ASSETS AND LABOR SUPPLY

PREPARED FOR THE DEPARTMENT OF
HEALTH, EDUCATION, AND WELFARE

JAMES P. SMITH

R-1728-HEW
MAY 1975
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This report was sponsored by the Office of the Assistant Secretary for Planning and Evaluation (ASPE) in the Department of Health, Education, and Welfare, under Grant No. 90088-D-74-01. Rand's main purpose in this research is to study female labor supply. In our proposal, we suggested that our research would have three central themes: (1) life cycle aspects of labor supply; (2) the dimensions of labor supply/labor force participation, weeks worked, and hours per week; and (3) long-term trends in female labor supply. In work nearing completion, Giora Hanoch has developed a theoretical model to deal with the issue of the dimensions of labor supply. James P. Smith, the author of the present report, has work under way on the time series pattern of female labor supply.

This report addresses some theoretical problems in life cycle models. It argues that many of the cross-sectional female labor supply functions that include assets or net worth as explanatory variables are seriously misspecified.
SUMMARY

This study examines the role of assets in labor supply functions. Although assets have been used with increasing frequency to measure the response of hours worked to nonwage-related income, I argue that it is incorrect to include them in a labor supply function. Using a simple life cycle model to examine the relationships one might expect between working hours and assets, I show that both are simultaneously determined by similar economic forces, and that the correlation between them should not be accepted as evidence of a causal sequence from assets to market work.

The savings behavior implied by the life cycle model is derived in Section II of this report. It is shown that savings are determined by a life cycle variation in the wage rates of family members and the interplay of interest rates and time preferences. The implied asset-hours relationship is discussed in four separate examples, each being a special case of a more general model and each highlighting one of the economic forces determining assets and market hours. Within each example, there results a statistical relationship between net assets and labor supply, but this association is independent of any wealth effect arising from assets to the amount of labor supplied.

In Section III, some empirical applications are attempted, using data from the 1967 Survey of Economic Opportunity. The most direct test consists of estimating a life cycle savings function. The signs of the coefficients seem to be generally consistent with the predictions of the model.
ACKNOWLEDGMENTS

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GLOSSARY OF SYMBOLS

$A_t$ Assets in period $t$
$A_0$ Initial assets
$E_{mt}, E_{ft}$ Total earnings in period $t$ by husband and wife
$k_t$ Share of full wealth accounted for by commodities consumed in period $t$
$M_t, F_t$ Amount of male (husband's) time and female (wife's) time spent in home production in period $t$
$N$ Number of periods in family's horizon (equal to life span)
$N_{mt}, N_{ft}$ Amount of husband's time and wife's time spent at work in period $t$
$P$ Lifetime price index of commodities
$R$ Family's level of full wealth
$r$ Interest rate
$S_t$ Savings in period $t$
$s_{it}$ Share of total cost of commodities in period $t$ accounted for by input $i$ ($i = m, t, or x$)
$T$ Number of hours in time period
$T^*$ Total time available over lifetime; equal to $NT$
$U$ Family utility
$V$ Number of working years in lifetime
$\bar{w}_m, \bar{w}_f$ Average discounted values of $w_{mt}$ and $w_{ft}$ over lifetime
$w_{mt}, w_{ft}$ Husband's and wife's wage in period $t$
$X_t$ Total quantity of market goods purchased in period $t$
$Z_t$ Level of consumption of commodities in period $t$

$\alpha$ Index of time preference
$\eta_t$ Wealth elasticity of consumption in period $t$
$\pi_t$ Shadow price of commodities in period $t$
$\sigma_c$ Elasticity of substitution in consumption between time periods
$\sigma_{ij}$ Allen partial elasticity of substitution in home production between inputs $i$ and $j$ ($i, j = m, f, and x$)
"Knowledge of the variation in consumption as between different households having different asset holdings would give little information as to how a household would react if its assets were increased unexpectedly by a given amount. ... this failing occurs because the observed asset holdings do not just happen to be there; instead they reflect the life plan of the individual."

I. INTRODUCTION

This research examines the role of assets in labor supply functions. Its ultimate aim is to increase our understanding about the determinants of female labor supply. To accomplish this goal, a necessary prerequisite is that the theoretical and statistical models be correctly specified. In recent work, variables measuring assets have been used with increasing frequency to measure the response of hours worked to nonwage-related income. Empirical labor supply studies have previously used an aggregate of all current period nonearnings income to estimate pure income effects. The estimated income slopes were disappointing because the income variable either had the wrong sign (positive), implying that leisure was an inferior good, or it was sufficiently small so that compensated own-wage slopes in the labor supply equation remained negative. In addition, the estimated income response exhibited considerable instability from study to study.

Many reasons were suggested to explain these failures. First, it was well known that the income measure was characterized by severe underreporting, leading, on standard errors in variables grounds, to biased income coefficients. Second, nonemployment income contains income that does not correspond to the theoretical construct. Unemployment compensation, disability insurance, and pension income, for example, are usually contingent upon the absence of market work. Since these were often included in total income, a spurious negative correlation between work and income was introduced. Third, data on assets and liabilities were unavailable. Inputed income flows from such assets are quite large for many individuals. A more detailed questioning of sources of income has been undertaken in recent microdata files, and in some surveys attempts were made to measure the market value of a

1See Greenberg and Kosters (1970); Fleisher, Parsons, and Porter (1973); and DaVanzo, DeTray, and Greenberg (1973).

2Such studies are too numerous to list. One example is Greenberg and Kosters (1970).
variety of assets and liabilities. Many estimates of the income response of hours worked are available, using these new data sources, but it is apparent that there is still little tendency for convergence across different studies.

I believe that the theoretical response to these empirical findings has been misplaced. Greenberg and Kosters (1970) assert that taste variation regarding the process of asset accumulation is a serious source of bias. They argue that some individuals prefer to acquire assets because of their conservative or risk-averse nature, and to achieve their desired asset levels, such individuals will tend to work longer hours. Greenberg and Kosters contend that it is necessary to purge assets of this taste factor before one uses them to estimate wealth effects. In their study, they develop statistical techniques that they hope will do precisely that.

Fleisher, Parsons, and Porter (1973) take a different approach. In their model, individuals can be temporarily out of equilibrium with more or fewer assets than they prefer. A person with excess assets will try to restore his position by working less, and one with deficient assets, by working more. Like Greenberg and Kosters, they attempt to control statistically for this secondary relation between assets and hours worked before using assets for their primary purpose—wealth effects.

While there may be truth in both of these views, much of the discussion concerning assets and labor supply appears to have concentrated on second-order effects, namely, taste variation or disequilibrium analysis. Theoretical treatment of the relation between assets and hours worked at a more elementary level seems appropriate, and this report is an attempt to provide that treatment. To a large extent, this represents, as the quote from Brumberg and Modigliani indicates, relearning many of the lessons from the older consumption-function literature and applying them to a new problem.

---

3 Three of the more prominent new data sets—The Survey of Economic Opportunity, The Income Dynamics Panel, and the National Longitudinal Survey—contain information on assets. For a complete list of the asset information available in all three, see Greenberg (1972).
The principal deficiency in many models is the neglect or lack of emphasis on the life cycle dimension. To understand the relationship between assets and hours of work, an adequate theory of asset accumulation and savings is needed. Assets have too often been forced into the confines of a one-period model, but it is only when one considers the multiperiod problem that it makes sense to speak of the asset and savings behavior of individuals. In this report, a simple life cycle model is used to explore the relationships that one might expect to find between working hours and assets. The model shows that both are simultaneously determined by similar economic forces, and that the correlation between them should not be accepted as evidence of a causal sequence from assets to market work.

The life cycle model is derived in Section II. Since the implications for variation in working hours and consumption have been explored in depth by Ghez and Becker (1972), Heckman (1971), and Smith (1973), they will not be discussed in great detail here. Rather, they will be used as intermediate inputs to derive the assets and savings-age profiles. The asset profile will be discussed in four separate examples--each being a special case of a more general model and each highlighting one of the economic forces determining assets and market hours. Within each example, a statistical relation between net assets and labor supply results, but this association is independent of any wealth effect running from assets to the amount of labor supplied.

In Section III, some empirical applications are made using data from the 1967 Survey of Economic Opportunity (SEO). First, the net worth age profiles derived from the SEO data are examined. The characteristics of these profiles tend to support the approach taken in this report. The most direct test of the model consists in estimating a life cycle savings function. Although the SEO savings data are quite poor, and the statistical significance of individual coefficients is consequently not overwhelming, the signs of coefficients seem generally consistent with the predictions of the model.
II. THE LIFE CYCLE MODEL

When viewed in a lifetime perspective, savings depends entirely on the consumption and income profiles. If the optimal levels of consumption and income are not coincident, savings allows them to differ by the transfer of funds between time periods. In order to know the optimal amount of savings, one must first derive the consumption and income profiles. The income profile, in turn, depends on the paths of market hours and wage rates. Ghez and Becker (1972), in an important work, developed a model that dealt with the life cycle consumption and hours profiles. In a previous paper (Smith, 1972), I extended their model to deal with the family context.

A model is developed in the appendix from which derived demand equations are obtained giving the amount of time and goods required in home production at every age.\(^1\) This model allows the intensity of market participation to vary with age, due to temporal variations in wages and other variables that elicit timing responses about the long-run desired levels. In deciding on the number of hours to supply to the market, an individual is actually confronted with two problems. Given his long-run or permanent values of wealth and his wages, he must determine the lifetime levels of market time and consumption of market goods. In addition, since he is faced with temporal variation in wages and other variables, he must decide on an optimal timing of hours and consumption. The intensity of market participation is not constant over the cycle, because factors are present that change the demand for commodities and the marginal costs of household production.

