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RESEARCH MEMORANDUM

RELIABILITY OF PROGRESS CURVES IN AIRFRAME PRODUCTION

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SUMMARY

The airframe manufacturing progress curve estimates direct labor per pound of airframe needed to manufacture the Nth airframe, from N, the cumulative number of planes of a given model produced at a given facility. The relation is customarily written as a linear function between the logarithm of direct labor per pound and the logarithm of the Nth airframe. Statistical tests of the similarity of the functions among various airframe manufacturers, on the basis of reported World War II data, have been made in this paper. An assessment has also been made of the reliability of predictions made with these curves.

The functions are shown to differ among the various airframe types and manufacturing facilities both in the amount and rate of change of required direct labor per pound of airframe.

Nevertheless for practical purposes it may be appropriate to use an average of individual progress functions. One such practical purpose would be the prediction of total direct labor requirements for the first 1000 airplanes of a particular model. The average error of prediction is shown to be about 25%. For the entire output of any particular airframe model produced in one facility the error of prediction is also 25%.

If specific curves are fitted to the past performance of a particular manufacturing facility in order to predict its future requirements, the margins of error of prediction average about 20%. All these margins of error, while averaging about 20-25%, represent specific errors which in .9 of the cases range between -40% and +70%.

An illustration of the possible practical significance of such errors is given.

Finally, functions with other variables, in addition to N, are briefly considered.

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RELIABILITY OF PROGRESS CURVES IN AIRFRAME PRODUCTION
- STATISTICAL REPORT -

General Problem & Hypotheses

The "progress function" or "learning curve" is one of the instruments of planning, scheduling and forecasting used in the aircraft industry and the Air Force. It is designed to express the relation between the amount of direct labor required to produce an airframe and the number of airframes produced. It associates the number of direct manhours per pound of airframe used in the production of a specific airframe with the number of airframes of that particular type produced in a specific production facility. The relationship, in general, indicates that the required number of direct manhours decreases as more airframes are produced.

Direct labor is the number of direct manhours that, so to speak, is congealed in the Nth airframe. It is that direct* labor that was expended in the production and fabrication of the component parts and their assembly into that particular airframe. The N of the Nth airframe is the cumulative number of airframes accepted up to and including the Nth airframe. N is not the rate of production per unit of time.

The form of the relationship between direct labor per pound (hereafter called m) of airframe for the Nth airframe, and the Nth airframe, is usually formulated as

$$1) \quad \log_{10} m = a + b \log_{10} N$$
$$\text{subject to } 10^a > 0 \quad \text{and} \quad -1 < b < 0$$

where a and b are parameters of the linear form. Graphically on double log paper the equation plots as a straight line with negative slope.

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* Defined below on page 4.

A statistical study of the reliability of this function for certain types of estimates is presented in this report. It is indisputable that lower direct labor costs occur as the number of items produced increases; the evidence on this point is overwhelming. Questions can be raised however: (1) How long does this reduction continue? (2) Can it be represented by a linear function on double log scale? (3) Does it fall at the same rate for all different airframe manufacturing facilities? (4) How reliably can one predict marginal and total labor requirements for a particular production facility from an industry average progress curve derived from the experience of all airframe manufacturers? (5) How reliably can a curve fitted to the experience of all bomber (fighter) production predict labor requirements for a specific type of bomber (fighter) produced in a particular facility? (6) How reliable is a single manufacturing plant's own early experience for predicting its later requirements for producing a particular type of airframe? (7) What may be the consequences of the margins of error involved in these estimating methods?

The general order of analysis follows the sequence of the above questions. These questions are investigated on the assumption that the estimates are made for a period in which general production conditions are the same as those which prevailed during World War II.

It must be emphasized that this study is concerned with the various types of estimates and predictions that might be made from the assumed linear form of the relationship. No attempt is made here to evaluate other forms of relationships that might be used for certain types of predictions. Nor is there any discussion here of the reasons for the decline in labor

requirements. Both of these questions may be analyzed in subsequent reports.

Source of Information

All information used was derived from the Source Book of World War II Basic Data; Airframe Industry, Vol. I, prepared by AAF Materiel Command, Wright Field (undated). The data reported in the Source Book were in turn derived from Aeronautical Monthly Progress Reports (AMPR's). The reliability of the AMPR's has been subject to a good deal of speculation and remains a moot point. The following description of the data is based entirely on the statements contained in the Source Book itself. The AMPR's provided data on acceptances, direct man-hours per unit and direct man-hour expenditure for the report month, subcontracting, etc. Prior to December 1942 direct man-hours were obtained from letter submitted by facilities or by district offices.

The following definitions were adopted by the AMPR's:

Direct man-hours per pound of airframe, m (on site plus off-site) is obtained by dividing direct unit manhours for the Nth airframe by its unit weight.[1]

Direct man-hours for the entire airframe is the "facility's best estimate of the total number of direct hours which would be required to perform the entire airframe manufacturing operation within the reporting facility".[2] This estimate is in turn the sum of two estimates; (1) "The estimated direct man-hours it would require to perform within the

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1. Source Book, p. 37.
2. Ibid, p. 37.

facility that part of the airframe...being produced outside the plant or plants of the reporting facility" and (2) direct man-hours per unit on site.[1]

Direct man-hours per unit on site are the "contractor's best estimate of (1) the direct unit hours expended within the reporting facility (including feeder plants) prior to acceptance on the last unit for which complete records are available in the report month, or (2) the average direct man-hours cost of the last lot produced for which complete records are available in the report month."[1] That is, the direct man-hours relate either to a single unit, or to an average of a lot -- in either case it is the last unit or last lot for which complete records are available. Man-hours per unit include all hours necessary to complete an airframe, whether these hours are spent during the month of completion (report month) or over a period of several months.

"Direct man-hours charged to a model normally are obtained from shop or work orders and not from payroll records."[1] Man-hours included are hours expended on the airframe manufacturing process, which includes machining, processing, fabricating, assembling, and installing all integral parts of the airplane structure, flight operations (but not test piloting), and reworking prior to acceptance.[1] Not included are hours expended in the production of raw stock, equipment items, spare parts and reworking after acceptance.[2] Direct man-hours are not the same as productive man-hours. The latter include also hours expended in mold loft, in jig fixture and tool production, in inspection, shipping, receiving, and warehousing.[3]

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1. Ibid, p. 37
2. Ibid, p. 23
3. Ibid, p. 1

It is important to note that the observations are the contractor's best estimates of the direct labor used. The methods of making these estimates varied considerably among the manufacturing facilities. It is believed that in some cases very crude estimates were presented. This does not affect the validity of the present study, which is designed to test the predictive utility of progress curves based on reported data. If the progress curves had been derived from exact data, their reliability might be either higher or lower. As long as present and future methods of obtaining data are basically similar to those used in the past, it makes no difference how they were obtained.

Cumulative plane number, N. Through April 1944 these are total acceptances for each model from a given manufacturing facility as reported to the Air Materiel Command Statistical Division in a "Special Historical Report of Airframe Weight", or in letters submitted by the facilities. Beginning with May 1944 the source of these data is the AMPR, #2, or the corrections thereto submitted by the facility or the district office.[1]

All model-facility combinations in the Source Book that satisfied the following criteria were used in the analysis:

1. More than 1,000 airframes of a given model were produced in the facility.
2. Data for airframes with N of less than 100 were available for the facility.
3. More than 60 percent of direct labor in any given month was on site production in the facility provided the cumulative N had reached 100.

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1. Ibid, p. 37

The model-facility combinations that satisfied these criteria were:

1. B-29 Boeing, Wichita
2. B-17 Boeing, Seattle
3. B-24 Ford, Willow Run
4. B-24 Con-Vult., Ft. Worth
5. B-25 N. American, Inglewood
6. B-26 Martin, Baltimore
7. A-20 (DB-7) Douglas, Santa Monica
8. A-30 Martin, Baltimore
9. A-26 Douglas, Long Beach
10. TBM Eastern, Trenton
11. P-40 Curtiss, Buffalo
12. P-39 Bell, Buffalo
13. P-51 (A-36) N. American, Inglewood
14. P-51 N. American, Dallas
15. RP-63 A & C Bell, Buffalo
16. FM1 Eastern, Linden
17. F6F Grumman, Bethpage
18. PT-13-17 (N2S) Boeing, Wichita
19. C-46 Curtiss, Buffalo
20. C-47 Douglas, Okla. City
21. AT-6 (SNJ) N. American, Dallas
22. AT-10 Beech, Wichita

The above facilities were classified into four groups: Bombers, fighters, trainers, and transports.

