is similar to the dependency of the input-output coefficients on the price. For a precise mathematical treatment of the energy portion of the model, see App. A.

EASTERN EUROPEAN SCENARIO

The first of several scenarios which we shall analyze with the HK model concerns Eastern Europe. Under the current agreement between the Soviet Union and the Eastern European nations, the Soviets determine oil prices according to a sliding price formula modification of the Bucharest Agreement.[1] The formula designates the price to Eastern Europe as a sliding five-year average of world market prices. For 1981, the first year of the Eleventh Five-Year Plan, the price will be equal to the average of the world market prices for 1976 through 1980.

The base case assumes that the Soviets will continue to use the sliding base formula for the 1980s. To calculate the price which the Soviets would charge under this assumption requires some historical data[2] and an estimate of what will happen to the world oil price during the 1980s. For the latter we take the "most likely" set of prices estimated by the U.S. Department of Energy.[3] The estimates suggest that the real world price for oil will increase at an annual rate of 3 percent during the period. Estimates of the likely volume of sales come from Planning Minister Nikolai Baibakov.[4]

Our alternative scenario to the base case assumes that the Soviets cease the subsidization of oil exported to Eastern Europe as of January 1, 1982, after which they sell it at the world market price. The higher price could easily lead to a reduction in the amount of oil imported by Eastern Europe from the Soviet Union. In this case it is assumed that the oil which would otherwise have gone to Eastern Europe is sold to someone else at the world market price. Thus, whether the oil is sold to Eastern Europe or elsewhere does not affect the economy.

The difference between the tradeoff curves for the base case and our alternative is depicted in Fig. 11 by solid and dashed lines. As can be seen, the difference is fairly small. In terms of our figure of merit, the difference in the level of defense spending in 1990 between
the two cases, assuming a 2 percent rate of growth of consumption in both, is only 6 percent.

MIDDLE EASTERN SCENARIOS

The purpose of analyzing the Middle Eastern scenarios is not so much to gain insight concerning the Middle East as to gain knowledge of the effects on the Soviet economy of an increase in the supply of oil beyond that in the base case, regardless of why this increase comes about.

The first of the Middle Eastern scenarios concerns Iran. We assume that the Soviet Union obtains control of Iranian oil, perhaps by military conquest or the use of coercion to obtain favorable trade agreements. We are ignoring the costs to the Soviets of undertaking such actions and measuring only the benefits. Assume Soviet control of Iranian oil is obtained on January 1, 1982, and the Soviets continue to produce the oil at the current rate, ignoring the possibility that the output of oil could be increased substantially. (Under the Shah, production was about 3.7 times as great as the 1981 estimated output of 70 million tons per year.[5]) We also assume that the Soviets sell this oil at the world market price and that the West does not retaliate against the seizure of the Iranian oil by an embargo of Soviet foreign trade or other means.

The result, as indicated in Fig. 11, is a substantial shift from the base case, denoted by the solid tradeoff curve, to the dashed tradeoff curve labeled Iran. As can be seen, with control of Iranian oil the Soviets would do better economically than they did during the

1970s. In terms of our figure of merit, the difference between the two curves is a substantial 29 percent. It is interesting to note that this 29 percent difference in the level of defense spending in 1990 (assuming a 2 percent rate of growth in consumption in both cases) is a measure of the military costs that the Soviets could afford to pay to absorb Iran.

The HK model can also examine more complicated Iranian scenarios. An example is a case where the West reacted to the takeover of Iranian oil by embargoing trade with the Soviets. We reemphasize that the case only measures the effect of a Soviet exogenous gain in oil output of 70 million tons per year. If this gain were obtained some other way, such as by unexpected discoveries of new oil fields in Russia, the impact on the economy would be the same (save for the second-order effect of the costs involved in exploiting the discovery).

The final scenario depicted in Fig. 11 is the same as the Iranian scenario except that in this case the Soviets gain control of the equivalent of not only Iranian oil, but that of Iraq, Kuwait, and the Neutral Zone as well, amounting to a gain of 195 million tons of oil per year. The difference between the resulting tradeoff curve and the base case is, according to our figure of merit, an enormous 66 percent.

Projections involving a particularly rapid growth of consumption such as in Fig. 11 cause problems with the agricultural sector of the HK model. This is because the actual production function for agriculture in the Soviet Union depends not only on the inputs of labor and two types of capital, as the model assumes, but also on land. Not including land makes it unrealistically easy to expand agricultural production in cases where there is a heavy demand for agricultural goods because of high consumption. Although this problem can reasonably be ignored for
the historical projections, for the 1980-90 projections—particularly projections such as the ones discussed here where consumption is likely to expand relatively rapidly—it cannot be ignored. The ideal solution would involve the estimation of an agricultural production function which includes land as an input. Due to funding and time constraints, however, we did not do this in this study. Instead, we dealt with the problem in a simpler fashion: We put an upper bound on the rate of growth of agriculture for the 1980-90 projection of 2.8 percent per year, assuming normal weather. We lower or increase this to the extent that weather departs from being normal. Our estimate was made partly by judgment and partly from econometric extrapolations of what the Soviets have accomplished in the past.

WORLD OIL PRICE

The final set of scenarios examines the impact of alternative assumptions concerning what the real rate of the increase in the price of oil will be. The U.S. Department of Energy has made a "most likely" estimate of 3 percent per year, which is what we have employed for our base case.[6] They also forecast a low and high of 0.3 percent and 5 percent, respectively. Figure 12 shows the resulting tradeoff curves for the 0.3 percent and 5 percent cases.

Increasing the world oil price is useful to the Soviet economy because in our base case they are a net oil exporter throughout the 1980s. Thus an increase in the world market price increases their earnings from foreign trade. Similarly, the previous scenarios, which all involved an increase in the supply of oil available to the Soviets, had their major effect on the economy by increasing the value of Soviet

exports. In all these cases the increased value of exports means that imports are increased correspondingly. The optimal mix of these imports depends on the point on the relevant tradeoff curve. The main qualitative difference between the scenarios which involve an increase in the price of oil and those which involve an increase in the supply of oil is that in the former case the domestic economy substitutes other resources for oil in production and consumption.
VIII. FOREIGN TRADE

In the final analysis, the purpose of foreign trade is the procurement of imported goods and services a) which are not produced within the country at all, b) are produced, as a result of whatever temporary reasons, in insufficient quantity and c) whose production within the country is more expensive than their purchase on the foreign market.[1]

The above quote from Smirnov in 1964 is still a valid description of the basic rationale behind Soviet foreign trade practices. But in more recent years, an additional element has entered into Soviet foreign trade decisions. The Soviet Union is facing a severe slowdown in the rate of growth of labor in the decade of the 1980s, and hence the need to increase productivity internally while deriving benefits from an "international division of labor" has become a major concern of Soviet leadership. The importance placed on foreign trade by Soviet leadership is probably best expressed in the following quote from Brezhnev's 25th CPSU Congress address:

We see foreign economic ties as an effective means of facilitating the accomplishment of both political and economic goals. The path of economic integration strengthens the power and unity of the socialist community's countries. Cooperation with developing countries facilitates a reorganization of their economic systems and public affairs on a progressive basis. Finally, economic, scientific, and technological ties with capitalist nations firmly establish and expand the economic base for the policy of peaceful coexistence.[2]

Long past are the days of Soviet autarchy. Soviet borrowing for

the purposes of buying Western technology and know-how was modest up until about 1974. From then to date, a combination of large orders for capital goods from the West and poor harvests requiring the purchase of foreign grain pushed the Soviets into a position of yearly increases in debt. The Soviet Union has actively entered into the arena of foreign trade and appears thoroughly committed to continuing to seek the economic benefits derived from trade with the world at large.

THE MODEL

The foreign trade portion of the HK model has three endogenous variables that all involve trade between the Soviet Union and the industrialized West: Soviet oil exports, agricultural imports, and MBMW (machine building metal working) imports. In the last half of the 1970s, oil was over 40 percent of the Soviet's exports to the West. The combined value of the agricultural and MBMW imports was over 50 percent of the total imports from the industrialized West.[3] The remainder of trade is handled exogenously by the model.

There are also parameters which measure credit and the relative efficiency of foreign compared with domestically produced capital. The amount of credit (defined to be the value of imports minus exports) available is in part determined by Western policy. Some of the following scenarios investigate the impact on the economy of a change in credit availability. A mathematical description of the foreign trade part of the model is given in App. A.

THE RELATIVE EFFICIENCY OF DOMESTIC AND IMPORTED CAPITAL

A topic of ongoing discussion in the economic literature is the marginal rate of return of imported (specifically Western) capital goods compared with capital produced in the Soviet Union.\[4\] Several researchers have attempted to quantify the relative rates of return, with widely divergent results. Their estimates range from one for Weitzman to 10 for Green and Levine, indicating that Western capital goods are between one and 10 times more efficient than capital produced in the Soviet Union.

The primary qualitative reason that Western capital is thought to be more efficient is because it embodies superior technology compared with Soviet capital. Another factor is superior quality control.

The impact on the economy of using differing values for the relative efficiency is shown in Fig. 13 to be substantial. The "low" tradeoff curve has an efficiency ratio between foreign and domestic capital of one, which means that there is no efficiency difference. The "high" curve has an estimate of 10. The base case is 1.5. The difference between the two curves in terms of the level of defense spending in 1990, assuming a 2 percent rate of growth of consumption in both cases, is 38 percent.

