ISSUES IN THE ANALYSIS AND DESIGN OF THE EXPERIMENTAL PORTION OF THE HEALTH INSURANCE STUDY

PREPARED UNDER A GRANT FROM THE OFFICE OF ECONOMIC OPPORTUNITY

JOSEPH P. NEWHOUSE

R-1484-OEO
JUNE 1974

Rand
SANTA MONICA, CA. 90406

The research reported herein was performed pursuant to a grant from the Office of Economic Opportunity, Washington, D. C. Reports of The Rand Corporation do not necessarily reflect the opinions or policies of the sponsors of Rand research.

Published by The Rand Corporation

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PREFACE

This report was prepared as part of the Health Insurance Study, which is supported by a grant from the Office of Economic Opportunity. The major part of the study is an experiment to test the effect of alternative health insurance plans; this report describes certain technical problems that have arisen in the design of the experiment. The objectives and methods of the experiment are described in a longer and less technical Rand study, R-965/1-OEO, *A Design for a Health Insurance Experiment*, which has also been published in *Inquiry*, March 1974. This report is a slightly revised version of a paper given at the Winter 1973 meetings of the American Statistical Association.

SUMMARY

This report discusses two issues that will be faced in the analysis of the data from the experimental portion of the Health Insurance Study and four issues in the statistical design of that study. The first analytical issue arises because the experimental plans are designed so that the consumer faces a specified coinsurance rate unless that would cause his out-of-pocket expenditures to exceed a given amount; if so, the coinsurance is eliminated. This maximum liability feature has been incorporated because of its policy interest; however, it creates an analytical problem. The consumer who has not exceeded the expenditure limit raises the probability that he will exceed the limit with each successive expenditure; as a result, the "true price" a consumer faces for services is less than the nominal price so long as the expenditure limit is not exceeded. It is proposed that this problem be treated by means of analyzing episodes of illness (rather than annual expenditures) and explicitly accounting for the probability of exceeding the expenditure limit; the implications of this proposal for the design are briefly discussed. The second analytical issue concerns the treatment of supplementary insurance. Should negative as well as positive supplementation be permitted, and how should analysis of supplementation decisions proceed (what should be the dependent variable)?

Four issues in the statistical experimental design of the study are presented. The first is how to determine the number of families to be allocated to any particular plan; this will be solved by use of the Conlisk-Watts model developed for the New Jersey Negative Income Tax Experiment. The second is how to determine the families who will be included in the experiment. This problem will be solved by using the Finite Selection Model developed by Morris; the model is briefly described. The third issue concerns how families will be allocated to insurance plans (experimental treatments); this will be done randomly, subject to a constraint that ensures near balance among the experimental treatments. The final issue is how to choose sites. This is solved by an extension of the Conlisk-Watts model and application of the Finite Selection Model.

ACKNOWLEDGMENTS

The author would like to acknowledge the assistance of Carl Morris in developing and clarifying the ideas contained in Sec. III, and the comments of Gus Haggstrom and Adele Massell on a draft.

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I. INTRODUCTION AND OBJECTIVES

This report discusses some selected issues in the design and analysis of the experimental portion of the Health Insurance Study (HIS). The objectives, methods of procedure, and significance of the experimental portion of the HIS are discussed in Newhouse (1974). Here the objectives of the experiment are briefly reviewed. Then I discuss two problems that the HIS research will have to face when the data from the experiment are analyzed, and the implications of those problems for the design. Finally, I describe four statistical design problems.

Stated at the most general level, the objective of the experiment is to advance the state of knowledge concerning the consequences of alternative ways of financing medical care services. Within this general mandate are several specific goals: first, measurement of own- and cross-price elasticities (insurance elasticity) of demand and their interactions with income. Measurement of price elasticity is a necessary condition for predicting utilization and cost under any particular insurance plan, and if the supply of services were perfectly elastic in the long run, it is sufficient to predict how insurance will affect the share of the nation's resources devoted to medical care. Measurement of the interaction of price elasticity with income will determine the distributional effects of any particular financing plan. In those plans requiring the consumer to make out-of-pocket payments, the plan limits maximum out-of-pocket loss to a certain percentage of income, since this is a potential policy option. The maximum out-of-pocket loss is called the Maximum Dollar Expenditure (MDE). Thus, price elasticity is to be measured within the context of this type of plan.