Statistical Analysis

Question 1: How long does the decline continue? In every case there was no evidence of any cessation of a decline. This conclusion is based on visual examination of the graphs presented in the Source Book. No elaborate statistical analysis appears to be needed to answer this question, given the available data. Whether or not the decline would cease for substantially larger N could not, of course, be determined.

Question 2: Does the progress curve correspond fundamentally to a linear function on double log scale? The purpose of this study is to evaluate the reliability of the learning curve as commonly used in its linear form. Furthermore a test for linearity would require specification of some alternative non-linear functional forms for comparison. Since it appeared that the observations would not be sufficient to give a very powerful test of the linear hypothesis with respect to some acceptable alternative, it was believed best to postpone such possible tests until more adequate observations were available. For the rest of

this study linearity is simply postulated.

The appropriateness of the linear function as a descriptive device for the accumulated data is indicated by the coefficients of correlation. These exceeded .90 in 16 of the model-facility combinations and exceeded .80 in the six other cases.

Question 3: Is the progress curve slope or height the same for all the model-facility combinations? For the first three categories of airframe the following hypotheses were tested for each category separately: H_1 — The k samples from the bombers ($k = 9$), fighters ($k = 8$), and trainers ($k = 3$) are samples from populations with constant height, a_o (unspecified). H_2 — The k samples are from populations with slope b_o (unspecified). Transports were not tested since there were only two acceptable model-facility combinations (hereafter called MFC's).

One difficulty in applying standard statistical tests to these hypotheses is that the residuals around the progress function are serially correlated. This reduces the number of degrees of freedom and almost always understates the size of the internally estimated error. Crude allowance can be made for this effect by assuming that the degrees of freedom are equal to a fraction of the number of observations. In this study the fraction is one-fourth, which is believed to err on the side of making it more difficult to deny the two hypotheses.

All the pertinent data and computations beyond the original quantities listed in the Source Book are given in Table A and B. Table C presents the analysis of variance of H_1 for each of the three categories. Table D summarizes the analysis of covariance for H_2 for each of the three categories.

Because of the qualifications expressed above about the available degrees of freedom, the critical F ratios for .05 and .01 probability are degrees of freedom estimated at one-fourth of the number of observations.

In every case the hypotheses H_1 and H_2 are very clearly denied. This means that question 3 has a negative answer. One may conclude that if a linear relationship between log m and log N exists it exists only uniquely for each particular MFC. The relationships differ in slope and height even among the various facilities producing the same general type of airframe (bombers, fighters, or trainers). The denial of H_1 and H_2 also constitutes a denial of homogeneity of the a_i and b_i where the MFC's are not classified according to bomber, fighter and trainer types.

This means that it is wrong to regard all the individual MFC's as having the same progress function. It is wrong in the sense that if there are linear functional relationships between log m and log N within individual MFC's, they do not have the same heights or slopes. But just as we do not require that everything be equal before considering them fundamentally alike for practical purposes, so one may talk of an average of the curves. Whether the use of the average as typical is appropriate or adequate can be judged only in terms of the margins of error resulting when one uses this averaging technique. It is these margins of error which will now be evaluated.

Margins of Error

The margin of error depends upon what is being predicted. One may predict the direct labor per pound of a given type of airframe or the cumulated direct labor requirements for the production of a given number

of airframes of a particular type. The latter was selected for study as more important. The margins of error will be relatively smaller for cumulative requirements than for marginal requirements, since variations in marginal requirements will offset each other and tend to cancel out when cumulated into a sum of direct labor requirements. It might be added that if one were to seek a method of estimating cumulated direct labor requirements, he would ordinarily obtain a prediction equation directly between cumulated direct labor and N, rather than between marginal direct labor, m, and N. This particular study, however, was directed toward an examination of the progress curve concept as postulated in the Source Book.

The margin of error also depends on the type of progress curve used, of which there are at least three: (1) An industry-wide average progress curve is one in which the a coefficient and b coefficient are obtained by combining all the data into one heterogeneous set. (2) The airframes can be classified on some basis, such as type of airframe (bomber, fighter, trainer), and for each class the a and b coefficients can be computed by pooling the data for that set. (3) The various MFC's can be kept separate and 'a' and 'b' can be derived for each from the early build-up part of its operations (to an approximate peak rate -- usually occurring 1 to 1 1/2 years after the tenth frame was produced). Questions 4, 5, and 6 deal with predictions made from each of these three progress curve types, respectively.

It is essential to note that because H_1 and H_2 were denied, which means that progress curves type (1) and (2) are really averages of heterogeneous concepts, there is no readily available method of deriving from the internal error variance the margin of error of any of the

estimates that might be made with these two types of curve. Therefore an alternative procedure was used for estimating the margin of error. For each facility predictions of direct labor requirements were made by means of the progress curves and compared with realizations as given in Table 3 of the Source Book.

Question 4: How reliable are the predictions derived from an industry-wide average progress curve? An industry-wide average progress curve was obtained by combining all the observations from the 20 selected MFC's (excluding transports) into one large sample. The resulting progress curve was integrated from zero to 1000 to obtain an estimate of the cumulated direct labor requirements for the first 1000 airframes in any MFC. Since these requirements are on a per pound basis and since weights of airframes differ from type to type the cumulated direct labor requirements per pound were multiplied by the weights of the airframes. Adjustments were made for changes in the weights of airframes due to modifications in design. Thus for each MFC an estimate was obtained of the required cumulated direct man-hours for the first 1000 airframes.[1] The prediction of direct labor requirements was confined to 1000 airframes because it was presumed that by the time 1000 airframes had been made the particular MFC could use its own experience for further prediction. Table E presents the resulting predictions and realizations. It will be seen that the absolute differences between predicted and actual values[2] average 25% of the actual. A graphic picture of the results is given in Figure 1.

1. In the cases of a few models the estimate had to be made for a range starting a little beyond the first plane and extending to the 1000th airframe. This was necessitated by the lack of check data for early production.
2. Weighted by actual man-hours.

Question 5: How reliable are the predictions derived from a general airframe-type progress curve? With the airframe-type curve (bomber, fighter, trainer) predictions for each MFC were made for both zero to 1000 airframes and for the entire run of airframes produced by the various MFC's. The bomber-type curve were obtained by combining the observations from the nine bomber MFC's into one large sample. That is, the observations were pooled but not the sample covariances. The fighter type coefficients were obtained by similarly combining observations on the 8 fighter types, and the trainer coefficients were obtained from the 3 trainer types. For each MFC prediction the corresponding type curve was integrated over the appropriate range. This integral, when multiplied by the weight of the plane, is the predicted cumulated direct labor requirement. All necessary data and computations are given in Tables A and B.

Predictions and realizations are given in Table E for the first 1000 airframes. The percentage of error is defined as the ratio of the difference between predicted and actual values to the actual. The weighted average of these errors (non-algebraic) is 25%. This failure to obtain a smaller margin of error by using type curves rather than the industry average curve suggests that there is no significant difference between the average a's and b's by airframe types. Table F contains the results for the complete run for each MFC. In this latter set of predictions the error again averages 26%. Figure 1 graphs the results for 0-1000 airframes and Figure II graphs the results for the complete runs.

Question 6: How reliable is a single MFC's own early build-up progress curve for predicting its subsequent direct labor requirements? For each particular MFC a progress curve was estimated from the build-up portion (usually lasting about one year) of its own production experience.

With this equation predictions were made of the direct labor requirements for the rest of the production run. Predictions were obtained by integrating the progress curve and multiplying by airframe weight. These predictions were then compared with realizations. Coefficients of the "build-up" progress curves are presented in Table G. The results of the predictions are presented in Table H. The average margin of error (ratio of absolute error to actual requirements) is about 22%. Inspection of the results does not indicate any correlation of the relative size of the error or prediction with either the number of airframes for which direct labor is predicted or the type of the airframe. Figure III is a graphic summary of the results.

