Measuring Technological Change of Heterogeneous Products

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Prepared under a grant from the National Science Foundation
This research was sponsored by the Science Indicators Unit of the National Science Foundation as part of its effort to develop better measures of science and research output. In the past decade, a number of practitioners, including one of the present authors, have worked with measures of technological change for heterogeneous products based on change of the product's performance characteristics. Such measures were potentially attractive to NSF because they are based on information that is often readily available and that can be manipulated in a straightforward manner. Furthermore, such measures avoid the complexity of delving into the complex ways that products are actually used—it is sufficient to measure the size, horsepower, speed, range, etc., of heterogeneous products. The principal task of the present study was to produce a better theoretical foundation for this work and to apply the theory in several case studies. Since hedonic price equations and cost-estimating relationships—both dealing with multi-characteristic products—bore striking similarities to the technological-change measures, an additional task was to derive an overarching theory that would encompass these two areas as well.

As the research progressed, we realized that the proposed measures of technological change encountered serious empirical problems—namely, that the typical lists of performance characteristics did not adequately capture the rate of technological change as assessed by alternative techniques. Our reluctant conclusion is that it may not be possible to use, unquestioningly, either the previously published technological-change measures or the modified versions developed in this study. Instead, it may be necessary to identify the more complex ways in which consumers actually use products to obtain unbiased measures of technological change.
SUMMARY

Technological-change measures of heterogeneous products must account for their diverse characteristics. The central task of this report is to develop and apply such measures. Technological change can be viewed and measured from two perspectives: the product producer's and the product user's. From the producer's perspective, the key feature of our approach is to assume that the output of heterogeneous products is related to a vector of product characteristics, and then to compute productivity as the ratio of output (characteristics) to inputs (resources or costs). Similarly, for the user, productivity is defined as the ratio of user-valued outputs to user inputs. Since hedonic price indexes, cost-estimating relationships, and technological-change measures represent products by their characteristics, we attempt to unify these several strands of research with a more general theory.

The following definition focuses on the heterogeneous nature of products: Technological change of a product, whose characteristics and prices change over time, is the change in total factor inputs required to produce the product, holding characteristics (output) constant. We then develop a theory of technological design and production to provide an explicit framework in which to place empirical measures of technological change, hedonic price indexes, and cost-estimating relationships. The theory specifies a set of $k$ relationships that constrain what can be produced at any particular time: $F_i(C, P, R, q) = 0, i = 1, \ldots, k$, where $C$ is average cost and $P, R,$ and $q$ are vectors of performance characteristics, factor inputs, and factor prices (assumed fixed to the firm). The number of constraints, $k$, limits the number of characteristics that the designer is free to specify. Given the firm's or market's choice structure, the equations can be solved as follows: $F(C, P, \ldots, P_{s}, q_1, \ldots, q_j) = 0$. This is a tradeoff surface generated by a constrained optimization process. From this surface, the designer must choose a point—a product. When the values of the variables in the producer's tradeoff function are chosen, the values of
the other technical and performance variables will be determined by the
set of constraining equations, for an exogenous set of factor prices.
If any of the constraint equations shift over time because of
technological change, a time variable should appear in that equation,
and the tradeoff function will also include time. The tradeoff function
can then be written as: \( C = C(P_1, \ldots, P_s, q_1, \ldots, q_j, t) \). This is
essentially the equation that is estimated empirically.

The same approach can be used to examine technological change of
the product user; e.g., an airline producing transportation services.
The analytics proceed in the same fashion as for the product producer,
but with the product as input rather than output. Technological change
in the production of user outputs can arise from a more productive
product, from other improved inputs, and from more efficient production
processes.

Technological-change equations were estimated for milling machines,
turbine-powered airliners, and turbine engines. The milling machine
sample spanned more than 100 years, with machine characteristics
available on more than 700 models, and prices on 270 of these
observations. Equations of deflated costs regressed against product
characteristics and time for the three products uniformly showed
negative technological change—prices increased over time, holding
performance constant. However, estimates based on user outputs (cost
per seat-mile, cost per pound of engine thrust, cost of machining a
given part) all showed positive technological change (declining costs).
These differences are attributed to missing characteristics in the
producer equation. If the importance of unmeasured variables that
contribute to the products' productivity increases over time, the
coefficient of the estimated time variable will be biased upward.
Although our results could also come from other sources (actual
technological decline, productivity increases in complementary inputs),
similar results were found in independent studies on computers—i.e.,
technological-change measures based on user outputs were higher than
those based on simple linear combinations of characteristics.
Based on our empirical findings, we conclude that the use of product characteristics as a shortcut to measures of value and performance is an uncertain undertaking and cannot be counted on to produce adequate measures of technological change. This same cautionary finding also applies to the estimation of hedonic price indexes and cost-estimating relationships.
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A collection of aircraft jet engine data was made available by Rand colleague John Birkler. Information on recent General Electric engines came from Robert Bell, General Electric Company (Los Angeles office). Aircraft lift-to-drag ratios were provided by John Hopkins, Lockheed Corporation, California Company; R. G. Steiner, Boeing Commercial Airplane Company, supplied standardized operating costs for recent Boeing and other aircraft.

Comments and advice on earlier drafts by our colleagues Bryan Ellickson and Dan Kohler helped us clarify, simplify, and improve the theoretical arguments. Although we did not accept the totality of their advice, their contributions were significant. Also, Jack Triplett of the Bureau of Labor Statistics, Department of Labor, stimulated the development of some of the report's arguments. Finally, Donald Buzzelli, National Science Foundation, has encouraged our own work and that of others in this area. His support has generated a better understanding of technological change.
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I. INTRODUCTION: RESEARCH TASKS

In a modern economy the generic name of a product— automobile, computer, milling machine, medical service—covers a dazzling array of variants and models. Within a product classification are found both expensive, high-performance items and cheaper versions of lesser quality and more limited performance, as well as models that emphasize one or another feature. Thus, most products are inherently heterogeneous.

A standard problem in the analysis of heterogeneous products is to distinguish advances of the entire product class over time from mere changes in the mix of the heterogeneous product. This corresponds to the typical economic problem of measuring movements along a surface versus shifts in the surface itself.

A satisfactory measure of technological change of a product must account for its diversity of characteristics. In this report, our central task is to develop and apply such measures. To do this, we employ the concept of productivity to compare the range of products produced in one period with those supplied at another time.

Economic productivity is the ratio of outputs to inputs, and total factor productivity measures are designed to account for all inputs or factors of production, measured in terms of resources or costs. The change in productivity over time is the rate of change of output less the rate of change of inputs.\(^1\) The key feature of our approach is that the output of a heterogeneous product is measured by a vector of that product's characteristics. The inputs required to produce the product are summarized in a measure of total factor resources, or costs.

In the course of this analysis we attempt to unify several strands of previous research: technological-change measures of products; hedonic price indexes; and cost-estimating relationships. Each of these three approaches represents heterogeneous products by their salient performance characteristics. In addition, the empirical relationships

\(^1\)If \(Q\) is output and \(R\) is inputs, productivity \((X)\) is: \(X = \frac{Q}{R}\). The percentage rate of change of productivity is then: \(\frac{\Delta X}{X} = \frac{\Delta Q}{Q} - \frac{\Delta R}{R}\), where \(\Delta\) is the differential with respect to time.
they yield are strikingly similar. One might therefore conjecture that these subjects are special cases of a more general theory; we show below that this is, indeed, the case.

A subsidiary question raised by this research is whether a relatively small set of definable, observable, and measurable characteristics yields appropriate and satisfactory measures of technological change, and whether variations in this set of characteristics are associated with changes in productivity as evaluated by the product's users. In our empirical investigations we find that this approach is sometimes deficient--apparently because of omitted characteristics in the set used to describe the product.

A DEFINITION OF TECHNOLOGICAL CHANGE FOR HETEROGENEOUS PRODUCTS

To focus directly on the heterogeneous nature of most products, we propose the following definition: Technological change of a product with many elements of performance, whose characteristics and prices change over time, is the change in total factor inputs required to produce the product, holding its characteristics (performance or output) constant.

Figure I shows the average resource cost of producing a product of given "performance" (whose definition is left unspecified for the moment) plotted for two time periods: $t_1$ and $t_2$. According to our definition of technological change, a vertically downward shift of the curve holds performance constant and demonstrates an improvement in technology. This type of change is usually referred to as a "process" improvement.

The definition of technological change can also be stated in an alternative form: the change in the characteristics (performance) of a product, holding its resource inputs constant. In Fig. 1, a horizontal shift to the right represents an improvement in performance at unchanged costs--a "product" improvement. It might appear that this is actually a different concept of technological change, and discussions of technological change often treat product and process changes as if they were quite separate concepts. In fact, they are alternative statements of the same underlying relationship. A "pure" change in process
Fig. 1 -- Measures of technological change: (A) cost reduction holding performance constant; (B) performance increase holding cost constant

technology is equivalent to a vertical translation of the curve, such that points on the curves at two points of time with the same value of performance (i.e., vertically displaced points) have the same slope. Likewise, pure product technological change corresponds to a horizontal translation of the curve. In an actual market we may well observe some cases of these pure forms of technological change as well as many mixed cases--improved or changed product characteristics produced with a different quantity of resources.

We will show below that much previous work in technological change measurement has been close to this basic concept; but without consideration of the definitional and theoretical implications, most of this work is deficient. Our definition of technological change for heterogeneous products also bears a close connection to hedonic price indexes and to cost-estimating relationships. In fact, a number of earlier studies in these fields can be carried one step further to generate measures of technological change.
TECHNICAL-CHANGE MEASURES, HEDONIC PRICE INDEXES, AND COST-ESTIMATING RELATIONSHIPS

Technical-change measures of heterogeneous products with many performance dimensions have often used a time variable as a proxy or index for the level of technology, which is usually assumed to increase monotonically over time.\(^2\) This time variable is then related to a set of performance characteristics in a regression equation. A sample observation is then the performance characteristics of a specific model of the product at that specific date:

\[ F(t) = F(P_1, \ldots, P_m, (p?)) \]  

[technological change measure];

where \( t \) is time, \( P_1, \ldots, P_m \) are the values of important performance characteristics, and \( p \) is price of the product. An alternative technique estimates separate cross-section equations by time period, and then measures the shift in the estimated surfaces.\(^3\) Price is not always included in these formulations; indeed, (as indicated by the question mark) it is most often left out.

Technology change analysts have developed several variants to this approach. For example, only those models with the highest ever-attained level of an important characteristic (i.e., record breakers) were included in the sample by Lienhard, who estimated the rate of change of thermal efficiency of steam power plants, the accuracy of clocks, the speed of vehicles, and the lowest temperatures reached by refrigerating devices.\(^4\)


Hedonic price index equations typically are used to generate a price index over time for heterogeneous products, holding performance constant. They have the form:

\[ p = F(P_1, \ldots, P_m, t) \]  \hspace{1cm} \text{[hedonic price index]}.

Frequently, the effect of time is represented by including a set of dummy variables, for separate time periods, rather than by a single continuous variable; the coefficients on the dummy variables yield a price index for each period, holding the performance variables \( P_1, \ldots, P_m \) constant. A large number of hedonic price index studies have examined such diverse products as tractors and automobiles, computers, people, and boilers. But only Ohta's study, to our knowledge, explicitly considers technological change in the sense defined above--and then, only in passing.

**Cost-estimating relationships** attempt to predict the cost of a product on the basis of past experience:

\[ C = F[P_1, \ldots, P_m, (t?)] \]  \hspace{1cm} \text{[cost-estimating relationship]};

where \( C \) is average cost. A time variable is sometimes included when it increases the predictive power of the equation.

