A Rand Computation Center Reference: Standard Atmosphere Calculations with JOSS

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ABSTRACT

This document describes a simple JOSS program that provides the values of temperature, pressure, and density of the U. S. Standard Atmosphere at any desired altitude from sea level to 700 km. It can be coupled with other JOSS programs where such values are needed as input quantities. For general use, the program permits the ready construction of model atmospheres and essentially replaces a version of MODAT (RM-4204-PR).

I. INTRODUCTION

This document describes a simple JOSS program that calculates the principal output parameters of the U. S. Standard Atmosphere.[1] The version available in the JOSS library, and detailed here, provides the standard values of atmospheric temperature, pressure, and density at any desired altitude between sea level and a height of 700 km. The program reproduces these standard atmosphere values to an accuracy of better than 1/2 percent over the complete altitude range, corresponding to a pressure variation of some twelve orders of magnitude. This accuracy is considerably in excess of our true knowledge of atmospheric conditions.

The program can serve two purposes. Primarily, it provides the user with accurate values of the standard atmosphere values without the necessity of looking up and interpolating tables, or approximating the standard atmosphere with inaccurate exponential or complex polynomial functions. Specifically, it can be coupled with other JOSS programs that require input values of atmospheric temperature, pressure, or density. For example, in the past, the program has been used in this way for satellite re-entry predictions.

A secondary use of the program is the construction of model atmospheres different from the U. S. Standard Atmosphere. For this use, it essentially replaces Version A of MODAT,[2] a digital computer program written in FORTRAN II.

II. PROGRAM DESCRIPTION

Execution and Example

An example of the use of the program is shown on page 3, and the program is reproduced on the two subsequent pages.

Start instruction is: "Do part 1."

Thereafter, the program demands the desired altitude level in kilometers, or fraction of kilometers, and provides the following output for this altitude:

Parameter	Unit	Letter Symbol
Altitude	kilometers	z
Temperature*	degrees absolute	t
Atmospheric Pressure	millibars	.001 · p
Atmospheric Density	grams per cc	s

Output Changes

To provide tabular output instead of single altitude level values: Instruct: "Do part 1."

(Ignore demand for altitude by pressing return key)

"Delete step 2.7."

"Type form 9."

"Do part 2 for z = ..."

Size Limitations

In its filed version, the program sets all necessary base data (for the U. S. Standard Atmosphere 1962) through parts 9 and 10. Once this is done, parts 1 and 9, and step 2.7 can be deleted.

The basic program consists only of parts 2, 3, and 4, a formula for the geopotential height h (step 9.1), and the stored base values. Start instructions are: "Do part 2 for z = ...," where z is the altitude at which values of temperature, pressure, or density are desired.

For specific applications, where it is known a priori that only certain altitude regions will be of interest, the necessary amount of base data (part 9) can be reduced as applicable. However, little advantage is gained in terms of computing time saved.

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* Kinetic below 90 km, molecular scale above.