There is another impact of foreign trade on the economy. By bringing technically advanced Western capital and goods into the economy, such trade accelerates the domestic rate of technological change, a factor that we identified in Sec. VI as having a major impact on the economy. While there is general agreement that this is the

Fig. 13—The relative efficiency of foreign and domestic capital

In the foreign trade case,[5] there is essentially no information on the qualitative, let alone quantitative, nature of this relationship, which is the main reason that no major model of the Soviet economy takes this relationship explicitly into account. Differential capital efficiencies can be used, as in the case of the HK model, to partially correct for the absence of this relationship, but not entirely. As a consequence, the importance of foreign trade to the economy is probably greater than is indicated by the major models of the Soviet economy, including ours.

CREDIT

The extension of credit to the Soviet Union by Western nations takes on many forms. From a policy perspective, credits guaranteed by Western governments are perhaps the most important, since this form of credit may be influenced by the foreign policy position of the creditor nation toward the Soviet Union. Although the United States is more inclined than other Western nations to use foreign trade practices as a foreign policy instrument, the potential for credit sanctions against the Soviet Union by the West does exist. The loss of long-term, low-interest loans (up to eight or nine years at about 7.5 percent interest) could curtail Soviet plans for the modernization of industry by the importation of Western capital goods and technology.

In addition to government-guaranteed credits, the Soviets have used credits extended by private firms, often in the form of compensation agreements. As of year-end 1978, the Soviets had an outstanding debt to Western commercial interests of approximately $4 billion. This figure includes promissory notes held by the Western firms against the Soviet Union.

The Soviets will probably need the continuation of credits from the West in the future more than they have in the past. The impending shortage of labor reinforces the need to increase total factor productivity within the Soviet economy, and Western credits will be useful to finance the importation of the needed technology and capital goods. The high prices for oil and gold in the late 1970s provided the Soviets with much-needed hard currency, improving their balance of payment situation; but the current relative stability of those markets
is likely to work against the Soviets. To complete the projects currently planned, the Soviets will need credit guarantees in the range of $2.5 billion per year for the next decade, a figure consistent with historic levels of credit availability. Continued bad harvests and diminishing productivity could drive the demand for credit up into the range of $5 billion per year.

Our base case assumption is that the Soviets during the 1980s will be able to obtain $2.4 billion per year of credit measured in terms of imports minus exports. This estimate is based on recent historical experience and some judgment. We suspect that better estimates of this parameter could be made, given more effort than was affordable in this study. We have also investigated, in Fig. 14, a low and a high value for credit of zero and $4.8 billion--twice the base case amount. As can be seen in that figure, the difference in the tradeoff curve between the low and the high case is small compared with the difference seen in a number of our previous comparisons. The difference as measured by our figure of merit is 6 percent.

It should be noted that the base case assumes that the relative efficiency of capital differential is a factor of 1.5 (the same number used by SOVMOD). [6] If a higher value is used, the impact of changes in credit is considerably greater. Also, as mentioned earlier, there is a downward bias in the estimate of the importance of credit because the effect which imports have on the rate of technological progress is not taken into account by this or any other major models of the Soviet economy. Addressing these issues in more detail than is possible in this study is a promising area for future research.

Fig. 14—Credit
IX. COMPOSITE SCENARIOS

There are a number of interesting scenarios that can be examined for useful insights which are composites of the preceding ones. We will look at two.

We take a "worst" case which has the most unfavorable scenario with respect to a number of factors previously discussed. We assume a low rate for total factor productivity growth (-0.75 percent), low credit (0), a low value for the ratio of the efficiency of imported and domestic capital (1.0), and a low rate of growth for the world oil price (0.3 percent).

We compare the "worst" case with a "best" case that assumes the high value for all those variables for which the "worst" case assumed the low value. Thus we take the rate of total factor productivity growth to be 0.94 percent, credit to be $4.8 billion per year, capital efficiency ratio to be 10, and the rate of growth of the world oil price to be 5 percent. It should be noted that the terms "worst" and "best" case are convenient labels only and are not meant to imply that these scenarios are lower and upper bounds on what could happen. There are a great number of factors not considered in this study which could have a beneficial or detrimental effect on the Soviet economy.

Figure 15 depicts the result. There is a dramatic difference between the two cases, most of which is due to the difference in the assumptions about the rate of total factor productivity growth. In the "best" case, the Soviet economy can perform not only as well as, but better than, the 1970s experience. Indeed, the economy can do almost as
well as it did during the 1960s; certainly there is no problem with "muddling through." In contrast, the "worst" case is a severe crisis. The Soviets would not be able to obtain the point we have defined as the minimum required economic performance to "muddle through." In fact, even at a zero percent rate of growth of defense spending, they cannot obtain the minimum rate of growth of consumption (1.9 percent) required to "muddle through." The Soviets would be faced with a situation in which they would be forced to take some combination of defense and consumption growth well below the minimum for "muddling through," which could lead to severe social and political difficulties.
X. CONCLUSIONS AND SUGGESTIONS FOR FUTURE RESEARCH

We conclude that if the base case is correct, the Soviet economy will "muddle through" the 1980s. They will not do as well as during the 1970s but will be able to continue to expand their defense spending in accordance with expectations, and maintain a rate of growth of consumption of at least 1 percent per capita.

If the "best" case is correct, the Soviets will do better than their 1970s experience. Indeed, they will have a decade of outstanding economic performance. If the "worst" case is correct, the Soviets will face a severe crisis. What the rate of total factor productivity growth will be during the next decade appears to be the key to whether the Soviet economy will do well or poorly.

The impact of poor weather, of ceasing to subsidize Eastern European oil, and of a different rate of growth for the world price of oil (either higher or lower than the base case) is small.

Increases in the supply of oil available to the Soviets, on the scale of current Iranian production, would have a major impact on the Soviet economy.

The relative efficiency of imported versus domestic capital is of substantial significance. The amount of credit extended by the West is of small importance if the relative efficiency of capital is equal to the base case value of 1.5, but it becomes considerably more important for higher relative efficiency values.

All of the above points are dominated by the question of whether the "Rosefielde-Lee world" is a better representation of reality than the "CIA world."
The obvious suggestion for further research is to study in depth the question of the "Rosefield-Lee world" versus the "CIA world," a subject which was only touched upon here. There is also a third "world," that advanced by Igor Birman and some of his fellow Soviet emigres. Birman argues that the actual situation in the Soviet Union as compared to the "CIA world" is one of relatively high current defense spending, low GNP, and low growth.

A comparison between the "Birman world" and the other two worlds would be useful for much the same reason as a comparison between the "Rosefield-Lee world" and the "CIA world." Such a comparison should examine the implications for the "Rosefield-Lee world" and the "Birman world" of many of the same factors that were considered in this study, such as technology, energy, and foreign trade. Besides examining what the next decade will be like for each of these worlds, the question of which world is the best representation of reality should be addressed using the model and whatever other information is available.

Another promising avenue for further research would be to produce similar optimal control models of the various Eastern European countries. This area of the world has recently come under intense scrutiny because of the Polish situation. Optimal control models have for the most part not been built for these countries. Once built, they could be linked with each other and our Soviet model to form one super model of the entire block of nations. The super model not only would allow questions of concern to a particular nation to be addressed, but also questions concerning interactions between the various nations. Thus, for example, the effect of Western policy toward Polish credit
could be examined not only for its impact on Poland, but also for its impact on Soviet options concerning Poland. In this context, it is important to remember that the model can be used not only to calculate tradeoffs between consumption and defense, but more generally between any two commodities.

An additional conclusion is that this project has demonstrated the usefulness of a new approach--optimal control theory--for the study of centrally planned economies in general and the Soviet Union in particular. Continued development of this approach can be expected to yield rich dividends.
Appendix A

A DETAILED DESCRIPTION OF THE HOPKINS-KENNEDY OPTIMAL CONTROL MODEL

In this appendix we first describe the core of the Hopkins-Kennedy (HK) model. We next describe a number of additions to and modifications of the model's core that were made to incorporate special features needed to investigate the issues of particular concern in this study. Examples of such special features include a parameter which measures the relative efficiency of Slav and non-Slav labor, a weather adjustment factor for the agricultural sector, special equations to incorporate the effects of increasing energy costs, and a foreign trade sector. The use of optimal control to find the tradeoff curve is then illustrated, followed by a discussion of the economic meaning of the first-order conditions and of the iterative computer method used to perform the optimization. The symbols used in this appendix are defined when they are first used in the text and, for easy reference, in Table A.1 at the end of the appendix.