The demand equations for home time (\(M_t\)) and market goods consumption derived in the appendix are\(^2,3\)

\(^1\)For a complete development of this model, see Smith (1973).
\(^2\)All symbols are defined in the Glossary of Symbols.
\(^3\)For expository simplicity, the model at this point is confined to a single-person family. For an extensive treatment of the family context, see Smith (1973). This assumption is relaxed in Section III.
\[
\frac{dM_t}{M_t} = -(s_{mt} \sigma _c + s_{xt} \sigma _{MX}) \frac{dw_{mt}}{w_{mt}} + \sigma _c (r - a), \quad (1)
\]

\[
\frac{dX_t}{X_t} = s_{mt} (\sigma _{MX} - \sigma _c) \frac{dw_{mt}}{w_{mt}} + \sigma _c (r - a). \quad (2)
\]

Equations (1) and (2) indicate that the work time and market goods consumption, given the parameters of the utility and production function, are determined by variations in the price of time \( w_{mt} \), the rate of interest, and the rate of time preference. To illustrate: As an individual's real wage increases over the life cycle, the amount of his time spent in the nonmarket sector will decline for two reasons. Because the price of one of the inputs is rising, the relative price of future commodities rises. The resulting decline in future consumption will, due to this scale effect, reduce the demand for home time. The magnitude of this effect (represented by \( s_{mt} \sigma _c \)) depends on the possibilities for intertemporal substitution (i.e., the larger \( \sigma _c \), the more elastic is the demand curve for commodities) and the share of time in total costs of nonmarket production.

In addition to this intertemporal substitution between commodities, there exists the possibility of substitution in the production process. As \( w_{mt} \) increases, market goods will be substituted for time. This effect \( (s_{mt} \sigma _{MX}) \) will also lead to a decline in the use of time as the real wage rises. It follows, then, that in those periods when the real wage is high, the model predicts, \textit{ceteris paribus}, that hours of market work will also be high. Note that in contrast to the traditional one-period labor-leisure choice, the sign of this effect is unambiguous. Since full wealth is fixed in this analysis, there are no income effects. It is, of course, the existence of income effects in the static theory that gives rise to the possibility of a negatively sloped curve of hours.

The roles of a positive interest rate and the rate of time preference are the standard Fisherian ones. By lowering the relative price of future commodities, a positive interest rate will raise future consumption levels for both market goods and nonmarket time. Time
preference for the future ($\alpha < 0$) will obviously have the same effect of increasing the desired levels of consumption in the future.

**ASSETS, SAVINGS, AND LABOR SUPPLY**

Equations (1) and (2) describe the age profiles of market goods consumption and hours of market work. If an exogenous wage pattern is assumed, the life cycle pattern of earnings is also given. \(^4\) These earnings and consumption profiles define the savings behavior of the individual at each age, and by appropriate accumulation of savings, the asset position of each point in the life cycle is determined.

Several potential motivations for savings are not included in this model. The miser’s motive in which assets provide utility directly, the precautionary motive in which attitudes toward risk must be considered explicitly, and the complications that arise because of durable goods are all heroically ignored. This does not reflect my assessment concerning the relative importance of these motives, but rather that the points I am attempting to make can be illustrated in a simpler world.

Savings is defined in the conventional manner as the difference between current income and current market-goods consumption. Thus

\[ S_t = E_t - X_t + rA_t, \]  

(3)

while net assets at any age $t$ equal

\[ A_t = A_0 + \int_0^t S_t \, dt. \]  

(4)

**CASE 1--THE FISHERIAN APPROACH**

Since Irving Fisher’s work (1965), economists have recognized that one factor determining the time pattern of consumption is a divergence between the rate of interest and the rate of time preference. In Case 1, it is the only factor allowed to influence the income, consumption, and savings profiles. The impact of other factors can be eliminated

\[^4\] This assumption is discussed in Section IV.
with two assumptions: (1) the wage level is constant over the cycle and the same for all individuals; and (2) initial and desired terminal assets \((A_n)\) are zero. Since all individuals have the same real wealth, the life cycle demand equations for goods and time simplify to

\[
\frac{dM_t}{M_t} = \frac{dX_t}{X_t} = \sigma_c (r - \alpha). \tag{5}
\]

Consumption and earnings are equal and age-invariant when the interest rate equals the subjective rate of time preference (Fig. 1). Of course, in this case, savings and net assets are identically zero at every age. If \(r\) and \(\alpha\) differ, the time paths of consumption and income are no longer coincident. For example, consider a more optimistic individual who does not discount the future as severely. Compared with the first individual, this optimist will consume commodities \((Z_t)\) in relatively greater proportions in the latter stages of the cycle. The derived demands for market goods and home time are also larger during the later ages, imparting a positive slope to the age profiles of consumption and household hours. Since wages are constant, the age earnings profile is determined exclusively by the hours profile, and market earnings will decline monotonically with age. The change in savings with age is

\[
dS_t = -\pi_t \frac{Z_t \sigma (r - \alpha) + r dA_t}{X_t}. \tag{6}
\]

In the early stages of the life cycle, earnings exceed consumption, and the savings generated become positive net assets. In succeeding periods, these assets produce income flows augmenting earnings. Net

---

5 Throughout, I am also assuming that \(r\) is constant. It is well within the spirit of Fisher's work to allow \(r\) be a function of the amount borrowed, but this complication is ignored.

6 Using Eq. (5), we have

\[
\begin{align*}
dS_t &= -M_{t/m_t} \frac{dM_t}{M_t} - X_t \frac{dX_t}{X_t} + r dA_t, \\
dS_t &= -(M_{t/m_t} + X_t) \sigma (r - \alpha) + r dA_t.
\end{align*}
\]
Fig. 1 — The Fisherian model: $r > \alpha$
assets will continue to grow until the income and consumption profiles cross and savings are zero. After this age, the individual dissaves and the rate of his dissavings increases until net assets are once again zero at the end of the life cycle.

In this Fisherian world, the empirical association between assets and labor supply depends on the life cycle stage. At younger ages, there exists, holding age constant, a positive correlation between assets and market work, but the sign of the correlation reverses for the older-age groups. Those individuals with a stronger future-time preference have larger assets at every age, but they work more only during the younger ages. Even if we confine ourselves to periods in which the relation between nonearnings income and market work remains positive, the size of an estimated income elasticity is a negative function of age.\(^7\) More importantly, the empirical relation between assets and work does not reflect a wealth effect at all, for the wealth of all individuals is identical by assumption. Taste variation among individuals in their respective rates of time preferences will determine the extent of the observed relation between assets and market work.

CASE 2—THE PRICE OF TIME

Chez and Becker, in their work on life cycle consumption, introduced another factor accounting for a nonconstant consumption age profile. Becker, and especially Chez, demonstrated that age variation in the price of time may also affect the time path of market consumption. We can isolate the pure life cycle price-of-time effect with the following simplifying assumptions: (1) the rate of interest, the rate of time preference, and initial and terminal assets are all zero; and (2), across all individuals, full wealth is constant. The second assumption

\(^7\)For individuals whose rate of present time preference exceeds the rate of interest, earnings will be relatively high and consumption low at the older ages. Net assets will always be negative, and once again assets and market hours are both low during the younger ages. In the text, I always consider the case where \(r\) exceeds \(\alpha\) because that appears to be true empirically.
follows if the arithmetic sum of life cycle wages is the same for all individuals. However, the age pattern of wage rates is allowed to vary among individuals. In this second case, savings are

\[ S_t = E_t - X_t, \]

\[ dS_t = \left[ w_{mt} N_{mt} + \sigma w_{mt} M_{mt} \right] \frac{dw_{mt}}{w_{mt}}. \quad (7) \]

Note that the savings profile is independent of the production elasticity of substitution \( \sigma_{MX} \). The increased consumption caused by the substitution of goods for time is matched dollar for dollar by additional earnings. This result holds because the elasticity of factor demand, the production effect only, is proportional to the share of the other factor, whereas the effect of this increased factor demand on savings is proportional to the share of the factor itself.  

---

Taking the differential of the saving equation gives

\[ dS_t = w_{mt} N_{mt} \frac{dw_{mt}}{w_{mt}} - w_{mt} M_{mt} \frac{dM_t}{M_t} - \frac{dX_t}{X_t} X_t. \]

If we substitute the life cycle demand for time and goods,

\[ \frac{dM_t}{M_t} = - (\sigma_{MX} s_t + \sigma c_{mt}) \frac{dw_{mt}}{w_{mt}}, \]

\[ \frac{dX_t}{X_t} = s_{mt} (\sigma_{MX} - \sigma c) \frac{dw_{mt}}{w_{mt}}, \]

\[ dS_t = \left[ w_{mt} N_{mt} + (\sigma_{MX} s_t + s_{mt} \sigma c_{mt})w_{mt} M_{mt} - X_t (\sigma_{MX} - \sigma c_{mt}) \right] \frac{dw_{mt}}{w_{mt}}, \]

\[ dS_t = \left[ w_{mt} N_{mt} + s_{mt} \sigma (M_{mt} + X_t) \right] \frac{dw_{mt}}{w_{mt}}. \]

---

An interesting special case of this model is when the intertemporal utility function is Cobb Douglas. In that case, a dollar per hour increase in the wage leads to a dollar per hour increase in savings.
In this second case, consumption and earnings are equal and stationary for individuals with flat wage profiles. An individual who faces a rising wage profile has an incentive to concentrate his market time in the future, giving his earnings profile a positive slope. Since the rising wage trend will also increase the relative price of future commodities, the consumption of future commodities will fall. Due to the scale effect, this will reduce the demand for future market-goods consumption. In addition, a higher wage provides an incentive to substitute market goods for time. If intertemporal commodity substitution outweighs production substitution, the market-goods age profile will have a negative slope.

