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A Correction for Middleton's Visible- and Infrared-Radiation Extinction Coefficients Due to Rain

C. C. Chen

A Report prepared for

UNITED STATES AIR FORCE PROJECT RAND

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PREFACE

As part of its continuing program of studies for the U.S. Air Force, The Rand Corporation attempts to examine the broad implications of the development and employment of precision-guided munitions (PGMs). Attenuation due to water droplets in the atmosphere (e.g., fog, clouds, and rain) plays a very important role in assessing the potential of optical sensors in many applications, including target acquisition and terminal homing guidance for PGMs.

The present report deals with the attenuation of visible and infrared radiation because of rain. An expression for the "extinction coefficient due to rain" is derived to replace an expression developed by W.E.K. Middleton (1952) which, under close examination, has been found to be erroneous. This report should be useful to the Air Force Cambridge Research Laboratories (AFCRL), to Air Force analysts interested in application studies of optical sensors, and to all workers in the field of infrared applications.

SUMMARY

This report describes how Middleton's simple expression for the visible- and infrared-radiation extinction coefficient due to rain is incorrect. By improperly using Stokes' law to obtain the terminal velocity of a raindrop through air, Middleton underestimated the extinction coefficient due to rain by nearly a factor of ten. A revised expression for this coefficient is derived here using empirically determined values for raindrop terminal velocity. The calculated values of the extinction coefficient thus obtained provide much closer agreement with the measured values.

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MIDDLETON'S VISIBLE- AND INFRARED-RADIATION EXTINCTION COEFFICIENTS DUE TO RAIN

The subject of infrared (IR) attenuation due to water droplets is extremely important in the application studies of IR sensors because the attenuation is so severe that it renders the IR sensors useless in most cases when fog, clouds, or rain are present. The atmospheric attenuation of IR radiation due to water vapor has been treated quite thoroughly in most books on IR technology,(1-3) but IR attenuation due to water droplets is usually glossed over quickly in such texts. Fortunately, the theory of scattering by water droplets has been treated quite thoroughly in the framework of the Mie scattering theory by Van de Hulst(4) and Deirmendjian.(5)

Middleton(6) derived a simple expression for the visible- and IR-radiation extinction coefficient due to rain as a function of only two parameters--the precipitation rate and the raindrop radius. The derivation of this coefficient is presented below.

The extinction coefficient due to rain for drops with radius a_i is defined as

$$(\gamma_i)_{ext} = n_i(\sigma_i)_{ext}, \tag{1}$$

where n_i is the number of raindrops per unit volume for drops with radius a_i, and where $(\sigma_i)_{ext}$ is the extinction cross section,

$$(\sigma_i)_{ext} = \pi a_i^2 (Q_i)_{ext}, \tag{2}$$

and $(Q_i)_{ext}$ is the wavelength-dependent extinction efficiency from the Mie theory.(4,5) Now, for $x = 2\pi a_i/\lambda >> 1$, $(Q_i)_{ext}$ approaches 2 as a limit.(4,5) The average raindrop size is about 0.05 cm. For IR wavelengths of interest, e.g., 3-5μ or 8-14μ, then $x \approx 300\text{-}1000$; thus $(Q_i)_{ext} = 2$. The extinction coefficient becomes

$$(\gamma_i)_{ext} \cong n_i 2\pi a_i^2. \tag{3}$$

If there are drops of various sizes, then the total extinction coefficient is just

$$\gamma_{ext} = \Sigma_i (\gamma_i)_{ext}. \tag{4}$$

It was found experimentally that the rain particle size distribution is a function of the total precipitation rate Z.(7) z_i is related to Z in the following manner:

$$z_i = \alpha_i Z, \tag{5}$$

where z_i is the rate of rainfall in cm (depth of water) per second for drops with radius a_i, and α_i is the fraction of total volume reaching ground contributed by drops of radius a_i. Also, since the measured values of the extinction coefficient due to rain are usually presented as a function of the total precipitation rate,(8) it is useful to express the calculated coefficient in terms of the same parameter. $(\gamma_i)_{ext}$ can be related to z_i by first noting that

$$n_i = \frac{\rho\, z_i/v_i}{\rho\, V_i}, \tag{6}$$

where ρ = density of water, 1 gm/cc
v_i = terminal velocity of raindrop size a_i, cm/sec
V_i = volume of each raindrop of radius a_i, $4\pi a_i^3/3$.