A second goal is to measure, as well as possible, the effects of alternative financing arrangements on health status. Whereas the first goal, the measurement of price elasticities, may be thought of as relating to the costs of a financing method, the second may be thought of as measuring certain of the benefits.

A third goal relates to understanding the consequences of increasing the demand for ambulatory services. Analysis shows that a national

health insurance plan could cause a substantial disequilibrium in the market for outpatient physician services. This in turn could lead to the activation of several kinds of mechanisms to equilibrate the market, including price increases, queueing, delays in appointments, change in case-mix seen by physicians, changes in revisit rates, and so forth. The extent to which each of these mechanisms operates will play an important role in determining who gets what kind of service for what kind of medical problem. The HIS will attempt to provide some information on how the burden of adjustment is distributed among these mechanisms.

A fourth goal is to measure the effect, if any, of prepaying the physician for his services rather than paying him on the basis of fee for service. A fifth goal is to find out how much additional private insurance families would buy if there were a public plan that required out-of-pocket payments for services (as Medicare does, for example). Finally, the HIS will attempt to learn as much as possible about the administrative problems and rules of operation that arise in health insurance plans, particularly those having income-related clauses.

To estimate the effect of price on utilization and health status, the HIS will give various health insurance plans to approximately 7500 individuals (in 2000 families) in four sites.[1] The insurance plans are structured so that the families pay a percentage of their bill (the coinsurance rate), which varies from zero to 100 percent. As mentioned above, if the family must pay something out of pocket, its expenditures are limited to a certain fraction of its income. The fraction varies as an experimental treatment; it is either 5 or 15 percent. In some other plans all outpatient care is free, but the family must pay a specified fraction of inpatient expenditures. Also, some individuals are to be enrolled in a Health Maintenance Organization, in which the physicians are prepaid. To ensure no adverse selection of insurance, families are randomly assigned to insurance plans and asked to give up their old insurance. In order that families will

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[1]There is also a control group that remains on existing insurance. Its purpose is described in Newhouse (1974).

not differentially refuse, all families are paid an amount equal to the maximum they could lose by giving up their old insurance. For example, if a family has complete coverage and is asked to accept a plan with a $1000 deductible, it is also offered $1000.

Observations on the utilization of the participants should establish the price elasticity of demand, as well as the effect of prepaying the physicians. The effect of insurance on health status is extremely difficult to assess because of the difficulty of measuring health status. In order to measure self-assessed health status, all of the participants will take periodic interviews; all of them will also take screening physical examinations at the end of the experiment to measure "objective" health status; some participants will take initial physical examinations.[1]

Measurement of the consequences of a disequilibrium in the market for ambulatory services is accomplished by selecting sites in which the physicians' workload varies. Although the range of variation in workload across communities may not include the workload that would be observed if free ambulatory coverage were instituted, it is the only method within the context of the experiment to obtain information on this important question.

The degree to which families will supplement not very generous insurance will be measured by permitting supplementation in the final year of the experiment. By that time the HIS will have an estimate of the actuarial value of the policy; supplementary insurance will be offered at varying rates to test the effect of alternative tax treatment of health insurance premiums. Such premiums are not now taxable income if paid by the employer, but it has been proposed that this treatment be changed (Feldstein and Allison, 1972; Mitchell and Vogel, 1973).

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[1] Some of the participants will not receive an initial physical in order to measure the effect, if any, of the physical on utilization.

II. SOME ISSUES IN ANALYZING DATA FROM THE EXPERIMENT

A principal issue the analyst of the experimental data will face is the treatment of price, given that price falls with total expenditure because of the MDE. Because of this feature traditional methods of analysis are inappropriate. Prior work in the field of medical analysis and demand analysis more generally has tended to analyze consumption (measured either in dollars or in physical units) per unit of time as a function of price. In these analyses price per unit is assumed to be constant. The theory underlying these analyses is standard economic theory, which assumes that the consumer optimizes so that he values the marginal unit at the marginal utility of income forgone to purchase it. With the MDE, however, there are two local optima. Figure 1 shows a two-commodity world of medical care and all other goods; I_1 and I_0 are indifference curves.[1] The kinked line is the budget line; after the consumer has consumed L units of medical care, he does not have to sacrifice more of other goods to obtain care. (The budget line is net of any taxes or premiums the consumer has paid to finance the insurance policy.) There are two local maxima, at A and B; in this diagram B is clearly the global maximum.