Cost-estimating relationships were developed during World War II to help predict the cost of military aircraft. Following the war, engineers found that similar relationships held for a wide variety of

---

manufactured products—from refrigerators to automobiles to electronics and tanks. A variable representing size was usually the most important determinant of cost, followed by one or two indicators of "quality": speed, temperature, etc. These relationships were derived by statistical means and engineering-cost estimations. Recent studies often use advanced statistical tools to estimate the relevant parameters, but the basic concepts remain quite similar to the simple, intuitively appealing ideas that stimulated their first use.

It is readily seen that these relationships are virtually identical. However, they have been used by different communities, for different purposes. The communities have been institutionally isolated from each other; the practitioners are drawn from different professions; they publish in different journals. Economists have been the major developers of the hedonic price index and publish their results in economic journals. They have developed a rich theory and often use advanced econometric techniques in their statistical analyses. The technology change community includes a few economists, but mainly a mix of scientists, engineers, and technology experts. They publish in journals devoted to science and technology. Their theory is ad hoc, but directly related to their main concern—the measurement and understanding of technological change. The statistical techniques vary from the clever to the naive to the crude. Cost estimators, often engineers, are more concerned with their clients than with the niceties of theory or estimation. They are looking for something that works: that is, an equation that predicts costs with sufficient accuracy to support planning and analysis. Despite this diversity of purpose and use, it will be shown below that all of these efforts can be derived from the same theoretical framework.

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II. THEORY OF TECHNOLOGICAL CHANGE

TWO PERSPECTIVES: PRODUCT PRODUCERS AND USERS

Technological change of a heterogeneous product can be viewed from two perspectives: the producer's and the user's. These perspectives can be modeled by two systems of relationships that map or transform "inputs" into "outputs":

\[ F(R_p, P, t) = 0 \quad [\text{producer transformation}] \quad (1) \]

\[ G(R_u, U, t) = 0 \quad [\text{user transformation}] \quad (2) \]

where \( P \) and \( U \) are vectors of "product" and "use" variables; \( R_p \) is a vector of factor inputs (resources) of the product producer; and \( R_u \) is a vector of resources consumed by the product user to produce the vector of outputs, \( U \). \( R_u \) will include as an input the output of the product producer as well as additional inputs such as labor, materials, and energy. For present purposes, it will be convenient to transform the resource vectors into scalars by calculating a cost figure from the quantity and price of each resource. This scheme can be diagrammed as in Fig. 2.

Technological change will then be \( -\partial R_p / \partial t \) in Eq. (1) and \( -\partial R_u / \partial t \) in Eq. (2), holding \( P \) and \( U \) constant. Alternatively, technological change can be defined as \( \partial P / \partial t \) and \( \partial U / \partial t \) in the two equations, holding the respective \( R \) vectors constant.

To lend concreteness to this formulation, consider transport aircraft. Some elements of \( P \) form the technical description of the product; they would include lift-to-drag ratio, structural efficiency, engine thrust, wing loading, and energy content of fuel. This subset of the \( P \) vector is the raw material of the engineers; its elements are of little concern to users. In contrast, other elements of \( P \) (which also

\[ \text{Productivity (X) is } X = Q/R, \text{ and } \Delta X/X = \Delta Q/Q - \Delta R/R; \text{ if } \Delta Q = 0, \text{ then the rate of change of productivity (holding output constant) is simply the negative of the rate of change of resource inputs.} \]
describe the product contribute to the product's value to its users. For aircraft, they include range, speed, payload, landing and takeoff distance, fuel consumption, and crew and maintenance requirements. The P vector is sometimes split into two separate vectors along the lines suggested above: "technical" characteristics and "performance" characteristics.² Although the performance characteristics contribute to the product's value, their value is derived from the activities that enter directly into users' profit (or utility) functions--i.e., the U vector; these include seat-miles or ton-miles, noise, pollution, and the costs of obtaining (or avoiding) these activities.

The measurement of U is ordinarily quite difficult. Definitions of the vector's elements are often scenario-specific; that is, they depend on the circumstances of each user. The value of an aircraft can vary

²See, for example, Alexander and Nelson, op. cit., pp. 9-10, and Ohta and Grilliches, op. cit., p. 333.
with route structure, geography, and climate. To take another example, computers may be ranked differently according to whether they are primarily used for running a payroll, supporting a text-processing system, or computing matrix inversions.

The specificity with which U is defined and measured by each user renders it highly individualistic. By contrast, the set of performance characteristics is a more universal description. Although not absolutely unchanging, it has greater stability over time and among producers and users than does U. Each user can apply its own algorithm to this set in order to establish a value for the product.

The technical characteristics set is also less universal. It can vary from producer to producer as each draws on its own experience, capabilities, research, and knowledge. It can also change over time as one technology replaces another. Computers, for example, which span several generations of technologies—vacuum tubes, transistors, integrated circuits—would require different sets of variables to describe their technical characteristics, yet could all be represented by the same set of performance characteristics.

Because of the great difficulty of defining and measuring U, analysts have often turned to a more accessible and stable product description: performance. Memory size and add speed are not only definable and measurable; they are also published! And they clearly contribute to the things that users value directly. So instead of focusing on system (2) above, technology-change analysts have turned to system (1). Or, they use (2) with P as a proxy for U. However, it has not often been recognized that although both transformations may yield valid, informative, and useful measures, these measures are quite different from each other. System (1), for example, would measure the changing productivity of aircraft producers in producing aircraft, whereas system (2) focuses on airlines producing air transportation. Furthermore, as will be shown in later sections, the hope that a relatively small number of performance variables will adequately define a heterogeneous product cannot be counted on empirically. In fact, all three case studies of the present research yielded inadequate estimates based on the producer's transformation.
THE BASIC MODEL: THE PRODUCER'S PERSPECTIVE

We begin by describing the production technology of a firm that produces a multi-characteristic product. It is assumed that marginal production costs are constant and that all firms share the same technology. (The latter assumption will be relaxed later.) The product is described by a bundle or vector of \( m \) characteristics: \( p_1, \ldots, p_m \). This vector includes performance characteristics, which indirectly enter the utility or profit function of the user of the product, as well as technical characteristics, which can be varied by the product designer in order to yield the performance that provides value to the product users.\(^3\)

Production of the product requires quantities of resources--input factors--designated \( r_1, \ldots, r_j \), with input factor prices of \( q_1, \ldots, q_j \). It is reasonable to assume that the input prices are given to the firm. One of the product characteristics of concern to the designer and producer is its cost.\(^4\) Because it features centrally in the following exposition, cost will be explicitly identified as similar to other elements in the performance vector that characterizes the product. This treatment of cost implies that in the technical description of a product's design, cost is symmetrical to the other performance characteristics. For the product design engineer, product cost and fuel consumption both flow from the same set of engineering-economic relationships.

A set of \( k \) relationships constrain what can be produced at any particular time. These constraints include engineering equations, rules of thumb, empirically determined relationships, and cost functions. They can be written in general, implicit form:

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\(^3\)It should be emphasized here that the distinction between performance and technical characteristics is somewhat arbitrary; it flows from the product design optimization process.

\(^4\)"Cost" is the average cost of producing quantity \( Q \) with performance \( P \).
\[ F_1(C, P, R, q) = 0 \]
\[ \vdots \]
\[ F_k(C, P, R, q) = 0 \]  

(3)

The number of constraints, \( k \), limits the number of characteristics that the designer is free to specify. Under certain conditions, the \( k \) equations of constraint define \( k \) of the total number of variables \( (z = m + j - k) \) as functions of the other \( z - k \) variables.\(^5\) Which of the variables are solved for depends on the uses of the analysis. If we consider the decision process in the production organization as hierarchical and separated into sequential steps, the engineers and designers could solve the system in a way that allowed them to present to the managers or to the marketplace a list of alternative product specifications. Given the firm's or consumers' choice of which product to produce, the solved equations would permit the designers to then calculate the required technical specifications. In order to do this, the system of equations (3) could be solved in the following way:

\[
P_{s+1} = f_1(C, P_1, \ldots, P_s, q_1, \ldots, q_j); \\
\vdots \\
P_m = f_m(C, P_1, \ldots, P_s, q_1, \ldots, q_j); \\
r_1 = f_{m+1}(C, P_1, \ldots, P_s, q_1, \ldots, q_j); \\
\vdots \\
r_j = f_{k-1}(C, P_1, \ldots, P_s, q_1, \ldots, q_j); \\
F(C, P_1, \ldots, P_s, q_1, \ldots, q_j) = 0
\]  

(4)

\(^5\)These conditions basically require that the Jacobian have full column rank and that the equations be continuous in a neighborhood.
Equation (5) is a tradeoff surface generated by a constrained optimization process. The tradeoff surface will have dimensionality \( d < m + j - k \). For the tradeoff surface shown by Eq. (5), and for factor input prices (q) fixed by input markets, the designers are free to specify any of the remaining characteristics, but one. This last characteristic is then a function of those specified. From this surface, the designer must choose a point—a product. When the values of the variables in the tradeoff function are chosen, the values of the other variables will be determined by the equation set (4) for given values of input factor prices.

If the tradeoff function is solved for cost (C) as a function of the other variables, we have essentially a supply curve of product characteristics; also note that Eq. (5) has the form of the typical cost-estimating relationship. If producers all dip into the same technology barrel and therefore face identical tradeoff surfaces, and if consumers vary in taste, the tradeoff surface also describes a hedonic price equation (with cost instead of price as the dependent variable).

As noted above, the dimensionality of the tradeoff surface is limited by the difference between the total number of variables and the number of constraints. Increasing the number of constraints reduces the freedom of the designer in making choices. When the dimensionality, \( d \), is small, the set of variables excluded from the tradeoff function may include performance variables as well as technical variables. Conversely, if \( d \) is large, the tradeoff function could well include both performance and technical variables and there would be more than one way of producing a product with a given set of performance measures at a given cost. Thus, in general, there is not a simple mapping from technical space to performance space.

The entire tradeoff surface—including cost—represents the state of the art of a given period. This is clearly seen in engineering handbooks on product design. Figure 3 is taken from a text on turbine engine technology; because it illustrates concretely the notion of

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6Note that cost (C) has been implicitly assumed to be one of the performance characteristics emphasized by consumers and designers. This need not always be the case. Indeed, it has been claimed that for some military products, the buyers have required maximum physical performance of essential characteristics, with costs allowed to take on whatever value was necessary to achieve the desired product.

technical constraints and a tradeoff surface, it merits closer examination. Along the surface that is projected onto the two-dimensional fuel-consumption/thrust plane, pressure ratio increases in the southeast direction, and temperature increases in the northeast direction. A joint choice of fuel consumption and thrust requires specific values of temperature and pressure. The variables of immediate interest to jet engine users are specific thrust (X), specific fuel consumption (SFC), weight (W), and cost (C). That is, the user has a utility function: \( U = U(X, \text{SFC}, W, C) \). Thermodynamic variables available to the designer include turbine inlet temperature (T) and overall pressure ratio (P). For a given mechanical and combustion efficiency level (E), thermodynamic calculations generate two equations relating fuel consumption and thrust to the thermodynamic variables: \( F_1(P, T, E, \text{SFC}) = 0 \) and \( F_2(P, T, E, X) = 0 \). However, thermodynamic optimization of the design cannot be separated from mechanical design and cost considerations. Thus, while high turbine temperatures are thermodynamically desirable, they involve the use of expensive alloys and cooled turbine blades, which increase engine complexity and cost. Similarly, the thermodynamic gains of increased pressure ratio also lead to increased weight, complexity, and cost because of the need for more compressor and turbine stages. More equations, therefore, are required to relate weights and costs to the technical variables: \( F_3(P, W_p) = 0 \), \( F_4(T, W_t) = 0 \), \( F_5(W, P, T, C) = 0 \) and \( W_p + W_t = W \). (\( W_p \) and \( W_t \) are the weights associated with higher pressures and temperatures.) In the fifth equation, a lower weight will cost more, holding other variables constant, as more expensive, lightweight materials are substituted for cheaper, heavier ones. Therefore, as one moves to the lower right in Fig. 3, gaining greater fuel economy (lower fuel consumption) and greater thrust, the technology constrains the designer to accept higher costs and weight because of the higher pressure ratio and temperatures that are required. The user-oriented tradeoff function would then be:

---

\(^9\)SFC and thrust, as graphed in Fig. 2, are scaled variables. SFC is pounds of fuel per pound of thrust per hour; specific thrust is pounds of thrust per pound of air flowing through the engine.