The profiles of such an individual are drawn in Fig. 2. Initially, consumption exceeds earnings, and this individual dissipates. Net assets become negative and decline further until savings are zero. In the later stages of the life cycle, positive savings occur as this individual finances his earlier consumption. At young ages, those with the steepest wage profile will have the lowest levels of net assets and market work. Once again, the sign of this correlation reverses for older groups where individuals now observed working more have fewer net assets.

These conclusions will not change if we consider the case in which production substitution is stronger than the ability to substitute between commodities over time. Although both consumption and earnings will now increase with age, earnings must rise at a more rapid rate than market-goods consumption. This follows because sufficient earnings are generated to finance the additional consumption due to the production effect alone. The scale effect will further increase earnings, but will lower consumption.

In this example the sign and the magnitude of the relation between assets and market work also change as one moves through the life cycle. The strongest positive (negative) correlation occurs at the youngest (oldest) ages. In terms of model specification, it would be inappropriate to include current wages and net assets as regressions in the labor supply equation. The net asset term would most likely capture (holding current wages constant) the effects of the past and future
CC = Consumption
EE = Earnings

Fig. 2 — Life cycle wage variation
wage streams. One should certainly not interpret the asset term as measuring wealth effects.

CASE 3--HUMAN CAPITAL VARIATION

The first two cases concerned the timing of work and expenditures that were induced by differential costs of economic activity at different ages. In those cases, we were attempting to follow individuals over their life cycles. In this third case, and the one that follows, the emphasis shifts to factors that represent permanent differences across individuals. Case 3 concerns human capital wealth variation and Case 4, nonhuman capital wealth variation.

In the previous example, all wage variation was a life cycle phenomenon. More realistically, some of the wage differences among individuals represent dispersion in wage levels and wealth (initial human capital). In Case 3, the savings behavior generated by nonlife cycle wage variation is examined. To isolate the variation in initial human capital, I shall employ the age neutrality assumption; i.e., if individual j's wage at t exceeds individual i's by \lambda percent, j's wages exceed i's by \lambda percent at all ages.

Since we want to examine variation in the demand for time and goods across individuals at a particular age, the one-period model is appropriate. 10 The change in savings induced by an increase in the initial human capital stock is

\[\frac{dM_t}{M_t} = (s_{x_t} n_t - s_{x} n_{MX}) \frac{dw}{w_{mt}},\]

\[\frac{dX_t}{X_t} = (n_t s_{x} + s_{mt} n_{MX}) \frac{dw}{w_{mt}},\]

\[dS_t = w \left( \frac{N}{mt} - M \frac{w_t}{mt} - \frac{dM_t}{M_t} - \frac{dX_t}{X_t} \right),\]

\[dS_t = [w \left( \frac{N}{mt} - M \frac{w_t}{mt} (n_t - \sigma_{MX}) s_{x} - X_t (n_t s_{x} + s_{mt} n_{MX}) \right) \frac{dw}{w_{mt}}],\]

\[dS_t = [w \left( \frac{N}{mt} - M \frac{w_t}{mt} s_{x} n_t - X_t s_{x} n_t \right) \frac{dw}{w_{mt}}].\]

10 Using Eq. (19) in the appendix, and assuming zero initial and terminal assets and that the household production function is homogeneous of degree one, we may write the demand for time and goods as
\[ dS_t = [w_{mt} N_{mt} - x_t \eta_t] \frac{dw_{mt}}{w_{mt}}. \] (8)

Note that savings at any age is independent of the consumption and production elasticities of substitution. The consumption term is eliminated, since we are going across profiles at the same point in the cycle. The production term does not appear because the increased market hours due to the production effect are sufficient to finance the increased market goods induced by the substitution in production.

Generally, if wealth elasticities are unity, the additional (dis) savings generated by a 100 percent increase in wage rates at every age equals the original level of (dis) savings. When individual period-consumption wealth elasticities differ from unity, the correlation between these elasticities and the original savings position must be considered. If wealth elasticities are higher in those periods when savings would have occurred, the additional savings induced will be smaller than the unit elastic case. If the wealth effect exceeds the substitution effect, individual j will work less than individual i and we will observe a negative correlation between hours and net assets. The association between assets and market work depends on the familiar conflict between the income and substitution effects, with a positive correlation implied if the substitution effect exceeds the income effect. Time-series evidence indicates that for many the income effect may dominate, but for women, the substitution effect appears stronger.\(^{11}\) Hence, it is not inconsistent with economic theory to find a negative empirical relation between assets and market work for men and a positive one for women.

This case may best be illustrated with an example: Two individuals have age invariant wages, but individual i's wage is higher than j's. If the rate of time preference equals the interest rate for individual i, consumption and earnings will be identical and savings and net assets will be zero. For individual j, if the wealth elasticity of consumption

\(^{11}\) See Mincer (1962).
is the same in every period (and, hence, equal to unity), his savings will also be zero:

\[ dS = w_{mt} N_{mt} - X_t = 0. \] (9)

However, if the wealth elasticity for future consumption exceeds that of present consumption (and therefore is greater than unity), positive savings results at young ages and dissavings during the older ages. This would lead to a positive asset level at every point in the cycle. The savings and asset positions of individuals reflect only their wage levels and profiles and supply no information that would enable us to measure wealth elasticity for leisure.

CASE 4—NONHUMAN WEALTH VARIATION

In the previous cases, initial and desired terminal assets were zero by assumption. In this final case, individuals differ in their initial asset holdings. However, empirical and theoretical problems are encountered before one may use these exogenous assets to estimate wealth effects in labor supply equations. Because it is difficult to identify assets that are completely exogenous and independent of life cycle behavior, these assets are of little help for current empirical work. Conceptually, \( A_0 \) corresponds to the discounted value at the beginning of the horizon (working life) of the entire stream of exogenous assets that one receives over a lifetime. Exogenous assets certainly should not be equated with assets actually held at the beginning of the horizon. Inheritances, which presumably qualify as part of \( A_0 \), are typically not received at young ages—indeed the actual receipt may occur a good deal later. To the extent that such future receipts are anticipated, their discounted value should be part of the asset variable. Clearly, reported net asset figures in most data sets omit most of these expected future receipts.

Even if adequate data on exogenous assets existed, interpreting their effects on labor supply is not straightforward. Although assets

\footnote{This is equivalent to allowing the rate of time preference to depend on the level of wealth.}
are usually received in a single year or over a relatively few time periods, their receipt will set in motion consumption and earnings patterns in all future years that will alter expected savings. The effect on savings of an increase in $A_0$ is

$$dS_t = -k_t \eta_t A_0 + r dA_t. \tag{10}$$

The savings behavior generated in the first period is

$$dS = (r - k_t \eta_t) A_0. \tag{11}$$

Savings will increase if the rate of return on assets exceeds the share of commodity consumption in the initial period times the wealth elasticity of consumption. In the initial period, if $\eta_t$ is in the vicinity of unity, $r$ is likely to exceed $k_t$ and savings will increase; but in later periods, savings must decrease in order to bring assets back to their terminal state. The profiles of two individuals with different initial assets are illustrated in Fig. 3. The reduction in labor supply that results from the increase in $A_0$ will be the same in all periods if $\eta_t$ is unity. Because future savings behavior of an individual is altered by the higher initial assets, the difference in

\[\text{Footnote 13: Once again, taking total differentials of the savings identity gives}\]

$$dS = \frac{dM_t}{mt} - \frac{dX_t}{X_t} + r dA_t,$$

where

$$\frac{dM_t}{M_t} = \frac{dX_t}{X_t} = \eta_t \frac{dA}{A R},$$

so

$$dS_t = -s_t \left( M_t + X_t \right) (\eta_t \frac{dA}{A R}) + r dA_t,$$

$$dS = -\eta_t Z_t \eta_t \frac{dA_0}{R} + r dA_t.$$

It is the difference between $A_0$ and $A^n e^{-rt}$ that is really important. If a dollar increase in $A_0$ increased transfers to future generations by $A^n e^{-rt}$, then the consumption, hours, and earnings profiles would be unaffected.
nonlabor income (rA_t) between any two people with different initial assets will change over the life cycle. Even in this model, the estimated wealth elasticity of leisure depends on the life cycle stage. No unique estimate of this wealth elasticity is possible if current assets are used.
III. SOME EXTENSIONS AND QUALIFICATIONS

The essential features of the savings model have now been set forth. Before the model can be tested empirically, it is necessary to consider a few potential problems. Although these will not affect the basic points made above, they are likely to be important in designing and interpreting empirical work.