Question 7: What are some possible consequences of these errors of estimate? The consequences of the errors of estimate in predicting cumulated direct labor requirements can be determined only in the context of some specific problem. As an illustration the following example is presented. If 1000 airframes have been produced and if a total of 5000 is to be produced, one may estimate the required amount of labor for the next 4000 airframes. Suppose that the slope of the progress curve had been computed to be -.32[1]. Now suppose it is discovered that the predicted amount of direct labor required was 20% less than that actually required. How many planes would have been produced by the time the predicted amount of direct labor had been used? Only 3100, or 22% less than the extra 4000 required. If instead the prediction had overstated the required amount, then utilization of the predicted amount would have resulted in 5030 airframes or 27% too many.[2] A review of the figures given in the earlier part of this report will indicate that the above

1. This is equivalent to an 80% progress curve.
2. See Appendix for mathematical derivation.

example is not an unusual one. In general an error in estimating direct labor requirements implies a greater discrepancy between actual and expected airframe production.

Alternative Progress Functions

Alternative relationships between direct labor per pound of airframe, cumulative N, Time and Rate of Production have been suggested and investigated with the present data. The results cast doubts on any of the alternatives being better fits than the usual progress curve. The principal reason that little improvement would be expected is the presence of very high correlation among Time, N and N. The various other relationships considered for each particular MFC were:

a. $\log m = a_2 + b_2 T$, where T is time

b. $\log m = a_3 + b_3 T + b_4 \Delta N$, where ΔN is rate of production per month

c. $\log m = a_4 + b_5 \log T + b_6 \log \Delta N$

d. $\log m = a_5 + b_7 T + b_8 \log \Delta N$

e. $\log m = a_6 + b_9 T + b_{10} \log N$

f. $\log m = a_7 + b_{11} \log N + b_{12} \log \Delta N$

Tables I-N list the values of the regression and correlation coefficients for each MFC.

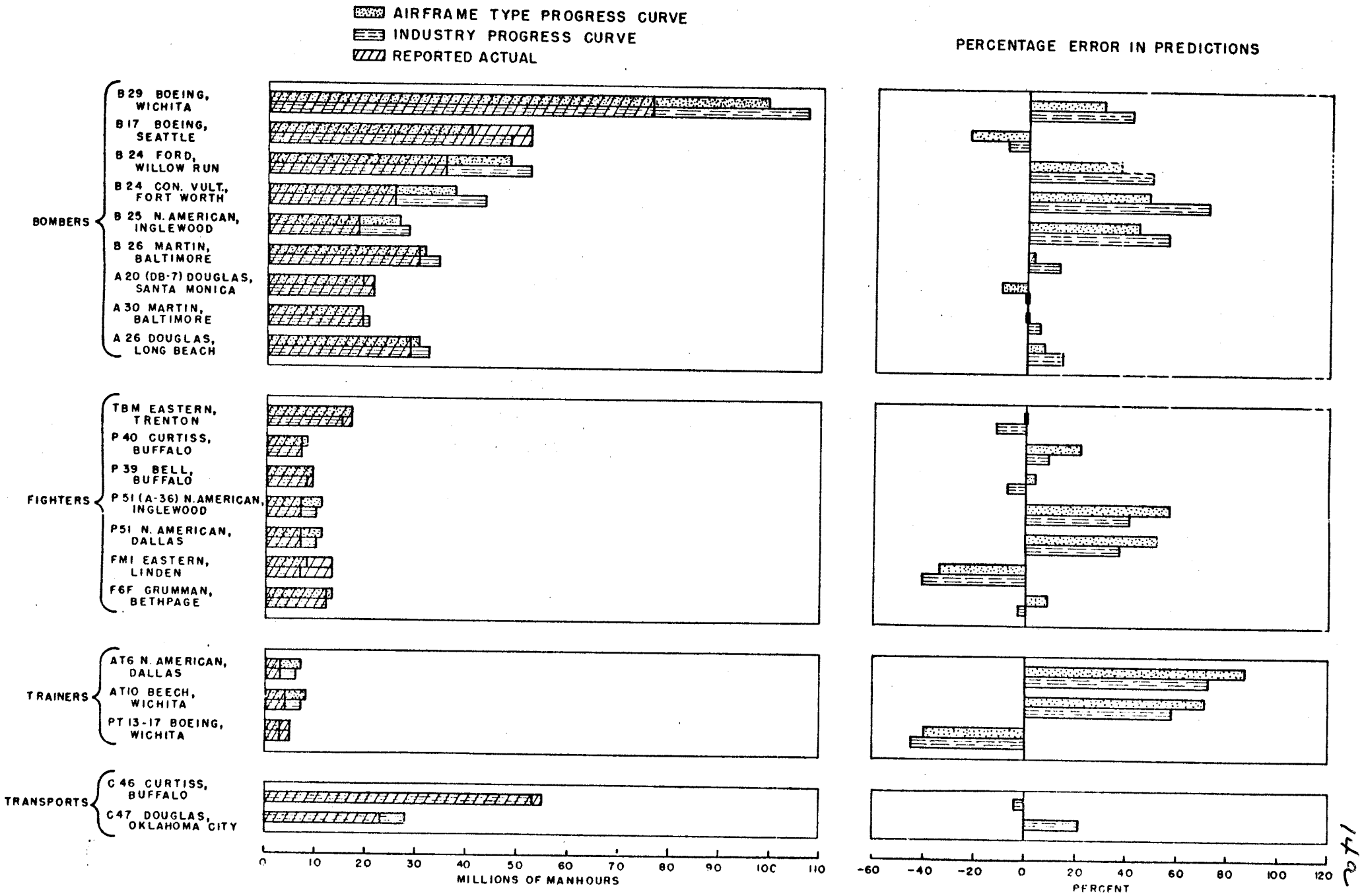
Conclusion

The preceding analysis has been concerned with the margin of error to be expected when estimating direct labor requirements and cumulative airframe production by the linear progress curve if the basic assumption of historical similarity of production conditions is fulfilled. The virtual certainty of non-fulfillment of some part of the basic assumption would increase the magnitude and seriousness of error. In each case there should be an investigation of the range of uncertainty in prediction (e.g., acceleration curves and program feasibilities, which

are in part derived from and based on "progress curves") before making decisions. This follows from the fact that reliable decisions can be made only among those alternative programs that are disparate beyond the range of uncertainty of error of estimate of the predictive method.