\(^9\)This discussion is taken from Horsley, op. cit., pp. 157-159.
\( F(C, X, SFC) = 0 \). The surface of Fig. 3 illustrates the designer's product possibility set or supply function from which a constrained maximization choice must be made. The shaded areas indicate typical choices for different classes of aircraft users: e.g., business jets and long-range subsonic transports.

**TECHNOLOGICAL CHANGE**

If any of the technical constraint equations in set (3) shift over time because of technological change, a time variable should appear in that equation. Without loss of generality, time-shift variables can be inserted in all the equations although, in practice, some of these will be of no effect. Nevertheless, if any of the equations include technological change, then the solution set (4) and the tradeoff function (5) will now also include time. The tradeoff function can then be written as:

\[
F(C, p_1, \ldots, p_s, q_1, \ldots, q_j, t) = 0; \quad (5')
\]

![Diagram](image)

*Fig. 3 -- Example of a turbine engine tradeoff surface in cost, performance, and technical characteristics*
or

\[ C = C(P_1, \ldots, P_s, q_1, \ldots, q_j, t). \]

We relate technological change to time here without going more deeply into the process that generates it. Others have modeled it as depending on research and development expenditures\(^{10}\) or cumulative production.\(^{11}\)

Shifts of a product's tradeoff surface over time have an intuitive appeal as an indicator of technological change. Returning to the jet engine illustration, the development of new nonmetallic, high-temperature materials (e.g., ceramics and carbon fiber materials) may over time permit higher temperatures at lower cost and weight. The two thermodynamic equations would not change, but the cost and weight equations as well as the tradeoff function would now require a time variable to represent these shifts in the technical relationships. Now, suppose that new knowledge of aerodynamics of turbine blades permitted increased pressure without a weight penalty. In this case, the entire technical constraint, represented above by \( F_3 \), would vanish; the dimensionality of the tradeoff function would now increase and weight could be added to the choice set: \( F'(C, X, SFC, W, t) = 0 \).\(^{12}\) We suspect that a great deal of technical change takes place in just this manner—that is, it increases the feasible set of performance characteristics. Empirically, this possibility raises particularly difficult issues because many of the additional characteristics of improved products are secondary or subsidiary to the principal characteristics that, in a sense, define the product. Ohta and Griliches, for example, use only five variables to describe automobiles, and Chow describes computers with four variables. A brief glance at the sales brochures for these products would indicate scores of features not included in the brief list of variables used in most research studies. Possibly, their omission could seriously bias measures of technical change.

\(^{10}\)Shishko, op. cit.


\(^{12}\)In other words, weight (in this example) is transformed from a technical to a performance characteristic.
TRADEOFF FUNCTIONS AND PRODUCTION FUNCTIONS

Standard economic theory treats production as a transformation function that converts resources (R) to outputs (Q): \( q(Q) = f(R) \), where \( R \) is a vector of individual inputs and \( Q \) is either a single, homogeneous product or (in more recent work) a vector of such products. In form, this relationship is quite close to the tradeoff function derived above. Why then do we not simply start off from the base of standard production theory instead of introducing the intermediate steps of technical constraints and interpreting the transformation as a tradeoff function?

There are several reasons for our preferred approach. The first flows from the multi-characteristic nature of the products. Users may value many features of a product. Producers, however, are constrained by technical limits and will in general be restrained in their ability to vary the characteristics sought by consumers. Standard production theory, which places no theoretical bounds on the dimensionality of output, is not well equipped to represent such products. When only homogeneous products are analyzed, dimensionality is not an issue; and it has not been considered to be a problem for multi-output studies. However, for heterogeneous goods, the number of descriptive and desirable characteristics is theoretically unlimited. Our approach described above provides a natural way to constrain the production transformation.

A second reason for our approach is the possibility that constraints will be changed or removed through technological advances, thus changing the dimensionality of the output vector. Although we do not make use of this feature directly in our empirical work, we believe that it is not only a better description of reality, but that it improves understanding of our results and those of other studies. Finally, the task of traditional production theory has been to elucidate the relationships among inputs—explanations of functional shares, substitution among resources, the productivity of separate inputs or their aggregate. Our focus is the heterogeneity of output; although the structure of equations is similar, the tradeoff function emphasizes the production of quantities of characteristics—not of products.
ALTERNATIVE DEFINITION OF PERFORMANCE: THE USER’S PERSPECTIVE

We now shift focus from the producer of a product to the user. The user combines a product with a given performance and price with a vector of additional resources ($R_u$) obtained at the vector of input prices ($q_u$). As discussed earlier, these resources can produce a vector of outputs ($U$), including those in the user's utility function or other kind of valuation function.

It should be noted that $U$ and $P$ may be identical: The producer's description of a product may enter directly into the user's evaluation. For example, a television set may be adequately described to all parties by its size, picture resolution, color intensity, and power consumption. A milling machine, though, can be used to produce a wide variety of parts, each of which emphasizes different characteristics: flexibility, low cost, high speed, manufacturing complexity, low labor usage, easy set-up. In such cases, the product performance characteristics vector ($P$) serves as a shorthand description to each user, who then applies his own calculus to the $P$ vector to generate a measure of the usefulness of the product for his mix of requirements. Indeed, we would argue that most heterogeneous consumer and producer goods are evaluated in this manner.13

For the most part, the analytics of the user's production technology proceeds in identical fashion to the approach developed above for the producer. The set of $y$ technical constraints faced by the product user will be:

$$F_1(X, U, R_u, q_u) = 0$$
$$\vdots$$
$$F_y(X, U, R_u, q_u) = 0$$

---

where $X$ is the cost to the user of producing the vector of user outputs ($U$); $R_u$ is the vector of inputs to the user firm; $q_u$ is the vector of input prices; input $r_1$ is defined as the multi-attribute product whose technology or productivity we are attempting to measure, and $q_1$ is its price. Solution of the system proceeds in the same manner as previously.

$$U_{d+1} = h_1(X, U_1, \ldots, U_d, q_1, \ldots, q_e);$$

$$U_{d+f} = h_f(X, U_1, \ldots, U_d, q_1, \ldots, q_e);$$

$$r_1 = h_{f+1}(X, U_1, \ldots, U_d, q_1, \ldots, q_e);$$

$$r_e = h_{f+e}(X_1, U_1, \ldots, U_d, q_1, \ldots, q_e);$$

$$H(X, U_1, \ldots, U_d, q_1, \ldots, q_e) = 0 \quad (8)$$

Equation (8) is the tradeoff surface of user tasks permitted by the set of technical constraints facing the user. Allowing these relationships to change over time, and solving for costs, yields:

$$X = X(U_1, \ldots, U_d, q_1, \ldots, q_e, t). \quad (9)$$

Productivity change is defined as $-\partial X/\partial t$, holding $U$ constant, or $\partial U/\partial t$, holding $X$ constant.

Productivity change in the production of $U$ can arise from several sources. (1) Product $r_1$ may become more productive; (2) the other inputs $r_2, \ldots, r_e$ may be more productive; or (3) the production process for $U$ may become more efficient, even with unchanged inputs. These possibilities raise the problem familiar to productivity analysts of attempting to attribute productivity change to a single source.
EMPIRICAL IMPLICATIONS OF THE PRODUCER-USER DICHOTOMY

The empirical work reported on below shows productivity for several products as viewed by users to be rising ($\frac{\Delta X}{\Delta t} < 0$), whereas productivity from the producer's perspective appears to be falling ($\frac{\Delta C}{\Delta t} > 0$). This difference could arise if (1) some of the other inputs into the user's technology had increased productivity, or (2) if the user's production process showed increased efficiency. It could also occur if both of these conditions held and if (3) productivity change of producers was, in fact, negative. However, this possibility raises another question with important empirical consequences. Is it possible that milling machines, commercial jet transport aircraft, and jet engines could all have declining productivity, as measured at the level of the producer? If this were really so, it would imply that producers in 1970 (say) could not re-produce their products of 1960 as efficiently as they had produced them in 1960. For one reason or another, it is certainly possible that such reverses could take place in the short run. But it is unreasonable that these trends would persist over periods measured in decades.

The most probable explanation for this phenomenon is missing variables in the estimated producer's tradeoff function. If the number and importance of the missing variables increased over time, the measured technological change would be biased downward. In general, we would then find that productivity change as measured by user characteristics would be greater than productivity change from the producer's perspective (even if the other sources of increased productivity were zero). The cost of machining a given part on different milling machines at different times would be affected by all changes to the milling machine--major, minor, and subsidiary changes. Improvements in convenience, loading and unloading speeds, set-up times, tool changing, and reliability would have an influence on total machining costs. Cumulative changes of this type could significantly affect productivity, even if the basic characteristics remained unchanged. We shall be returning to this point below.
TECHNOLOGICAL CHANGE WITH HETEROGENEOUS PRODUCERS

It was explicitly assumed above that producers of a product shared the same technology so that the tradeoff surface was identical for all firms. Empirical estimates of cost or price as a function of performance characteristics would then trace out the tradeoff surface. For many products, one could reasonably argue that producers are little different from each other in their inherent technological capabilities, especially over the long run, with mobility of technical staff, dissemination and copying of products, and publication of patents and other technical information. However, market strategies will tend to lead to specialization and the carving out of market niches. The large investments required to compete across the board often drive firms to specialized positions. Experience in a narrow area would then enhance the knowledge, technology, and real cost structure of the array of firms in an industry, but only in their regimes of specialization. Thus, whether through market strategy or unique company qualities (geography, personality, random variability), some heterogeneity on both sides of the market is likely to be present.

Under conditions of producer heterogeneity, market transactions will lie on an envelope of individual producer tradeoff surfaces, as described in the hedonic context by Rosen.\(^1\) This is illustrated in Fig. 4, where cost is plotted against one performance characteristic, \(P_1\), all others held constant. Lines \(E_1\) and \(E_2\) are the envelopes of individual producer's curves (A, B, C) for time periods 1 and 2. Measurement of technological change can proceed as previously, only now the interpretation will be of industry performance rather than that of a single firm. Market choices are constrained by what is technically possible. As the set of the possible is altered, so too will be the choices that are made.

THE NECESSITY FOR INCLUDING COST OR PRICE

The measure of technological change defined above departs from one branch of the literature by focusing on the resource cost of achieving performance. The most important reason for including costs is that higher levels of performance can generally be attained through the expenditure of more resources. This is illustrated in Fig. 5. At time period $t_0$, performance characteristic $P_1$ can be traded off for $P_2$, as shown by the curve $C_0(t_0)$. However, that curve is drawn for a constant value of cost $C_0$. If greater resource costs are incurred, higher levels of performance are possible--at a price. By using a more complex design, more expensive materials, or better workmanship, more of $P_1$ can be obtained without giving up any of $P_2$, as shown on $C_1(t_0)$. If there were a distribution of tastes and demands in the market (as shown by the points A to G), products could be found throughout the $P_1 - P_2$ space. If then, in a later period 1, technology improved to the level indicated by the dashed lines, higher performance would be available at the same time.