Substituting Eq. (6) into Eq. (3), there results

$$(\gamma_i)_{ext} = \frac{3}{2} \frac{z_i}{v_i} \frac{1}{a_i} \qquad \text{cm}^{-1}. \tag{7}$$

Middleton(6) used Stokes' law to obtain the following expression for the terminal velocity of a raindrop through air:

$$v = \frac{2a^2 \rho g}{9\mu}, \tag{8}$$

where, as before, ρ = density of the droplet, 1 gm/cc
a = radius of the raindrop, cm
g = acceleration due to gravity, 980 cm/sec^2
μ = viscosity of the air, 180×10^{-6} gm/cm-sec.

Substituting Eq. (8) into Eq. (7), a simple expression for $(\gamma_i)_{ext}$ is obtained:

$$(\gamma_i)_{ext} = 1.25 \times 10^{-6} \frac{z_i}{a_i^3} \text{ cm}^{-1}. \tag{9}$$

Middleton's expression, Eq. (9), has sometimes been used. Thus, for example, using Eq. (9) and the rainfall data of Laws and Parsons,(7) Hudson(3) obtained extinction coefficients through numerical integration and calculated the transmittance of a 1.8-km path through four different kinds of rain. These results are presented in Table 1. Hudson then concluded that IR attenuation due to rain is not generally serious enough to interrupt the intended mission of IR systems. However, measured extinction coefficient versus rain rate(8) showed a much larger extinction coefficient, by almost a factor of 10, than the one calculated by Hudson using Middleton's expression. The measured values are presented in Table 2. According to the measured values, any rain heavier than medium, or rainfall rate greater than about 10 mm/hr, would render the IR sensors almost useless along a 1.8-km path because only about 6.7 percent of IR radiation can pass through. Along a 10-km path the transmittance through a light rain would be 0.1 percent, rather than 52 percent, as Hudson would predict.

The discrepancy between calculated values of the extinction coefficient due to rain according to Middleton's expression and the measured values is so great that a thorough examination of Middleton's expression

Table 1

CALCULATED TRANSMITTANCE OF A 1.8-km PATH THROUGH RAIN[a]

Condition	Rainfall Rate (mm/hr)	Extinction Coefficient (km^{-1})	Transmittance of a 1.8-km Path
Light rain	2.5	0.065	0.88
Medium rain	12.5	0.1675	0.74
Heavy rain	25	0.24	0.65
Cloudburst	100	0.538	0.38

[a] From Ref. 3.

Table 2

MEASURED VALUE OF EXTINCTION COEFFICIENT THROUGH RAIN[a]

Condition	Rainfall Rate (mm/hr)	Extinction Coefficient (km^{-1})	Transmittance of a 1.8-km Path
Light rain	2.5	0.69	0.29
Medium rain	12.5	1.6	0.056
Heavy rain	25	2.0	0.0275

[a] From Ref. 8.

was undertaken. It was found that, in the course of deriving the extinction coefficient in Ref. 6, a serious mistake was made in using Stokes' law to obtain the terminal velocity of a raindrop through air. The mistake consisted of not recognizing that Stokes' law applies only to a sphere moving very slowly through a viscous fluid at rest. In mathematical terms, Stokes' law applies only when the Reynolds number (R_e) is smaller than 1.(9,10) The Reynolds number is defined as

$$R_e = \frac{\rho' vL}{\mu}, \qquad (10)$$

where ρ' = density of the fluid medium (in this case, air)

v = velocity of the object moving through the fluid

L = characteristic length of the object

μ = viscosity of the fluid.