THE CORE OF THE HK MODEL

The model has 21 sectors:

1. Ferrous metallurgy
2. Nonferrous metallurgy
3. MBMW (Machine building metal working)
4. Forest products
5. Soft goods  
6. Processed foods  
7. Construction materials  
8. Coal and peat  
9. Oil  
10. Gas  
11. Electric power  
12. Chemicals  
13. Paper and pulp  
14. Construction  
15. Agriculture  
16. Transportation and communication  
17. Trade and distribution  
18. Industry not elsewhere classified (NEC) and other  
19. Weapons production  
20. Military services  
21. Other  

There are three factor inputs in the model: labor and two types of capital, machinery and structures. The sum across all sectors of the labor used in a given period is equal to the total amount available; that is

\[(A.1) \quad N^T_T = \sum_{i=1}^{21} N^T_i \]

\[T = 1, \ldots, 10\]
\[i = 1, \ldots, 21\]
where \( i \) = subscript referring to sector \( i \)

\( N_i^T \) = vector with a component for each sector \( i \) which represents the supply of labor for that sector in period \( T \)

\( \bar{N}_T \) = the exogenously determined total supply of labor for period \( T \)

\( T \) = a superscript referring to time period \( T \)

\( \bar{T} \) = the number of time periods in the projection

The labor input is measured in efficiency units, such that:

\[
(A.2) \quad L_i^T = (1 + \lambda)^T \bar{N}_i^T
\]

\[
L_i^T \geq 0
\]

\[
T = 1, \ldots, \bar{T} \\
i = 1, \ldots, 21
\]

where \( L_i^T \) = vector with a component for each sector \( i \) which represents the supply of labor measured in efficiency units for that sector in period \( T \)

\( \lambda \) = the rate of labor augmenting technical change

Gross output of every sector is determined by the factor inputs used by the sector through a production function:

\[
(A.3) \quad X_i^T = f_i^T (L_i^T, S_i^T, M_i^T)
\]

\[
T = 1, \ldots, \bar{T} \\
i = 1, \ldots, 21
\]

where \( f_i^T \) = production function relating factor inputs to gross outputs for sector \( i \) in period \( T \)

\( M_i^T \) = vector with a component for each sector \( i \) which represents the stock of machinery for sector \( i \) in period \( T \)
\[ S^T = \text{vector with a component for each sector } i \text{ which represents the stock of structures for sector } i \text{ in period } T \]

\[ X^T = \text{vector with a component for each sector } i \text{ which represents gross output for sector in period } T \]

For the moment we can think of this set of production functions as time-invariant translog[1] with parameters constrained to ensure constant returns to scale. Later, when we incorporate the special features concerning energy, these production functions will become more complicated. We have

\[
\ln X_i^T = \ln g_{0i} + g_{1i} \ln L_i^T + g_{2i} \ln M_i^T + g_{3i} \ln S_i^T \\
+ b_{1i} (\ln L_i^T)^2 + b_{2i} (\ln L_i^T) (\ln M_i^T) + b_{3i} (\ln L_i^T) (\ln S_i^T) \\
+ b_{4i} (\ln M_i^T)^2 + b_{5i} (\ln M_i^T \ln S_i^T) + b_{6i} (\ln S_i^T)^2
\]

\[ M_i^T \geq 0, \quad S_i^T \geq 0, \quad L_i^T \geq 0, \quad g_{ki} \geq 0 \]

\[ g_{1i} + g_{2i} + g_{3i} = 1 \]

\[ b_{1i} + 1/2 b_{2i} + 1/2 b_{3i} = 0 \]

\[ b_{4i} + 1/2 b_{2i} + 1/2 b_{5i} = 0 \]

\[ b_{6i} + 1/2 b_{3i} + 1/2 b_{5i} = 0 \]

\[ T = 1, \ldots, T \]

\[ i = 1, \ldots, 21 \]

\[ k = 1, \ldots, 3 \]

[1] For more information on translog production functions, see Ernst Berndt and Laurits Christensen, 1973. Also see Laurits Christensen et al., 1971, pp. 255-256.
where \( b_k = \text{coefficients, } k=1, \ldots, 6 \)
\[ g_k = \text{coefficients, } k=0, \ldots, 3 \]

Net output is derived from gross output by subtracting the amount of output which is used as intermediate input in the production of other goods. The amount of sector \( i \) consumed as intermediate input in a given year in the production of one unit of gross output of sector \( i \) is given by an input-output coefficient \( a_{ij}^T \). The set of input-output coefficients is for the moment assumed to be a function only of time. The functional dependence of the set of input-output coefficients will become more complicated when the features of the model related to energy are incorporated. We have

\[
(Y^T_i = X^T_i - \sum_{j=1}^{21} a_{ij}^T X^T_j)
\]

\[ Y^T_i \geq 0 \]

\[ T = 1, \ldots, \bar{T} \]
\[ i = 1, \ldots, 21 \]

where \( a_{ij}^T \) = the input-output coefficient which gives the amount of good \( i \) needed to produce one unit of gross output of good \( j \) in period \( T \)

\( Y^T \) = net output vector

Capital is assumed to be nontransferable between sectors. Once capital has been allocated to one sector by investment, it cannot be moved to another. The capital accumulation relations are
(A.6)

\[
\begin{align*}
S_{i}^{T+1} & = (1 - \delta_{S}) S_{i}^{T} + \Delta S_{i}^{T} \\
M_{i}^{T+1} & = (1 - \delta_{M}) M_{i}^{T} + \Delta M_{i}^{T} \\
\Delta S_{i}^{T} & \geq 0, \Delta M_{i}^{T} \geq 0 \\
T & = 1, \ldots, \bar{T} \\
i & = 1, \ldots, 21
\end{align*}
\]

where \( \delta_{S}, \delta_{M} \) = the depreciation rates of structures and machinery, respectively

\( \Delta S_{i}^{T}, \Delta M_{i}^{T} \) = gross investment vectors in (or gross addition to) structures and machinery, respectively

Note that the structure of Eqs. (A.6) implies a one-year investment gestation lag.

The total amount of new structures built is assumed to be equal to the net output of the construction industry (sector 14) minus the amount of new structures used for consumption and defense. In other words, the construction industry produces the new structures. A similar relation holds for new machinery. The total new installation of machinery is set equal to the net output of the MBMW sector (sector 3) less the output used for consumption and defense. Mathematically, we have

(A.7)

\[
\begin{align*}
\sum_{i=1}^{21} \Delta S_{i}^{T} & = Y_{14}^{T} - C_{14}^{T} - D_{14}^{T} \\
\sum_{i=1}^{21} \Delta S_{i}^{T} & = Y_{3}^{T} - C_{3}^{T} - D_{3}^{T} \\
T & = 1, \ldots, \bar{T}
\end{align*}
\]
where $c^T = \text{the consumption vector}$

$d^T = \text{the defense vector}$

There is finally a set of material balance equations which set net output equal to the sum of consumption plus defense plus investment. That is,

$\mathbf{y}_i^T = \mathbf{c}_i^T + \mathbf{d}_i^T + \mathbf{I}_i^T$

$T = 1, \ldots, \bar{T}$

$i = 1, \ldots, 21$

where $I^T = \text{the gross investment vector}$

The sets of Eqs. (A.1) through (A.8) constitute the production relations in the model.

There are certain variables in these production relations that are under the control of decisionmakers. These are the allocation of labor and the two types of investment goods among the sectors in each period. Each set of such allocations will map out a possible course for the economy. The model employs an optimizing procedure to find the subset of all courses for the economy which are technically efficient. Each of these technically efficient courses corresponds to a point on the production possibility frontier which is the tradeoff curve that we wish to calculate.

More specifically, we maximize, subject to constraints, the discounted sum of consumption over a time horizon of length $\bar{T}$ for various constant rates of growth of defense spending. The constraints imposed are:
the production relations (A.1) to (A.8) above
a terminal capital stock
the labor supply for each period measured in efficiency units
the initial capital stock

The terminal capital stock for each project was determined judgmentally, taking into account the results of HK model runs over a very long time period in which the capital labor ratio converged to a golden-rule-like value. In maximizing the discounted sum of consumption, we must specify the proportion of each sector's output in the consumption bundle in each year. This was done using Engel-like curves, to be discussed below. The actual tradeoff curves shown in the text were derived in the following way. A rate of growth of defense spending was chosen, and the maximum discounted sum of consumption subject to this defense constraint was calculated by the model. The average rate of growth of consumption over the projection period was then calculated, and the point corresponding to this rate of growth of consumption and the specified rate of growth of defense was plotted as one point of the tradeoff curve. The procedure was repeated for other rates of growth in defense spending, generating as many points as were necessary to graph the tradeoff curve.