15 None of the following studies purporting to measure technological change include costs: Alexander and Nelson, op. cit., Shishko, op. cit., Dodson, op. cit., Lienhard, op. cit.
old cost. Observations that ignored costs would not be able to accurately measure technological change between periods.

An estimated tradeoff function that excluded costs would be adequate only if the distribution of deflated costs remained constant over time. In essence, technology would be measured by the second definition given earlier: an increase in performance, holding resources constant. In this case, it would not be necessary to hold resources constant by statistical means, since there would not be any variation. This is shown in Fig. 5, where cost is plotted against an index of performance. If all marketed products have the same cost (for example, $C_1$ in Figs. 4 and 5), then the estimate of movement of the tradeoff surface would not require a price variable. Similarly, if the distribution of prices remained constant (as shown by the distributions in Fig. 6), technological change could be measured without considering price. However, it seems doubtful that these conditions are met in practice.
Fig. 6 -- Cost-performance tradeoff showing restrictions permitting exclusion of costs from technology measures

Another interpretation of regression equations of the form $t = f(P)$ is that these measure the shift of one of the technical constraints. Horsley tests exactly this kind of movement by regressing specific fuel consumption and specific thrust against temperature, pressure, and time.\footnote{Horsley, \textit{op. cit.}, pp. 165-169; Horsley clearly notes that he is testing changes in technical thermodynamic relationships and is not surprised to see a time coefficient not significantly different from zero in his equations that regress one characteristic against others and time.} For this approach to be a valid estimation, the equation must be structured as a technical constraint and not as a shift in the tradeoff function.
III. EMPIRICAL MEASURES OF TECHNOLOGICAL CHANGE

The theory of technological-change measures was applied to case studies on milling machines, aircraft turbine engines, and turbine-(jet-)powered civil transport aircraft. Several motives lay behind the selection of these cases. Milling machines reached their modern form in the 1860s and since then have been the subject of continuous evolutionary change. As the most versatile of machine tools, milling machines were an essential element in the development of modern manufacturing methods. The long history of the product and its importance in economic growth made it a choice candidate for testing several hypotheses on technological change. Jet engines, on the other hand, represent a "high technology" product—one that has enabled aircraft to break into new regimes of speed and operating efficiency. Furthermore, several previous studies have been conducted on jet engines, so that data were readily available and comparisons with the earlier studies could be made. Jet-powered passenger aircraft allow one to examine the effects of technological change of subsystems (engine and airframe) on the total aircraft system.

MILLING MACHINES

Milling machines attained their developed form in the early 1860s with the Brown and Sharpe Universal model. Until the widespread diffusion of numerical control in the 1970s, the milling machine retained its basic form. Even today, 40-year-old designs are being updated by the addition of computer-controlled numerical features. Stripping away the outer panels often reveals virtually the same castings and structures as on 1930s machines.

Milling machines remove metal by means of a rotating, multiple-tooth cutter; the workpiece is secured to a table whose motion relative to the cutter is controlled in combinations of longitudinal, transverse, and vertical feeds. Moreover, some designs permit rotation of the table around the vertical axis as well as rotation of the workpiece around the longitudinal axis. Furthermore, the spindle that carries the revolving
tool can sometimes be translated or even rotated. In general, then, there are 12 possible dimensions of movement: three translational and three rotational for the piece being machined; and the same number for the milling tool. Some of the earliest universal machines had five or six degrees of movement of the work and one or two of the tool. Some modern numerical control machines can be programmed to move simultaneously in five dimensions. Control, therefore, is an essential characteristic of milling machines. Control can be further described as manual, power, or numerically controlled.

Throughput or cutting ability is another central feature. The volume of metal that can be removed per unit time is a function of the size and rigidity of the machine, the power delivered to it, and the ability to cool the work as energy is delivered to it during the milling process. Cutting ability also depends critically on the milling cutter: its strength, hardness, and cutting lifetime.

Precision is also a desirable characteristic. However, this feature did not change much until recent years. Milling machine users did not demand precision much beyond .001 inches, and most machines could achieve that level. Throughput increased drastically while precision remained more or less constant. It was generally acknowledged that heavy cuts at the limits of the machine would leave a rough finish, but the required precision could be obtained with a final light cut. In the 1970s, however, precision of .0001 inches and better became routine; but because of the past absence of variability of this attribute, it does not figure in the statistical analysis.

Data were collected on more than 700 milling machines from the first Brown and Sharpe Universal to present-day models. Prices were obtained on about 270 models. The data were gathered from the sales records and files of Cincinnati Milacron (formerly Cincinnati Milling Machine Co.), Bridgeport Machines Company, the Smithsonian Institution, and National Archives records of the U.S. arsenals. A microfilm copy of Brown and Sharpe sales ledgers from 1866 to 1900 was also used,\(^1\) and Brown and Sharpe catalog information came from the Rhode Island Historical Society. In addition, the Lockheed Corporation provided

\(^1\)These were kindly provided by Professor Roderick Floud, Birkbeck College, London.
purchase records of milling machines from 1960 to 1975. However, there remain gaps in some time periods, and it is not known how representative the sample is of the universe of milling machines. Nevertheless, the main companies and their products are represented in the sample, and there is good coverage of several cross-sections over the entire period.

An input cost index for machine tools was developed for three input factors: labor, capital, and materials. Based on value share of total costs from Census of Manufacturers data from 1905 to the present, labor was given a weight of .45, materials .35, and capital rental costs .20. Wage rates were for skilled workers; materials prices were based on the Wholesale Price Index for "Metals and Metal Products." The capital rental rate used the interest rate on railroad bonds or the Moody's Corporate Aaa rate. A depreciation rate of 10 percent (an assumed lifetime of 10 years) was added to the interest rate. The interest rate plus depreciation was then used to multiply a cost index for producers' durable equipment.

Despite the large effort put into the collection of the information, the statistical analysis was disappointing in some respects, and yet revealing in others. Every estimated equation shows declining productivity—that is, increasing resource costs, holding performance constant. Alternative functional forms, choices of variables, treatment of time, various deflating procedures, and separate equations for companies and narrow time periods produced more or less similar results. Real costs increased in every 20-year period and throughout the 110-year span.

In tests of functional form, the logarithm of deflated cost was consistently the best dependent variable, and the logarithm of a size variable (weight, horsepower, table length) yielded the most acceptable statistics for the independent variables. For the other independent variables, log and linear transformations performed about equally well. Time was entered in linear, quadratic, and logarithmic form, as well as in separate dummy variables for each decade. The simple linear version

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2See the appendix for a complete description of the cost index.

3Factor shares varied somewhat over the period, but since there does not seem to have been any secular trend in the shares, a fixed-weight cost index was used.
yielded the most acceptable results. Cross-products of time and the independent variables were also tested, but were insignificant. Typical equations are shown in Table 1 (t-statistics in parentheses under the coefficients).

These equations were developed in an iterative method. As mentioned above, size, control, and movement are the principal functional areas that give value to milling machines. Several variables are associated with size. Table length and width are good indicators of the size of the piece that can be machined. Weight measures the bulk of the machine and is associated with rigidity—necessary for maintaining accuracy. Power is a measure of the energy applied in the cutting process—metal removal is a function of the energy applied by the cutting tool to the workpiece. All of these variables are highly correlated, and several alternative combinations were tried in the statistical analysis. The weight was always the most significant and stable of these variables, although power makes the most engineering sense in that it is most closely associated with throughput, or metal removal per unit time.

Until the introduction of electric motors with rated horsepower, explicit power figures were rarely given for milling machines. However, standard formulas had been developed and used to calculate the power delivered to the cutter and workpiece. The formulas were based on the diameter and speed of the pulleys used to drive the machines, and on the width of the leather belt that drove the pulleys. It was generally assumed that the belt could pull 50 pounds per inch of width. Since one horsepower equals 550 foot-pounds per second, calculation of horsepower is then:

\[ HP = \pi \cdot D \cdot (rpm/60) \cdot W \cdot 50/550 \]

\[ ^6 \text{The simple correlation coefficients between the logs of weight and horsepower and between weight and table length were .85 and .88.} \]

\[ ^5 \text{See, for example, The Cincinnati Milling Machine Company, } A \text{ Treatise on Milling and Milling Machines, Cincinnati, 1916, pp. 182-183.} \]
Table 1
EQUATIONS OF LOGARITHM OF DEFLATED MILLING MACHINE PRICES
(In 1967 dollars)

<table>
<thead>
<tr>
<th>Independent Variable</th>
<th>Equation (1)</th>
<th>Equation (2)</th>
</tr>
</thead>
<tbody>
<tr>
<td>( R^2 )</td>
<td>.952</td>
<td>.907</td>
</tr>
<tr>
<td>SEE</td>
<td>.199</td>
<td>.293</td>
</tr>
<tr>
<td>Number of observations</td>
<td>244</td>
<td>256</td>
</tr>
<tr>
<td>Intercept</td>
<td>33.3</td>
<td>30.6</td>
</tr>
<tr>
<td>Year</td>
<td>.0186</td>
<td>.0200</td>
</tr>
<tr>
<td></td>
<td>(22.3)</td>
<td>(17.9)</td>
</tr>
<tr>
<td>Bridgeport</td>
<td>-.908</td>
<td>-1.14</td>
</tr>
<tr>
<td></td>
<td>(10.4)</td>
<td>(9.7)</td>
</tr>
<tr>
<td>Power controls</td>
<td>.062</td>
<td>.175</td>
</tr>
<tr>
<td></td>
<td>(3.4)</td>
<td>(7.4)</td>
</tr>
<tr>
<td>Total controls</td>
<td>.024</td>
<td>-.006</td>
</tr>
<tr>
<td></td>
<td>(2.2)</td>
<td>(.4)</td>
</tr>
<tr>
<td>( \ln ) weight</td>
<td>.736</td>
<td></td>
</tr>
<tr>
<td></td>
<td>(26.7)</td>
<td></td>
</tr>
<tr>
<td>( \ln ) power</td>
<td></td>
<td>.412</td>
</tr>
<tr>
<td></td>
<td></td>
<td>(13.1)</td>
</tr>
</tbody>
</table>

NOTES:
Bridgeport = dummy variable for Bridgeport Machines;
Year = year of machine description;
Power = horsepower of machine;
Power controls = number of dimensions of workpiece and tool movement controlled by power or numerical means;
Total controls = number of dimensions of workpiece and tool movement controlled by manual, power, or numerical means;
Weight = weight of machine, pounds.
where HP is horsepower, D is pulley diameter in feet, rpm is revolutions per minute, and w is belt width in inches. This formula was used when horsepower figures were not provided in the source material.

Control was measured in several ways: by entering each possible dimension that was controlled as a dummy variable, by counting separately the number of workpiece movements and tool movements, by counting the total number of movements, and by counting the number of movements under power or numerical control. This last measure was always significant and dominated all of the other measures.

Range of motion in several directions was also examined, but the variables describing the movement of table or tool were not consistently significant, mainly because they were highly correlated with the size variables. Other variables examined included speeds of motion of the workpiece and tool.

Early in the statistical analysis, it was found that different companies tend to specialize in different sizes of machines. Bridgeport, for example, markets a range of smaller millers. Since the sample of companies changed over time, this specialization could lead to spurious results; therefore, dummy variables were included for the main companies, with the variable for Bridgeport being consistently significant.

The equations shown above formed the basis of a stepwise regression. The stepwise routine then picked from all the remaining variables those with the highest significance and continued this process until no statistically significant variables were excluded. No other variable was statistically significant! This result is consistent with those in Floud's study of the British machine-tool industry. Floud related the cost of production to the weight of a wide variety of machine tool types in a single company, for separate time cross-sections. For the years 1868 to 1894, the simple correlation coefficient between cost and weight lay between .95 and .98. In the present study, the correlation over time between the logs of deflated cost and weight is .90.