FIG. I

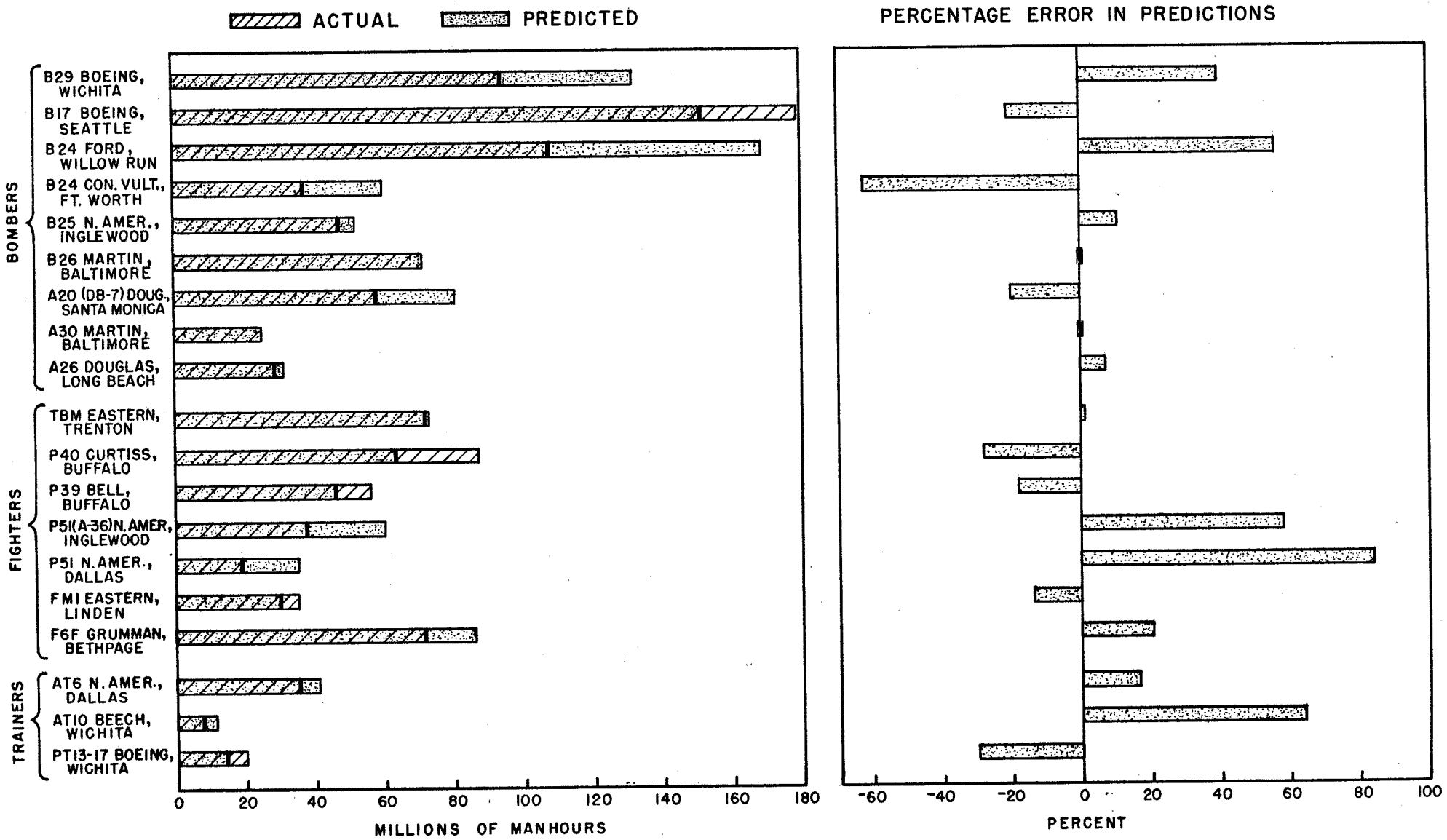
PREDICTED AND ACTUAL DIRECT LABOR REQUIREMENTS FOR FIRST 1000 AIRFRAMES
PREDICTIONS FROM INDUSTRY AVERAGE AND AIRFRAME TYPE



14a

FIG. II

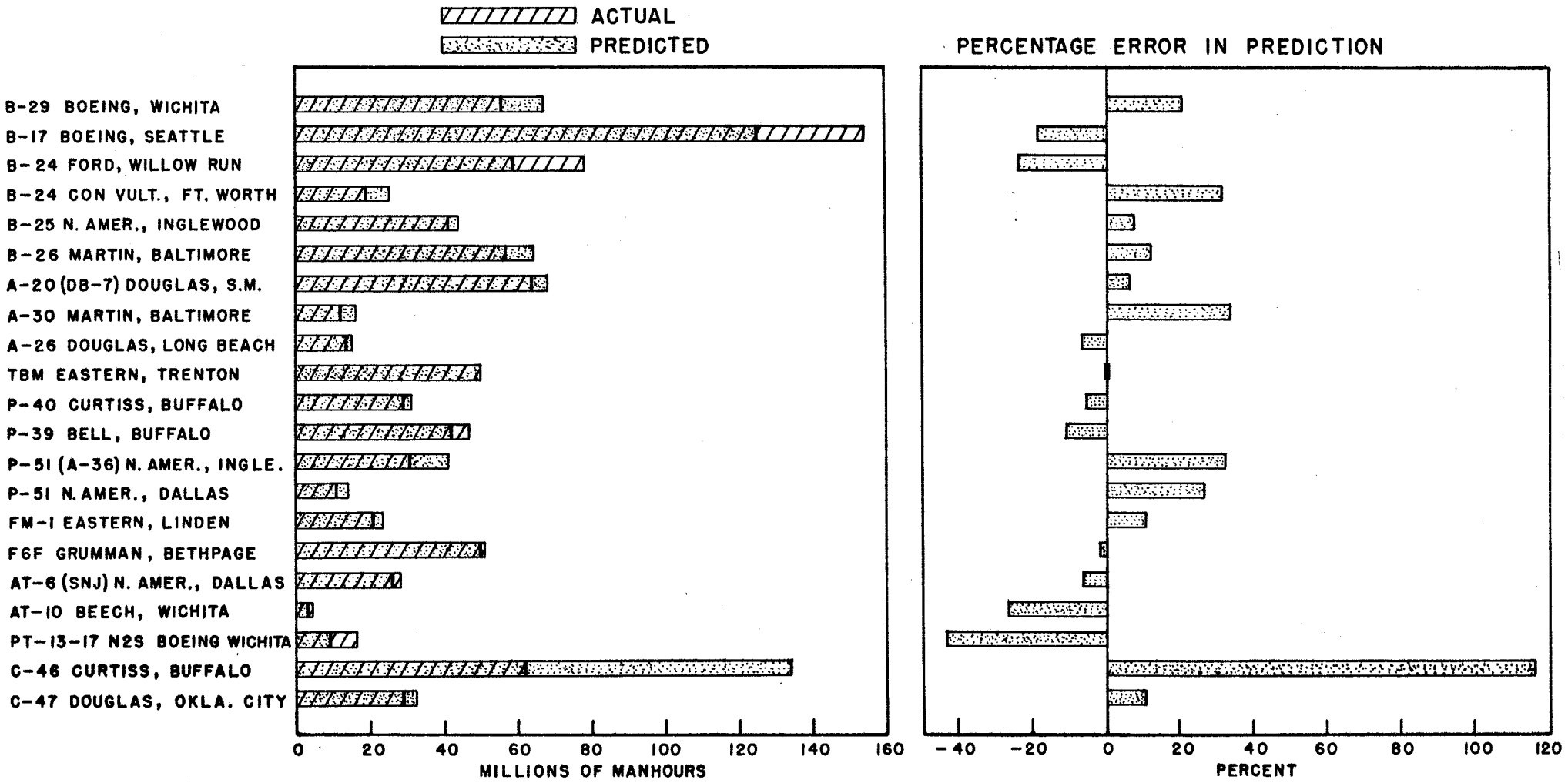
PREDICTED AND ACTUAL DIRECT LABOR REQUIREMENTS FOR TOTAL PRODUCTION PREDICTED BY AIRFRAME TYPE PROGRESS CURVES



SOURCE: TABLE F

FIG. III

PREDICTED AND ACTUAL DIRECT LABOR REQUIREMENTS (AFTER PRODUCTION BUILDUP); PREDICTIONS BY EACH MODEL FACILITY COMBINATION'S PROGRESS CURVE DURING BUILDUP PERIOD



SOURCE: TABLE H

TABLE A

Summary of Data and Computations

X_{ij} = $\log_{10}$ of number of airframes (N) for ith month of jth sample ($i = 1,\ldots n_j$) ($j = 1,\ldots..k$)