SPECIAL FEATURES OF THE HK MODEL

The model used in this study consists of the core described above, with additions and modifications made to incorporate the special features described below.
Relative Labor Efficiency

To incorporate the effects of the differing rates of growth of the Slav and non-Slav parts of the population and of the differential labor efficiency of these two groups, the supply of labor variable was decomposed into two parts such that

\[ L^T = \overline{LNS}^T + \bar{\gamma} \overline{LS}^T \]

\( T = 1, \ldots, \bar{T} \)

where \( \overline{LNS}^T \) = the exogenously determined number of non-Slav workers in period \( T \)

\( \overline{LS}^T \) = the exogenously determined number of Slav workers in period \( T \)

\( \bar{\gamma} \) = the relative efficiency of Slav compared with non-Slav workers

Energy

Four of the 21 sectors concern energy: coal and peat (sector 8), oil (9), gas (10), and electric power (11). To incorporate the effects of resource exhaustion in the exhaustible energy sectors (sectors 8, 9, and 10), the production function for each of these three sectors was modified by including a resource exhaustion parameter which is modeled as a negative rate of Hicks neutral technical change. In addition, maximum gross production constraints were placed on each of these sectors in each period. Thus, for these sectors, the production functions (Eqs. (A.3)) are replaced by:
(A.10)

\[ \begin{align*}
  x_i^T &= \exp(-\bar{\omega}_i T) f_i^T (L_i^T, S_i^T, M_i^T) \\
  x_i^T &\leq \bar{x}_i^T \\
  i &= 8, 9, 10 \\
  T &= 1, \ldots, \bar{T}
\end{align*} \]

where \( \bar{\omega}_i \) = the rate of Harrod neutral technical deterioration in sector \( i \)

\( \bar{x}_i^T \) = the exogenously set maximum gross output of sectors \( i = 8, 9, 10 \) in period \( T \)

Because of these cost increases in the domestic energy sector, and because of the fact that the price of imported oil can change (as is discussed in more detail later), the model was modified to allow for substitution of other factors for energy inputs. As presented so far, the model has fixed input-output coefficients, \( a_{ij}^T \). We generalize this, allowing the input-output coefficients to change as a function of the relative price of energy. We have

(A.11)

\[ \begin{align*}
  a_{ij}^T &= \delta_i \hat{a}_{ij}^T + (1 - \delta_i) a_{ij}^{T-1} \\
  i &= 8, \ldots, 10 \\
  j &= 1, \ldots, 21 \\
  T &= 2, \ldots, \bar{T}
\end{align*} \]

where \( \delta_i \) = the depreciation rate applicable to energy using capital in sector \( i \)

\( \hat{a}_{ij}^T \) = the optimal input-output coefficient relating input of sector \( i \) into production of sector \( j \) in period \( T \)

\( \hat{a}_{ij}^T \) is derived from the equation:
(A.12)
\[
\frac{\hat{a}_{ij}^T}{\bar{a}_{ij}^T} = (p_i^T)^\hat{\delta}_i
\]
\[i = 8, \ldots, 10\]
\[j = 1, \ldots, 21\]
\[T = 1, \ldots, \bar{T}\]

where \(\hat{a}_{ij}^T\) = the base input-output coefficient, i.e., the coefficient before adjustment for energy prices

\(p_i^T\) = the relative cost of input \(i\) in period \(T\), which will be exactly defined in the subsection on model solution

\(\hat{\delta}_i\) = the elasticity of substitution of energy input \(i\) for other inputs

The process of adjustment of input-output coefficients to changes in energy prices can be summarized as

1. Begin with the historically estimated input-output coefficient, \(\hat{a}_{ij}^T\).
2. Multiply it by the relative price of energy source \(i\), raised to the elasticity of substitution, to derive the optimal coefficient, \(\hat{a}_{ij}^T\).
3. Since energy is used with capital equipment with a depreciation rate of \(\delta_i\), the actual input-output coefficient at time \(T\), \(a_{ij}^T\), is a convex combination of the coefficients applicable to capital surviving from earlier periods, \(a_{ij}^{T-1}\), and \(\hat{a}_{ij}^T\) with weights \((1 - \delta_i)\) and \(\delta_i\), respectively.

The HK model is formulated and solved as though the input-output coefficients were fixed and exogenous. These coefficients are then adjusted during the model solution process according to the values of
the $P_i^T$ (the relative cost of energy) generated as the model is being solved. (The exact process of generation of the $P_i^T$ will be given in the subsection on model solution.)

**Foreign Trade**

Foreign trade is incorporated into the HK model by altering the material balance equations (Eqs. (A.8)) to take into account imports and exports. Foreign trade is broken down into two categories, exogenous trade and endogenous trade. The endogenous trade has three components: imports of MBMW from the industrialized West; oil exports to the industrialized West; and agricultural imports from the industrialized West. For sectors other than 3 (MBMW), 9 (oil) and 15 (agriculture), the material balance equation can be written as

\[(A.13)\]

\[ Y_i^T = C_i^T + D_i^T + I_i^T + E_i^T \]

\[ i = 1, 2, 4, \ldots, 8, 10, \ldots, 14, 16, \ldots, 21 \]

\[ T = 1, \ldots, \bar{T} \]

where $E_i^T$ = the exogenous net export vector

The remaining material balance equations are

\[(A.14)\]

\[ Y_3^T = C_3^T + D_3^T + I_3^T + E_3^T - \eta (MBMW)^T \]

\[ I_3^T \geq I_3^T \]

\[ (MBMW)^T \geq 0 \]
where \( I^T \) = the exogenously set minimum investment for period \( T \)

\[ MBMW^T = MBMW \text{ imports from the industrialized West} \]

\( \eta = \) a coefficient which represents the extent to which MBMW produced in the industrialized West is more efficient than domestically produced MBMW

\[ (A.15) \]

\[ Y^T_9 = C^T_9 + D^T_9 + I^T_9 + E^T_9 + O^T \]

where \( O^T = \) oil exports to the industrialized West

\[ (A.16) \]

\[ Y^T_{15} = C^T_{15} + D^T_{15} + I^T_{15} + E^T_{15} - AG^T \]

\[ AG^T \geq 0 \]

where \( AG^T = \) agricultural imports from the industrialized West

Finally, there is a balance of trade constraint with the industrialized West:

\[ (A.17) \]

\[ AG^T + MBMW^T = P^T_{w}O^T + CR^T \]

where \( CR^T = \) credit from the industrialized West for an excess of imports over exports

\( P^T_{w} = \) world price of oil in period \( T \)
Consumption

It is assumed in each year that consumption occurs in fixed proportions; i.e.,

\[ \mathbf{C}_i^T = \gamma^T \mathbf{c}_i^T \]

where \( \gamma^T \) = consumption index which gives the number of units of consumption in period \( T \)
\( \mathbf{c}_i^T \) = vector where each component represents the amount of good in one unit of consumption in period \( T \)

We adjust these proportions during solution, however, to take into account Engel-like effects and the impact of increasing energy costs.

The Engel-like effects are incorporated via

\[ \mathbf{C}_i = \mathbf{g}_i + \mathbf{h}_i (\sum_{j=1}^{21} \mathbf{C}_j) \]

where \( \mathbf{g}_i \), \( \mathbf{h}_i \) = vectors of parameters

At each iteration in the solution process, the previous iteration's value of \( \sum_{j=1}^{21} \mathbf{C}_j^T \) is used to derive \( \mathbf{C}_i^T \).

The proportions \( \mathbf{C}_i^T \) are also allowed to vary with energy costs as are the input-output coefficients. The equations, which parallel those for the input-output coefficients, are
(A.20) 
\[ \hat{c}_i^T = \delta_i \hat{c}_i^{T-1} + (1 - \delta_i) \hat{c}_i^T \]
\[ i = 8, \ldots, 10 \]
\[ T = 2, \ldots, \bar{T} \]

\[ \frac{\hat{c}_i^T}{\bar{c}_i^T} = (P_i^T) \hat{\gamma}_i \]
\[ i = 8, \ldots, 10 \]
\[ T = 1, \ldots, \bar{T} \]

where \( \hat{c}_i \) = vector where each component represents the optimal amount of good \( i \) in one unit of consumption in period \( T \)

\( \bar{c}_i^T \) = vector where each component represents the amount of good \( i \) in one unit of consumption in the base period

\( \hat{\gamma}_i \) = the elasticity of substitutes of consumption good \( i \) for other goods

The model is formulated and solved as though the \( \bar{c}_i^T \) are exogenous and fixed. During the process of the solution iterations the \( \bar{c}_i^T \) are adjusted, as is indicated by the (A.19) and (A.20) sets of equations.

**OPTIMAL CONTROL FORMAT**

The optimal control problem consists of maximizing the discounted sum of consumption over the interval \( \bar{T} \) subject to a number of constraints, all of which have been discussed and stated mathematically, save for two which were only discussed. These are the terminal capital constraints which state that the total summed across all sectors of each of the two types of capital at the end of the final period must equal the terminal value for the two types of capital stock. We have
where $\bar{S} =$ aggregate stock of structures at the end of the terminal period
$\bar{M} =$ aggregate stock of machinery at the end of the terminal period

The maximization problem which is solved by the model, including a
restatement of all of the relevant constraints which were discussed above, is

(A.22)

Maximize

$$\sum_{T=1}^{T} \bar{S}^T \gamma^T$$

subject to: \[ i' S^{T+1} = \bar{S} \]
\[ i' M^{T+1} = \bar{M} \]
\[ C^T = \gamma^T C^T \]
\[ X^T \leq \bar{X}^T \]
\[ \pi^T, F^T = C \pi^T \]
\[ (I^T - A^T) X^T = C^T + I^T + D^T + E^T + A F^T \]
\[ I^T \geq I^T \]
\[ X_i^T = f_i^T(L_i^T, S_i^T, M_i^T) \]
\[ S^{T+1} = (1-\delta_s) S^T + (\Delta S)^T \]
\[ e_S' I^T = i' (\Delta S)^T \]
\[ M^{T+1} = (1 - \delta_M) M^T + (\Delta M)^T \]
\[ e_M' I^T = i' (\Delta M)^T \]
\[ \bar{L}^T = i' L^T \]

$T = 1, \ldots, \bar{T}$
where $A^T = 21 \times 21$ matrix of the input-output coefficients $a_{ij}^T$ for the period $T$

e_M = \text{vector which has zeros for every component except for number 3 which is equal to 1}

e_S = \text{vector which has zeros for every component except for number 14 which is equal to 1}

$F^T = \text{vector of endogenous net exports to the industrialized West in period } T. \text{ It has zeros for all of its 21 components (which represent the 21 sectors) except for 3, 9, and 15 which are } -\text{MBMW}^T, 0^T, \text{ and } -\text{AG}^T, \text{ respectively.}$

$i = \text{the "summer" vector--it has one for each component.}$

$\pi^T = \text{the vector of foreign exchange costs of exports and imports with the industrialized West for period } T. \text{ All}$

$\text{components } = 1 \text{ except for component 8 which equals } p_w^T.$

$\Lambda = 21 \times 21$ diagonal matrix with one for all diagonal components except for (3,3)--the MBMW sector--which has $\eta.$

$\delta = 1/(1+r^\gamma)$, where $r^\gamma$ is the assumed social rate of discount.