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The statistical results indicate that technological change in milling machines, measured by total factor productivity holding characteristics constant, declined over much of the period examined, by about 2 percent per year. These results are robust and resist challenge from alternative equation formulations and lists of independent variables.

The apparent decline in technology in milling machines requires explanation, especially since it is inconsistent with many expert opinions on machine-tool progress. It is also inconsistent with objective evidence. A 1949 article used a number of cases to highlight the improved productivity of new machine tools. Of course, these cases were selected to prove a point, but they are illuminating, nonetheless. One case described a 36-inch boring mill that sold for $1200 in 1890, whereas the same model cost $19,735 in 1949. Using a typical machining job for comparison, a one-day's output of the 1949 machine would have required 60 machines in 1890. Moreover, it would have taken 30 operators to run the machines in 1890 compared with one in 1949. (This information, together with other examples, is shown in Table 2.) Note that the author has conveniently held the performance of the machine constant by measuring the output of the same part per unit time. Some other points are worth considering in this example. First, in order to be productive to a user, the machine tool must be combined with other inputs (labor) to produce the output. For the user, it may be possible to decrease total costs by reducing the complementary input while the price of the machinery actually rises. Second, as suggested above, productivity can be estimated by two levels--at the level of the machine producer, and at that of the machine user. In the examples of Table 2, it is the user's definition of performance (U) and cost that is being measured.

As shown in the first example in Table 2, from 1890 to 1949 the nominal cost of machine tools to produce the same output actually fell at a compound rate by .67 percent per year (from $60,000 to $20,000). At the same time, the machine-tool cost index rose by 1.62 percent per

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### Table 2
PRODUCTIVITY CHANGE IN MACHINE TOOLS

<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Price per machine ($)</td>
<td>1200 20,000 440 8960</td>
<td>1950 4425 1525</td>
<td>4070 7920</td>
<td>15,250</td>
<td></td>
</tr>
<tr>
<td>Machines per job</td>
<td>50 1 30 1</td>
<td>6 1 3.54 1</td>
<td>1 1 0.66</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Total cost of machines ($)</td>
<td>60,000 20,000 13,200 8960</td>
<td>11,700 4425 5400</td>
<td>4070 7920</td>
<td>10,065</td>
<td></td>
</tr>
<tr>
<td>Change in total cost (%/yr)</td>
<td>-0.67 -0.64 -3.80</td>
<td>-1.12 +2.4</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Machine tool cost index (1967=1.00)</td>
<td>0.210 .573 .218 .573 .308 .550 .308 .550 .297 .550</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Change in input costs (%/yr)</td>
<td>+1.62 +1.62 +2.35 +2.35</td>
<td>+3.45 +4.0</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Change in machine productivity (%/yr)</td>
<td>+2.29 +2.26 +6.15 +7.2</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Number of operators</td>
<td>30 1 50 1</td>
<td>3 0.5 1.77 0.5</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Wage rate index (1967=1.00)</td>
<td>.0566 .509 .0566 .509 .188 .488 .188 .488</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Total labor cost (operators times wage index)</td>
<td>3.70 .509 2.83 .509 .564 .244 .333 .244</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Change in total labor cost (%/yr)</td>
<td>-2.00 -2.82 -3.3 -1.2</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Change in wage rate (%/yr)</td>
<td>+3.73 +3.73 +3.9 +3.9</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Change in labor productivity (%/yr)</td>
<td>+5.73 +6.55 +7.2 +5.1</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Capital share</td>
<td>0.65 0.20 0.35 0.30</td>
<td>0.65 0.70</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Labor share</td>
<td>0.55 0.80 0.65 0.70</td>
<td>0.65 0.70</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Change in users' total factor productivity (%/yr)</td>
<td>+4.38 +5.69 +6.83 +4.61</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
year. Resource use in the production of the machine tool therefore changed by: -2.29 percent per year (-.67 - 1.62).\footnote{As described above, we define productivity change as the negative of $\Delta R/R$ for constant performance, where $R$ is an aggregate measure of factor inputs (i.e., real cost or price) and $\Delta$ represents the derivative with respect to time. However, when only nominal cost or price is available, $R$ can be calculated by deflating the nominal quantity by a price index: $R = p/I$, where $p$ is nominal price and $I$ is an index of weighted input prices. The percentage change in total factor productivity is then: $-\Delta R/R = \Delta I/I - \Delta p/p$.}

To compute the total costs of the machine-tool user, labor costs must be added to machinery costs. Here we find that while wage rates rose by nine times, labor usage fell by a factor of 30. The total nominal cost of labor (number of operators times the wage rate) fell by 2 percent per year. Since wage rates were rising at a 3.73 percent rate, total labor productivity rose by 5.73 percent per year. Total factor productivity for the machine-tool user is a weighted sum of the labor and capital productivity changes, using as weights the value share of labor and capital in the total production costs of the particular parts.\footnote{Instead of the price of the machine tool, the cost of the machine to the user should be its rental price, computed as the purchase price times the prevailing market rate of return plus the depreciation rate. This was used in computing factor shares, but for convenience the price shown in Table 2 was used to compute total factor productivity. The differences in the computations are insignificant.} The table indicates that technological change in the selected cases was substantial over the period examined.

The primary difference in the two approaches is that the machine-tool examples measure user output and the equations measure performance characteristics. Therefore, the central assumption that characteristics are adequate proxies for output must be questioned. The milling machine is only one component of a system that also includes the milling cutters and machine operators. Major changes have taken place in cutter technology as industry developed high-speed tool steels, carbides, diamonds, and other cutting materials; in addition, industrial laboratories gained a better understanding of the physics and geometry of metal removal. These produced changes in the milling tool that called forth complementary changes in the machine--greater power,
increased rigidity and strength, higher operating speeds, and greater automaticity. For example, when a machining job on a low-power, slow machine took all day, an additional ten minutes to place the workpiece on the machine and remove it was not of great concern. But as labor became relatively more expensive and as the total machining time for the same job was reduced to 15 minutes, reducing set-up time and other lost time due to operator interventions became more important. The machines were therefore given greater automatic control and more power operations, and were made more convenient to operate. Because of all these effects, machines became more capable—and more expensive. But the total machine-cutter-operator system was more productive. If all these changes were reflected in machine characteristics in our database, we would be able to account for the productivity change. However, since productivity gains estimated by our equations are negative, we are led to consider the possibility that not all the relevant variables have been available.

Another possibility is that, despite the selected examples given in Table 2, milling machine producer technology did indeed decline. Gains to the user would then come about from greater efficiency in other inputs—cutter, labor, materials—and in the user's organization of production. Although gains from all these sources probably did occur, it seems unlikely that the efficiency of producing milling machines actually declined, especially since machine-tool producers would themselves have been beneficiaries of the other sources of improved productivity (i.e., more productive labor, capital, materials, and organization).\footnote{An empirical test of declining productivity would be that used machines sell at a premium; that is, after accounting for physical deterioration, used machines would be priced higher than new ones. Casual examination of used machine-tool prices in the 1960s and 1970s does not indicate that such premiums exist.}

An additional explanation is that the milling machine cost index may drastically understate the increase in factor prices. Mismeasurement more likely runs in the other direction, though, since increased productivity of factors over time probably occurred, whereas the implicit assumption in constructing the cost index was of no change.
AIRCRAFT

The aircraft sample comprised all commercial turbine-powered transports from the Boeing 707 and Hawker-Siddeley Comet in 1958 to the latest Boeing models 757 and 767 and the Airbus Industry A310--24 aircraft altogether. Variables were chosen to emphasize airframe performance as well as total system (airframe and engine) characteristics. Airframe characteristics include structural efficiency (e.g., the ratio of payload to airframe weight), aerodynamic efficiency (lift-to-drag ratio), operating costs less depreciation and fuel, landing distance, and size characteristics (payload, structural weight, number of seats, cabin volume). The price of the airframe was calculated as the airplane price minus the price of the engines. This price was deflated by an airframe cost index developed by the United States Air Force.\textsuperscript{11} Operating costs less fuel and depreciation were deflated by the same index. This variable captures mainly the cost of airframe maintenance and crew costs. Structural efficiency measures were intended to capture improvements in materials, structures, and production technology. Landing distance (as opposed to takeoff distance, which depends mainly on available thrust) depends on structural features such as high-lift devices (which add weight and cost), brakes, and thrust-reversers. This variable is important because shorter landing distance allows service to be extended to more airports.

Several earlier studies on cost-estimating relationships for airframes and one study on structural efficiency gave us a starting point for our statistical analyses. The authors of one recent cost study on military aircraft attempted to overcome shortcomings of earlier work that related cost to only two variables, weight and speed. They concluded, however: "Our search for other explanatory variables that would improve the accuracy of estimates was less fruitful than we had hoped. The variations in cost that are not explained by weight and speed are not explained by any other objective indexes that we could

find.12 An unpublished Rand study in the early 1970s that attempted to assess improvements in airframe state-of-the-art (excluding costs) concluded that the data "do not indicate any clearcut improvement in airframe state-of-the-art with time. The results would indicate that whatever improvement has occurred is small enough to be lost in the noise level of the corrected data."13

Despite these findings, we believed that an improved theoretical understanding would provide the key to a more complete view of the producers' airframe tradeoff surface. However, after including a size variable, none of the other variables--not even speed--was significant in the airframe cost equations. (The cost-estimating studies referred to above included military fighter aircraft with a much wider variability in speed than the present sample.) The following is a typical equation:

\[ \ln \text{Airframe cost} = -9.13 + 0.91 \ln \text{airframe weight} \]

\[ (15.4) \]

\[ + 0.0039 \text{ time}; R^2 = 0.93, \text{ SEE} = 0.19, N = 24 \]

\[ (3.1) \]

where Airframe cost = cost of aircraft minus engines, deflated by airframe cost index (1967 = 1.00);

Airframe weight = weight of airframe (aircraft empty weight minus engine weight), in pounds;

Time = aircraft certification date; quarter years since 1942, third quarter.

The coefficient on the time variable indicates a real cost increase of about 1.6 percent per year. Other equations were in the same range.

Statistical analysis of the total aircraft system produced almost identical results to the airframe case. As shown in Table 3, after accounting for a size variable, no other variable was consistently significant. In the several equations that were estimated, the time

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13 T. F. Kirkwood, personal communication.
Table 3

EQUATIONS OF LOGARITHM OF DEFLATED AIRCRAFT PRICE
(In 1967 dollars; 24 observations)

<table>
<thead>
<tr>
<th>Independent Variable</th>
<th>Equation 1</th>
<th>Equation 2</th>
<th>Equation 3</th>
<th>Equation 4</th>
</tr>
</thead>
<tbody>
<tr>
<td>( R^2 )</td>
<td>.889</td>
<td>.953</td>
<td>.940</td>
<td>.955</td>
</tr>
<tr>
<td>SEE</td>
<td>.220</td>
<td>.147</td>
<td>.166</td>
<td>.147</td>
</tr>
<tr>
<td>Intercept</td>
<td>-8.73</td>
<td>-8.79</td>
<td>-8.63</td>
<td>-8.74</td>
</tr>
<tr>
<td>&amp;n empty weight</td>
<td>.916</td>
<td>.875</td>
<td>.959</td>
<td>.899</td>
</tr>
<tr>
<td></td>
<td>(11.0)</td>
<td>(18.8)</td>
<td>(14.4)</td>
<td>(17.3)</td>
</tr>
<tr>
<td>Ratio: total useful</td>
<td></td>
<td></td>
<td>-.539</td>
<td>-.183</td>
</tr>
<tr>
<td>weight to airframe</td>
<td></td>
<td></td>
<td>(4.2)</td>
<td>(1.0)</td>
</tr>
<tr>
<td>weight</td>
<td>Date</td>
<td></td>
<td>.0050</td>
<td>.0037</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>(5.3)</td>
<td>(2.6)</td>
</tr>
</tbody>
</table>

NOTES:
Date = aircraft certification date; quarter-years since 1942, third quarter.
Empty weight = aircraft weight without fuel and payload, in pounds.
Total useful weight = engine weight plus useful weight (maximum takeoff weight minus empty weight).
Airframe weight = empty weight minus weight of all engines.

coefficients showed deflated price increasing from 1 to 2 percent per year.

