RAND Research Brief
EDUCATION

Developing a Strategic Research and Development Program in Mathematics Education

Improving proficiency in mathematics and eliminating the gaps in proficiency among social groups is and has been the goal of many public and private efforts over the past decade and a half. States and national professional organizations have developed standards for mathematics proficiency and assessments intended to measure the degree to which students attain such proficiency. Various programs have been instituted to attract and retain more effective teachers of mathematics. New curricular materials have been developed along with training and coaching programs intended to provide teachers with the knowledge and skills needed to use those materials. However, these efforts have been supported by limited and uneven research and research-based development, which is part of the reason for the limited success of these efforts.

The Office of Educational Research and Improvement (OERI, now the Institute of Education Sciences) charged the RAND Mathematics Study Panel with developing a research agenda to address the most pressing issues in mathematical proficiency. The panel was composed of mathematics educators, mathematicians, psychologists, policymakers, and teachers. The panel's report (RAND Mathematics Study Panel, Deborah Loewenberg Ball, Chair, *Mathematical Proficiency for All Students: Toward a Strategic Research and Development Program in Mathematics Education*, RAND, MR-1643-OERI, 2003) proposes a long-term, strategic program of research and development in mathematics education. In the short term, the program is designed to produce knowledge that would support efforts to improve the quality of mathematics teaching and learning with the teachers and materials that are now in place or that will become available over the next several years. More important, over 10 to 15 years, the program would build a solid base of knowledge for the design and development of effective instructional practice. That instructional practice, in turn, would enable the dual goals of increased levels of proficiency and equity in attaining these levels of proficiency.

The work of the Mathematics Study Panel was part of a larger RAND effort to suggest ways that education research and development can be made more rigorous, cumulative, and usable. (For a report on R&D in reading education, see RAND Reading Study Group, Catherine Snow, Chair, *Reading for Understanding: Toward an R&D Program in Reading Comprehension*, RAND, MR-1465-OERI, 2002.)

FOCUS AREAS FOR A LONG-TERM RESEARCH AND DEVELOPMENT PROGRAM

Although agreement on the broad goals for mathematics proficiency is widespread, the details of those goals and the means for achieving them are often the subject of disputes among educators, mathematicians, education researchers, and members of the public. These disputes center on the content that should be taught and how it should be taught, and they seem to be based more often on ideology than on evidence. Amid this debate, U.S. schools are expected to provide more and better opportunities for students to learn mathematics. Yet, many schools lack the key resources needed to do so. For example, there is an acute shortage of qualified mathematics teachers, and many widely used curriculum programs and assessment instruments are poorly matched with increasingly demanding instructional goals. While there is considerable policy-level pressure to seek "research-based" alternatives to existing programs and practices, the education and research communities lack rigorous evidence about the degree to which alternative existing or proposed curriculum and instructional practices effectively support all students' learning of mathematics.

Despite more than a century of efforts to improve school mathematics in the United States, investments in research and development have been virtually nonexistent. The limited resources that likely will be available for mathematics education research and development in the near future make it necessary to focus those resources on a limited number of topics. The panel selected three domains in which both proficiency and equity in proficiency present substantial challenges, but where past work would allow for some immediate progress:

- Developing teachers' mathematical knowledge in ways that are directly useful for teaching
- Teaching and learning skills used in mathematical thinking and problem-solving
- Teaching and learning of algebra from kindergarten through the 12th grade (K–12).

These are only the starting points for addressing mathematics proficiency problems. Fundamental problems would remain and would be the subject of work in the longer-term collective effort envisioned by the study panel.

Developing Teachers' Mathematical Knowledge for Teaching

The quality of mathematics teaching and learning depends on what teachers do with their students, and what teachers *can* do with their students depends on their knowledge of mathematics. However, the nature of the knowledge required for successful teaching of mathematics is poorly specified, and the evidence concerning the mathematical knowledge that is needed to improve instructional quality is surprisingly sparse. The same is true for the ways in which teachers' knowledge requirements for effective teaching may differ for diverse student populations. For these reasons, the panel proposed a programmatic focus on three areas in which to conduct fruitful lines of research on the knowledge needed for teaching:

- Developing a better understanding of the mathematical knowledge needed for the actual work of teaching
- Developing improved means for making useful and usable mathematical knowledge available to teachers
- Developing valid and reliable measures of the mathematical knowledge teachers have.

Teaching and Learning Mathematical Practices

The term "practices" refers to the specific things that successful mathematics learners and users *do*. Justifying claims, using symbolic notation efficiently, defining terms precisely, and making generalizations are examples of mathematical practices. Although competent use of mathematics depends on the ways in which people approach, think about, and work with mathematical tools and ideas, the study panel hypothesizes that these practices are often not systematically cultivated in school, although they may be picked up by students at home or in other venues outside of school. Moreover, it is likely that students with poorly developed mathematical practices will have difficulties in learning mathematics.

Although past studies have investigated how students engage in particular practices, less is known about how these practices develop over time, how individual practices interact with one another, and what this means for the consequent requirements for teachers' own knowledge and practices in mathematics. The panel proposed that new research be undertaken to systematically address the question of how mathematical practices can be characterized, taught, and learned. In sum, work in this focus area would accomplish the following:

- Develop a fuller understanding of specific mathematical practices, including how they interact and how they matter in different mathematical domains
- Examine the use of these mathematical practices (e.g., practices that are used in various aspects of schooling, students' out-of-school practices, or practices employed by adults in their everyday and work lives) in various settings
- Investigate ways in which these specific mathematical practices can be developed in classrooms and the role these practices play as a component of a teacher's mathematical resources.