Y_{ij} = $\log_{10}$ of direct labor per pound (m) of Nth airframe

j	Model Facility Combination (MFC)	no. of months n_j	$\bar{X}_j$	$\bar{Y}_j$	$\sum_i (X-\bar{X}_j)(Y-\bar{Y}_j)$	$\sum_i (X-\bar{X}_j)^2$	$\sum_i (Y-\bar{Y}_j)^2$
	Bombers						
1	B29 Boeing, Wichita	29	2.39055	.21958	- 7.07967	13.07530	3.89334
2	B17 Boeing, Seattle	53	3.01342	.19471	-12.78098	26.46808	6.53935
3	B24 Ford, Willow Run	32	3.19243	-.16110	- 9.66374	18.15253	5.38342
4	B24 Con Vult, Ft. Worth	21	2.79004	.01215	- 1.88999	3.52311	1.04726
5	B25 N. Amer., Inglewood	40	2.79174	.11084	- 3.92989	18.39503	1.02686
6	B26 Martin, Baltimore	49	2.92967	.13123	- 6.24223	16.46687	2.61667
7	A20 Douglas, S. M.	51	2.94629	.25462	- 6.66702	19.46477	2.95151
8	A30 Martin, Baltimore	32	2.70846	.24832	- 1.99458	6.32976	.91616
9	A26 Douglas, Long Beach	11	2.54338	.29768	- 1.18027	3.36022	.42240
	Within sample totals	-	-	-	-51.42837	125.23567	24.79197
	Total sample	318	2.86138	.14725	-55.60084	139.15419	29.71182
	Fighters						
10	TBM Eastern, Trenton	30	3.10854	.24263	- 4.97647	15.19139	1.67317
11	P40 Curtiss, Buffalo	54	3.53743	.24786	- 3.13698	22.04390	.64323
12	D39 Bell, Buffalo	42	3.14535	.28694	- 6.04679	26.94483	1.41427
13	P51 No. Amer, Inglewood	45	3.21869	.03858	- 5.39520	15.07918	2.51940
14	P51 No. Amer, Dallas	21	3.07116	-.00982	- 3.11269	7.68082	1.40582
15	RP63 Bell, Buffalo	8	2.54156	.30728	- .40268	1.29794	0.12544
16	FM1 Eastern, Linden	32	3.13038	.29918	- 6.40178	13.79663	3.02507
17	F6F Grumman, Bethpaze	32	3.41592	.07667	- 5.42154	15.87386	1.82191
	Within sample totals	-	-	-	-34.90413	117.90855	12.69833
	Total sample	264	3.24065	.18436	-35.38149	129.47640	16.11514
	Trainers						
18	AT6 No. Amer, Dallas	52	3.49985	.01651	- 3.68499	26.36360	.70333
19	AT10 Beech, Wichita	16	2.67094	.11772	- 1.69323	5.35816	.58006
20	PT13-17 Boeing, Wichita	60	3.29319	.31733	-11.99147	26.98049	5.57605
	Within sample totals	-	-	-	-17.36969	58.70225	6.86449
	Total sample	128	3.29936	.17009	-18.50085	67.11315	9.43884
	Transports						
21	C46 Curtiss, Buffalo	40	2.54941	.31958	- 4.81209	16.20536	1.97441
22	C47 Douglas, Oklahoma City	28	2.96840	.07812	- 6.78549	14.64911	3.24972
	Within sample totals	-	-	-	-11.59758	30.85447	5.22413
	Total sample	68	2.72194	.22016	-12.67951	33.74591	6.18438
	Industry						
	Within sample totals	-	-	-	-115.29977	332.70094	49.57895
	Total sample	778	3.04995	.16997	-121.77594	405.67841	61.84021

TABLE B

Model Facility - Combination Progress Curve Statistics

j	Model Facility Combination (MFC)	MFC progress curve intercept a_j	MFC progress curve slope b_j	$\sum_1^n y'^2_{.x}$	s_b	$r_{y.x}$	Range of No. of airframes (N lower	Range of No. of airframes (N upper
	Bombers							
1	B29 Boeing, Wichita	1.514	-.5414	.0600	.0130	-.992	0	1606
2	B17 Boeing, Seattle	1.649	-.4828	.3576	.0163	-.972	45	6949
3	B24 Ford, Willow Run	1.538	-.5323	.2438	.0212	-.977	0	8238
4	B24 Con Vult, Ft. Worth	1.508	-.5364	.0333	.0223	-.984	79	1927
5	B25 N. Amer, Inglewood	.707	-.2136	.1872	.0164	-.904	0	3180
6	B26 Martin, Baltimore	1.241	-.3790	.2503	.0180	-.951	0	3677
7	A20 Douglas, S. M.	1.263	-.3425	.6679	.0265	-.880	0	5685
8	A30 Martin, Baltimore	1.100	-.3151	.2376	.0389	-.828	0	1566
9	A26 Douglas, Long Beach	1.191	-.3512	.0078	.0161	-.991	0	1107
	Within sample totals	1.322	-.411	3.6723	.0093	-.96	0	8238
	Total sample	1.291	-.400	7.4958	.0131	-.93		
	Fighters							
10	TBM Eastern, Trenton	1.258	-.3275	.0429	.0103	-.987	7	7190
11	P40 Curtiss, Buffalo	.751	-.1423	.1968	.0108	-.833	9	13686
12	D39 Bell, Buffalo	.992	.2244	.0572	.0073	-.980	0	9407
13	P51 No. Amer, Inglewood	1.190	.5577	.5890	.0301	-.875	0	9872
14	P51 No. Amer, Dallas	1.234	.4052	.1443	.0315	-.947	0	4650
15	RP 63 Bell, Buffalo	1.095	.3102	.0005	.0025	-.998		1030
16	F11 Eastern, Linden	1.751	.4640	.0545	.0115	-.991	23	5715
17	F6F Grumman, Bethpaze	1.245	-.3421	.0334	.0084	-.991	22	12211
	Within sample totals	1.144	-.296	2.3658	.0089	-.95	0	13686
	Total sample	1.070	-.273	6.4466	.0138	-.88		
	Trainers							
18	AT6 No. Amer, Dallas	.505	-.1397	.1933	.0121	-.853	0	12811
19	AT10 Beech, Wichita	.961	.3160	.0450	.0245	-.960	19	1700
20	PT13-17 Boeing, Wichita	1.781	-.4444	.2464	.0126	-.978	20	8419
	Within sample totals	1.146	-.296	1.7249	.0162	-.93	0	12811
	Total sample	1.080	-.276	4.3388	.0227	-.86		
	Transports							
21	C46 Curtiss, Buffalo	1.076	-.2969	.5455	.0297	-.851	0	2526
22	C47 Douglas, Oklahoma City	1.453	-.4632	.1067	.0167	-.984	0	5190
	Within sample totals	1.243	-.376	.8648	.0207	-.96	0	5190
	Total sample	1.243	-.376	1.4202	.0253	-.94		
	Industry							
	Within sample totals	1.227	-.347	9.6210	.0062	-.95	0	13.686
	Total sample	1.086	-.300	15.2857	.0070	-.88		

TABLE C

Analysis of Variance Test of:

H_1: The samples from each category (bombers, fighters, transports) are from populations with equal intercepts; A_o (unspecified).

Category	Source of Variation	Sum of Squares (a)	Degrees of freedom (b)	Mean square (a/b)	F
Bombers	(1) among MFC	26.24	8	3.28	
	(2) within MFC	1.3400	300	.00447	
	(1) ÷ (2)				733
Fighters	(1) among MFC	22.1007	7	3.157	
	(2) within MFC	.88806	248	.00358	
	(1) ÷ (2)				882
Trainers	(1) among MFC	46.042	2	23.021	
	(2) within MFC	.2757	122	.00226	
	(1) ÷ (2)				11,371

$F_{.01}$ exceeded in all cases

$$\frac{\dfrac{\sum_j n_j a_j - \dfrac{(\sum_j n_j a_j)^2}{\sum_j n_j}}{k-1}}{\dfrac{\dfrac{\sum_j s_{y.x} \sum_i X_{ij}^2}{\sum_i x_{ij}^2}}{\sum_j (n_j - 2)}}$$

TABLE D

Analysis of Variance Test of:

H_2: The samples from each category (bombers, trainers, fighters) are from populations with equal slopes, β (unspecified).