With the information in the aircraft data base, it was also possible to estimate a performance function for the user—the airline industry. Several user-valued variables were examined: cost per passenger-seat-mile, cost per payload-ton-mile, and cost per ton-mile of useful weight.\(^{14}\)

\(^{14}\)Useful weight is defined as maximum takeoff weight minus empty weight. This weight can be divided between payload and fuel (payload and range), depending on the requirements of the user.
In order to estimate these user cost figures, nominal user cost was first calculated as the sum of operating costs (minus fuel and depreciation), fuel, and capital. The logarithm of nominal user costs was deflated by an input factor cost index.\textsuperscript{15} A current-weighting scheme was adopted because of the very sharp swings in relative prices of fuel and other costs over the sample period. (The cost-weight of fuel, for example, increased from 20 percent to 50 percent.)

Operating and fuel costs per hour were obtained from the Civil Aeronautics Board's Form 41, submitted quarterly by U.S. airlines for each aircraft type.\textsuperscript{16} The second, third, and fourth years of operating experience were averaged to obtain the figures included in the data base.\textsuperscript{17} This procedure was meant to bypass the introductory phases of a new aircraft's operations, when costs have not yet settled into a stable pattern. Capital cost per hour was based on aircraft price, using a Boeing accounting convention that assumes a 15-year life, 10-percent salvage value, and 8-hour operation per day, 365 days per year. Capital cost was deflated by the Air Force aircraft cost index;\textsuperscript{18} fuel cost was deflated by the Producer Price Index for jet fuel (kerosene, for earlier years). In order to transform these hourly costs to a per-mile basis, they were divided by economical cruise speed in miles per hour.

\textsuperscript{15}We deflated nominal costs (C) by an input cost index I to calculate \(\ln R = \ln C - \ln I\). Nominal costs are equal to the sum of the nominal cost elements: operating costs, fuel costs, and capital costs \((C = \Sigma C_i)\); the input cost index is: \(I = \sum W_i \ln p_i\), where \(p_i\) is the price index of each cost element and \(W_i\) is a weight equal to the value share of each cost element in total nominal cost \((W_i = C_i / \Sigma C_i)\). The dependent variable in the airline equations was thus defined as: \(\ln R = \ln \Sigma C_i - \sum W_i \ln p_i\). This treatment is appropriate for an underlying translog production function, whose factor shares may change over time. See Douglas W. Caves, Laurits R. Christensen, and W. Erwin Diewert, "The Economic Theory of Index Numbers and the Measurement of Input, Output, and Productivity," \textit{Econometrica}, Vol. 50, No. 6, November 1982.

\textsuperscript{16}Early British aircraft operating costs were obtained from British European Airways aircraft type costs from the years 1962 to 1969.

\textsuperscript{17}In a few cases, however, only the first year's figures were available.

\textsuperscript{18}Fatkin, \textit{op. cit.}. 
Deflated user costs per mile were then used as dependent variables, with independent variables of time and a measure of size (to account for economies of product scale). These equations are shown in Table 4, with time entered as a quadratic. A plot of seat-mile costs (in 1967 dollars) versus year of aircraft certification is shown in Fig. 7.

The costs plotted in Fig. 7 show that deflated seat-mile costs fell rapidly from the first jet-powered commercial passenger aircraft in the late 1950s to the introduction of the wide-body fleets in the early 1970s. Seat-mile costs declined from about 1.3 cents (1967 dollars) to 0.6 cents—an annual productivity growth rate of about 7.5 percent.

Table 4

<table>
<thead>
<tr>
<th>Independent Variable</th>
<th>Dependent Variable</th>
<th>A</th>
<th>B</th>
<th>C</th>
</tr>
</thead>
<tbody>
<tr>
<td>R²</td>
<td>.891</td>
<td>.858</td>
<td>.875</td>
<td></td>
</tr>
<tr>
<td>SEE</td>
<td>.160</td>
<td>.218</td>
<td>.182</td>
<td></td>
</tr>
<tr>
<td>Intercept</td>
<td>4.22</td>
<td>4.76</td>
<td>-3.14</td>
<td></td>
</tr>
<tr>
<td>Date</td>
<td>.017</td>
<td>-.039</td>
<td>-.033</td>
<td></td>
</tr>
<tr>
<td></td>
<td>(1.9)</td>
<td>(3.1)</td>
<td>(3.2)</td>
<td></td>
</tr>
<tr>
<td>Date squared</td>
<td>-.000068</td>
<td>.000135</td>
<td>.00010</td>
<td></td>
</tr>
<tr>
<td></td>
<td>(1.7)</td>
<td>(2.5)</td>
<td>(2.2)</td>
<td></td>
</tr>
<tr>
<td>ln empty weight</td>
<td>-.752</td>
<td>-.433</td>
<td>-.181</td>
<td></td>
</tr>
<tr>
<td></td>
<td>(10.1)</td>
<td>(4.2)</td>
<td>(2.1)</td>
<td></td>
</tr>
</tbody>
</table>

NOTES:
A = log deflated cost per ton-mile useful weight.
B = log deflated cost per ton-mile payload.
C = log deflated cost per seat-mile.
Date = aircraft certification date; quarter-years since 1942, third quarter.
Empty weight = aircraft weight without fuel and payload, pounds.
Fig. 7 -- Seat-mile costs over time, jet-powered turbine aircraft, 1958-1983 (In 1967 dollars)

Part of this growth, however, can be attributed to the increased size of the aircraft, as the weight term in the equations of Table 4 makes clear. The quadratic date expression in the seat-mile equation (column C, Table 4), for example, shows productivity increasing (real costs declining) by 5 percent in 1967 (the 100th quarter), but leveling out to zero growth by the mid-1980s. One reason for this apparent halt to
productivity gains could be the higher fuel costs that have motivated aircraft buyers in the 1980s to spend more for a fuel-efficient product. 19

Comparison of the time trends of user-oriented (airline) outputs (Table 4) with the movement of deflated prices in the performance characteristics equations (Table 3) shows increasing productivity in the former and declines in the latter. 20 These results are similar to those found in machine tools. Productivity estimates based on product characteristics may not adequately reflect the total value of the product to users, partly because of the non-inclusion of all the relevant characteristics. For example, seat-mile costs are a function of the number of seats, operating costs, fuel costs, and speed, as well as of aircraft price. To test this, we estimated equations with price as the dependent variable, and these other variables as independent variables, with the notion of capturing the relevant user-oriented variables. These equations are shown in Table 5. The coefficients on the time variables continue to show positive signs. Moreover, only in Eqs. 3 and 4 do operating cost and fuel cost have the expected negative sign. (Buyers should be willing to pay more for more efficient aircraft, which are also more costly to produce.) The coefficients of these variables, though, in general are unstable and insignificant. One could conclude, therefore, that simple linear approximations to user value functions do not necessarily guarantee accurate measures of productivity—even if most of the relevant variables are included.

19A cost index with fixed 1980s weights would emphasize the preferences of this period and would tend to show productivity continuing to increase. On the other hand, a base-weighted index with a 1950s weighting would exaggerate the apparent productivity slowdown. The current-weight scheme used here falls between these two alternatives.

20Costs per ton-mile of useful weight show a weakly increasing trend, peaking in 1974 (column A, Table 4).
Table 5  
EQUATIONS OF LOGARITHM OF DEFATED AIRCRAFT PRICE  
AND USER VARIABLES  
(In 1967 dollars; 19 observations) 

<table>
<thead>
<tr>
<th>Independent Variable</th>
<th>Equation</th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>1</td>
<td>2</td>
<td>3</td>
</tr>
<tr>
<td>$R^2$</td>
<td>.921</td>
<td>.933</td>
<td>.949</td>
<td>.944</td>
</tr>
<tr>
<td>SEE</td>
<td>.204</td>
<td>.195</td>
<td>.177</td>
<td>.171</td>
</tr>
<tr>
<td>Intercept</td>
<td>-4.98</td>
<td>-12.2</td>
<td>-4.22</td>
<td>-8.77</td>
</tr>
<tr>
<td>ln fuel cost</td>
<td>.367</td>
<td>.321</td>
<td>-.150</td>
<td>-.198</td>
</tr>
<tr>
<td></td>
<td>(1.1)</td>
<td>(1.0)</td>
<td>(.4)</td>
<td>(.6)</td>
</tr>
<tr>
<td>ln operating cost</td>
<td>-.157</td>
<td>.266</td>
<td>-.16</td>
<td>-1.07</td>
</tr>
<tr>
<td></td>
<td>(.57)</td>
<td>(.7)</td>
<td>(.4)</td>
<td>(.34)</td>
</tr>
<tr>
<td>ln speed</td>
<td>.209</td>
<td>1.12</td>
<td>-.57</td>
<td></td>
</tr>
<tr>
<td></td>
<td>(.16)</td>
<td>(.8)</td>
<td>(.4)</td>
<td></td>
</tr>
<tr>
<td>ln seats</td>
<td>.937</td>
<td>.65</td>
<td>.267</td>
<td></td>
</tr>
<tr>
<td></td>
<td>(6.2)</td>
<td>(2.8)</td>
<td>(.9)</td>
<td></td>
</tr>
<tr>
<td>ln empty weight</td>
<td></td>
<td>.859</td>
<td>1.03</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>(1.9)</td>
<td>(4.4)</td>
<td></td>
</tr>
<tr>
<td>Date</td>
<td>.0060</td>
<td>.0017</td>
<td>.0034</td>
<td></td>
</tr>
<tr>
<td></td>
<td>(1.5)</td>
<td>(.4)</td>
<td>(1.1)</td>
<td></td>
</tr>
</tbody>
</table>

NOTES:
Fuel cost = fuel cost per hour of operation, 1967 dollars.
Operating cost = operating cost per hour of operation, not including fuel and depreciation, 1967 dollars.
Speed = economical cruise speed, miles per hour.
Seats = number of seats in standard configuration.
Empty weight = aircraft weight without fuel and payload, pounds.
Date = aircraft certification date; quarter-years since 1942, third quarter.
AIRCRAFT TURBINE ENGINES

Data were collected on 44 U.S. military and civil aircraft turbine engines. Price information was missing on eight of these, mainly early military engines. Price data that were used were the marginal prices of the 1000th engine, when such figures were available. Otherwise it was the price within a year or so of its qualification. The engine model for which data were collected was the first of the type that passed the military or civil qualification tests. The date of qualification was used as the time variable. The principal performance characteristics that were examined were thrust, weight, speed, and fuel consumption; the technical characteristics included turbine inlet temperature, pressure ratio, and air flow.