Note that any concave function of net outputs can be maximized by the model. In particular, the model can be used to determine if any given set of $21 \times \bar{T}$ net outputs, one for each sector in each of $\bar{T}$ periods, is feasible. To do this one would change the objective function from maximize $\sum_{T=1}^{\bar{T}} \delta^T Y^T$ to maximize the scalar $Z$ subject to all of the constraints of the previous problem plus the constraint that

(A.23) $Y_i^T \geq Z \bar{Y}_i^T$ for every $i = 1, \ldots, 21$

$T = 1, \ldots, \bar{T}$

where $\bar{Y}_i^T = \text{the net output of sector } i \text{ in period } T \text{ which is being tested for feasibility}$

If the resulting maximized value of $Z \leq 1$, then and only then is the set of net outputs being tested feasible.
Returning to the maximization problem of interest here, we employ Lagrangian analysis to obtain the solution. The relevant Lagrangian expression is

\[(A.24)\]

\[
\xi = \sum_{T=1}^{\bar{T}} \left( \delta \gamma T \right) + \theta_s (\bar{\gamma}^{T+1} - \bar{\gamma}) + \theta_m (\bar{\gamma}^{T+1} - \bar{\gamma}) + \sum_{T=1}^{\bar{T}} \mu^T (C^T - \gamma^T \bar{C}^T) + \sum_{T=1}^{\bar{T}} \nu^T (\bar{X}^T - X^T) + \sum_{T=1}^{\bar{T}} \pi^T (\pi^T F^T - CR^T) + \sum_{T=1}^{\bar{T}} p^T ((I - \bar{A}^T)X^T - C^T - \bar{I}^T - D^T - E^T - \bar{A}^T) + \sum_{T=1}^{\bar{T}} x^T (\bar{I}^T - \bar{I}^T) \]
\[ + \sum_{T=1}^{\bar{T}-1} \sum_{i=1}^{21} q_i^T (r_i^T, s_i^T, m_i^T) - x_i^T \]  
(A.24.9)

\[ + \sum_{T=1}^{\bar{T}-1} \rho_s^T \left[ (1 - \delta_s^T) s^T + (\Delta s)^T - s^{T+1} \right] \]  
(A.24.10)

\[ + \sum_{T=1}^{\bar{T}} r_s^T \left[ e_s^T - v^T \Delta s^T \right] \]  
(A.24.11)

\[ + \sum_{T=1}^{\bar{T}-1} \rho_m^T \left[ (1 - \delta_m^T) m^T + (\Delta m)^T - m^{T+1} \right] \]  
(A.24.12)

\[ + \sum_{T=1}^{\bar{T}} r_m^T \left[ e_m^T - v^T \Delta m^T \right] \]  
(A.24.13)

\[ + \sum_{T=1}^{\bar{T}} w^T (l^T - v^T l^T) \]  
(A.24.14)

\[ T = 1, ..., \bar{T} \]

where $\xi$ = the Lagrangian expression

$\theta_s^T, \theta_m^T, v^T, T, p^T, x^T, q^T, r_s^T, r_m^T, w^T$ = Lagrange multipliers for every value of $T$
The first order conditions are:

(A.25)

\[
\frac{\partial \xi}{\partial Y^T} = \delta^T - (\mu^T)'(C^T) = 0 \\
(A.25.1)
\]

\[
\frac{\partial \xi}{\partial C^T} = \mu^T - p^T = 0 \\
(A.25.2)
\]

\[
\frac{\partial \xi}{\partial T^T} = -p^T + r_s^T e_s + r_m^T e_m \leq 0 \\
(A.25.3)
\]

\[
- (p^T + r_s^T e_s + r_m^T e_m)'(I_T - I_T) = 0
\]

\[
\frac{\partial \xi}{\partial X^T} = -u^T + (I - \Lambda^T)'p^T - q^T = 0 \\
(A.25.4)
\]

\[
\frac{\partial \xi}{\partial F^T} = \tau^T \pi^T - \Lambda p^T \leq 0, \quad F^T, \quad (\tau^T \pi^T - \Lambda F^T) = 0 \\
(A.25.5)
\]

\[
\frac{\partial \xi}{\partial S^T} = (q^T)(M_{PS})^T - [p_{S^T}^{T-1} - (1 - \xi_S^T)p_{S^T}^T] = 0 \\
(A.25.6)
\]

\[
T = 2, \ldots, \bar{T}
\]

\[
\frac{\partial \xi}{\partial S^T} = \theta_s^{T+1} - \rho_s^{T+1} = 0 \\
(A.25.7)
\]

\[
\frac{\partial \xi}{\partial (\Delta S)^T} = \rho_s^T - r_s^T \leq 0 \\
- (\rho_s^T - r_s^T)'(\Delta S)^T = 0 \\
(A.25.8)
\]

\[
\frac{\partial \xi}{\partial M^T} = (q^T)(M_{PM})^T - [(\rho_M^T - (1 - \xi_M^T)p_M^T] = 0 \\
(A.25.9)
\]

\[
T = 2, \ldots, \bar{T}
\]

\[
\frac{\partial \xi}{\partial M^T} = \theta_m^{T+1} - \rho_m^{T+1} = 0 \\
(A.25.10)
\]

\[
\frac{\partial \xi}{\partial (\Delta M)^T} = \rho_m^T - r_m^T \leq 0 \\
- (\rho_m^T - r_m^T)'(\Delta M)^T = 0 \\
(A.25.11)
\]

\[
\frac{\partial \xi}{\partial L^T} = (q^T)((M_{PL})^T - w^T) = 0 \\
(A.25.12)
\]
where $q^T = N \times N$ diagonal matrix with $q_i^T$ in the $i$th slot

$\text{MPL}^T = \text{the marginal physical product of labor vector}$

$\text{MPS}^T = \text{the marginal physical product of structure vector}$

$\text{MPM}^T = \text{the marginal physical product of machinery vector}$

We are maximizing a linear expression subject to linear and
strictly concave constraints. Thus, as long as one or more of the
strictly concave constraints (such as the production functions) are
binding, which for the scenarios we deal with is always the case, then
any solution to the system of first-order equations is unique and
represents a global maximum.

INTERPRETATION OF FIRST-ORDER CONDITIONS AND SOLUTION METHOD

We will now proceed with an interpretation of these (rather
complex) first-order conditions of the optimal control problem. $q^T$ is
the marginal value (in terms of contribution to the objective function,
namely the sum of discounted consumption) of having additional gross
output of any good in any period. Equations (A.24.6)-(A.24.12) then
simply say that the marginal contribution of any factor to this goal
must be equal in each sector. For example, labor used in sector $j$, $L_j^T$
makes a marginal physical contribution of $\text{MPL}_j^T$ to gross output, and
thus a marginal value contribution of $q_j^T$ to the objective function.
This contribution must be equal in each sector $j$, or else labor could be
reallocated to increase the objective function. In fact, the
contribution must be equal to the Lagrange multiplier, interpreted as
the marginal value of labor in that period, or the "shadow wage." The
situation is slightly different with regard to capital, due to its
nonshiftability (or, in other terms, the "irreversibility of investment") which translates into a constraint that investment allocations $\Delta S$ and $\Delta M$ must be nonnegative.

Our discussion will focus on investment in structures, but everything said will hold for machinery as well. Equation (A.24.6) states that the marginal value product of structures in any sector $j$ $q_j^T (MPS)_j^T$ must equal the rate of return to holding capital in that sector. Rates of return must then be equalized across sectors if investment allocations to each sector are strictly positive (A.24.8). This is exactly analogous to the labor case: If rates of return (and thus marginal value products) were unequal, investment could be reallocated to increase the objective function. However, if there is no new investment in a sector (i.e., if $\Delta S_j^{T-1}$ is zero), then the marginal value product can be below that in other industries, since no allocation of capital out of that sector is feasible. Equation (A.22.7) gives us an additional valuation: The value of capital in period $T + 1$ is simply equal to that specified in the objective function.

So far the first-order conditions have been familiar in economic terms: They state that the marginal value contribution of factors must be equalized across sectors. The other conditions determine just how high supplies of these factors should be; i.e., how many resources should be devoted to output of the consumption goods in any given period and how many devoted to production of capital goods, so that additional consumption goods can be produced in later periods. In other words, there are two general ways to increase the discounted sum of total consumption. One is to devote labor resources directly to producing consumption goods in some given year, say $T_1$, contributing $\delta_{T_1}$ to the
objective function. This use of labor, however, naturally entails a
cost in terms of forgone output of investment goods. An alternative way
the labor can be used is to produce capital goods; when installed,
capital goods increase the capital stock of the economy, and thus make
it possible to produce more consumption goods in a later period, say $T_2$,
contributing $\delta_{T_2}$ to the objective function. An optimal economic
allocation scheme, of course, just balances these strategies so that
maximum cumulative discounted consumption is attained. First-order
conditions (A.24.1)-(A.24.4) guarantee this in the following way.