THE FAMILY CONTEXT

The arguments advanced thus far would not be altered substantially if we extended the model to the family context. If there are two working members in the family (the husband and wife), savings is redefined as

\[
S_t = w_{mt} N_{mt} + w_{ft} N_{ft} - X_t + rA_t. \tag{12}
\]

Using the life cycle demand equations for male time, female time, and market goods, one can show that the change in savings is

\[
dS_t = \left[ w_{mt} N_{mt} + w_{mt} M_{tc} \right] \frac{dw_{mt}}{w_{mt}} + \left[ (w_{ft} N_{ft} + w_{ft} F_{tc}) \right] \frac{dw_{ft}}{w_{ft}} - \pi_{tc} (r - \sigma). \tag{13}
\]

Using the life cycle demand equations for male time, female time, and market goods,

\[
\frac{dM_t}{M_t} = s_{mt} (\sigma_{MM} - \sigma_c) \frac{dw_{mt}}{w_{mt}} + s_{ft} (\sigma_{MF} - \sigma_c) \frac{dw_{ft}}{w_{ft}} + \sigma_c (r - \alpha),
\]

\[
\frac{dF_t}{F_t} = s_{mt} (\sigma_{MF} - \sigma_c) \frac{dw_{mt}}{w_{mt}} + s_{ft} (\sigma_{FF} - \sigma_c) \frac{dw_{ft}}{w_{ft}} + \sigma_c (r - \alpha),
\]

\[
\frac{dX_t}{X_t} = s_{mt} (\sigma_{XM} - \sigma_c) \frac{dw_{mt}}{w_{mt}} + s_{ft} (\sigma_{XF} - \sigma_c) \frac{dw_{ft}}{w_{ft}} + \sigma_c (r - \alpha).
\]

Concentrating on the husband's wage term, we have
The wife's wage coefficient is identical to the husband's, with the appropriate relabeling of subscripts. The absolute magnitude of a dollar change in male wage should exceed that of a dollar change in a female wage. The male wage effect is larger, because of the relative home specialization of females, as long as the elasticity of substitution between time periods is not large. The female coefficient would exceed that of the male only if \( \sigma_c > (N_{mt} - N_{ft})/(F_t - M_t) = 1 \). Empirical evidence suggests that \( \sigma_c \) is less than one, so the male wage coefficient will be higher than that of the female.\(^2\)

The insights that the family context provides for understanding asset accumulation are not captured in this simple model. One neglected factor is the influence of children on a family's savings behavior. A complete life cycle theory that includes the optimal timing and spacing pattern of children is outside the scope of this report, but I will sketch out some of the issues involved and present tentative empirical findings in Section III.

**SECULAR CHANGES**

In the derivation of the life cycle model, I assumed that lifetime full wealth did not change from one age to the next. Because the model will be tested using simulated cohorts derived from cross sections, this assumption must be relaxed. The negative correlation between age and savings implied by the life cycle argument could be

\[
\frac{dS_t}{S_t} = [w_{mt}N_{mt} + S_M \sigma_c (N_{mt} - N_{ft} - F_t - M_t) + X_t] \\
- s_{mt} (\sigma_M M_{mt} + \sigma_M X t + \sigma_M M_{mt}) \frac{dM_{mt}}{w_{mt}} + \ldots
\]

\[
\frac{dS_t}{S_t} = [w_{mt} N_{mt} + w_{mt} M_{mt} \sigma_c] \frac{dM_{mt}}{w_{mt}} + \ldots
\]

\(^2\)For an attempt to estimate \( \sigma_c \), see Ghez and Becker (1972).
negated if the dissimilarities among cohorts are large. In any cross-sectional survey, two individuals who differ in age are not only observed at alternative points in their progress through the life cycle experience, but they are also members of distinct cohorts. The measured age difference captures both a movement along a life cycle hours path and across the profiles of different cohorts; the age thus becomes, in part, a cohort index.

Perhaps the most systematic cohort bias results from secular trends in wages and real wealth that make younger cohorts in any cross-section wealthier than their predecessors. To illustrate: If wages increase over time by \( \lambda \) percent, \(^3\) nominal wealth \((R)\) will decline by \( \lambda \) percent as we move toward older ages in a cross-section (or equivalently from younger to older cohorts). \(^4\) Real wealth will decline by less than \( \lambda \) percent, since the higher wages of the younger cohorts increase the cost to them of producing a given amount of household commodities. For each new cohort, household production costs \((P)\) rise by \( s_{mt} \lambda \) percent. \(^5\)

\(^3\)The point made in the text does not depend on the assumption that the secular rises in male and female wage rates are identical. Although the rates of growth of male and female wages can differ, it is only essential that these secular growth rates be constant. I am also assuming that all initial wealth is in the form of human capital.

\(^4\)Full wealth is defined as

\[
R = T \int_0^N w_{mt} e^{-rt} dt,
\]

\[
\frac{dR}{R} = T \frac{dw_{mt}}{w_{mt}} \int_0^N w_{mt} e^{-rt} dt,
\]

\[
\frac{dR}{R} = -\lambda.
\]

\(^5\)For a change in price in any simple period, we have

\[
\frac{dp}{p} = \sigma \frac{-1}{\gamma} \frac{d\tau}{\gamma} \left( \frac{1 - \sigma}{\gamma} \right) \frac{d\tau}{\gamma} ;
\]

and summing across all ages,
To illustrate: If time accounts for 75 percent of the cost of household production, and if wages rise over time by 4 percent per year, real wealth will increase at an annual rate of 1 percent.

Including the secular rise in real wages, the savings function can be rewritten as

$$dS_t = \frac{dw}{w_{mt}} \left[ N_{mt} \hat{w} N_{mt} + \sigma_c \hat{w} M_{mt} \right]$$

$$- \pi_t \left[ \sigma_c (r - \alpha) - \lambda [s_{xt} + s_{mt} \sigma_c] + r \right] dA_t.$$

Secular increases in real wealth will bias the age term in our regression towards positive values ([s_{xt} + s_{mt} \sigma_c] is necessarily

$$\frac{dp}{p} = \frac{\sigma_c - l}{w_{mt}} \left( \int_0^N e^{c_{mt} \pi t} \left[ \sigma_c \right] \left[ s_{mt} \right] e^{-rt} dt \right),$$

$$\frac{dp}{p} = \frac{dw}{w_{mt}} \int_0^N k_t s_{mt} dt,$$

$$\frac{dp}{p} = \frac{dw}{w_{mt}} s_{mt}.$$

6. Rewriting the consumption demand equation, we obtain

$$Z_t = \mathcal{R}(\sigma_c - 1) \pi_t - \sigma_c e^{(r - \alpha) \sigma_c},$$

$$\frac{dZ_t}{Z_t} = -\lambda + (1 - \sigma_c) s_{mt} \lambda + \sigma_c (r - \alpha) - \sigma_c \frac{dw}{w_{mt}},$$

$$\frac{dZ_t}{Z_t} = -\sigma_c s_{mt} \frac{dw}{w_{mt}} + \sigma_c (r - \alpha) - \lambda [s_{xt} + s_{mt} \sigma_c].$$
positive). If this bias offsets the life cycle effect, the age coefficient in a savings function need not be negative. There are two reasons for this bias. As we increase age, real wealth now decreases by $s_{xt}^\lambda$ percent, depressing consumption and increasing saving. In addition, the life time price index falls by $s_{mt}^\lambda$ percent. Holding $\pi_c$ constant, as we do in Eq. (14), this increases the relative cost of consuming as we increment age by a year, reducing consumption and increasing saving. Note, however, that the wage coefficient in Eq. (14) is not affected by the presence of secular growth.

It is also of interest to investigate the impact of secular growth on observed cross-sectional savings-age profiles. To do so, it is necessary to calculate the total effect of the wage change. The total effect is

$$dS_t = -S_t^\lambda.$$ (15)

As a new cohort enters with proportionately higher wages at every age, his savings will be proportionate to the level of the previous cohort.  

---

7 An age term is included in the regression because we first integrate Eq. 14 before estimate. The term $-\pi Z_t [\sigma (r - \alpha) - \lambda[s_{xt} + s_{mt}^\sigma]]$ became the coefficient of the age term.

8 Taking differentials of the savings function,

$$dS_t = -\lambda w_{mt} + \pi Z_t [s_{mt} - s_{xt} - \sigma s_{mt}^c + \sigma s_{c mt}^c] \lambda + r dA_t,$$

$$dS_t = -\lambda w + \pi Z_t [s_{mt} + s_{xt}] \lambda + r dA_t,$$

$$dS_t = X_t - w_{mt}^c N + r dA_t.$$

9 The difference between this exercise and the previous one is that in the regression, if the true life cycle wage change is $dw_{mt}/\omega_{mt}$, the observed change in a cross-section will be $dw_{mt}/\omega_{mt} - \lambda$. This is the measured wage change with age and is captured in the wage coefficient in the regression. To get the total effect on the savings profiles, we must add the wage coefficient to the secular change in wages.
The change in savings with age will be more negatively sloped when savings were originally positive, and more positively sloped at ages when dissavings occur. This is illustrated in Fig. 4, where the savings profiles of two consecutive cohorts are drawn. Figure 4 illustrates the savings profile of Cases 1 and 2. At those ages with positive savings, observed savings will decrease more rapidly (or increase less rapidly) than the true life cycle age effect. Correspondingly, when dissavings occur, dissaving will increase less rapidly (or decrease more rapidly) than the true life cycle profile.

RETIREMENT

An important reason for saving is that one's consumption horizon usually exceeds one's working horizon. In order to consume during retirement, it is necessary to consume at a rate less than earnings during the working period. The existence and size of pension funds suggests that savings for retirement is of great practical importance. Indeed, this motive for savings was the only one present in the original Brumberg-Modigliani life cycle model. In their framework, leisure was not an object of choice, there were no inheritances or bequests, and the interest rate and rate of time preference were zero. Under these assumptions, the following consumption, saving, and asset functions result (if the earnings stream is constant):

\[
\begin{align*}
X_t &= \frac{V}{N} E_m t', \\
S_t &= \frac{N - V}{N} E_m t', \\
A_t &= \frac{t(N - V)}{N} E_m t',
\end{align*}
\]

Before retirement

\[
\begin{align*}
X_t &= \frac{V}{N} E_m t', \\
S_t &= \frac{N - V}{N} E_m t', \\
A_t &= \frac{t(N - V)}{N} E_m t',
\end{align*}
\]

After retirement

\[
\begin{align*}
X_t &= \frac{V}{N} E_m t', \\
S_t &= \frac{V}{N} E_m t', \\
A_t &= \frac{V(N + 1 - t)}{N} E_m t',
\end{align*}
\]
Fig. 4 — Savings profiles of two consecutive cohorts
where $V$ is the number of working years, $N$ is the number of consuming years, and $E_{mt}$ is earnings in a given year. The profiles resulting from this model are illustrated in Fig. 5. The constant rate by which assets grow during the working period depends on the proportion of the horizon in which one works. These assets are then drawn down during the retirement period.