Statistical results of our sample broadly duplicated results reported in earlier studies. However, we were looking specifically for the effect of time on deflated costs, holding performance constant. A previous study found deflated engine costs to be rising over time. In the present analysis, we find the time variable to be insignificantly different from zero, or slightly positive. Table 6 displays the results of several alternative combinations of variables. As shown in Table 6, deflated engine prices increased at a rate of about 0.35 to 0.53 percent per quarter-year, holding performance constant. Thrust, speed, and fuel consumption exhibited stable coefficient estimates and were statistically significant (specific fuel consumption had the smallest t-statistic). Neither thrust-to-weight ratio nor weight was ever significant when thrust was in the equation, as shown in Eq. 2 for weight. In past studies on technology measures, turbine inlet temperature was often a very important variable. As a variable not directly valued by users, its inclusion was explained as substituting for excluded user variables. However, when price is included in the equations, as in Table 6, turbine inlet temperature ceases to be significant (see Eq. 3).


\(^{22}\)Alexander and Nelson, *op. cit.*
Table 6
EQUATIONS OF LOGARITHM OF DEFLATED ENGINE PRICES
(In 1967 dollars; 36 observations)

<table>
<thead>
<tr>
<th>Independent Variable</th>
<th>Equation 1</th>
<th>Equation 2</th>
<th>Equation 3</th>
</tr>
</thead>
<tbody>
<tr>
<td>$R^2$</td>
<td>.954</td>
<td>.954</td>
<td>.956</td>
</tr>
<tr>
<td>SEE</td>
<td>.216</td>
<td>.220</td>
<td>.216</td>
</tr>
<tr>
<td>Date</td>
<td>.0053</td>
<td>.0051</td>
<td>.0035</td>
</tr>
<tr>
<td></td>
<td>(3.0)</td>
<td>(2.2)</td>
<td>(1.4)</td>
</tr>
<tr>
<td>$\ln$ thrust</td>
<td>.721</td>
<td>.748</td>
<td>.687</td>
</tr>
<tr>
<td></td>
<td>(10.7)</td>
<td>(10.5)</td>
<td>(9.1)</td>
</tr>
<tr>
<td>Mach number</td>
<td>.437</td>
<td>.434</td>
<td>.428</td>
</tr>
<tr>
<td></td>
<td>(4.0)</td>
<td>(3.9)</td>
<td>(3.9)</td>
</tr>
<tr>
<td>$\ln$ specific fuel consumption</td>
<td>-.304</td>
<td>-.306</td>
<td>-.326</td>
</tr>
<tr>
<td></td>
<td>(1.4)</td>
<td>(1.4)</td>
<td>(1.5)</td>
</tr>
<tr>
<td>$\ln$ weight</td>
<td>- .026</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>(.2)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\ln$ turbine inlet temperature</td>
<td></td>
<td></td>
<td>.602</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>(1.0)</td>
</tr>
</tbody>
</table>

NOTES:
Date = date of engine certification; quarter-years since 1942, third quarter.
Thrust = maximum takeoff thrust without after-burner, pounds.
Mach number = maximum speed specified for engine.
Specific fuel consumption = pounds of fuel consumed per pound of thrust per hour at maximum takeoff thrust.
Weight = engine weight, pounds.
Turbine inlet temperature = maximum design temperature of gas entering the turbine, degrees Rankine.
In a manner similar to the treatment of user-oriented aircraft cost, a user jet engine cost was also calculated. This was defined as the cost of fuel and capital per pound of thrust per hour of operation. The basic idea here is that users buy engines for one principal purpose: their propulsion capabilities, i.e., thrust. We then want to know the resources required to achieve this output. Costs were calculated at takeoff performance values. Although figures for cruise performance would be more appropriate to the analysis, such figures are highly variable—depending on specific speeds and altitudes. For this reason, takeoff values are widely used and compared in the aviation industry. Moreover, they are indicative of the broader range of engine performance.

Fuel costs were obtained from the specific fuel consumption figures multiplied by the price per pound of jet fuel in 1967 (10.8 cents per gallon, 6.8 pounds per gallon) and by the producer price index for jet fuel. Capital cost calculations are more conjectural. Turbine engines typically undergo overhauls that restore them to an "as new" condition. The engine's lifetime could therefore be extended indefinitely. However, an earlier study found that average overhaul costs were about 20 percent of original engine price. Therefore, the engine's market value the day before overhaul would be 80 percent of its original price. We then assumed that, without overhaul, the value would decline at a straight-line rate; the lifetime of the engine would thus be five times the average time before overhaul (ATBO). Based on reported data, we divided the engines into three classes: For military supersonic engines, the ATBO was estimated at 1000 operating hours, and total life therefore was 5000 hours; for military subsonic models, ATBO was 2000 hours, with a 10,000-hour life; and civil subsonic engines were estimated at 3000 hours ATBO, and a 15,000-hour life. The price of the engine was then divided by the estimated life to obtain capital cost per operating hour. The sum of capital cost and fuel cost per hour was

\[ \text{Cost per hour} = \frac{\text{Capital Cost}}{\text{ATBO}} + \frac{\text{Fuel Cost}}{\text{ATBO}} \]

\[ \text{Cost per hour} = \frac{\text{Capital Cost}}{\text{Lifetime}} + \frac{\text{Fuel Cost}}{\text{Lifetime}} \]

\[ \text{Cost per hour} = \frac{\text{Capital Cost}}{\text{Estimated Life}} + \frac{\text{Fuel Cost}}{\text{Estimated Life}} \]

---

\[ ^{23} \text{J. R. Nelson, Life Cycle Analysis of Aircraft Turbine Engines,} \]

\[ ^{24} \text{Ibid., pp. 34-39, 60-63.} \]

\[ ^{25} \text{This procedure is equivalent to assuming an infinite engine life} \]

\[ ^{26} \text{and calculating overhaul costs on a per-hour basis by dividing by ATBO.} \]
divided by thrust to calculate user cost. These estimated costs were
deflated by a current-weighted input price index calculated in a fashion
identical to the aircraft case.

Deflated user costs were regressed against time to determine the
rate of productivity change; thrust was also included to account for
product scale effects. Several other variables were also examined.
Speed is a desirable characteristic that necessitates added strength and
weight—and cost. Weight itself is an undesirable characteristic; the
trend over time has been for the ratio of thrust to weight to increase.
Speed and weight were therefore added to the equations. It was also
believed that military engines may be more advanced or possess different
levels of unmeasured characteristics than civil designs. A military
dummy variable was therefore tested, but was never significant. These
results are shown in Table 7.

Neither weight nor thrust-to-weight ratio was ever significant; but
speed was always highly significant, stable, and important. Most
important, deflated user costs declined over time at a rate of about
half a percent per quarter, or 2 percent per year. Once again we
observe rising user-oriented productivity measures, whereas price-
performance productivity equations show declining trends.

CONVERSION OF HEDONIC PRICES AND COST-ESTIMATING RELATIONSHIPS
to MEASURES OF TECHNOLOGICAL CHANGE

The definition of technological change as the change in total
factor productivity, holding performance constant, can be applied to
different types of data. In this section, we demonstrate the
application of the definition to hedonic price indexes and cost-
estimating relationships. Hedonic price equations produce price indexes
by equating nominal price to a vector of product characteristics and a
time function. The price indexes measure price changes, holding
performance constant. It is a simple matter to deflate the price index
for each year by a factor input cost index to generate measures of
technological change.

Several studies, for example, have examined hedonic prices for
automobiles. Table 8 transforms hedonic prices for automobiles
Table 7

LOGARITHM OF DEFATED JET ENGINE USER COSTS
(In 1967 dollars; 36 observations)

<table>
<thead>
<tr>
<th>Independent Variable</th>
<th>Equation 1</th>
<th>2</th>
</tr>
</thead>
<tbody>
<tr>
<td>R^2</td>
<td>.880</td>
<td>.881</td>
</tr>
<tr>
<td>SEE</td>
<td>.135</td>
<td>.137</td>
</tr>
<tr>
<td>Intercept</td>
<td>-2.52</td>
<td>-2.55</td>
</tr>
<tr>
<td>Date</td>
<td>-.0046</td>
<td>-.0049</td>
</tr>
<tr>
<td>(6.5)</td>
<td>(4.2)</td>
<td></td>
</tr>
<tr>
<td>ln thrust</td>
<td>-.255</td>
<td>-.191</td>
</tr>
<tr>
<td>(7.7)</td>
<td>(1.8)</td>
<td></td>
</tr>
<tr>
<td>Mach number</td>
<td>.500</td>
<td>.496</td>
</tr>
<tr>
<td>(11.3)</td>
<td>(10.7)</td>
<td></td>
</tr>
<tr>
<td>ln weight</td>
<td>- .031</td>
<td>( .3)</td>
</tr>
</tbody>
</table>

NOTE: Variables are defined in text and in Table 6.

estimated by Ohta and Griliches into technological change indexes by deflating the nominal prices by a factor input cost index. The data show a growth of technological change in automobiles of about 2.5 percent per year from 1955 to 1971.

Two separate studies on computers illustrate variations in measures obtained from using performance (P) and use (U) characteristics. A 1967 study by Chow estimated hedonic price indexes for computers based on performance characteristics. Again, we use the same approach to

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26This cost index was compiled for machine tools for the present study; the cost structures of automobiles and machine tools are sufficiently alike to permit its use here. See appendix for details on the construction of the index.

Table 8

TECHNOLOGICAL CHANGE IN AUTOMOBILES, 1955-1971

<table>
<thead>
<tr>
<th>Year</th>
<th>Hedonic Price Index (1967=100) (p)</th>
<th>Input Cost Index (1967=100) (I)</th>
<th>Technological Change Index I/p</th>
</tr>
</thead>
<tbody>
<tr>
<td>1955</td>
<td>98.1</td>
<td>71.5</td>
<td>.729</td>
</tr>
<tr>
<td>1956</td>
<td>100.2</td>
<td>76.7</td>
<td>.765</td>
</tr>
<tr>
<td>1957</td>
<td>100.5</td>
<td>80.4</td>
<td>.800</td>
</tr>
<tr>
<td>1958</td>
<td>98.7</td>
<td>81.5</td>
<td>.826</td>
</tr>
<tr>
<td>1959</td>
<td>97.5</td>
<td>84.1</td>
<td>.862</td>
</tr>
<tr>
<td>1960</td>
<td>98.3</td>
<td>85.7</td>
<td>.872</td>
</tr>
<tr>
<td>1961</td>
<td>99.9</td>
<td>86.3</td>
<td>.864</td>
</tr>
<tr>
<td>1962</td>
<td>102.6</td>
<td>87.2</td>
<td>.850</td>
</tr>
<tr>
<td>1963</td>
<td>101.3</td>
<td>88.3</td>
<td>.871</td>
</tr>
<tr>
<td>1964</td>
<td>100.0</td>
<td>90.6</td>
<td>.906</td>
</tr>
<tr>
<td>1965</td>
<td>97.4</td>
<td>93.0</td>
<td>.954</td>
</tr>
<tr>
<td>1966</td>
<td>98.6</td>
<td>96.7</td>
<td>.981</td>
</tr>
<tr>
<td>1967</td>
<td>100.0</td>
<td>100.0</td>
<td>1.000</td>
</tr>
<tr>
<td>1968</td>
<td>103.1</td>
<td>105.2</td>
<td>1.021</td>
</tr>
<tr>
<td>1969</td>
<td>104.1</td>
<td>111.8</td>
<td>1.074</td>
</tr>
<tr>
<td>1970</td>
<td>106.6</td>
<td>120.0</td>
<td>1.126</td>
</tr>
<tr>
<td>1971</td>
<td>113.3</td>
<td>123.9</td>
<td>1.093</td>
</tr>
</tbody>
</table>


compute technological change as in the automobile example.\textsuperscript{28} In this case, the annual rate of increase of technological change was 21 percent for the years 1954 to 1965, as shown in Table 9.