The problem caused by the kinked budget line, although somewhat novel from the point of view of empirical demand analysis, is reasonably tractable. Consumers can be assumed to have a utility function of a specified kind; then by observing expenditure choices, one can infer the parameters of the function.[2] Knowing the function, one can predict the consequences of any price structure.

What makes experimental data difficult to analyze is that the

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[1]The only unusual thing about them is that they turn up, indicating that the consumer has negative value for certain levels of medical care consumption. This may be because it takes increasing amounts of time (assumed to have increasing value) to consume more care or because sufficient exposure to medical care may actually decrease health status, through increasing the risk of infection or iatrogenic disease, relative to the possible benefits of care.

[2]This suggestion was made by Kenneth Arrow.

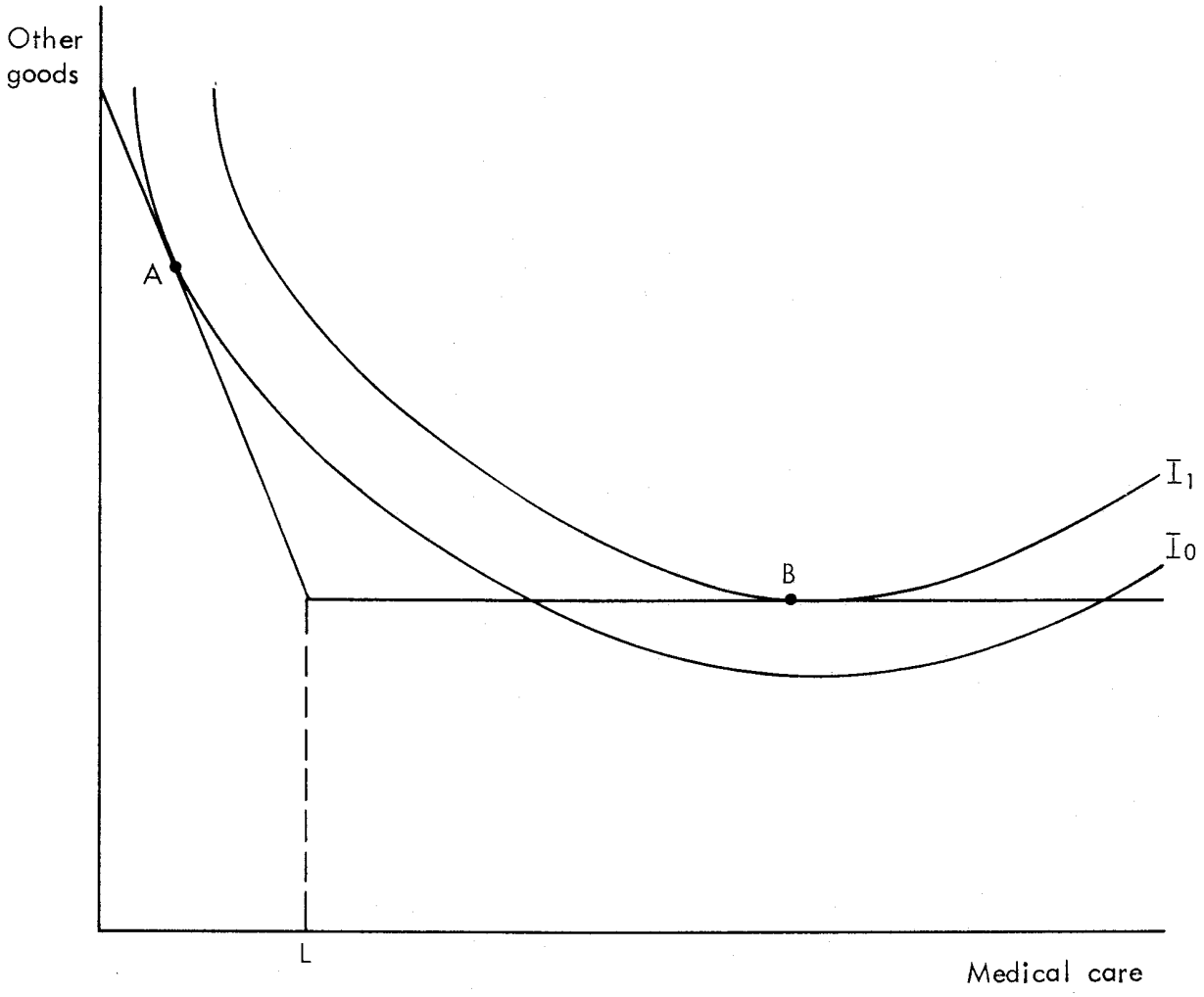

Fig. 1 — Indifference curves and budget line for medical care and other goods

consumer will typically face several choices during the expenditure accounting period, none of which taken singly could cause him to exceed the deductible, but all taken together may. Thus, when making his initial choices, the consumer is operating under uncertainty about what the marginal price at the end of the accounting period will be. (See Keeler, Newhouse, and Phelps, 1974 for a theoretical model of this problem.)

The plan for modeling this situation is to associate all expenditures with an illness episode,[1] then to explain an individual's expenditures on each illness episode using as explanatory variables the insurance plan the consumer has, the consumer's expectations regarding expenditures on this episode and future expenditures, the amount of expenditure the consumer must make before his coinsurance rate changes, and a set of demographic variables such as income. Expected expenditures on this episode are separated from expected expenditures on future episodes, because the consumer has information about this episode when he begins therapy that he does not have about future episodes. Expenditures on this episode are a function of diagnosis (using extraneous data);[2] and expenditures on future episodes are a function of the individual's age, sex, general health status (including any chronic conditions), and so forth.[3] The theory underlying an episodic model is discussed at length in Phelps (1973), although Phelps does not treat the case of a price that falls with expenditure.

The resulting equation (together with an error term) generates a distribution of expenditure per episode per individual; the experimental data will also generate a distribution of episodes. If these distributions are analytically tractable, they may be convoluted and mean individual expenditure per year (or other accounting period) predicted.