In the case of a sphere, the characteristic length is its diameter. For an assumed spherically shaped raindrop falling through air, $\rho' = 1.2929 \times 10^{-3}$ gm/cc, $\mu = 180 \times 10^{-6}$ gm/cm-sec. By using the criterion that $R_e \leq 1$, it can be shown that Eq. (8) is applicable for calculating the water droplet terminal velocity only when the droplet radius is equal to or less than 40μ. It is found in Ref. 7 that raindrop size less than 250μ contributes less than one percent of the total volume for all kinds of rainfall rates even to as low as 0.25 mm/hr. This demonstrates the strength of the conclusion that Stokes' law may not be used to calculate the raindrop velocity through air.

Table 3 presents values of raindrop terminal velocity obtained experimentally,(11) together with those obtained by Eq. (8). For comparison purposes, Table 3 also lists some experimental values of the terminal velocity of fall for water droplets in stagnant air (not necessarily raindrops).(12-15) The experimental values of raindrop terminal velocity as observed in Ref. 11 agree very well with those of water droplets as observed in Ref. 12.

Table 3

RAINDROP TERMINAL VELOCITY

Raindrop Radius, cm	Measured Terminal Velocity, m/sec		Calculated Terminal Velocity from Eq. (8), m/sec
	Best[a]	Gunn, et al.[b]	
0.025	2.1	2.06	7.55
0.05	3.9	4.03	30.2
0.075	5.3	5.4	68
0.10	6.4	6.49	121
0.125	7.3	7.4	189
0.15	7.9	8.06	225
0.175	8.35	8.5	369
0.20	8.70	8.83	484
0.225	9.0	9.0	610
0.25	9.2	9.09	755.5
0.275	9.35	9.15	915
0.30	9.5		1090
0.325	9.6		1280

[a] From Ref. 11

[b] From Refs. 12-15.

If Middleton had used the experimentally determined terminal velocity of the raindrops instead of Stokes' law, he would have obtained a more nearly correct expression for the visible and IR extinction coefficient due to rain.

Using the rainfall data of Laws and Parsons(7) and the raindrop terminal velocity values quoted by Goldstein,(11) we can perform numerical integration over Laws and Parsons' rainfall distribution to obtain the total extinction coefficients for different kinds of rain. Using Eqs. (4), (5), and (7), we present below a sample calculation of the extinction coefficient for medium rain with a rainfall rate of 12.5 mm/hr.

The values of α_i for different kinds of rain are presented in Table 4. The details of such a simple calculation are presented in Table 5. Similar calculations are performed to obtain the extinction coefficients for the other three kinds of rain listed in Table 1. The results are presented in Table 6. By comparing Table 6 and Table 2, it can be observed that Eq. (7) is fairly good in obtaining a correct value for the rain extinction coefficient in heavy rains, but still rather optimistic (by roughly 50 percent) in predicting transmission through light rains.

From this short report, we conclude that Middleton's expression for visible and IR extinction due to rain is incorrect and should not be used in estimating the IR attenuation through rain. In estimating the extinction coefficient, measured values of raindrop velocity (rather than values calculated using Stokes' law) should be inserted into Eq. (7).

Table 4

SIZE-DISTRIBUTION OF RAINDROPS[a]
(Tabulated value is α_i, the fraction of total volume having radius a_i for each rainfall rate, Z)

a_i, cm	Z, mm/hr			
	2.5	12.5	25	100
0.025	0.07	0.03	0.02	0.01
0.050	0.28	0.12	0.08	0.04
0.075	0.33	0.25	0.18	0.09
0.100	0.19	0.25	0.24	0.14
0.125	0.08	0.17	0.20	0.17
0.150	0.03	0.10	0.13	0.18
0.175	0.01	0.04	0.08	0.15
0.200	0.01	0.02	0.03	0.09
0.225		0.01	0.02	0.06
0.250		0.01	0.01	0.03
0.275			0.01	0.02
0.300				0.01
0.325				0.01

[a] From Ref. 7.