The number $q_j^T$ represents the marginal contribution of gross output
of the $j$th good to the value of the objective function, and is also the
marginal cost of increasing gross output of that good. This
interpretation is just a different way of looking at condition
(A.24.12), recalling that Lagrange multiplier $w^T$ gives the marginal
contribution of additional labor supplies to the objective function.

Equation (A.24.4) then indicates that $p^T$ is a vector which contains
the marginal costs of increasing the net output of each of the goods,
where cost is again measured in terms of forgone value in the objective
function. This marginal cost must be equated to the marginal benefit of
each of these net outputs in increasing the objective function, and Eqs.
(A.24.1) and (A.24.2) guarantee this. They state that the marginal cost
of producing the composite consumption good in period $T$ should be $\delta^T$,
since it contributes to the objective function with a marginal benefit
of $\delta^T$. The marginal cost of producing structures in any period $T$ should
equal $r_s^T$, the Lagrange multiplier of the constraint relating
construction output and structure stocks; it is simply the marginal
contribution of additional investment in structures to the objective
function. $r_m^T$ has an identical interpretation.
Finally, constraint (A.24.5) states that the marginal cost of obtaining net outputs of any good from foreign sources, $T^{T} \pi_{1}^{T}$, should be equal to its domestic marginal cost, subject to the efficiency differential discussed above.

It is necessary, of course, to develop an algorithm for finding the actual values of the variables that solve these first-order conditions. Our algorithm works in this way.

1. Begin with initial guesses of the proportions of the investment allocation vectors (the $\Delta H^{T}$ and $\Delta S^{T}$) and the patterns of the net output vectors (the $\gamma^{T}$ and the $I^{T}$).

2. In the first period, which has the fixed capital stocks with which the model starts, find the level of the net output vectors, the $\gamma^{T}$ and the $I^{T}$, (with the proportions given) that uses all the available labor. Then distribute the net output of investment goods to the various sectors according to the investment allocation proportions already given. This gives sectoral specific capital stocks for the next period. Repeat this procedure for all periods.

3. Compute the $p^{T}$, $q^{T}$, $r^{T}$, $p^{T}$, and $w^{T}$. Adjust the investment allocation patterns so that a sector's share is increased if its marginal value product is above average, and vice versa. Adjust the final goods output pattern (the $\gamma^{T}$ and $I^{T}$) so that an investment good is increased if $r_{i}^{T} > p_{i}^{T}$, and vice versa.

4. By reiterating the above procedure a sufficiently large number of times, the first-order conditions can be solved to any desired degree of accuracy.
### Table A.1
DEFINITIONS OF SYMBOLS USED IN APPENDIX A

$A_T = 21 \times 21$ matrix of the input-output coefficients $a_{ij}^T$ for the period $T$

$a_{ij}^T = \text{the input-output coefficient which gives the amount of good } i \text{ needed to produce one unit of gross output of good } j \text{ in period } T$

$\hat{a}_{ij}^T = \text{the optimal input-output coefficient}$

$\tilde{a}_{ij}^T = \text{the base input-output coefficient, i.e., the coefficient before adjustment for energy prices}$

$AG_T = \text{agricultural imports from the industrialized West in period } T$

$b_k = \text{coefficients of the general translog production function Eq. (A.4) where } k = 1, \ldots, 6$

$\beta = \text{vector of parameters associated with Eq. (A.19)}$

$C_T = \text{vector in which each component represents the amount of good } i \text{ consumed in period } T$

$\bar{C}_T = \text{vector in which each component represents the amount of good } i \text{ in one unit of consumption in period } T$

$\hat{C}_T = \text{vector in which each component represents the optimal amount of good } i \text{ in one unit of consumption}$
\( \mathbf{C}^T \) = vector in which each component represents the amount of good \( i \) in one unit of consumption in the base period

\( \mathbf{CR}^T \) = credit from the industrialized West for an excess of imports over exports in period \( T \)

\( \mathbf{D}^T \) = the defense vector

\( \mathbf{E}^T \) = the exogenous net export vector

\( \mathbf{e}_M \) = vector which has zero for every component except for number 3 which is equal to 1

\( \mathbf{e}_S \) = vector which has zeros for every component except for number 14 which is equal to 1

\( \mathbf{F}^T \) = vector of endogenous net exports to the industrialized West in period \( T \). It has zeros for all of its 21 components (which represent the 21 sectors) except for 3, 9, and 15, which are \( \mathbf{MBMW}^T \), \( \mathbf{O}^T \), and \( -\mathbf{AC}^T \), respectively

\( f_{i}^T \) = production function relating factor inputs to gross outputs for sector \( i \) in period \( T \)

\( \mathbf{g}_k \) = coefficients of the general translog production function Eq. (A.4), where \( k = 0, \ldots, 3 \)

\( \mathbf{g} \) = vector of parameters associated with Eq. (A.19)

\( \mathbf{I}^T \) = the gross investment vector

\( \mathbf{I}^T \) = the exogenously set minimum investment for period \( T \)
\( i \) = subscript referring to sector \( i \)
\( L^T \) = the supply of labor vector, where labor is measured in efficiency units
\( \overline{LNS}^T \) = the exogenously determined number of non-Slavic workers in period \( T \)
\( \overline{LS}^T \) = the exogenously determined number of Slavic workers
\( M^T \) = vector with a component for each sector \( i \) which represents the stock of machinery for sector \( i \)
\( \bar{M} \) = aggregate stock of machinery at the end of the terminal period
\( \Delta M^T \) = the gross investment vector in (or gross addition to) machinery
\( MBMW^T \) = MBMW imports from the industrialized West
\( MPL^T \) = the marginal physical product of labor vector
\( MPM^T \) = the marginal physical product of machinery vector
\( MPS^T \) = the marginal physical product of structures vector
\( N^T \) = the supply of labor vector, where labor is measured in terms of the number of workers
\( \overline{N}^T \) = the exogenously determined total supply of labor for period \( T \)
\( O^T \) = oil exports to the industrialized West

\( p^T_i \) = the relative cost of input i

\( p^T_w \) = world price of oil

\( p^T \) = Lagrange multiplier for period T associated with Eq. (A.23.7)

\( q^T \) = Lagrange multiplier associated with Eq. (A.24.9)

\( q^T_i \) = NxN diagonal matrix with \( q^T_i \) in the \( i^{th} \) slot

\( r^T_M \) = Lagrange multiplier associated with Eq. (A.23.13)

\( r^T_S \) = Lagrange multiplier associated with Eq. (A.23.11)

\( S^T \) = the stock of structures vector

\( S^\tilde{\alpha} \) = aggregate stock of structures at the end of the terminal period

\( \Delta S^T \) = the gross investment vector in (or gross addition to) structures

\( T \) = superscript referring to time period T

\( T \) = the number of time periods in the projection

\( \tilde{w}_i \) = the rate of Harrod neutral technical deterioration in sector i

\( w^T \) = Lagrange multiplier associated with Eq. (A.23.14)

\( X^T \) = the gross output vector
\( \bar{X}_i^T \) = the exogenously set maximum gross output of sectors \( i = 8, 9, 10, 15 \) in period \( T \)

\( Y^T \) = the net output vector

\( \bar{Y}^T \) = the net output vector being tested for feasibility

\( Z \) = scalar associated with Eq. (A.23)

\( \Lambda \) = 21 x 21 diagonal matrix with 1 for all diagonal components except for (3, 3)--the MBMW sector--which has \( \eta \)

\( \gamma^T \) = consumption index which gives the number of units of consumption in period \( T \)

\( \bar{Y} \) = the relative efficiency of Slav compared with non-Slav workers

\( \hat{\gamma}_i \) = the elasticity of substitution of consumption good \( i \) for other goods

\( \Theta_M \) = Lagrange multiplier associated with Eq. (A.23.3)

\( \Theta_S \) = Lagrange multiplier associated with Eq. (A.23.2)

\( \tau \) = the "summer" vector--it has one for each component

\( \pi^T \) = the vector of foreign exchange costs of exports and imports with the industrialized West. All components = 1 except for component 8 which equals \( p^T \)

\( \lambda \) = the rate of labor augmenting technical change
\[ \eta = \text{coefficient which represents the extent to which MBMW produced in the industrialized West is more efficient than domestically produced MBMW} \]

\[ \rho_M^T = \text{Lagrange multiplier associated with Eq. (A.23.12)} \]

\[ \rho_S^T = \text{Lagrange multiplier associated with Eq. (A.23.10)} \]

\[ \mu^T = \text{Lagrange multiplier associated with Eq. (A.23.4)} \]

\[ v^T = \text{Lagrange multiplier associated with Eq. (A.23.5)} \]

\[ \tau^T = \text{Lagrange multiplier associated with Eq. (A.23.6)} \]

\[ x^T = \text{Lagrange multiplier associated with Eq. (A.23.8)} \]

\[ \delta_i = \text{the elasticity of substitution of energy input } i \] for other inputs

\[ \delta_i = \text{the depreciation rate applicable to energy using capital in sector } i \]

\[ \delta_M = \text{the depreciation rate of machinery} \]

\[ \delta_S = \text{the depreciation rate of structures} \]

\[ \delta = \frac{1}{1 + r^*}, \text{where } r^* \text{ is the assumed social rate of discount} \]

\[ \zeta = \text{the Lagrangian expression} \]
Appendix B

THE DATA

This appendix is divided into two major parts--data needed for the core of the Hopkins-Kennedy (HK) model, and data required for historical verification, special model features, or scenarios discussed in the text. The special model features and scenarios concern the "Rosefielde-Lee world," labor supply, energy, foreign trade, and Engel-like curves.