Category	Source of Variation	Sum of Squares (a)	Degrees of freedom (b)	Mean square (a/b)	F
Bombers	(1) among individual regression coefficients	1.57685	8	.19711	
	(2) within sample-individual regression coefficient-residuals (1) ÷ (2)	2.09592	300	.00698	28.2
Fighters	(1) among individual regression coefficients	1.2457	7	.17811	
	(2) within sample individual regression coefficients-residuals (1) ÷ (2)	1.11903	248	.00451	39.49
Trainers	(1) among individual regression coefficients	1.24	2	.62008	
	(2) within sample individual regression coefficient-residuals (1) ÷ (2)	.4847	122	.00397	156.2

$F_{.01}$ exceeded in all cases

$$F = \frac{\dfrac{\sum_j \left(\sum_i xy\right)^2 - \left(\sum_j \sum_i xy\right)^2}{\sum_j \sum_i x^2}}{k-1} \Bigg/ \frac{\sum_j \sum_i y^2 - \dfrac{\left(\sum_i xy\right)^2}{\sum_i x^2}}{\sum_j n_j - 2k}$$

TABLE E

Predictions of Direct Labor Requirements for First 1000 Planes (less N_o) by Industry Progress Curve and by Airframe Type Progress Curve

j	Model Facility Combination	N_o	Manhours (Millions) Predicted by Industry Curve	Manhours (Millions) Predicted by Airframe type curve	Manhours (Millions) Actual Reported	$\frac{Pr - Ac}{Ac}$ (Percent) Industry Curve	$\frac{Pr - Ac}{Ac}$ (Percent) Airframe type curve
	Bombers:						
1	B29 Boeing, Wichita	0	107	99	76	+ 41	+ 30
2	B17 Boeing, Seattle	45	48	40	52	- 8	- 23
3	B24 Ford, Willow Run	0	52	48	35	+ 49	+ 37
4	B-24 Con.Vult., Ft. Worth	79	43	37	25	+ 72	+ 48
5	B25 N.Amer., Inglewood	0	28	26	18	+ 56	+ 44
6	B26 Martin, Baltimore	0	34	31	30	+ 13	+ 03
7	A20 (DB7) Douglas, S.M.	0	21	19	21	+ 0	- 10
8	A30 Martin, Baltimore	0	20	19	19	+ 5	0
9	A26 Douglas, Long Beach	0	32	30	28	+ 14	+ 07
	error per bomber facility					29	22
	weighted average* per bomber facility					.29	.24
	Fighters:						
10	TBM Eastern, Trenton	7	15	17	17	- 12	0
11	P40 Curtiss, Buffalo	9	7	8	7	+ 9	+ 22
12	P39 Bell, Buffalo	0	8	9	9	- 7	+ 4
13	P51 (A36) N.Amer., Ingle.	0	10	11	7	+ 41	+ 57
14	P51 N.American, Dallas	0	10	11	7	+ 37	+ 52
15	RP63 A&C Bell, Buffalo	-	-	-	-	-	-
16	FM1 Eastern, Linden	23	7	8	13	- 41	- 34
17	F6F Grumman, Bethpaze	22	12	13	12	- 3	+ 9
	error per fighter facility					21	25
	weighted average* per fighter facility					20	21
	Trainers:						
18	AT6 N.American, Dallas	0	6	7	3	+ 73	+ 87
19	AT10 Beech, Wichita	19	7	8	4	+ 58	+ 71
20	PT13-17 Boeing, Wichita	20	3	3	5	- 45	- 40
	error per trainer facility					59	66
	weighted average* per trainer facility					56	62
	Transports:						
21	C46 Curtiss, Buffalo	0	53	-	55	- 04	-
22	C47 Douglas, Okla. City	0	28	-	23	+ 22	-
	weighted average* per transport facility					09	-
	All Facilities:						
	error per facility (non-algebraic average)					28	29
	weighted* error per facility (non-algebraic)					25	25

* Weighted by actual manhours.
- Not computed.

TABLE F

Predicted and Actual Direct Labor Requirements for Total Production
Based on Airframe Type Progress Curves

j	Model Facility Combination	N_o	N_z	Direct Manhours: Predicted based on type curve	Direct Manhours: Reported Actual	$\frac{P_r - A_c}{A_c}$
				(Millions)		(Percent)
	Bombers					
1	B29 Boeing, Wichita	0	1606	132	94	+ 40
2	B17 Boeing, Seattle	45	6949	151	179	- 21
3	B24 Ford, Willow Run	0	8238	169	108	+ 56
4	B24 Con. Vult., Ft. Worth	79	1927	60	37	- 62
5	B25 N. Amer., Inglewood	0	3180	52	47	+ 11
6	B26 Martin, Baltimore	0	3677	71	71	0
7	A20 (DB-7) Douglas, S.M.	0	5685	58	81	- 20
8	A30 Martin, Baltimore	0	1566	25	25	0
9	A26 Douglas, Long Beach	0	1107	31	29	+.07
	Error per bomber facility					24
	Weighted error* per bomber facility					27
	Fighters					
10	TBM Eastern, Trenton	7	7190	73	72	+ 01
11	P40 Curtiss, Buffalo	9	13686	63	87	- 28
12	P39 Bell, Buffalo	0	9407	46	56	- 18
13	P51 (A-36) N. Amer., Ingle.	0	9872	60	38	+ 58
14	P51 N. Amer., Dallas	0	4650	35	19	+ 84
15	RP63 A&C Bell, Buffalo	-	-	-	-	-
16	FM1 Eastern, Linden	23	5715	30	35	- 14
17	F6F Grumman, Bethpaze	22	12211	86	71	+ 21
	Error per fighter facility					.32
	Weighted Error* per fighter facility					25
	Trainers					
18	AT6 N. Amer., Dallas	0	12811	41	35	+ 17
19	AT10 Beech, Wichita	19	1700	11	7	+ 64
20	PT13-17 Boeing, Wichita	20	8419	14	20	- 30
	Error per trainer facility					37
	Weighted error* per trainer facility					27
	Total - all facilities					
	Error per facility					29
	Weighted error* per facility					26

* Weighted by actual manhours.
- Not computed.

TABLE G

First and Second Portion Progress Curves
Individual Model Facility Combinations

j	Model Facility Combination (MFC)	Range of product 1st portion	Progress curve for 1st portion intercept	slope	Progress curve for 2nd portion intercept	slope	Range of Production for 2nd portion
1	B29 Boeing, Wichita	0- 267	1.41	.48	2.21	.78	268- 1606
2	B17 Boeing, Seattle	45- 385	1.87	.58	1.45	.43	386- 6949
3	B24 Ford, Willow Run	0- 769	1.69	.61	1.62	.55	770- 8238
4	B24 Con Vult. Ft.Worth	79- 711	1.25	.43	1.94	.68	712- 1927
5	B25 N. Amer., Ingle.	0- 242	.64	.19	1.14	.35	243- 3180
6	B26 Martin, Baltimore	0- 409	1.17	.35	1.54	.47	410- 3677
7	A20mDouglas, S.M.	0- 623	1.14	.30	-	-	624- 5685
8	A30 Martin, Baltimore	0- 651	.88	.21	-	-	652- 1566
9	A26 Douglas, Long Beach	0- 402	1.24	.38	1.20	.35	403- 1107
10	TBM Eastern, Trenton	7-1335	1.23	.32	1.86	.49	1336- 7190
11	P40 Curtiss, Buffalo	9-8494	.81	.16	.13	.08	8495-13686
12	D30 Bell, Buffalo	0-1073	1.05	.26	.99	.22	1074- 9407
13	P51 N.Amer., Ingle.	0- 910	.77	.21	-	-	911- 9872
14	P51 N.Amer., Dallas	0-1248	1.04	.32	3.19	.97	1249- 4650
15	RP63 Bell, Buffalo	-	-	-	-	-	-
16	FM1 Eastern, Linden	23-1215	1.65	.42	2.14	.57	1216- 5715
17	F6F Grumman, Bethpaze	22-2097	1.27	.35	1.43	.39	2098-12211
18	AT6 N.Amer., Dallas	0-2089	.52	.15	.97	.26	2090-12811
19	AT10 Beech, Wichita	19- 560	1.11	.39	-	-	561- 1700
20	PT13-17 Boeing, Wichita	20- 720	2.19	.63	1.37	.33	721- 8419
21	C46 Curtiss, Buffalo	450	.63	.06	-	-	451- 2526
22	C47 Douglas, Okla. City	1384	1.50	.48	.90	.31	1385- 5190

- Not computed.