Table 5

CALCULATION OF THE EXTINCTION COEFFICIENT FOR RAIN
(Medium rain conditions, rainfall rate, 12.5 mm per hr)

a_i, cm	z_i (cm/sec)	v_i (cm/sec)[a]	z_i/v_i, 10^{-7}	γ_i (km^{-1})[b]
0.025	1.05×10^{-5}	210	0.5	0.3
0.05	4.2×10^{-5}	390	1.078	0.323
0.075	8.75×10^{-5}	530	1.65	0.33
0.10	8.75×10^{-5}	640	1.37	0.206
0.125	5.95×10^{-5}	730	0.815	0.0975
0.150	3.5×10^{-5}	790	0.443	0.0443
0.175	1.4×10^{-5}	875	0.168	0.0144
0.20	0.7×10^{-5}	870	0.0805	0.00605
0.225	0.35×10^{-5}	900	0.0388	0.00258
0.25	0.35×10^{-5}	920	0.038	0.00228
				$\Sigma = 1.32611$

[a] From Table 3.

[b] From Eq. (5).

Table 6

CALCULATED VALUES OF EXTINCTION COEFFICIENTS THROUGH RAIN

Condition	Rainfall Rate (mm/hr)	Extinction Coefficient (km^{-1})	Transmittance of a 1.8-km Path
Light rain	2.5	0.42	0.47
Medium rain	12.5	1.32	0.093
Heavy rain	25	2.14	0.0214
Cloudburst	100	5.84	$< 10^{-4}$

REFERENCES

1. Jamieson, J. A., R. H. McFee, G. N. Plass, R. H. Grube, and R. G. Richards, *Infrared Physics and Engineering*, McGraw-Hill Book Co., Inc., New York, 1963.

2. Wolfe, W. L. (ed.), *Handbook of Military Infrared Technology*, Office of Naval Research, Dept. of Navy, Washington, D.C., 1965.

3. Hudson, R. D., Jr., *Infrared System Engineering*, John Wiley and Sons, Inc., New York, 1969.

4. Van de Hulst, H. G., *Light Scattering by Small Particles*, John Wiley and Sons, Inc., New York, 1957.

5. Deirmendjian, D., *Electromagnetic Scattering on Spherical Polydispersions*, The Rand Corporation, R-456-PR (April 1969); also American Elsevier Publishing Company, New York, 1969.

6. Middleton, W.E.K., *Vision Through the Atmosphere*, University of Toronto Press, 1952.

7. Laws, J. O., and D. A. Parsons, "The Relation of Raindrop Size to Intensity," *Trans. Am. Geophysical Union*, Vol. 24, 1943, pp. 452-460.

8. Rensch, D. B., and R. K. Long, "Comparative Studies of Extinction and Backscattering by Aerosols, Fog, and Rain at 10.6μ and 0.63μ," *App. Opt.*, Vol. 9, No. 7, July 1970, pp. 1563-1573.

9. Schlichtiny, H., *Boundary Layer Theory*, 4th ed., McGraw-Hill Book Co., Inc., New York, 1960.

10. Landau, L. D., and E. M. Lifschitz, *Fluid Mechanics*, Addison-Wesley Co., Inc., Reading, Mass., 1959.

11. Goldstein, Herbert, "Attenuation by Condensed Water," in D. E. Kerr (ed.), *Propagation of Short Radio Waves*, McGraw-Hill Book Co., Inc., New York, 1951, p. 671.

12. Gunn, Ross, and G. D. Kinzer, "The Terminal Velocity of Fall for Water Droplets in Stagnant Air," *J. of Meteorology*, Vol. 6, 1949, pp. 243-248.

13. List, R. J., *Smithsonian Meteorological Tables*, Sixth Revised Edition, The Smithsonian Institute, Washington, D.C., 1958, p. 396.

14. Letestu, S., *International Meteorological Tables*, Secretariat of the World Meteorological Organization, WMO-No. 188, TP 94, Geneva, 1966, Table 4.16.

15. Wobas, H. B., F. W. Murray, and L. R. Koenig, *Calculation of the Terminal Velocity of Water Drops*, The Rand Corporation, P-4564, January 1971.