CORE OF THE MODEL DATA

To use the core of the HK model, the following input data are required: a set of input-output coefficients, the parameters for the translog production function for each sector and each year, the net outputs of each sector in the base year, and the amount of labor measured in efficiency units for each year. The data on the labor supply are covered in the second part of this appendix.

Input-Output Coefficients

The input-output coefficients used by the core of the model for the 1980s are assumed to be constant with respect to time. The basic data source for these coefficients is the 1972 set of input-output coefficients calculated by Treml et al. for the Soviet "material" sphere of production.[1] Treml calculated these for 56 sectors. We have aggregated these data into 18 sectors. The relationship between the

Treml sectors and our sectors is shown in Table B.1. These 18 sectors taken together comprise Soviet net material product. To obtain what the West calls GNP, three sectors were added: defense, military services, and other.

The input-output coefficients for intermediate goods can be expressed as a $21 \times 21$ matrix. It is useful to conceptually divide this matrix into four quadrants, viz.:

Table B.1

THE TREML SECTORS AGGREGATED TO BECOME HK MODEL SECTORS, BY SECTOR

<table>
<thead>
<tr>
<th>HK Model Sectors</th>
<th>Treml Sectors</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. Ferrous metallurgy</td>
<td>1,2</td>
</tr>
<tr>
<td>2. Nonferrous metallurgy</td>
<td>1,2</td>
</tr>
<tr>
<td>3. Machine building metal working</td>
<td>9-26</td>
</tr>
<tr>
<td>4. Forest products</td>
<td>34-36, 38</td>
</tr>
<tr>
<td>5. Soft goods</td>
<td>40-43</td>
</tr>
<tr>
<td>6. Processed foods</td>
<td>44-49</td>
</tr>
<tr>
<td>7. Construction materials</td>
<td>39</td>
</tr>
<tr>
<td>8. Coal and peat</td>
<td>3,7</td>
</tr>
<tr>
<td>9. Oil</td>
<td>4,5</td>
</tr>
<tr>
<td>10. Gas</td>
<td>6</td>
</tr>
<tr>
<td>11. Electric power</td>
<td>8</td>
</tr>
<tr>
<td>12. Chemicals</td>
<td>27-33</td>
</tr>
<tr>
<td>13. Paper and pulp</td>
<td>37</td>
</tr>
<tr>
<td>14. Construction</td>
<td>51</td>
</tr>
<tr>
<td>15. Agriculture and forestry</td>
<td>52-53</td>
</tr>
<tr>
<td>16. Transportation and communication</td>
<td>54</td>
</tr>
<tr>
<td>17. Trade and distribution</td>
<td>55</td>
</tr>
<tr>
<td>18. Industry not elsewhere classified and other branches</td>
<td>50,56</td>
</tr>
</tbody>
</table>

[2] Our sectors 1 and 2 (ferrous metallurgy and nonferrous metallurgy) were calculated from Treml sectors 1 and 2 (metallurgy and industrial metal products) by first aggregating the Treml sectors and then disaggregating the result in proportion to the net outputs of ferrous and nonferrous metallurgy as given in Greenslade, 1976.
Quadrant I represents the Net Material Product portion of the Soviet economy. Its input-output coefficients are found from the Treml data, as has been discussed. As a simplification, we assumed that the outputs of the three non net material output sectors (defense, etc.) are not used as inputs into any of the other sectors, including themselves. This assumption means that all of the coefficients in quadrants III and IV of our matrix are taken to be zero. The values of the coefficients in quadrant II represent the inputs of goods of the net material output sectors into the production of goods of the other three sectors. These coefficients had their values assigned by analogy. In particular, it was assumed that the technology (and hence the coefficients) used in the production of a unit of output of the defense sector is the same as that used in the production of MBMW (Machine Building Metal Working). The technologies for the mainly service-type activities of the military service and other sectors were assumed to be convex combinations of the technologies for the transportation and communication sector and the trade and distribution sector, with weights being (0.5, 0.5) and (0.25, 0.75), respectively.
In addition to the input-output coefficients for intermediate goods used to produce a unit of another good, there are also input-output coefficients which give the amount of one of the factors (labor, structures, or machinery) used to produce a unit of a good. The Treml data for 1972 yield the combined value of all three factors in a unit of output for each of the net material product goods. To divide the combined 1972 value for all three factors into that which can be attributed to each factor separately, we used data from Treml et al., which were calculated for 1966.[3] The factor input-output coefficients for the three non net material product goods were found by using the same analogies as were used for the determination of the intermediate good input-output coefficients.

**Production Function Parameters**

We have for each sector a translog production function with three factors: labor, structures, and machinery. Specifically,

\[
\ln X_i^T = \ln g_{0i} + g_{1i} \ln L_i^T + g_{2i} \ln M_i^T + g_{3i} \ln S_i^T \\
+ b_{1i} (\ln L_i^T)^2 + b_{2i} (\ln L_i^T) (\ln M_i^T) + b_{3i} (\ln L_i^T) (\ln S_i^T) \\
+ b_{4i} (\ln M_i^T)^2 + b_{5i} (\ln M_i^T \ln S_i^T) + b_{6i} (\ln S_i^T)^2
\]

\[
M_i^T \geq 0, S_i^T \geq 0, L_i^T \geq 0, g_{ki} \geq 0
\]

\[
g_{1i} + g_{2i} + g_{3i} = 1
\]

---

\[ b_{1i} + \frac{1}{2} b_{2i} + \frac{1}{2} b_{3i} = 0 \]
\[ b_{4i} + \frac{1}{2} b_{2i} + \frac{1}{2} b_{5i} = 0 \]
\[ b_{6i} + \frac{1}{2} b_{3i} + \frac{1}{2} b_{5i} = 0 \]

where \( b_k \) = parameters where \( k = 1, \ldots, 6 \)
\( g_k \) = parameters where \( k = 0, \ldots, 3 \)
\( i \) = subscript referring to sector \( i \)
\( L^T \) = the supply of labor vector where labor is measured in efficiency units
\( M^T \) = vector with a component for each sector \( i \) which represents the stock of machinery for sector \( i \)
\( S^T \) = the stock of structures vector
\( T \) = superscript referring to time period \( T \)
\( \bar{T} \) = the number of time periods in the projection
\( X^T \) = the gross output vector

The \( g \)'s and the \( b \)'s are the production function parameters that we need to determine to use the production functions in the model. These core-of-the-model production functions (and hence the parameters) are independent with respect to time. When certain of the special model features and/or scenarios are used, this is no longer true. In these cases the parameters must be solved for separately for each time period---a procedure which is a simple extrapolation of that which is used here.
To find the g's and b's we first derive the equations for labor's share, structure's share, and machinery's share of the value of net output. These are simply the first derivative of the log of output of the translog production function with respect to the log of the factor in question. We have

\begin{align*}
\alpha_{Li} &= g_{1i} + 2b_{1i} \ln L_i^T + b_{2i} \ln M_i^T + b_{3i} \ln S_i^T \\
\alpha_{Mi} &= g_{2i} + 2b_{2i} \ln L_i^T + b_{4i} \ln M_i^T + b_{5i} \ln S_i^T \\
\alpha_{Si} &= g_{3i} + b_{3i} \ln L_i^T + b_{5i} \ln M_i^T + 2b_{6i} \ln S_i^T \\
&\quad i = 1, \ldots, 21
\end{align*}

where \( \alpha_L, \alpha_M, \alpha_S \) = the shares of labor, machinery, and structures, respectively.

The empirical values of the factor shares can be calculated from our input-output coefficient data. For example, the value of labor's share for sector \( i \) is equal to the labor input-output coefficient normalized such that the value of net output (1 - the sum of the intermediate good coefficients for that sector) is equal to 1. The machinery and structure's shares are calculated similarly.

Given the empirical values of the factor shares, it is clear from Eqs. (B.2) that knowledge of the "b" parameters is sufficient to calculate the value of the "g" parameters. The problem is thus reduced to finding the b's.

The cross and own elasticities of substitution are related by the well-known equations:
\[(B.3) \quad \alpha_{Li} U_{Lli} + \alpha_{Mi} U_{Mli} + \alpha_{Si} U_{SlL} = 0
\]
\[\quad \alpha_{Li} U_{Mli} + \alpha_{Mi} U_{MMi} + \alpha_{Si} U_{SlM} = 0
\]
\[\quad \alpha_{Li} U_{Sl} + \alpha_{Mi} U_{SMi} + \alpha_{Si} U_{SSI} = 0
\]
\[\quad i = 1, \ldots, 21
\]

where \(U\) = the elasticity of substitution between the two inputs indicated by subscripts.