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[1]There are two types of episodes. An acute episode represents the consumer's response to a random loss of health; an acute episode in general will terminate within a fairly short period of time, either because the underlying pathology is self-limiting or because medical intervention has cured the problem. A second type of episode is chronic; a chronic problem in general requires medical intervention to maintain a kind of health and is not expected to terminate. The chronic episode therefore lasts for the entire accounting period. (A chronic condition in remission that "flares up" will be treated as an acute episode.) For analytical purposes it differs from an acute condition in that expenditures may be assumed to be better foreseen.

[2]We will estimate expected expenditure by using mean expenditure for that diagnosis (if possible mean expenditure conditional on a particular plan). Obviously there will have to be some grouping or classification.

[3]If possible, generalized least squares will be used to allow for non-zero covariances among family members.

If they are not analytically tractable, a simulation may be performed to predict mean expenditure per year. If there are family related clauses in the insurance plan (for example, family deductibles), individual expenditures must be aggregated to the level of the family.

The problem of price per unit falling with quantity and the proposed solution of analysis by episode have several implications for the design. The most obvious is that data must be gathered to permit the definition of illness episodes and to link medical services to them. This implies a degree of cooperation on the part of the physicians in filling out claims forms; it also implies assuring ourselves that the application of the definition of an episode is reliable. A second implication is to minimize the number of discontinuities or kinks in the price line in order to simplify modeling the uncertainty the consumer faces. This has been done by limiting all plans to at most one change of price; that is, one coinsurance rate will apply to family expenditures until expenditures have reached a certain percentage of family income, after which there is no coinsurance. (There is no plan with a deductible followed by a non-zero coinsurance rate, followed by some other limit.) A third implication is to stipulate that there be no carryover of unreimbursed expenditures from one accounting period to another. At one extreme there could have been a moving average accounting period; this would mean there would be no coinsurance so long as the rate of expenditures exceeded a certain amount. Besides creating a perverse incentive to consume services (unless based on the rate of unreimbursed expenditure), this arrangement would be extremely difficult to analyze. A modification was considered at some length, to permit carryover of unreimbursed expenditures occurring at the end of the accounting period to the next accounting period. The intent of this is to prevent someone from having to satisfy an expenditure limit twice in the same illness episode, which could happen if the episode occurred near the end of a fixed accounting period. The empirical complications such a clause would introduce were not worth the value of having it.