Teaching and Learning Algebra in Kindergarten Through 12th Grade

Coordinated studies of goals, instructional approaches, curricula, student learning, teachers' opportunities to learn, and policy signals—within a content domain—can be used to systematically investigate how various elements of instruction and instructional improvement affect student learning of that domain. The panel proposed research and development related to algebra, and the improvement of proficiency in algebra, as the initial domain in which to work for several reasons. First, algebra is foundational in all areas of mathematics because it provides the tools (i.e., the language and structure) for representing and analyzing quantitative relationships, for modeling situations, for solving problems, and for stating and proving generalizations. Second, many states now require students to demonstrate substantial proficiency in algebra in order to graduate from high school, a requirement that is creating challenges for students and teachers alike. Third, algebra is a "gatekeeper" course. Without proficiency in algebra, students cannot access a full range of educational and career options, and this curtailment of opportunities often falls most directly on groups that are already disadvantaged.

Despite the strong history of work in this area, there is little evidence about what is happening today in algebra classrooms; how innovations in algebra teaching and learning can be designed, implemented, and assessed; and how policy decisions shape student learning and affect equity. Because most studies have focused on algebra at the high school level, not much is known about younger students' learning of algebraic ideas and skills. In addition, little is known about what happens when algebra is viewed as a K–12 subject, what happens when it is integrated with other subjects, or what happens when it emphasizes a wider range of concepts and processes. Three major components frame the recommended research agenda in algebra:

- Analyses and comparisons of curriculum, instruction, and assessment
- Studies of relationships among teaching, instructional materials, and learning
- Studies of the impact of policy contexts on equity and student learning.

BUILDING THE INFRASTRUCTURE FOR A COORDINATED PROGRAM OF RESEARCH AND DEVELOPMENT

Drawing on the work of the National Research Council and other groups, the RAND Mathematics Study Panel proposed several criteria to judge whether a mathematics research and development program is likely to meet high standards of rigor and usefulness.

One set of criteria deals with the strategic framing, design, and conduct of relevant projects. A high-quality program of research and development should respond to pressing practical needs. It should build on existing research and be informed by relevant theory. Research methods should be appropriate to the investigation of a particular question and reflect the theoretical stance taken by the investigator. A coordinated program of research and development would also support groups of researchers to investigate significant questions from various theoretical and conceptual frames using methods that are consistent with both the questions and the frames.

A second set of criteria concerns the kinds of communication, information-sharing, and critiquing that are vital to building high-quality knowledge and evidence-based resources for practice. To support the synthesis, replication, and generalization of results to other settings, researchers and developers must make their findings public and available for critique through broad dissemination to appropriate research, development, and practice communities.

Because solutions to the problems identified in the study panel's report are not the province of any single community of experts, it will be important to build a community of multidisciplinary professionals who have wide-ranging experience and expertise. In addition, a research and development program meeting the criteria just listed will require a significant design and management effort.

The panel recommended that OERI establish an overarching group, the Mathematics Education Research Panel, whose responsibility would be to assess the progress of the program as a whole, synthesize the program's results, and suggest any necessary new initiatives. In addition, three subpanels would provide planning and guidance for each of the three focus areas of the program—mathematical knowledge for teaching, mathematical practices, and algebra. The membership of these subpanels should represent a wide range of viewpoints and disciplines.

The RAND panel also recommended the creation of a peer review system that encompasses individuals with high levels of expertise in both relevant subject matter and research methods to ensure that quality proposals are supported.

Investment in a research infrastructure will contribute significantly to the quality of the program. Key research infrastructure elements include the development of common measures that can be used to gather evidence across projects, deliberate nurturing of new scholars and developers, and support of opportunities for communication among and between researchers and practitioners.

CONCLUSIONS

The program described by the RAND Mathematics Study Panel is both ambitious and strategic. Shaped by hypotheses on what sort of efforts will yield payoffs in increased mathematical proficiency for all students, it is a program that will have high scientific rigor and an emphasis on the usability of the knowledge that it produces. The program will involve unprecedented scrutiny, testing, and revision of instructional interventions, building evidence on how those interventions work and what it takes to make them effective.

RAND research briefs summarize research that has been more fully documented elsewhere. This research brief summarizes work done within RAND Education and the RAND Science and Technology Policy Institute for the U.S. Department of Education's Office of Educational Research and Improvement. The work is documented in Mathematical Proficiency for All Students: Toward a Strategic Research and Development Program in Mathematics Education, *RAND Mathematics Study Panel, Deborah Loewenberg Ball, Chair, MR-1643-OERI, 2003, 112 pp., $20.00, ISBN: 0-8330-3331-X, available from RAND Distribution Services (Telephone: 310-451-7002; toll free 877-584-8642; FAX: 310-451-6915; or email: order@rand.org). Building on more than 25 years of research and evaluation work, RAND Education has as its mission the improvement of educational policy and practice in formal and informal settings from early childhood on. A profile of RAND Education, abstracts of RAND documents, and publication ordering information may be viewed at www.rand.org. Publications are distributed to the trade by NBN. RAND® is a registered trademark. RAND is a nonprofit institution that helps improve policy and decisionmaking through research and analysis; its publications do not necessarily reflect the opinions or policies of its research sponsors.*

RAND

1700 Main Street, P.O. Box 2138, Santa Monica, California 90407-2138 • Telephone 310-393-0411 • FAX 310-393-4818
1200 South Hayes Street, Arlington, Virginia 22202-5050 • Telephone 703-413-1100 • FAX 703-413-8111
201 North Craig Street, Suite 202, Pittsburgh, Pennsylvania 15213-1516 • Telephone 412-683-2300 • FAX 412-683-2800

RB-8023-OERI (2003)