Generalizing the simple Brumberg-Modigliani model to include leisure, interest rates, time preferences, and wage rates becomes complicated. Because wage rates no longer measure time values during the retirement period, obtaining explicit equations is difficult and one must rely on heuristic discussion of the implications. One issue I will ignore is that the decision to retire may be endogenous. A complete model should predict the optimal length of working life as a function of institutional factors (social security) and the factors emphasized in my model—interest rate, time preference, and wage rates. But for simplicity, I will assume that retirement decision is exogenous—i.e., that an individual is constrained by fate to work zero hours at all ages after 65. The weakly separable utility function (CES) makes the solution to this problem relatively simple.10 The retirement and preretirement periods can be separated, because the ratio of marginal utilities between any two periods during the working interval depends only on consumption in those two periods and is independent of consumption during the retirement period. As long as prices in the working interval are unchanged, rates of growth of consumption and leisure during the preretirement periods remain unaltered. Because the income transfer from the retirement period is reduced, the levels of consumption of goods and leisure will decline by the same proportionate amount, decreasing earnings at every age during the working life. With the separability assumption, the analysis for the working period is equivalent to that of an increase in bequests at the end of working life. Savings during working life will then rise at each age, with the largest

---

10. The assumption of weak separability can easily be questioned. One would think that the years close to retirement would be affected more.
Fig. 5—Retirement profiles
increases occurring at those ages where the consumption of goods and leisure are the greatest. The change in savings during the working life is

$$dS = \pi Z_t \frac{d\lambda}{\lambda} + \tau dA_t,$$

where $d\lambda/\lambda$ is the proportion of full lifetime income that must be transferred from the working period to the retirement period.

The profiles of consumption of goods and leisure during the retirement period are affected. Since wage rates no longer measure the value of time, prices become endogenous. The profiles of consumption of market goods will be flatter than if one were free to choose his working hours. For example, if the consumption of goods were rising, this rise would be partly offset as the rising ratio of goods to time reduced the value placed on consumption. Similarly, if goods consumption were falling, the decline in consumption would be mitigated by the rising value placed on goods. If the weakly separable assumption is a reasonable approximation, it is clear that the empirical problem will be reduced if the analysis is confined to the preretirement period. In my empirical work, regressions are run using observations that never extend beyond age 65.
IV. EMPIRICAL TESTS

Because of the demands it places on currently available data, the life cycle model is difficult to test empirically. The model combines Cases 1 and 2. Since it deals with the timing of participation and expenditures, we must be able to follow people through at least part of their lifetime experience. Moreover, the data source should contain information on assets, savings, labor force participation, and wage rates. Obviously no existing data set meets all of these criteria, and some concessions and simplifications are necessary. The ideal data to use would be observations on the same individuals over a number of years. But the absence of extensive panel data forces us to simulate data with the more available cross-sectional information. Essentially this involves creating a synthetic cohort from the cross section. First, the sample is stratified by age of the family unit.\(^1\) Then, within every age group, mean values of all variables are calculated. In the absence of secular growth, the observed variation between these age cells corresponds to the expected life cycle variation for any cohort if a cohort's expectation is unbiased on average. Using the first two cases as the life cycle model, and aggregating over all families, we have\(^2\)

\[
dS = a_1 \frac{dw_{mt}}{w_{mt}} + a_2 \frac{dw_{ft}}{w_{ft}} + a_3.
\]

Upon integrating,

\[
S = a_1 \log w_{mt} + a_2 \log w_{ft} + a_3 \text{ age},
\]

---

\(^1\)In my empirical work, the sample is stratified by age of the husband.

\(^2\)A number of difficulties are encountered in this step. First, the aggregation problems are well known. Second, in the integration, we are implicitly assuming that the coefficients are locally constant.
where $a_1$ and $a_2$ are the male and female wage coefficients derived above, $a_3$ is the age coefficient that captures the interplay of interest rates and time preferences. If $r > \alpha$, $a_3$ will be negative. The wage terms should both be positive, with the male wage coefficient exceeding that of the female.

The 1967 Survey of Economic Opportunity (SEO) sample was selected because it contains data on the asset and debt positions of families. Age, sex, race, educational attainment, and family relationship data are given for each individual in the family. Adult members were questioned about their earnings, health, and labor force behavior. Another advantage of the SEO data is that it is possible to analyze at one time the savings, consumption, and leisure decisions. Because of the identity linking the three, family consumption can be computed as a residual from the savings and income data.

**LIFE CYCLE PROFILES**

The life cycle profiles of assets and debts obtained from the SEO seem consistent with the implications of the model.\(^3\) Graphs (a) and

\(^3\) More recently, other data sets, such as the National Longitudinal Survey ("Parnes" data) and the Income Dynamics Panel, have included information on assets.

\(^4\) The general picture emerging from the SEO does not conflict with data from those surveys whose primary purpose was to collect accurate asset and savings information for families. The following table gives the net-worth-age relationship from the Survey of Consumer Finances. The age trend is similar to the SEO profiles.

**NET WORTH-AGE, DECEMBER 31, 1962**

<table>
<thead>
<tr>
<th>Age of Household Head</th>
<th>Net Worth ($)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Less than 25</td>
<td>557</td>
</tr>
<tr>
<td>25-34</td>
<td>4,831</td>
</tr>
<tr>
<td>35-44</td>
<td>14,792</td>
</tr>
<tr>
<td>45-54</td>
<td>22,237</td>
</tr>
<tr>
<td>55-64</td>
<td>32,511</td>
</tr>
<tr>
<td>65 and over</td>
<td>30,124</td>
</tr>
</tbody>
</table>

(b) of Fig. 6 show the mean net worth holdings of families, stratified by husband's age. For the all-white sample, asset accumulation continues into the mid-fifties. Although the profile becomes quite erratic after this point, there is apparently some tendency for the net worth position to decline slightly after retirement. The peak in the net worth profiles certainly occurs at a later point in the life cycle than the peak in the earnings profiles. The obvious life cycle asset accumulation evident in these graphs is consistent with the underlying motivations stressed in this report, namely, that the wealth owned at any point is the consequence of past savings behavior. The model highlighted two factors that underlie life cycle savings: (1) age-related wage increases that increased the amount saved and (2) a pure age term that implied reductions in savings with age. These profiles suggest that the wage effect dominates the age term so that savings and net worth rise initially with age. As an individual ages, the increases in

---

5 Net worth consists of the sum of assets held in the form of business, land, home, car, bank accounts, government bonds, stocks, personal loans, and other assets minus debts in the form of business, home, land, car, clothing, fuel, medical, bank, and other debts. The other assets are boats, royalties, and commodity contracts. Not included as assets are household furnishings, clothing, many consumer durables (refrigerators, televisions, etc.), cash, pension benefits, inheritances, life insurance policies, and human capital investments.

6 The sample consisted of white and black nonfarm, married, spouse-present families. The asset variables in these graphs are 3-year moving averages. Since the probability of being included in the original tape was not identical across families, these means were constructed using the probability of sample inclusion as the weight for the family.

7 The life cycle model predicts the following order to peaks in profiles: working hours, earnings, wage rates, consumption.

8 It was proved that

\[ dS_t = \frac{dω}{w_{mt}} [w_{mt} N_{mt} + c w_{mt} M_t] - \pi Z c (r - α) . \]

I am assuming throughout that \( r > α \), based on the empirical evidence that follows.
Fig. 6 — Net worth and components of net worth of white and black families
wages diminish and the age effect begins to rival and eventually dominate the wage term as savings and net worth both decline.

Because it concentrated on the aggregate savings rate and not on the savings portfolio, the life cycle model offers no insights concerning the composition of assets. Essentially, I viewed the family as solving its savings problem by a two-stage maximization procedure. In the first stage, the family selects its optimal level of savings and the implied pattern of asset accumulation. The determinants of savings in this first stage are the factors emphasized in this report. In the second stage, such factors as liquidity, capital transactions costs, and consumer durable purchases become important ingredients of the theory. I am assuming that these factors can be safely ignored in the first stage.

Graphs (c) and (d) of Fig. 6 show the age pattern and form in which assets are held. Automobiles are the most popular type of savings for the young, and dominate other assets for the first few years. Automobile net worth is relatively age invariant, and it quickly becomes a minor component of wealth. Another characteristic of young households is the relative abundance of debts. Car, clothing, fuel, and bank debt do not increase much over the cycle, so for the young these debts are large in relation to total wealth. Consumer durable purchases are also concentrated among younger families. Quite often these purchases are the initial form in which savings takes place, and we know from other data sources that they tend to decline with age.\(^9\) During the middle years of the life cycle, housing and land dominate the asset portfolio. Both gross housing assets and mortgage debt rise with age, with mortgage debt peaking at an earlier age (40) than gross assets (53). At this stage, asset holdings become more diverse as families begin to accumulate assets that are financial and liquid in nature (stocks, bonds, etc.) These financial assets also have a considerable age trend and are most important during the older ages. Among older families,

\(^9\)In the 1970 Survey of Consumer Finances, the proportion of families having expenditures on household durables in 1969 are, by age of head,
automobile wealth declines as ownership becomes less frequent. After retirement, business assets are generally converted into financial assets and these financial assets rival home and land wealth.

The graphs of Fig. 6 provide useful comparisons between racial groups. White-black differences in economic well-being are typically made using income as the index of economic position. But the relative disadvantage of black families using net worth as the welfare indicator far exceeds that obtained when income is used; the average net worth of white families is approximately four times that of black families. Net worth is also more unevenly distributed among groups than income. Homeownership comprises a larger share of net worth for the less-educated and black families. The most striking contrast between white and black families occurs in the financial net worth profile. The age increases observed for whites are absent in the black profile. Indeed, throughout much of the life cycle, near net assets held in this form are negative for black families. The consequences of this lack of liquidity in the portfolio of blacks have, unfortunately, received little attention from economists.