TABLE H

Predicted and Actual Direct Labor Requirements for 2nd Portion of Production
Based on Individual Model Facility Combination Progress Curves (1st Portion)

J	Model facility combination	Second Portion Range of Cumulative Production	Predicted Direct Labor Manhours based on MFC Curve of First Portion	Reported Actual	$\frac{P_r - mA_c}{A_c}$
			(Millions)	(Millions)	(Percent)
1	B-29 Boeing, Wichita	268 - 1606	67	56	+ .20
2	B-17 Boeing, Seattle	386 - 6949	125	154	- .19
3	B-24 Ford, Willow Run	770 - 8238	59	78	- .24
4	B-24 Con Vult., Ft. Worth	712 - 1927	25	19	+ .31
5	B-25 N. Amer., Inglewood	243 - 3180	44	41	+ .07
6	B-26 Martin, Baltimore	410 - 3677	64	57	+ .12
7	A-20 (DB-7) Douglas, S. M.	624 - 5685	68	64	+ .06
8	A-30 Martin, Baltimore	652 - 1566	16	12	+ .31
9	A-26 Douglas, Long Beach	403 - 1107	14	15	- .07
10	TBM Eastern, Trenton	1336 - 7190	50	50	.00
11	P-40 Curtiss, Buffalo	8495 - 13686	29	31	- .06
12	P-39 Bell, Buffalo	1074 - 9407	42	47	- .11
13	P-51 (A-36) N. Amer., Ingle.	911 - 9872	41	31	+ .32
14	P-51 N. Amer., Dallas	1249 - 4650	14	11	+ .26
15	RP-63 A&C Bell, Buffalo	-	-	-	-
16	FM1 Eastern, Linden	1216 - 5715	23	21	+ .10
17	F6F Grumman, Bethpaze	2098 - 12211	50	51	- .02
18	AT-6 (SNJ) N. Amer., Dallas	2090 - 12811	26	28	- .07
19	AT-10 Beech, Wichita	560 - 1700	3	4	- .27
20	PT-13-17 (N2S) Boeing, Wichita	721 - 8419	9	16	- .44
21	C-46 Curtiss, Buffalo	451 - 2526	134	62	+1.16
22	C-47 Douglas, Okla. City	1385 - 5190	32	29	+ .10
	Error per facility				.21
	Weighted error per facility				.22

- Not computed.

TABLE I

Regression and Correlation Statistics of

$\log m = a_2 + b_2 T$

	Plane and Plant	a_2	b_2	r_2
1	B-29 Boeing, Wichita	.87	-.043	-.99
2	B-17 Boeing, Seattle	.89	-.020	-.94
3	B-24 Ford, Willow Run	.49	-.040	-.89
4	B-24 Con-Vult., Ft. Worth	.48	-.036	-.98
5	B-25 N. Amer., Inglewood	.38	-.013	-.96
6	B-26 Martin, Baltimore	.51	-.015	-.92
7	A-20 (DB-7) Douglas, S.M.	.63	-.014	-.88
8	A-30 Martin, Baltimore	.49	-.015	-.81
9	A-26 Douglas, Long Beach	.76	-.036	-.97
10	T-B-M Eastern, Trenton	.64	-.026	-.94
11	P-40 Curtiss, Buffalo	.37	-.0045	-.64
12	P-39 Bell, Buffalo	.58	-.014	-.90
13	P-51 (A-36) N. Amer., Ingle.	.42	-.017	-.91
14	P-51 N. Amer., Dallas	.44	-.041	-.97
15	RP-63 A&C Bell, Buffalo	.55	-.053	-.97
16	FM-1 Eastern, Linden	.81	-.031	-.94
17	F6F Grumman, Bethpaze	.47	-.024	-.90
18	At-6 (SNJ) N. Amer., Dallas	.18	-.0062	-.80
19	At-10 Beech, Wichita	.41	-.035	-.84
20	PT-13-17 (N2S) Boeing, Wichita	.75	-.014	-.81
21	C-46 Curtiss, Buffalo	.68	-.017	-.91
22	C-47 Douglas, Okla. City	.63	-.038	-.91

TABLE J

Regression and Correlation Statistics of

$$\log m = a_3 + b_3 T + b_4 \Delta N$$

	Plane and Plant	a_3	b_3	b_4	R_3
1.	B-29 Boeing, Wichita	.85	-.032	-.0028	-.99
2.	B-17 Boeing, Seattle	.88	-.016	-.00091	-.96
3.	B-24 Ford, Willow Run	.60	-.030	-.0011	-.93
4.	B-24 Con-Vult., Ft. Worth	.50	-.033	-.00064	-.98
5.	B-25 N. Amer., Inglewood	.39	-.013	-.00025	-.96
6.	B-26 Martin, Baltimore	.56	-.012	-.0017	-.94
7.	A-20 (DB-7) Douglas, S.M.	.62	-.012	-.00047	-.89
8.	A-30 Martin, Baltimore	.60	-.011	-.0033	-.86
9.	A-26 Douglas, Long Beach	.75	-.021	-.0024	-.98
10.	T-B-M Eastern, Trenton	.69	-.0058	-.0015	-.96
11.	P-40 Curtiss, Buffalo	.45	-.0031	-.00047	-.80
12.	P-39 Bell, Buffalo	.58	-.012	-.00013	-.90
13.	P-51 (A-36) N. Amer., Ingle:	.38	-.0071	-.00081	-.24
14.	P-51 N. Amer., Dallas	.48	-.038	-.00034	-.98
15.	RP-63 A&C Bell, Buffalo	.58	-.041	-.00068	-.98
16.	FM-1 Eastern, Linden	.93	-.027	-.0010	-.97
17.	F6F Grumman, Bethpaze	.57	-.017	-.00054	-.94
18.	At-6 (SNJ) N. Amer., Dallas	.22	-.0059	-.00017	-.82
19.	At-10 Beech, Wichita	.48	-.031	-.00093	-.88
20.	PT-13-17 (N2S) Boeing, Wichita	.82	-.013	-.00071	-.85
21.	C-46 Curtiss, Buffalo	.64	-.0092	-.0021	-.93
22.	C-47 Douglas, Okla. City	.66	-.033	-.00056	-.93

TABLE K

Regression and Correlation Statistics of

$$\log m = a_4 + b_5 \log T + b_6 \log \Delta N$$

	Plane and Plant	a_4	b_5	b_6	R_4
1.	B-29 Boeing, Wichita	1.29	- .73	-.19	-.97
2.	B-17 Boeing, Seattle	1.47	- .45	-.33	-.94
3.	B-24 Ford, Willow Run	1.14	-1.023	-.074	-.98
4.	B-24 Con-Vult., Ft. Worth	.97	- .84	-.040	-.98
5.	B-25 N. Amer., Inglewood	.56	- .40	+.018	-.90
6.	B-26 Martin, Baltimore	.95	- .48	-.12	-.95
7.	A-20 (DB-7) Douglas, S. M.	1.21	- .18	-.38	-.87
8.	A-30 Martin, Baltimore	.91	- .31	-.20	-.89
9.	A-26 Douglas, Long Beach	1.027	- .63	-.044	-.99
10.	T-B-M Eastern, Trenton	.76	- .83	+.17	-.99
11.	P-40 Curtiss, Buffalo	.87	- .13	-.20	-.90
12.	P-39 Bell, Buffalo	.88	- .42	-.036	-.97
13.	P-51 (A-36) N. Amer., Ingle.	1.046	- .19	-.35	-.91
14.	P-51 N. Amer., Dallas	.57	- .76	+.058	-.96
15.	RP-63 A&C Bell, Buffalo	.66	- .40	-.060	-.10
16.	FM-1 Eastern, Linden	1.17	- .86	+.041	-.99
17.	F6F-Grumman, Bethpaze	.83	- .65	-.013	-.99
18.	At-6 (SNJ) N. Amer., Dallas	.34	- .28	+.018	-.87
19.	At-10 Beech, Wichita	.74	- .47	-.12	-.97
20.	PT-13-17 (N2S) Boeing, Wichita	1.50	- .71	-.11	-.98
21.	C-46 Curtiss, Buffalo	1.04	- .43	-.77	-.92
22.	C-47 Douglas, Okla. City	1.060	- .90	-.017	-.96