Sample Program

```
Recall atmos from library.
Done.
Do part 1.
                U. S. STANDARD ATMOSPHERE, 1962

   Minimum output version for values of atmospheric temperature,
atmospheric pressure, and atmospheric density at any desired
altitude level between 0 km and 700 kilometers.
   Can be coupled to other JOSS programs as desired.
   [Reference:  R-1550/8  A Rand Computation Center Reference:
    Standard Atmosphere Computations with JOSS]

Altitude in kilometers = 3.125

Alt[km]    Temp[oK]   Press[mb]    Dens[g/cc]
  3.125     267.847    6.901 02     8.976-04

Altitude in kilometers = 17.75

Alt[km]    Temp[oK]   Press[mb]    Dens[g/cc]
 17.750     216.650    7.867 01     1.265-04

Altitude in kilometers = 612.2

Alt[km]    Temp[oK]   Press[mb]    Dens[g/cc]
612.200    2604.239    3.025-09     4.047-16

Altitude in kilometers = _
I'm at step 1.3.
Delete step 2.7.
Type form 9.
Alt[km]    Temp[oK]   Press[mb]    Dens[g/cc]
Do part 2 for z=0(100)400,402.25,500,700.
   .000     288.150    1.013 03     1.225-03

100.000     210.650    3.007-04     4.972-10

200.000    1400.879    1.335-06     3.320-13

300.000    1830.650    1.886-07     3.590-14

400.000    2160.650    4.038-08     6.511-15

402.250    2166.585    3.913-08     6.292-15

500.000    2420.650    1.098-08     1.580-15

700.000    2700.650    1.194-09     1.541-16
```

```
Type all.

1.01 Type "                    U. S. STANDARD ATMOSPHERE, 1962".
1.02 Line.
1.03 Type "   Minimum output version for values of atmospheric temperature,'
1.04 Type "atmospheric pressure, and atmospheric density at any desired".
1.05 Type "altitude level between 0 km and 700 kilometers.".
1.06 Type "   Can be coupled to other JOSS programs as desired.".
1.065 Type "   [Reference:  R-1550/8  A Rand Computation Center Reference: '
1.066 Type "    Standard Atmosphere Computations with JOSS]".
1.07 Line.
1.1 Do part 9.
1.3 Demand z as "Altitude in kilometers".
1.33 Type "Sorry; only between 0 and 700 km." if z>700 or z<0.
1.35 To step 1.3 if z>700 or z<0.
1.4 Line.
1.5 Type form 9.
1.7 Do part 2 for z=z.

2.1 Set i=first[i=20(1)41:Z(i)<=z<Z(i+1)].
2.2 Do part 3 for j=21(1)i if i\=20.
2.3 Do part 4.
2.4 Type z,t,(.001*p),s in form 10.
2.5 Line.
2.7 To step 1.3.

3.1 Set l=[T(j)-T(j-1)]/[H(j)-H(j-1)].
3.2 Set P(j)=P(j-1)*[T(j-1)/T(j)]**[m*g*10**5/(R*l)] if l\=0.
3.3 Set P(j)=P(j-1)*exp[[m*g*10**5]*[H(j-1)-H(j)]/[R*T(j-1)]] if l=0.
3.4 Delete P(j-1) if j\=21.

4.1 Set L=[T(i+1)-T(i)]/[H(i+1)-H(i)].
4.2 Set t=T(i)+L*[h-H(i)] if L\=0.
4.3 Set t=T(i) if L=0.
4.4 Set p=P(i)*[T(i)/t]**[m*g*10**5/(R*L)] if L\=0.
4.5 Set p=P(i)*exp[[m*g*10**5]*[H(i)-h]/[R*T(i)]] if L=0.
4.6 Set s=m*p/[R*t].

9.1 Let h=[X*z]/[X+z].
9.11 Set P(20)=1.01325*10**6.
9.21 Set R=8.31432*10**7.
9.22 Set X=6356.766.
9.23 Set g=980.665.
9.24 Set m=28.965.
9.32 Set T(20)=288.15.
9.321 Set T(21)=216.65.
9.322 Set T(22)=216.65.
9.323 Set T(23)=228.65.
9.324 Set T(24)=270.65.
9.325 Set T(25)=270.65.
9.326 Set T(26)=252.65.
9.327 Set T(27)=180.65.
9.328 Set T(28)=180.65.
9.329 Set T(29)=210.65.
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9.330 Set T(30)=260.65.
9.331 Set T(31)=360.65.
9.332 Set T(32)=960.65.
9.333 Set T(33)=1110.65.
9.334 Set T(34)=1210.65.
9.335 Set T(35)=1350.65.
9.336 Set T(36)=1550.65.
9.337 Set T(37)=1830.65.
9.338 Set T(38)=2160.65.
9.339 Set T(39)=2420.65.
9.340 Set T(40)=2590.65.
9.341 Set T(41)=2700.65.
9.342 Set T(42)=1.
9.420 Set H(20)=0.
9.421 Set H(21)=11.
9.422 Set H(22)=20.
9.423 Set H(23)=32.
9.424 Set H(24)=47.
9.425 Set H(25)=52.
9.426 Set H(26)=61.
9.427 Set H(27)=79.
9.528 Set Z(28)=90.
9.529 Set Z(29)=100.
9.530 Set Z(30)=110.
9.531 Set Z(31)=120.
9.532 Set Z(32)=150.
9.533 Set Z(33)=160.
9.534 Set Z(34)=170.
9.535 Set Z(35)=190.
9.536 Set Z(36)=230.
9.537 Set Z(37)=300.
9.538 Set Z(38)=400.
9.539 Set Z(39)=500.
9.540 Set Z(40)=600.
9.541 Set Z(41)=700.
9.542 Set Z(42)=701.
9.6 Do step 10.1 for k=20(1)27.
9.7 Do step 10.2 for k=28(1)42.

10.1 Set Z(k)=[X*H(k)]/[X-H(k)].
10.2 Set H(k)=[X*Z(k)]/[X+Z(k)].

Form 9:
Alt[km]    Temp[oK]    Press[mb]    Dens[g/cc]

Form 10:
___.___    ____.___    .........    .........
```

Mathematical Basis

The mathematical basis for the construction of model atmospheres in general is described in RM-4204-PR,[2] and for the construction of the U. S. Standard Atmosphere 1962 in reference (1).