<table>
<thead>
<tr>
<th>Age (yr)</th>
<th>Proportion of Families with Household Durable Expenditures</th>
</tr>
</thead>
<tbody>
<tr>
<td>25-34</td>
<td>57</td>
</tr>
<tr>
<td>35-44</td>
<td>53</td>
</tr>
<tr>
<td>45-54</td>
<td>47</td>
</tr>
<tr>
<td>55-64</td>
<td>39</td>
</tr>
<tr>
<td>65 and over</td>
<td>25</td>
</tr>
</tbody>
</table>

Household durable expenditures include televisions, washing machines, cooking ranges, refrigerators, dryers, dishwashers, air conditioners, furniture, sewing machines, stereos, radios, and tape recorders. (See 1970 Survey of Consumer Finances, p. 73.)

10 Separate regressions were not run for blacks because of the quality of the data.

11 Projector (1968) reports that the Gini coefficient for assets was .67, for the same group, the Gini coefficient for income was .45 (see the 1970 Survey of Consumer Finances). The erratic nature of the black profile may be attributed to the greater sampling variability caused by smaller sample sizes.
Clearly the age pattern of individual components of net worth can differ drastically from the aggregate relationships, and limiting the empirical work to some subset of total savings can give very misleading results. In the empirical work, the most aggregate definition of savings available is used.

REGRESSION ANALYSIS

Although suggestive, these asset profiles provide no real test of the life cycle savings model. To obtain a stronger test, regressions were run using as regressors wage rates of family members, age, and the number of children under 7 years. Actual savings measures did not exist in the SEO, so they were constructed by taking first differences of the asset series. One disadvantage of this method is that assets contain considerable measurement error. The original savings data were so dominated by noise that no statistically significant results were possible. Therefore, I smoothed the asset profiles by taking 3- and 4-year moving averages, and then first-differenced the moving average series. To compute savings, we are essentially taking linear approximations 3 or 4 years apart in the asset age profile.\textsuperscript{12} Because assets will reappear in the savings computation 3 or 4 years later, some (negative) serial correlation is present. As is well known, standard errors must therefore be treated with more than the usual skepticism.

An examination of the residuals clearly indicated that they tended to fan out as age increased. The sample was divided into two equal age groups, and separate regressions were run for ages 19-42 and 43-65. An F-test on the ratio of the sum of squared residuals from the two regressions was performed, and the hypothesis of constant variances was rejected at the 5-percent confidence limit.\textsuperscript{13} The presence of heteroscedastic residuals is not surprising because the savings measures are computed from assets, and the error in assets appears proportional to

\textsuperscript{12} Using 3-year moving averages, savings at age $t = \frac{(A_{t+2} - A_{t-1})}{3}$.

\textsuperscript{13} The F-value for (18,18) degrees of freedom is 2.25. The computed F was 3.06.
the level of assets. In the regressions reported in this report, all observations were weighted by the inverse of assets.

In previous work, I analyzed the leisure hours profiles of both men and women. Regressions of family consumption and home hours of males and females are reported in Tables 1 and 2. One severe limitation

Table 1

FAMILY SAVINGS
Independent Variables

<table>
<thead>
<tr>
<th>Equation</th>
<th>Husband's Earnings</th>
<th>Husband's Hourly Wage</th>
<th>Wife's Hourly Wage</th>
<th>Age</th>
<th>Children under 7 years</th>
<th>Constant</th>
<th>( R^2 )</th>
</tr>
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<td>1</td>
<td>2976^b</td>
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<td>-879.1^b</td>
<td>-73.09</td>
<td>-743.5</td>
<td>-23090</td>
<td>.13</td>
</tr>
<tr>
<td></td>
<td>(2.00)</td>
<td></td>
<td>(.32)</td>
<td>(2.02)</td>
<td>(1.40)</td>
<td>(2.07)</td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>---</td>
<td>2998^b</td>
<td>739.4^b</td>
<td>-47.63</td>
<td>-366.7</td>
<td>-236.7</td>
<td>.10</td>
</tr>
<tr>
<td></td>
<td>(1.53)</td>
<td></td>
<td>(.29)</td>
<td>(1.55)</td>
<td>(.77)</td>
<td>(1.40)</td>
<td></td>
</tr>
<tr>
<td>3</td>
<td>3216^b</td>
<td>---</td>
<td>-2016.0^b</td>
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<td>-247.0</td>
<td>.22</td>
</tr>
<tr>
<td></td>
<td>(2.87)</td>
<td></td>
<td>(.93)</td>
<td>(2.40)</td>
<td>(1.21)</td>
<td>(28.9)</td>
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<tr>
<td>4</td>
<td>.4553</td>
<td>---</td>
<td>793.0</td>
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<td>-296.5</td>
<td>-344.4</td>
<td>.21</td>
</tr>
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<td></td>
<td>(2.74)</td>
<td></td>
<td>(.82)</td>
<td>(2.19)</td>
<td>(.80)</td>
<td>(.27)</td>
<td></td>
</tr>
<tr>
<td>5</td>
<td>---</td>
<td>4664^b</td>
<td>-2255.0</td>
<td>-62.07</td>
<td>-377.4</td>
<td>-1724</td>
<td>.20</td>
</tr>
<tr>
<td></td>
<td>(2.42)</td>
<td></td>
<td>(.68)</td>
<td>(2.42)</td>
<td>(.70)</td>
<td>(.95)</td>
<td></td>
</tr>
<tr>
<td>6</td>
<td>---</td>
<td>1573</td>
<td>617.1</td>
<td>-37.96</td>
<td>-315.6</td>
<td>-902.5</td>
<td>.15</td>
</tr>
<tr>
<td></td>
<td>(2.17)</td>
<td></td>
<td>(.61)</td>
<td>(1.63)</td>
<td>(.08)</td>
<td>(.58)</td>
<td></td>
</tr>
</tbody>
</table>

NOTE: The first two equations used 3-year moving averages of the asset data, and the last four equations used 4-year averages.

a t-values are indicated in parentheses.
b Variables entered in log form.

An alternative would have been to use log savings as the dependent variable. The negative values for savings precluded such use.

Other weights (the reciprocal of log male hourly wage or log male earnings) were used as well. The results were similar to those reported in the text.

See Smith (1972).
Table 2
FAMILY CONSUMPTION
Independent Variables$^a$

<table>
<thead>
<tr>
<th>Dependent Variable</th>
<th>Log Husband's Hourly Wage</th>
<th>Log Wife's Hourly Wage</th>
<th>Age</th>
<th>Children under 7 Years</th>
<th>Constant</th>
<th>$R^2$</th>
<th>D.W.</th>
</tr>
</thead>
<tbody>
<tr>
<td>Log consumption 1</td>
<td>.7192 (2.68)</td>
<td>.0929 (.22)</td>
<td>.0080 (2.30)</td>
<td>.0193 (.30)</td>
<td>7.89 (30.7)</td>
<td>.68</td>
<td>2.18</td>
</tr>
<tr>
<td>Log consumption 2</td>
<td>.7832 (3.05)</td>
<td>.0452 (.11)</td>
<td>.0105 (3.15)</td>
<td>.0151 (.24)</td>
<td>7.87 (32.05)</td>
<td>.78</td>
<td>2.10</td>
</tr>
</tbody>
</table>

NOTE: Consumption definition 1 is defined as the difference between family income minus savings. Savings was defined as net worth at age $t$ minus net worth at age $t-1$. Family income equals male earnings plus female earnings plus all nonlabor income flows received in that period. Consumption definition 2 equals consumption 1 plus an imputed income return to the net worth held in the previous period. An interest rate of 5 percent was used.

$^a$t-values are indicated in parentheses.

of the empirical work is that the principal explanatory variables, wage rates and children, are themselves endogenous and part of the life cycle decisionmaking process of the family. It is both beyond the quality of the data and current theory to treat all of these decisions simultaneously. The pattern of observed wages is not exogenous but results from a process of human capital accumulation over one's lifetime. One can show that the predictions regarding the hours and consumption profiles are not substantially affected by including human capital, because both investment time and consumption time will fall as the wage rate rises. The predictions of the model without human capital are reinforced by the decline in the investment time; hours of market work will increase as the wage rate rises over the life cycle. However, the omission of human capital investments leads to underreporting of savings among young families, and the overall savings-age relationship

$^{17}$See Ghez and Becker (1972); Heckman (1971).
is distorted. Human capital theory and empirical evidence indicate that savings used to finance human capital are large at the youngest ages and decline throughout the life cycle. My savings profiles, which exclude human capital investments, overstate the decrease in savings with age.

The severe measurement problem encountered with the savings data is evident in the low $R^2$ and in the lack of significance for some of the variables. Still, the empirical estimates basically support the life cycle model. In the savings function, all variables have the theoretically expected sign except the female wage variable, which has an insignificant coefficient. The male wage variable has the predicted positive sign in the savings function. However, the absolute magnitude of the coefficient seems below what we would expect. The male wage coefficient has the correct negative (positive) effect in the male labor supply (consumption) equations.

I anticipated difficulty in estimating an independent effect for female wages. The true life cycle variation in female wages is small compared with that of male wages, so it should play a smaller role in explaining the timing of expenditures and participation. Moreover, during any week, approximately 60 percent of the married women are not working. Each mean female wage is based on fewer observations than the mean male wage, and because of this, the female wage is probably less reliable as a statistic describing the true wage of working individuals. Another problem is that the value of time (shadow home wage) of nonworking women is not necessarily equal to the observed wage of workers. Ruben Gronau has pointed out that for population subgroups in which a large proportion are not working, the observed wage distribution represents only one section of the total wage-offer distribution. The unobserved section of the wage-offer distribution has been rejected by job seekers as unacceptable. As Gronau demonstrates, the observed wage may

---

18 Variation in nonmarket productivity biases the savings-function wage coefficient upward. If $\lambda$ is the percentage increase in nonmarket productivity, $dS = (1 - \sigma_c)\pi e^{c\lambda}$. Empirical estimates suggest that $\sigma_c < 1$. (See Ghez and Becker (1972).)