In a 1963 dissertation,\textsuperscript{29} Knight measured the frequency of various computer operations encountered in a mix of over 100 typical "scientific" type computer problems for several of the most common scientific computers then in use. He then estimated the contribution of

\textsuperscript{28} An input cost index for military avionics was used to deflate prices. Fatkin, \textit{op. cit.}

\textsuperscript{29} Kenneth E. Knight, \textit{A Study of Technological Innovation—The Evolution of Digital Computers}, Ph.D. dissertation, Graduate School of Industrial Administration, Carnegie Institute of Technology, 1963.
<table>
<thead>
<tr>
<th>Year</th>
<th>Knight Price Index (1967=1.00)</th>
<th>Chow Price Index</th>
<th>Input Cost Index (1967=1.00)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1950</td>
<td>100.0</td>
<td></td>
<td>.576</td>
</tr>
<tr>
<td>1951</td>
<td>100.0</td>
<td></td>
<td>.640</td>
</tr>
<tr>
<td>1952</td>
<td>60.0</td>
<td></td>
<td>.674</td>
</tr>
<tr>
<td>1953</td>
<td>60.0</td>
<td>23.6</td>
<td>.687</td>
</tr>
<tr>
<td>1954</td>
<td>43.3</td>
<td>21.5</td>
<td>.716</td>
</tr>
<tr>
<td>1955</td>
<td>43.3</td>
<td>18.3</td>
<td>.725</td>
</tr>
<tr>
<td>1956</td>
<td>30.0</td>
<td>16.8</td>
<td>.759</td>
</tr>
<tr>
<td>1957</td>
<td>14.7</td>
<td></td>
<td>.813</td>
</tr>
<tr>
<td>1958</td>
<td>23.9</td>
<td>11.7</td>
<td>.847</td>
</tr>
<tr>
<td>1959</td>
<td>18.0</td>
<td>7.76</td>
<td>.870</td>
</tr>
<tr>
<td>1960</td>
<td>11.8</td>
<td>6.53</td>
<td>.894</td>
</tr>
<tr>
<td>1961</td>
<td>8.20</td>
<td>4.98</td>
<td>.900</td>
</tr>
<tr>
<td>1963</td>
<td>4.92</td>
<td>3.04</td>
<td>.917</td>
</tr>
<tr>
<td>1964</td>
<td>2.48</td>
<td>2.48</td>
<td>.930</td>
</tr>
<tr>
<td>1965</td>
<td>1.17</td>
<td></td>
<td>.942</td>
</tr>
<tr>
<td>1966</td>
<td>1.00</td>
<td>1.00</td>
<td>.966</td>
</tr>
<tr>
<td>1967</td>
<td>.92</td>
<td>1.027</td>
<td></td>
</tr>
<tr>
<td>1968</td>
<td>.406</td>
<td>1.067</td>
<td></td>
</tr>
<tr>
<td>1969</td>
<td>.369</td>
<td>1.117</td>
<td></td>
</tr>
<tr>
<td>1970</td>
<td>.311</td>
<td>1.173</td>
<td></td>
</tr>
<tr>
<td>1971</td>
<td>.260</td>
<td>1.225</td>
<td></td>
</tr>
<tr>
<td>1972</td>
<td>.234</td>
<td>1.279</td>
<td></td>
</tr>
<tr>
<td>1973</td>
<td>.190</td>
<td>1.362</td>
<td></td>
</tr>
<tr>
<td>1974</td>
<td>.173</td>
<td>1.500</td>
<td></td>
</tr>
<tr>
<td>1975</td>
<td>.157</td>
<td>1.545</td>
<td></td>
</tr>
<tr>
<td>1976</td>
<td>.140</td>
<td>1.622</td>
<td></td>
</tr>
<tr>
<td>1977</td>
<td>.120</td>
<td>1.694</td>
<td></td>
</tr>
</tbody>
</table>


**NOTES:** Price indexes are nominal ("then year") amounts. Chow index linked to Knight index for 1965.
each operation to the time required to work through the problem. From these data, he was able to formulate an algorithm--defined as a measure of computer "power"--that related about 20 performance characteristics to the time taken for a typical scientific task. He also made the same analysis of nine typical "commercial" computer tasks such as inventory control, accounting, and payroll. Knight's measure of computer power corresponds to our definition of user tasks (U). Knight applied these scientific and commercial computer-power algorithms to 225 computers introduced from 1944 to 1963. For these same computers, he collected the rental cost and estimated an equation of the form:

$$\ln(1/C) = a + b \ln U + \sum c_i t_i,$$

where $C$ is the nominal cost per second of operation, $U$ is the measure of computer power based on the performance algorithm, and $t_i$ are dummy variables for each year of his sample. In subsequent publications, Knight updated the analysis through 1979.\footnote{Kenneth E. Knight, "Changes in Computer Performance," Datamation, September 1966; "Evolving Computer Performance, 1963-1967", Datamation, January 1968; "A Functional and Structural Measurement of Technology," paper prepared for Workshop on Technology Measurement, Dayton, Ohio, October 1983.} Hedonic price indexes, based on the user task algorithm, were calculated from the coefficients on the time variables; these are presented in Table 9. Productivity change from 1954 to 1965--the same years as the Chow sample--was about 27 percent per year. (Productivity estimates are shown in Table 10.) For virtually the same sample of computers over the same years as in the Chow study, technological change, holding performance constant, was about seven percentage points, or 30 percent greater based on U than on P. Although one could argue that these two estimates are "in the same ball park," we suggest that this evidence is consistent with the other empirical comparisons between performance characteristics and user tasks shown above.

Cost-estimating relationships that include time variables can be treated in the same way as hedonic price equations to yield measures of technological change. One example will be chosen to illustrate this.
Table 10
TECHNOLOGICAL CHANGE FOR COMPUTERS, 1950-1979

<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Avg price change, %/yr</td>
<td>-18.5</td>
<td>-24.2</td>
<td>-19</td>
<td>-30</td>
<td>-11</td>
<td>-21.4</td>
</tr>
<tr>
<td>Avg input cost change, %/yr</td>
<td>+ 2.5</td>
<td>+ 2.5</td>
<td>+ 3.5</td>
<td>+ 2.7</td>
<td>+ 5.5</td>
<td>+ 3.8</td>
</tr>
<tr>
<td>Avg technological change, %/yr</td>
<td>+21.0</td>
<td>+26.7</td>
<td>+22.5</td>
<td>+32.7</td>
<td>+16.5</td>
<td>+25.2</td>
</tr>
</tbody>
</table>

point. A study of acquisition costs of aircraft turbine engines estimated the average cost of 1000 engines by the following equation (renaming the coefficients to be consistent with definitions in this report):\textsuperscript{11}

\[
\ln C = -8.34 + .705 \ln \text{Thrust} + .457 \ln \text{Mach} + .0067 t' + .018 \xi t'; R^2 = .95,
\]

where Thrust = maximum thrust, and Mach = maximum flight envelope Mach number. The variable represented here as time (t') was actually the estimate obtained from an equation with time (measured in quarter-years) as a dependent variable: \( t' = f(P) \). The variable included in the cost equation was obtained by substituting the value of the performance characteristics of each engine into the equation for t'. On the average, t' equals t so that it can be interpreted as "time" in the cost equation. The variable \( \xi t' \) is the residual from the t' equation, with an average value of zero. The authors deflated cost by an input cost index based on the cost of labor.\textsuperscript{12} Interpreting t' as time yields an

\textsuperscript{11}Nelson and Timson, op. cit., p. 39.

\textsuperscript{12}This cost index overstates the amount of input cost increases because materials, which amount to about one-third of engine costs, rose at a somewhat slower rate than labor.
estimated *increase* in deflated costs, holding performance constant. Factor productivity thus fell by at least 2.7 percent per year. A second equation of the same form estimating cumulative production costs of 1000 engines generated similar results—i.e., deflated costs rose at an annual rate of 1.8 percent.\(^{33}\)

\(^{33}\)Nelson and Timson, *op. cit.*, p. 47.
IV. CONCLUSIONS

Two principal conclusions emerge from this study. (1) We have derived measures of technological change that make theoretical and intuitive sense; that can be applied to past studies and data; that can be used to interpret and better understand several commonly used techniques; and that can be applied to new data. (2) The use of product characteristics as a shortcut to measures of value and performance is an uncertain undertaking and cannot be counted on to produce adequate measures of technological change. This latter point requires additional comment.

Heterogeneous products are describable by a large number of attributes whose relative importance changes over time. Additionally, new characteristics are added, while others are eliminated. For purposes of convenience, a relatively small subset of attributes are often sufficient to place a product within a class, the members of which are subject to much fine-grained, detailed analysis by potential users. Statistical analyses of products that are based on a few selected characteristics, therefore, by necessity can only say things about movements of the central core of characteristics. If a significant amount of technological change takes place in the fine detail, the standard analysis will miss it. However, since user outputs are a function of all the product's characteristics (as well as of other inputs), productivity measures based on user outputs are more likely to capture the totality of technological change than are measures based on product characteristics alone.

Statistical biases produced by unmeasured characteristics whose importance grows over time affect not only measures of technological change, but also hedonic price estimates and cost-estimating relationships. In general, when the values of the missing variables are positively correlated with time, the coefficient of the time variable will be biased upward. Statistical tests are not able to discern whether this bias exists or what its size is without recourse to independent information. For cost-estimating relationships, this
problem may not be severe because the attention is not on the time coefficient, but on the predicted cost. So long as the structure of correlations persists, the predicted values should be unaffected by the biased coefficient estimate.

Are technological-change measures based on product characteristics, then, a useful concept? As defined by the tradeoff function, they can tell interesting but perhaps incomplete stories. As shown above for computers, milling machines, jet engines, and commercial jet aircraft, characteristics cannot be counted on to be a proxy for output. Measures based on characteristics do not provide an easy entry into a full understanding of technological change. However, they do provide information that, when combined with additional clues, may yield useful results.

It should finally be noted that user output is not usually single-dimensional. Measures such as seat-cost per mile may be of central importance, but they are not the only contributors to profit or utility. For aircraft, buyers and their customers routinely consider noise, safety, pollution, comfort, convenience, flexibility, range, reliability, and numerous other attributes. It is fitting to conclude with the reminder that we live in a multi-attribute world; the mapping from a set of attributes as seen from one viewpoint to another set as seen and evaluated by others is a central analytical feature of this world.

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1Hedonic price indexes and technology change measures may be more reliable for automobiles than for the other products reviewed here because the characteristic set describing automobiles was quite stable during the analyzed periods.
Appendix

DERIVATION AND SOURCES OF MILLING-MACHINE FACTOR-COST INDEX

WAGES


MATERIALS


CAPITAL RENTAL COST

Capital rental is calculated as the price of capital equipment times the nominal interest rate plus depreciation:

\[ R_t = p_t (r_t + d_t); \text{ an average lifetime of 10 years is assumed, so } d_t = .10. \]

CAPITAL PRICE


1887-1889, 1927-1928: These gaps in coverage were filled by assuming the same year-to-year change as shown in the Materials series "Metals and Metal Products" and then applying these changes to the last years of the available indexes.
INTEREST RATES


TOTAL FACTOR COST INDEX

All series were indexed to 1967 = 100 by simple linking. Wages (W), Materials (M), and capital rental (R) were combined to generate factor costs (C) using the following weights: \( C = .45W + .35M + .20R \). These weights were derived from *Census of Manufacturers* figures as reported in *Economic Handbook of the Machine Tool Industry, 1981-1982*, National Machine Tool Builders' Association, McLean, Va. Figures for "Metal Cutting Machine Tool Industry, SIC 3541" on "Value Added by Manufacture," "Cost of Material," and "Value of Shipments" are from p. 63. "Total Payroll" is from p. 235. Average weights for the period 1937 to 1977 were used. Comparisons were also made with the 1905 *Census of Manufacturers*, which did not reveal any long-term trends in the factor weights.