TABLE L

Regression and Correlation Statistics of

$$\log m = a_5 + b_7 T + b_8 \log \Delta N$$

	Plane and Plant	a_5	b_7	b_8	R_5
1.	B-29 Boeing, Wichita	.96	-.037	-.12	-.99
2.	B-17 Boeing, Seattle	1.10	-.012	-.24	-.98
3.	B-24 Ford, Willow Run	1.38	-.023	-.51	-.98
4.	B-24 Con-Vult., Ft. Worth	.52	-.035	-.029	-.98
5.	B-25 N. Amer., Inglewood	.42	-.012	-.031	-.96
6.	B-26 Martin, Baltimore	.69	-.013	-.14	-.94
7.	A-20 (DB-7) Douglas, S.M.	.87	-.0097	-.19	-.89
8.	A-30 Martin, Baltimore	.77	-.012	-.20	-.89
9.	A-26 Douglas, Long Beach	1.070	-.016	-.32	-.10
10.	T-B-M Eastern, Trenton	1.17	-.013	-.32	-.99
11.	P-40 Curtiss, Buffalo	.92	-.0024	-.26	-.90
12.	P-39 Bell, Buffalo	.83	-.0072	-.18	-.96
13.	P-51 (A-36) N. Amer., Ingle.	.76	-.010	-.23	-.94
14.	P-51 N. Amer., Dallas	.80	-.036	-.18	-.98
15.	RP-63 A&C Bell, Buffalo	.82	-.042	-.16	-.98
16.	FM-1 Eastern, Linden	1.51	-.024	-.38	-.99
17.	F6F Grumman, Bethpaze	1.22	-.015	-.36	-.98
18.	At-6 (SNJ) N. Amer., Dallas	.42	-.0053	-.11	-.85
19.	At-10 Beech, Wichita	.94	-.023	-.33	-.94
20.	PT-13-17 (N2S) Boeing, Wichita	1.33	-.012	-.32	-.91
21.	C-46 Curtiss, Buffalo	.72	-.015	-.055	-.91
22.	C-47 Douglas, Okla. City	.77	-.038	-.076	-.93

TABLE M

Regression and Correlation Statistics of

$$\log m = a_6 + b_9 T + b_{10} \log N$$

	Plane and Plant	a_6	b_9	b_{10}	R_6
1.	B-29 Boeing, Wichita	1.24	-.020	-.30	-.99
2.	B-17 Boeing, Seattle	1.84	+.0053	-.61	-.97
3.	B-24 Ford, Willow Run	1.56	+.0013	-.55	-.98
4.	B-24 Con-Vult., Ft. Worth	1.13	-.014	-.33	-.99
5.	B-25 N. Amer., Inglewood	.39	-.013	-.0038	-.96
6.	B-26 Martin, Baltimore	1.10	-.0033	-.30	-.95
7.	A-20 (DB-7) Douglas, S.M.	.95	-.0077	-.17	-.89
8.	A-30 Martin, Baltimore	.91	-.0056	-.21	-.84
9.	A-26 Douglas, Long Beach	1.25	+.0054	-.40	-.99
10.	T-B-M Eastern, Trenton	1.15	-.0055	-.27	-.99
11.	P-40 Curtiss, Buffalo	.93	+.0035	-.22	-.86
12.	P-39 Bell, Buffalo	1.011	+.00079	-.24	-.98
13.	P-51 (A-36) N. Amer., Ingle.	.49	-.015	-.032	-.91
14.	P-51 N. Amer., Dallas	.81	-.026	-.18	-.99
15.	RP-63 A&C Bell, Buffalo	1.13	+.0032	-.33	-.10
16.	FM-1 Eastern, Linden	1.60	-.0062	-.38	-.99
17.	F6F Grumman, Bethpaze	1.22	-.0010	-.33	-.99
18.	At-6 (SNJ) N. Amer., Dallas	.43	-.0020	-.10	-.86
19.	At-10 Beech, Wichita	1.090	+.012	-.40	-.97
20.	PT-13-17 (N2S) Boeing, Wichita	2.10	+.0069	-.61	-.99
21.	C-46 Curtiss, Buffalo	- .0021	-.043	+.47	-.94
22.	C-47 Douglas, Okla. City	1.51	+.0036	-.50	-.98

TABLE N

Regression and Correlation Statistics of

$$\log m = a_7 + b_{11} \log N + b_{12} \log \Delta N$$

	Plane and Plant	a_7	b_{11}	b_{12}	R_7
1.	B-29 Boeing, Wichita	1.52	-.57	+.034	-.99
2.	B-17 Boeing, Seattle	1.60	-.39	-.13	-.98
3.	B-24 Ford, Willow Run	1.55	-.52	-.028	-.98
4.	B-24 Con-Vult., Ft. Worth	1.52	-.52	-.022	-.99
5.	B-25 N. Amer., Inglewood	.69	-.26	+.089	-.91
6.	B-26 Martin, Baltimore	1.27	-.34	-.075	-.96
7.	A-20 (DB-7) Douglas, S. M.	1.30	-.22	-.21	-.89
8.	A-30 Martin, Baltimore	1.26	-.26	-.19	-.90
9.	A-26 Douglas, Long Beach	1.22	-.25	-.16	-.99
10.	T-B-M Eastern, Trenton	1.22	-.37	+.074	-.99
11.	P-40 Curtiss, Buffalo	.97	-.079	-.19	-.90
12.	P-39 Bell, Buffalo	.99	-.23	+.014	-.98
13.	P-51 (A-36) N. Amer., Ingle.	1.25	-.19	-.27	-.93
14.	P-51 N. Amer., Dallas	1.12	-.46	+.12	-.95
15.	RP-63 A&C Bell, Buffalo	1.05	-.33	+.048	-1.00
16.	FM-1 Eastern, Linden	1.63	-.52	+.13	-.99
17.	F6F Grumman, Bethpaze	1.18	-.37	+.067	-.99
18.	At-6 (SNJ) N. Amer., Dallas	.45	-.16	+.052	-.86
19.	At-10 Beech, Wichita	1.023	-.28	-.078	-.96
20.	PT-13-17 (N2S) Boeing, Wichita	1.85	-.42	-.078	-.98
21.	C-46 Curtiss, Buffalo	1.010	+.057	-.52	-.88
22.	C-47 Douglas, Okla. City	1.47	-.46	-.011	-.98

Appendix*

The basis for the calculations on page 12 is as follows.

Labor expended in the production of the first n_o items is (A_o). Total labor (A_e) required for the production of a total required number (n_r) is estimated by the formula:

$$A_e = A_o \left(\frac{n_r}{n_o}\right)^{1-m}$$

The total labor actually required for the production of n_r is some quantity $A_r = A_e$.

Assuming the formula is correct, but that the estimation of the value of m is in error, what would have been the actual production (n_a) had the estimated labor (A_e) been expended. This is given by

$$\log n_a/n_r = \frac{-(\log n_r/n_o)\ (\log A_r/A_e)}{(1 - m)\ (\log n_r/n_o) + \log A_r/A_e}$$

where the logarithms are taken to any convenient base.

In terms of the additional labor estimated (B_e) and the additional labor required (B_r) to build the additional $n_r - n_o$ items,

$$\frac{A_r}{A_e}\ \frac{A_o + B_r}{A_o + B_e} = 1 + \left[1 - \left(\frac{n_o}{n_r}\right)^{1-m}\right] \frac{B_r - B_e}{B_e} \quad .$$

In terms of P, the "slope of the progress curve",

$$m = \frac{-\log P}{\log 2} \quad .$$

The ratio of the difference between actual and required output to the required additional output is

$$\frac{n_a - n_r}{n_r - n_o} = \frac{5}{4}\ \left(\frac{n_a}{n_r} - 1\right) \quad .$$

* This mathematical derivation was made by H. Germond.

Example:

Suppose $n_r = 5\ n_o$, and $\underline{P}$ is erroneously estimated to be 80 percent. Then

$$m = \frac{-\log 0.80}{\log 2} = 0.321928 \quad ,$$

$$\frac{A_r}{A_e} = 1 + \left[1 - (0.2)^{0.678072}\right] \frac{B_r - B_e}{B_e}$$

$$= 1 + 0.664225 \ \frac{B_r - B_e}{B_e} \quad ,$$

$$\log_{10} n_a/n_r = \frac{-\ 0.69897 \log_{10} A_r/A_e}{0.47395 + \log_{10} A_r/A_e}$$

and

$$\frac{n_a - n_r}{n_r - n_o} = \frac{5}{4} \left(\frac{n_a}{n_r} - 1\right) \quad .$$

Suppose, now, the estimated labor for the production of the $n_r - n_o$ additional units is found to be in error by 20 percent, depending upon (1) $B_e = 1.2\ B_r$, or (2) $B_e = 0.8\ B_r$. Carrying out the computation yields the results given in the text.