19 The absolute magnitude of the male wage coefficient in the consumption function is approximately the same size as Ghez and Becker's estimates.
change, due to a selectivity bias, without any alterations in the wages offered by firms. For example, in time periods when there are young children in the family, the implicit home wage increases, and many women will leave the labor force. Indeed, it is only the women receiving the highest wage offers in the distribution who will remain in the labor force. Only part of the observed life cycle variation in female wages reflects a real change in their market opportunities. In both the savings and consumption functions, female wages had no discernable effect. The female own-wage coefficient in the females hours equation is negative--a result consistent with the model. The positive female wage coefficient in the male function suggests that male time and female time are substitutes.

Age has the predicted negative sign in the savings function. A year increase in age reduces savings by approximately $60. As pointed out above, secular growth in real wages biases the age term toward positive values, so obtaining the negative age term is encouraging. The consumption and hours equations are consistent with the savings function. The age term captures the effect of a divergence between interest rates and time preferences. An interest rate larger than the rate of time preference leads to a positive coefficient in the consumption and home hours equations. With the possible exception of male hours, my empirical estimates support this. A strong confirmation of the life cycle approach is the consistancy in the results across the savings consumption and leisure regression.

The effect of children under 7 years of age on savings is negative, although the t-values are not very high. One problem with this variable is that children are concentrated in a relatively small part of the life cycle. Notice that as we go from a 3- to a 4-year moving average, the children's variable becomes smaller in absolute size. The smoothing process tends to dampen the children's variable, since we are approximating the savings function over a wide enough age range that we are soon outside the ages of concentrated child bearing.

The presence of children in the household may affect savings by altering consumption expenditures or money income. Preschool children apparently have a depressing effect on the market participation of
females, but have the opposite effect of increasing male working hours (Table 3).\(^{20}\) Evaluating at the mean market hours and wages for males and females, I find an increase of 116 male market hours and $405 in male earnings and a decrease of 294 female market hours and $636 in female market earnings\(^{21}\)—a net decrease of approximately $231 in family earnings and a reduction in savings through the reduced earnings.

However, a simple count of the number of young children at home cannot be expected to serve as a measure of the many changes that occur during the life cycle in those characteristics of family structure that determine a family member's labor market behavior. I defined a group of variables measuring the fraction of women at each age with children at home in a set of mutually exclusive child-age categories (Table 4). For males, the presence of children in any age category tended to increase their market work, but for females it appears that the factor intensity of children might well switch as the child proceeds through his aging process. Although parents with preschool children are consuming a relatively (wife) time intensive commodity, these children become less time intensive as they age, so that the presence of an older child makes household consumption more goods intensive than consumption in childless families. Through the mechanism of changes in earnings, the presence of children may produce smaller savings at younger ages and larger savings at the older ones.

The relationship between family size and household savings has long been a popular subject in the demographic and development literature, and the postulated negative correlation is thought to be a contributing factor in limiting capital formation and growth. But the theoretical grounds for this hypothesis are not particularly convincing.\(^{22}\) The models employed by economic demographers were couched in

\(^{20}\) See Smith (1972).

\(^{21}\) The mean male and female hourly wages are 3.47 and 2.17. The mean male and female hour times are 6604 and 8201.

\(^{22}\) Indeed, Irving Fisher, to whom we owe much of our existing theory of capital and savings, hypothesized that increasing family size would increase savings. Not only does regard for one's offspring lower impatience, but the increase of offspring has in part the same effect. So far as it adds to future needs rather than to immediate needs, it
<table>
<thead>
<tr>
<th>Dependent Variable</th>
<th>Log Husband's Hourly Wage</th>
<th>Log Wife's Weekly Wage</th>
<th>Age</th>
<th>Children Under 7 Years</th>
<th>Asset/Debt Ratio</th>
<th>Log Net Worth</th>
<th>Constant</th>
<th>R²</th>
</tr>
</thead>
<tbody>
<tr>
<td>Log husband's home hours</td>
<td>-.1040 (.68)</td>
<td>.0202 (.82)</td>
<td>.00014 (.67)</td>
<td>-.0177 (4.60)</td>
<td>---</td>
<td>---</td>
<td>8.92 (524.5)</td>
<td>.75</td>
</tr>
<tr>
<td>Log wife's home hours</td>
<td>.0444 (2.74)</td>
<td>-.0396 (1.50)</td>
<td>.00057 (2.54)</td>
<td>.0358 (3.68)</td>
<td>---</td>
<td>---</td>
<td>8.95 (491.7)</td>
<td>.82</td>
</tr>
<tr>
<td>Log husband's home hours</td>
<td>-.0281 (.97)</td>
<td>-.003 (.13)</td>
<td>.0095 (2.83)</td>
<td>-.0209 (4.52)</td>
<td>---</td>
<td>-.0213 (2.94)</td>
<td>9.02 (233.0)</td>
<td>.80</td>
</tr>
<tr>
<td>Log wife's home hours</td>
<td>.1023 (3.10)</td>
<td>-.0574 (2.13)</td>
<td>.0012 (3.14)</td>
<td>.0367 (9.18)</td>
<td>---</td>
<td>-.0164 (1.99)</td>
<td>9.03 (2.06)</td>
<td>.83</td>
</tr>
<tr>
<td>Log husband's home hours</td>
<td>-.0792 (4.37)</td>
<td>.034 (1.41)</td>
<td>-.0008 (1.75)</td>
<td>-.0236 (5.23)</td>
<td>.00127 (2.26)</td>
<td>---</td>
<td>8.89 (457.0)</td>
<td>.81</td>
</tr>
<tr>
<td>Log wives' home hours</td>
<td>.0692 (3.53)</td>
<td>-.0258 (.99)</td>
<td>-.0004 (.77)</td>
<td>.0230 (6.15)</td>
<td>.00126 (2.07)</td>
<td>---</td>
<td>8.93 (424.3)</td>
<td>.84</td>
</tr>
</tbody>
</table>

* ^t-values are indicated in parentheses.
Table 4

EFFECT OF CHILDREN'S AGE ON WORKING TIME OF THEIR MOTHERS

<table>
<thead>
<tr>
<th>Sample Group</th>
<th>Child Groups, by Age (years)</th>
<th>&lt;6</th>
<th>6-13</th>
<th>&gt;13</th>
<th>6-13</th>
<th>&gt;13</th>
<th>6-13</th>
<th>&gt;13</th>
</tr>
</thead>
<tbody>
<tr>
<td>All whites</td>
<td></td>
<td>-</td>
<td>-</td>
<td>?</td>
<td>?</td>
<td>+</td>
<td>+</td>
<td>-</td>
</tr>
<tr>
<td>All blacks</td>
<td></td>
<td>?</td>
<td>?</td>
<td>+</td>
<td>?</td>
<td>+</td>
<td>?</td>
<td>?</td>
</tr>
<tr>
<td>College whites</td>
<td></td>
<td>-</td>
<td>-</td>
<td>?</td>
<td>?</td>
<td>+</td>
<td>-</td>
<td></td>
</tr>
<tr>
<td>High school whites</td>
<td></td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>?</td>
<td>+</td>
<td>?</td>
<td></td>
</tr>
<tr>
<td>Elementary school</td>
<td></td>
<td>?</td>
<td>-</td>
<td>?</td>
<td>-</td>
<td>+</td>
<td>?</td>
<td>?</td>
</tr>
</tbody>
</table>

NOTE: + indicates effect is to increase hours of work.
- indicates effect is to decrease hours of work.
? indicates t-value less than 1.

a one-period framework, and children were assumed to increase the consumption "needs" of the family. But if savings results from a divergence between the desired expenditure and income profiles, the effect of children on the absolute level of lifetime consumption is far less critical than their impact on the timing of consumption over the life cycle. Children undeniably have a large impact on the composition of goods consumed, increasing the consumption of food and housing and reducing the consumption of stereos and travel. Their effect on total consumption expenditures is far less clear. The relative market goods intensity of the commodity of child services, compared with all other household commodities, is basically an empirical issue in the same way that the relative wife and husband time intensity of children is an empirical issue. As we find for female time, the relative goods intensity of children may well be a function of the age of children. My

operates, like a descending income stream, to diminish impatience. Parents whose families are increasing often feel the importance of providing for future years far more than parents in similar circumstances but with small families. Consequently, an increase in family size, other things being equal, will reduce the rate of interest.
empirical results (Table 2) show an insignificant positive effect of the number of children under 7 years on family consumption.\textsuperscript{23} Apparently, the main avenue through which children influence savings is through the labor supply side\textsuperscript{24} rather than the consumption expenditure side.\textsuperscript{25}

Although I have argued that it is inappropriate to introduce assets in labor supply regressions, I included them in the male and female home-hours regressions to observe what effect they have on the empirical specification. Notice that contrary to the view that assets serve as a measure of wealth and should have a positive sign in the home-hours regressions, increasing assets increase market work. This probably reflects the positive serial correlation of work time with past work, producing the current assets. Also, including assets reduces the male wage coefficient in the male home-hours equation. The total male wage

\textsuperscript{23}Two studies dealing with this empirical issue were those of Ghez and Becker (1972) and Landsberger (1973). Ghez and Becker estimate that doubling family size would increase consumption by 25 percent. Landsberger took into account the age structure of children and reported a reduction in consumption if the number of young children under 6 years increased, but an increase in consumption when the number of children older than 13 years increased. Unfortunately, both studies have serious defects; Ghez ignored the age structure of children, and Landsberger did not include household age as an explanatory variable. Landsberger's finding could simply reflect the positive consumption-age relationship implied by the life cycle theory.