It follows that given the cross elasticities of substitution, the own elasticities \((U_{LL}, U_{MM}, U_{SS})\) can be derived. The cross elasticities are necessarily symmetric, and thus only three of the elasticities remain to be determined. For the purposes of this study we chose a value of 1.0 for each of these, which is the same assumption that is used by the leading econometric model of the Soviet Union, SOVMOD. We expect in future work to examine this issue more closely and will probably adopt cross elasticities eventually which are less than 1.0.

Given the elasticities and factors shares, the \(b\)'s can be calculated and thus our task completed by using the relationship:

\[(B.4) \quad \begin{pmatrix}
0 & \alpha_{Li} & \alpha_{Mi} & \alpha_{Si} \\
\alpha_{Li} & 2b_{L1} + (\alpha_{Li})^2 - \alpha_{Li} & b_{21} + \alpha_{Li} \alpha_{Mi} & b_{31} + \alpha_{Li} \alpha_{Si} \\
\alpha_{Mi} & b_{21} + \alpha_{Li} \alpha_{Mi} & 2b_{41} + (\alpha_{Mi})^2 - \alpha_{Mi} & b_{51} + \alpha_{Mi} \alpha_{Si} \\
\alpha_{Si} & b_{31} + \alpha_{Li} \alpha_{Si} & b_{51} + \alpha_{Mi} \alpha_{Si} & 2b_{61} + (\alpha_{Si})^2 - \alpha_{Si}
\end{pmatrix} = \]


\[
\begin{pmatrix}
0 & 1 & 1 & 1 \\
1 & U_{LLi} & U_{LMi} & U_{LSi} \\
1 & U_{MLi} & U_{MMi} & U_{MSi} \\
1 & U_{SLi} & U_{SMi} & U_{SSI} \\
\end{pmatrix}^{-1}
i = 1, \ldots, 21
\]

**Base Year Net Outputs**

To estimate the net output for each of the 21 sectors in the model for the base year, 1979, we used a combination of aggregated 1979 estimates of Soviet GNP[4] and appropriately disaggregated older indices of GNP.[5] The CIA estimates were converted from the aggregated form (consumption, investment, etc.) to the disaggregated form (the 21 sectors used in the model) according to proportions which were estimated for 1975. The result was value added by sector for 1979. Our input-output coefficients were then utilized to obtain the desired net outputs.

**HISTORICAL VERIFICATION, SPECIAL MODEL FEATURES, AND SCENARIO DATA**

In addition to the data needed for the core of the HK model, this study requires data for certain special subjects. Some data are given in the text; more are provided in what follows.

Historical Verification

The ability of the HK model to replicate historical observations was tested by using 1960 as a base year and projecting forward to 1975. Net outputs for 1960 were calculated from data in Greenslade, 1976[6] and from information on input-output coefficients to be discussed momentarily. The net outputs for the defense sectors were obtained judgmentally by reallocating the net output of certain sectors, leaving total GNP unchanged and so that the resulting ratio of defense to GNP is equal to CIA estimates. Input-output tables are available not only for 1972, as previously discussed, but for 1959 and 1966 as well.[7] For the years between those for which we have input-output tables (1960 to 1965 and 1967 to 1971) the input-output coefficients were calculated by assuming a constant rate of change in the value of the coefficients. All years after 1972 were assumed to have the same coefficients as 1972.

Lee Data

The defense spending and consumption data used in our Lee historical projections came from Lee, 1979.[8] The rate of Lee total factor productivity growth for this period was calculated from data on output,[9] labor supply,[10] and capital.[11] To obtain the Lee rate of

[10] See the immediately following subsection on labor supply.
[11] Wharton Econometric Forecasting Associates, Inc. maintains a large econometric model of the Soviet Union named SOVMOD. We thank Dr. Bond for generously providing us with access to the data bank for SOVMOD.
total factor productivity growth for our projections, we assumed that the difference between the Lee rate of total factor productivity growth in the historical period we examined (1960-75) and the CIA rate of total factor productivity growth did and will remain the same in later years. We also assumed that the rate of inflation in the Lee world was zero.

Labor Supply

As is discussed in the text, we are using the term Slav labor to apply to all workers who live in the predominantly Slav republics of the Soviet Union and non-Slav labor to refer to those Soviet workers who live elsewhere. The Slav republics are RSFSR, Ukraine, Belorussia, Moldavia, Latvia, Lithuania, and Estonia. The non-Slav ones are Georgia, Armenia, Azerbaijan, Kazakhstan, Kirgizstan, Tadzhikstan, Turkmenistan, and Uzbekistan.

For our projections into the 1980s we took the initial labor force to be that of our base year. The rate of growth of the Slav and non-Slav components of the labor force for the 1980s was assumed to be the same as the rate of growth of the Slav and non-Slav able-bodied populations. Projections by republic of the able-bodied population (defined as males 16 to 59 years of age, females 16 to 54 years of age) have been compiled by Feshbach.[12] They were converted into projections of the able-bodied population for what we have defined to be Slavs and non-Slavs.

The historical projections also require as an input the supply of Slav and non-Slav labor. This was assumed to change at the same rate as employment for the two groups.[13] The level of employment was set to be

consistent in 1972 with the 1972 input-output table.

**Foreign Trade**

The model divides foreign trade into two major variables, endogenous foreign trade and exogenous foreign trade. For our projections into the future the value of exogenous foreign trade is an input into the model, while the value of endogenous foreign trade is calculated by the model.

Regression analysis was used to obtain our estimates of the value of exogenous foreign trade, defined as equal to total net exports save for three deletions--oil exports from the Soviet Union to the industrialized West and agriculture plus MBMW (machine building metal working) imports from the industrialized West. Data for this regression were obtained for the period 1960 to 1977 from the SOVMOD data bank.[14] The log of exogenous foreign trade was calculated for 1960 to 1977 from the SOVMOD data and regressed on time and a dummy variable. The resulting percentage rate of increase was used for our projections into the future.

In the case of our historical projections, the value of total net exports was taken from the SOVMOD data and used as an input.

**Energy**

To model the effects of resource exhaustion in the coal and peat, oil, and gas sectors, the equivalent of -2 percent Harrod neutral rate of technical change was assumed. This number was determined for the project by Richard Nehring, an expert on Soviet energy. He also

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[14] WEFA SOVMOD data bank, courtesy of Dr. Bond.
provided maximums for the gross outputs for these three sectors. These maximums are given by sector and by year in Table B.2.

Engel-Like Curves

As discussed in App. A, the model uses Engel-like curves to take into account the shift in the pattern of consumption among the various sectors as total consumption rises. Such curves are similar to Engel curves which specify how the allocation of total income to the purchase of various goods changes as income rises.

The general formula for the Engel-like curves used is given in App. A as Eq. (A.19) and is repeated here:

Table B.2
MAXIMUM GROSS OUTPUT ESTIMATES FOR THE NONRENEWABLE ENERGY SECTORS

<table>
<thead>
<tr>
<th>Year</th>
<th>Coal and Peat (Millions of Metric Tons)</th>
<th>Gas (Billions of Cubic Meters)</th>
<th>Oil (Millions of Barrels per Day)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1980</td>
<td>481</td>
<td>435</td>
<td>12.01</td>
</tr>
<tr>
<td>1981</td>
<td>478</td>
<td>470</td>
<td>12.06</td>
</tr>
<tr>
<td>1982</td>
<td>475</td>
<td>505</td>
<td>12.04</td>
</tr>
<tr>
<td>1983</td>
<td>473</td>
<td>540</td>
<td>11.96</td>
</tr>
<tr>
<td>1984</td>
<td>471</td>
<td>570</td>
<td>11.81</td>
</tr>
<tr>
<td>1985</td>
<td>470</td>
<td>600</td>
<td>11.69</td>
</tr>
<tr>
<td>1986</td>
<td>470</td>
<td>630</td>
<td>11.25</td>
</tr>
<tr>
<td>1987</td>
<td>470</td>
<td>660</td>
<td>10.85</td>
</tr>
<tr>
<td>1988</td>
<td>475</td>
<td>690</td>
<td>10.24</td>
</tr>
<tr>
<td>1989</td>
<td>480</td>
<td>720</td>
<td>9.30</td>
</tr>
<tr>
<td>1990</td>
<td>485</td>
<td>750</td>
<td>8.52</td>
</tr>
</tbody>
</table>
\begin{equation}
C_i = g_i + b_1 \left( \sum_{j=1}^{21} C_j \right)
\end{equation}

where \(b, g = \) parameters
\(C = \) consumption
\(i, j = \) subscripts referring to sectors \(i\) and \(j\), respectively.

Due to the nature of the available data, consumption was aggregated into four sectors. Each of these was regressed on total consumption in accordance with Eq. (B.5), resulting in estimates of the parameters \(g\) and \(b\). The data were for 1960 through 1978 and were taken from the SOVMOD data bank.\[15\] The values of \(g\) and \(b\) were assumed to be the same for all consumption sectors which were combined to form a given aggregate. The aggregates were food, soft goods, durables, and services.

\[15\] WEFA SOVMOD data bank, courtesy of Dr. Bond.
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