The design should provide a hedge if episodic analysis proves infeasible. The hedge is that an analysis of covariance model (basically

estimation of means across plans adjusted for demographic differences) should yield reasonably precise estimates of a few dissimilar plans. This can be done by minimizing the number of plans. There are now 16 plans and around 8000 family years to be allocated to them. This should yield estimates of the effects of insurance to plus or minus 10 percent (Newhouse, 1974). If, however, episodic analysis proves infeasible, sequential design will permit even greater concentration of families among a smaller number of plans. The current time schedule calls for enrollment of 500 families in the fall of 1974, the next 500 families nine months later, the next 500 families six months later, and the last 500 families three months after that. As a result, concentration of families in a few plans will be feasible, should it prove necessary.

A second set of analytical issues in connection with supplementary behavior concerns the design and analysis of the supplemental portion of the experiment. At what terms should supplementary insurance be available? Should both positive and negative supplementation be permitted? Should an individual be allowed to vary his coinsurance rate, his expenditure limit, or both? What is the dependent variable for analysis and what are the explanatory variables?

Since the overall objective is to understand the demand for supplementary insurance, the terms at which such insurance can be purchased will reflect the rates at which it would be available in the marketplace, if a national plan were enacted that resembled the plan of the HIS. Therefore supplementation will be permitted at different premiums, reflecting both the tax treatment of insurance premiums and various loading charges (the excess of premiums over payout). Insurance premiums are not now taxable income if the employer pays them, which means there is a subsidy equal to the marginal tax rate from the purchase of insurance. Loading charges vary from 100 percent for some individual insurance to 6 percent for individuals in the largest group. The HIS will offer insurance at a small number of loading charges, ranging from 100 percent down to minus 30 percent or so, reflecting a tax subsidy and a very low loading. The actuarial value will be adjusted for the age and sex mix of the family, since the private market is likely to take account of such differences.

No decision has been made on whether both positive and negative supplementation will be allowed. If negative supplementation were not allowed, an individual could purchase more generous insurance, but not less generous.[1] To allow positive supplementation is sufficient to provide data on the degree to which individuals would purchase additional insurance if the government mandated a particular plan. To allow both positive and negative supplementation is to attempt a broader study of the demand for insurance. If negative supplementation is allowed, nothing is sacrificed in terms of measuring what would happen if only positive supplementation were allowed (since anyone who negatively supplements would just be assumed not to supplement if such were not allowed). However, negative supplementation raises an ethical and a practical issue, because allowing negative supplementation on the basis of actuarial value raises the possibility that an individual could be worse off ex post.[2] (For example, an individual with experimental full coverage who chose to change to a $500 deductible and received $200 (if that were the actuarial value of the difference) could have a bill of $500 and be out of pocket $300.) If an individual were to be worse off ex post, it would usually be to his advantage to withdraw from the experiment and return to his old insurance. Current HEW guidelines on research with human subjects require that withdrawal be permitted. Withdrawal under these conditions would obviously defeat the experiment. To prevent withdrawal, one can make lump sum payments to those who have generous insurance plans in an amount large enough to compensate them for their worst case.[3] (In the above example, all

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[1] For example, an individual who has an experimental policy with a $1000 deductible who had previous insurance would be paid $1000 to participate. In the last year, he could change the deductible to $500 and be paid, say, $800. If negative supplementation is not allowed, an individual with experimental full coverage would not be allowed to choose a deductible of $500 (plus a money payment).

[2] To allow negative supplementation of ungenerous plans also raises the issue of accurate calculation of actuarial values; the HIS will not be well placed to determine, say, the actuarial value of increasing a deductible from $1000 to $2000.