Essentially, the program is based on the perfect gas law, the hydrostatic equation, and the concepts of molecular scale temperature and geopotential height. The structural input data [part 9] are molecular scale temperature values at a series of geometric or geopotential reference heights, and the relation between geometric and geopotential heights [part 10 and step 9.1] that expresses the variation of the gravitational field of earth over the applicable altitude regions.

From these base data, the program (a) determines the reference height interval applicable to the desired altitude [step 2.1], (b) integrates atmospheric pressure from sea level to the applicable lower reference height [part 3], (c) integrates further upward from this reference height to the desired altitude [part 4], and (d) calculates temperature, pressure, and density at this altitude.

For research uses, this formulation of the program permits the easy construction of other model atmospheres -- for other planets as well as for earth -- by simple changes of the base data. Ad hoc models for satellite re-entry[3] can be derived this way,[4] portions of other reference atmospheres can be duplicated, such as the U. S. Standard Atmosphere Supplements 1966,[5] or atmospheric parameters measured empirically by rockets and satellites can be compared to the standard values.

In addition, other parameters, such as lapse rates, scale heights, and engineering quantities, can be obtained by requesting print-out or by simply adding the appropriate formulas (let statements) to the basic program (see also reference 2).

For example (at altitude z):

Geopotential Temperature Lapse Rate:	Symbol L
Geopotential Altitude:	Symbol h
Gravitational Acceleration A(20):	Let A(20) = g · [X/(X + z)] * 2.

Geopotential Pressure Scale Height A(21):

Let A(21) = [R · t/(m · g)] · 10 * (-5).

Geopotential Density Scale Height A(22):

Let A(22) = t/{[(m · g/R) · 10 * 5] + L}.

Geometric Pressure Scale Height A(23):

Let A(23) = A(21) · g/A(20).

Geometric Density Scale Height A(24):

Let A(24) = A(22) · g/A(20).

III. PROGRAM COUPLING

Symbol Substitution

The basic program in its general version uses the following 18 letters, where only the underlined ones are requisite symbols:

g h <u>i</u> <u>j</u> k ℓ m p s t z ; <u>H</u> L <u>P</u> R <u>T</u> X <u>Z</u> .

The following letters can be substituted for without causing program difficulties:

Letter	Appearance in Steps
g	3.2 3.3 4.4 4.5 9.23
h	4.2 4.5 9.1
k	9.6 9.7 10.1 10.2
ℓ	3.1 3.2 3.3
m	3.2 3.3 4.4 4.5 4.6 9.24
p	2.4 4.4 4.5 4.6
s	2.4 4.6
t	2.4 4.2 4.3 4.4 4.6
z	1.3 1.33 1.35 1.7 2.1 2.4 9.1
L	4.1 4.2 4.3 4.4 4.5
R	3.2 3.3 4.4 4.5 4.6 9.21
X	9.1 9.22 10.1 10.2

By substituting A(1), A(2), ... A(12) for these symbols, the program then makes use of only 7 letters (out of 52 available).

Coupling

To couple with other JOSS programs, after possible symbol duplication has been checked:

Delete part 1.

Delete step 2.7

Delete forms 9 and 10

Store base data through "Do part 9."

Provide desired altitude input as symbol z [in kilometers] and instruct to "Do part 2 for z = z."

Delete step 2.4, and link desired output quantity (e.g.: atmospheric density s) with whatever use is to be made of the program.

Engineering Quantities

Unit changes of the input quantity altitude in kilometers can be made by relating the new input quantity (e.g. altitude in feet) to z. For example,

Let z = [altitude in feet] · 3.048 · 10 * (-4).

Unit changes of the output quantities (e.g. density in grams per cubic centimeter) can be made similarly. For example,

Let [density in pounds per cubic foot] = s · 62.428.

REFERENCES

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4. Schilling, G. F., Density and Temperature Variations in the Intervening Layer, P-3598, The RAND Corporation, Santa Monica, 1967.
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