\textsuperscript{24}Children may also affect earnings profiles by changing the wages received by men and women. One determinant of the incentives to invest in market-oriented activities is the amount of expected future market work. Moreover, the rate at which one's human capital depreciates may, in part, be a function of the duration and continuity of one's market participation. We might anticipate that the presence of children may indirectly increase male market wages because of the added incentives for them to invest. Mincer and Polacheck (1974) estimate that the net depreciation of female human capital caused by the birth of a first child is 1.5 percent. They also found that the depreciation increased with the level of female education. Although their results cannot yet be regarded as conclusive, this potential avenue of causation from children to future wages, earnings, and savings should be more carefully explored.

\textsuperscript{25}Attempts at including variables that measured the number of children in other age groups produced so much multicollinearity that no useful results were obtained.
effect should include the male wage coefficient plus the asset coefficient times the change in assets induced by the higher wage.\footnote{Another problem is that male wages and assets are so collinear that the significance of the male wage is substantially reduced.}

Another role that the existence of other family members may play has been suggested by Jacob Mincer. According to the Permanent Income hypothesis, adjustments to transitory changes in income take place via saving behavior. But Mincer states:

If assets are low or not liquid and access to the capital market costly or nonexistent, it might be preferable to make the adjustment to transitory income on the money income side rather than on the money expenditure side. \ldots\ A transitory increase in labor force participation of the wife may well be an alternative to dissaving, asset accumulation, or increasing debt. One useful empirical implication of this hypothesis for labor force behavior is that they should be inversely related to the level of family assets, both in the life cycle and short-run sense.\footnote{Mincer (1962), p. 76.} Mincer rationalizes the counter-cyclical responses of the Labor Force Participation Rates of black wives (the only secondary labor force group that so behaves) on the basis of this hypothesis. But he did not have the data necessary to test his theory, and to my knowledge it has not been tested since his study. However, the empirical test suggested by Mincer does not seem to be a powerful or discriminating one because of the simultaneity involved in asset and labor force adjustments. An alternative is to argue that the asset-debt ratio is a good proxy for the family's borrowing position. Families with high ratios of assets to debts face lower borrowing rates and will tend to work less. Table 3 shows some support for this hypothesis. Increasing the ratio of household assets to debts reduces the time worked for both men and women. The absolute reduction in market hours for females exceeds that of males.
Appendix
MATHEMATICAL MODEL

Let the family maximize lifetime utility

\[
U = \left[ \int_0^N Z_t c^{(\sigma - 1)/\sigma} e^{-\alpha t} c^{\sigma/(\sigma - 1)} \right] dt
\]  
(A.1)

with the production function, and subject to the following time and money expenditure constraints:

\[
Z_t = f(X_t, M_t, P_t)
\]  
(A.2)

where \( t_o \) is homogenous of degree 1,

\[
M_t + N_{mt} = P_t + N_{ft} = T
\]  
(A.3a)

\[
\int_0^N X_t e^{-rt} dt = \int_0^N (w_{mt} N_{mt} + w_{ft} N_{ft}) e^{-rt} dt + A_o
\]  
(A.3b)

\[
R = \int_0^N \pi_t Z_t e^{-rt} dt = T \int_0^N (w_{mt} + w_{ft}) e^{-rt} dt + A_o
\]  
(A.4)

When the family maximizes utility function (A.1) subject to budget constraint (A.4), the following must hold between consumption in period \( t \) and \( t + j \):

\[
\frac{-dZ_{t+j}}{Z_t} = \left( \frac{Z_{t+j}}{Z_t} \right)^{1/\sigma} e^{\alpha_j} = \frac{\pi_t}{\pi_{t+j}} e^{\gamma_j}
\]  
(A.5)

Therefore consumption in any period \( t + j \) can be expressed as

\[
Z_{t+j} = Z_t \left( e^{(r-\alpha)j} \frac{\pi_t}{\pi_{t+j}} \right)^{\sigma_c}
\]  
(A.6)
and since
\[ R = \int_{-t}^{N} \pi_t Z_t e^{-rt} dt = \int_{-t}^{N-t} \pi_{t+j} Z_{t+j} e^{-r(t+j)} d_j, \tag{A.7} \]
we may substitute (A.6) into (A.7):
\[ R = Z_t \int_{-t}^{N-t} \left( \frac{\pi_t e^{-rt}}{\pi_{t+j} e^{-r(t+j)}} \right)^{1-\sigma} \quad e^{-\alpha c(t+j)} \quad d_j \tag{A.8} \]
or
\[ R = Z_t \int_{-t}^{N-t} \left( \frac{\pi_t e^{-rt}}{\pi_{t+j} e^{-r(t+j)}} \right)^{1-\sigma} \quad e^{-\alpha c t} \quad dt. \tag{A.9} \]
Define the lifetime price index \( P \) as follows:
\[ P = \left[ \int_{-t}^{N-t} \left( \frac{\pi_t e^{-rt}}{\pi_{t+j} e^{-r(t+j)}} \right)^{1-\sigma} \quad e^{-\alpha c t} \quad dt \right] \frac{1}{1-\sigma} \tag{A.10} \]
Then
\[ Z_t = \frac{R}{P} \left( \frac{\pi_t}{\pi_{t+j}} \right)^{-\sigma} e^{(r-\alpha)\sigma c t}, \]
which is equivalent to Eq. (5) in the text.

\( R \) and \( P \) are constant over the life cycle, so
\[ \frac{dZ_t}{Z_t} = -\sigma \frac{d\pi_t}{\pi_t} + \sigma_c (r - u) \tag{A.11} \]
and

$$\frac{d\pi_t}{\pi_t} = s_{mt} \frac{dw_{mt}}{w_{mt}} + s_{ft} \frac{dw_{ft}}{w_{ft}}. \quad (A.12)$$

The demand for male home time is

$$\frac{dM_t}{M_t} = \frac{dZ_t}{Z_t} - \left( s_{ft} \sigma_{MF} + s_{xt} \sigma_{MX} \right) \frac{dw_{mt}}{w_{mt}} + s_{ft} \sigma_{MF} \frac{dw_{ft}}{w_{ft}}. \quad (A.13)$$

Finally, by substituting Eq. (A.12) into (A.11), and (A.11) into (A.13), we obtain the demand function for home time described in the text:

$$\frac{dM_t}{M_t} = -(s_{mt} \sigma_C + s_{ft} \sigma_{MF} + s_{xt} \sigma_{MX}) \frac{dw_{mt}}{w_{mt}}$$

$$+ s_{ft}(\sigma_{MF} - \sigma_C) \frac{dw_{ft}}{w_{ft}} + \sigma_C (r - \alpha), \quad (A.14)$$

and the demand for goods is

$$\frac{dX_t}{X_t} = s_{mt}(\sigma_{MX} - \sigma_C) \frac{dw_{mt}}{w_{mt}} + s_{ft}(\sigma_{FX} - \sigma_C) \frac{dw_{ft}}{w_{ft}} + \sigma_C (r - \alpha). \quad (A.15)$$

The pure one-period model results when there are no interperiod price effects. Using the age neutrality assumption, the percentage difference in R among families is

$$\frac{dR}{R} = \left( \frac{T}{R} \right) _w \frac{dw_{mt}}{w_{mt}} + \left( \frac{T}{R} \right) _f \frac{dw_{ft}}{w_{ft}} + \left( \frac{dA_o}{A_o} \right) \left( \frac{A_o}{R} \right), \quad (A.16)$$

i.e., a weighted average of the percentage changes in male wages, female wages, and the initial assets of families. The weights are the
shares in total full wealth of male human capital wealth, female human capital wealth, and all nonhuman forms of wealth. The percentage change in the lifetime price index may be expressed as

\[ \frac{dP}{P} = \frac{dw_{mt}}{w_{mt}} \bar{S}_M + \frac{dw_{ft}}{w_{ft}} \bar{S}_F. \]  \quad (A.17)

Thus,

\[ \frac{dM_t}{M_t} = \frac{dA_o}{A_o} A_o R + \left[ \left( \frac{T^m_{mt}}{R} - \bar{S}_M \right) + s_{mt} \sigma_{MM} \right] \frac{dw_{mt}}{w_{mt}} \]

\[ + \left[ \left( \frac{T^f_{ft}}{R} - \bar{S}_F \right) + s_{ft} \sigma_{MF} \right] \frac{dw_{ft}}{w_{ft}} \]

or

\[ \frac{dM_t}{M_t} = \frac{dA_o}{A_o} A_o R + \left( \frac{E_m}{R} + s_{mt} \sigma_{MM} \right) \frac{dw_{mt}}{w_{mt}} + \left( \frac{E_F}{R} + s_{ft} \sigma_{MF} \right) \frac{dw_{ft}}{w_{ft}}. \]  \quad (A.18)

If we drop the assumption of unitary income elasticities implied by the CES, this equation generalizes to

\[ \frac{dM_t}{M_t} = \eta_t \frac{dA_o}{A_o} A_o R + \left( \eta_t \frac{E_m}{R} + s_{mt} \sigma_{MM} \right) \frac{dw_{mt}}{w_{mt}} \]

\[ + \left( \eta_t \frac{E_F}{R} + s_{ft} \sigma_{MF} \right) \frac{dw_{ft}}{w_{ft}}, \]  \quad (A.19)

which is the standard version of the one-period labor-supply model derived in a number of sources.
BIBLIOGRAPHY