[3] These payments may be held in an escrow account and made conditional on completion of the experiment.

those with full coverage could be paid $300, and those who wanted a $500 deductible could receive an additional $200.) Whether this is sufficiently expensive so as not to be worthwhile depends upon the actuarial values involved; the higher the actuarial value, the less must be paid for worst case compensation. As noted above, no decision has yet been made on negative supplementation.

Given that positive supplementation at least will be allowed at various loading charges, the question arises as to what kind of supplementary behavior should be permitted. The tentative answer to this question is that individuals should be permitted to choose coinsurance rates of 0, 25, and 50 percent, so long as their choice is less than their experimental rate, and that they should be allowed to set their MDE at 5 percent of income. (The zero coinsurance rate is equivalent to a zero MDE.) The basic reason for allowing variation in both dimensions (coinsurance and MDE) is that the private insurance market would offer such variation, and it is important to understand how much consumers will choose to vary each dimension, because changes in the different dimensions will have different implications for demand.

Choice of explanatory variables will be based on theoretical work related to consumer choice of insurance; unfortunately, this literature is not yet very far advanced.[1] Nevertheless, from the work that has been done it is clear that the consumer's choice will be a function of the distribution of the consumer's expected expenditures, his permanent income, and the price of supplementation. Depending upon these variables, the consumer chooses an optimal MDE-coinsurance pair from the pairs that are open to him. Each of the two dimensions of choice will be analyzed separately. Although the choices in each dimension are interdependent, theory is not powerful enough to specify restrictions that exclude a variable from one equation and not from the other. Hence, structural equations are not identified and only a reduced form equation can be estimated. Since the choice of supplementary insurance structured by the HIS is discrete, methods developed by Nerlove and

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[1] Theoretical beginnings may be found in Arrow (1973a, 1973b), and Phelps (1973).

Press (1973) for estimation with polytomous dependent variables will be used.

An alternative to analyzing choice of MDE and coinsurance is to analyze the risk the consumer bears. Provided suitable measures of risk can be found, this offers the possibility of testing hypotheses related to risk-bearing behavior. Analysis of risk per se, however, is not sufficient for policy purposes, because there may not be any convenient way of moving from a measure of risk to a unique structure of insurance. The structure is important for policy purposes, because it will affect demand (except in special cases). Even if there were a way to move from risk to a structure, it appears more efficient to work directly with the structure, if that is what one is interested in. A similar argument can be made for not measuring supplementation by the amount of the supplementary premium that the consumer pays.

III. ISSUES IN STATISTICAL DESIGN

Among the many possible issues in the statistical design of the experiment, four will be discussed here: the choice of the number of individuals to be assigned to any particular plan, selection of participating individuals from the community, assignment of individuals to plan, and choice of sites in which to experiment.

The number of individuals to be assigned to any plan will be determined by use of the Conlisk-Watts model developed for the New Jersey Negative Income Tax Experiment (Conlisk and Watts, 1969). This model assumes that one is interested in estimating a vector of coefficients β in a model:

$$y = X\beta + \varepsilon \ , \tag{1}$$

where $E(\varepsilon) = 0$,
$V(\varepsilon) = \sigma^2 I$.

In this case y is utilization or expenditure by individual; the X matrix consists of a set of prices (as determined by insurance plans) or a set of dummy variables representing insurance plans (in an analysis of covariance model). The estimator of β is:

$$b = (X'X)^{-1} X'y,$$
$$V(b) = \sigma^2 (X'X)^{-1} .$$

The admissible regressor rows of X are specified. A budget constraint is assumed and cost per regressor row (insurance plan) is given. The model then chooses the number of observations to be assigned to any design point such that

$$\Phi = \text{tr}(WV(Pb)) \text{ is minimized} \ , \tag{2}$$

where W is a diagonal matrix with weights on the diagonal, P is any arbitrary set of vectors, but most frequently equals either I or X,

and tr is the trace of the matrix. Φ is therefore a linear combination of the variances of the regression coefficients.

Apart from specifying W and P, the major issue to be resolved in this step is the choice of X. Within this issue there are in turn two subissues. One is the choice of plans or, equivalently, the admissible rows of X. Since the model is free to allocate no observations to any design point, this choice really concerns which design points are constrained to have no observations assigned to them. The second is the issue of what functional form to choose.

Choice of design points can be thought of as first determining the number of design points that are not constrained to be zero (that is, determining the maximum number of insurance plans) and then determining what those design points are. Although constraining fewer points to be zero will lead in general to a smaller value of Φ, there are two costs to considering larger numbers of design points. The first is a computational cost. The second follows from the desire to hedge, discussed above, by maintaining the viability of the analysis of covariance model. More design points degrade the precision of this model. As mentioned above, there are initially 16 design points; they are described in Newhouse (1974).

Several possible functional forms will be considered. It is envisioned that a decision-theoretic approach to choice of functional form will be used, so that functional form will be chosen that minimizes expected loss. (See Conlisk, 1973.)

A secondary issue related to the design of plans is the possibility of truncating the MDE below 5 or 15 percent of income for greater efficiency in the estimation of price elasticities. (For example, the MDE could be 15 percent of income or $1000, whichever is less.) Truncation may represent a gain because the method of compensating families in the experiment for potential losses in welfare generally requires that they receive a lump sum equal to their MDE. The question then arises: What is the potential net gain from placing an absolute dollar ceiling on the MDE? It would have the effect of eliminating price variation in certain high ranges of expenditure, thus reducing the ability to measure price elasticities in this range, while increasing the

experimental budget available to allocate to lower ranges of expenditure. The optimal level of the MDE therefore depends on the relative interest in the estimation of elasticities at different levels of expenditures. This problem has been structured so that the Conlisk-Watts model may be applied to it. Rows of the X matrix will represent plan-total expenditure pairs. (Total expenditure is specified as a range, such as $500-$1500.) Then each plan-total expenditure pair is associated with the conditional probability of that observation, given that the individual is assigned to that plan. The constraint is then placed on selection of regressor rows such that all rows associated with a particular plan must be selected if any are. The MDE (together with the truncation factor) enters the model by changing the costs of various experimental plans, as well as the probabilities of obtaining expenditures in various intervals. By running the model with alternative MDEs (and given weights for expenditure-elasticity interactions), one obtains a set of truncations that minimizes Φ. For example, the optimum might be the lesser of 15 percent of income or $1200 in the 100 percent coinsurance plans, but the lesser of 15 percent of income or $600 in the 25 percent coinsurance plan.

Individuals will be chosen for this experiment by means of the Finite Selection Model (FSM) developed by Morris (forthcoming a) for this experiment. This model is similar to the Conlisk-Watts model in its objectives but quite different in its underlying assumptions. The Conlisk-Watts model assumes that the regressor rows (the rows of the X matrix) come from discrete space, but that there is an infinite population to sample from. This assumption is appropriate for selection of treatments but less so for selection of families. By contrast, the FSM assumes that the regressor rows may come from continuous space, but that there is a finite population to sample from. For example, individuals have associated with them a vector of demographic characteristics that are continuous or nearly so (age, family, size, income, education, for example). There are, however, only a finite number of individuals to choose to participate.

More formally, the FSM assumes that one is interested in estimating equation (1) from a subset of size n of all N available families, $n < N$.

The objective function is given by (2). If there are n observations, (2) can be rewritten as:

$$\Phi_n = \text{tr } D(X'X_n)^{-1} = \text{tr}DS_n \ , \tag{3}$$

where the subscript n indicates that there are n rows in X, and D equals P'WP. In this case the X matrix consists of individual characteristics such as age, income, and education and does *not* include the experimental insurance plan (though it could if that were desirable; see below). Suppose an (n + 1)st observation is to be added that reduces Φ_n as much as possible for its cost. That is, we wish to maximize

$$(\Phi_n - \Phi_{n+1}(x))/c(x) \ , \tag{4}$$

where Φ_{n+1} is conditional on using an X matrix equal to $\begin{pmatrix} X_n \\ x' \end{pmatrix}$ and x' is a row vector of characteristics of the (n + 1)st family. c(x) is the cost of a family with characteristics x. An algebraic identity gives

$$S_{n+1} = S_n - \frac{S_n xx' S_n}{1 + x'S_n x} \ .$$

Hence,

$$\begin{aligned} \Phi_{n+1}(x) &= \text{tr}DS_{n+1} \\ &= \text{tr}DS_n - \frac{\text{tr}DS_n xx'S_n}{1 + x'S_n x} \\ &= \Phi_n - \frac{\text{tr } x'S_n DS_n x}{1 + x'S_n x} \\ &= \Phi_n - \frac{x'S_n DS_n x}{1 + x'S_n x} \ , \end{aligned}$$

since $x'S_n DS_n x$ and $x'S_n x$ are scalars.

Substituting, (4) equals:

$$\frac{x'S_n DS_n x}{c(x)[1 + x'S_n x]} . \qquad (5)$$

Given a list of unchosen individuals, (5) may be computed for each x and the maximizing x chosen. The procedure can be repeated until n is such that $\Sigma_{i=1}^{n} c_i = C$, where C is a budget constraint.

The stepwise algorithm implied by the successive use of (5) has led to an optimal or nearly optimal set of families, but if it did not do so, substitutions and corrections could be applied at the end of the selection by using a similar algorithm until a satisfactory list is obtained. Although the literature contains no discussion of the use of this algorithm on the Φ objective function in this context of costs and variances, there is experience and theory for using a related algorithm to determine "D-optimum" subsets (choosing subjects to maximize det $|X_n'X_n|$), and the experience there has been favorable (Harville, 1973; Johnson, 1973; Mitchell, 1973; Wynn, 1972).

The FSM will be used to choose the families that will participate in the experiment, but the families will be "randomly" allocated to experimental treatments (plans) and the control group. Although the FSM can, in principle, select optimal family-plan combinations (given a model to be estimated), random allocation offers some protection against omitted variables. (That is, one can be reasonably sure that any such variable will be balanced among the treatment groups.) By allocating randomly one pays a price in efficiency of estimation (if there are no omitted variables). The price paid for this protection can be kept small by making the random allocation subject to a constraint of near orthogonality between the demographic and plan variables, ensuring near balance among the treatment groups.[1]

The Conlisk-Watts model and the FSM may also be applied to site selection. Morris (forthcoming b) has proposed a generalization of the

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[1]The idea contained in this paragraph was suggested by Bradley Efron.

Conlisk-Watts model that can be used to determine the optimal number of sites, given that there are fixed costs of operating in each site. A random effects model is assumed for city-specific coefficients $\beta_i \sim N_k(\beta, T)$, where k is the dimensionality of the β vector. T is therefore the between-city variance-covariance matrix. For simplicity, assume that the same design points are to be used in each city and that the cost of a design point does not depend on city. The model then minimizes V:

$$\begin{aligned} V &= \text{between site variance} + \text{within site variance} \\ &= (1/K)(v + \Phi) \ , \end{aligned} \tag{6}$$

subject to

$$C = K(C_0 + \sum_{j=1}^{m} c_j n_j) \ , \tag{7}$$

where there are K sites, $v = tr(DT/\sigma^2)$, Φ is defined by (2) for the observations within each site, C_0 is the fixed cost of operating in any site (for opening an office, running a field staff, and so forth), c_j is the cost of an observation at the jth design point (or insurance plan), and n_j represents the number of observations at the jth design point in each city.

The solution to this problem is quite simple. Define $C_K = (C_T/K) - C_0$ as the amount per city available to spend on design points after paying the fixed costs per site C_0. Then, following the standard Conlisk-Watts procedure, minimize (2) subject to a budget constraint of C_K. This defines $\Phi^{(K)}$, where the superscript indicates that Φ depends on K. The optimal K is the integer K^* that minimizes $(1/K)(v + \Phi^{(K)})$ and can be determined by enumeration in this case.

After the optimal number of cities is determined in this fashion, the FSM will be used to select actual cities. The variable of interest across cities is the workload of physicians; the FSM will tend to select extreme values. A city-specific cost index will also be entered

in the FSM, so that it will tend to select cheaper cities. The first site is Dayton, Ohio. There will be a regional constraint; the second site will be in the West and the third site in the South. No decision has been made on the location of the fourth site.

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