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RESEARCH MEMORANDUM

LUNAR INSTRUMENT CARRIER - ATTITUDE STABILIZATION

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SUMMARY

In this report the various attitude disturbances acting on a moon rocket are considered and their magnitudes are estimated. It is shown that the attitude-stabilization requirements can be satisfied by spinning the vehicle at about 80 rpm around its roll axis. Effects of vehicle shape and mass distribution on the attitude problem are examined, and design restrictions are indicated.
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INTRODUCTION

The free-flight trajectory of the Lunar Instrument Carrier is to culminate in a non-destructive landing on the moon's surface. This landing is to be accomplished by reducing the vehicle's approach velocity of about 9000 ft/sec to an actual touch-down velocity in the range 0-500 ft/sec, the deceleration force being developed by a braking rocket fired downward along a thrust line essentially coincident with the vehicle roll axis. It is required, then, that the vehicle approach the moon in such an attitude that its roll axis is aligned along the predetermined direction of its velocity vector, with the braking-rocket nozzle directed toward the moon.

This requirement can be fulfilled by orienting the vehicle to the landing attitude immediately after powered flight from the earth's surface and imparting to it a roll-axis spin rate sufficiently high to maintain this initial orientation through the earth-moon transit period (about 2.5 days) in the face of the disturbances that may act on it.

The significant sources of attitude disturbance are:

- Initial misalignment
- Initial pitch/yaw rates
- Solar radiation torque
- Gravitational gradient of the earth

There is also the possibility of attitude disturbance due to meteor impact, but this is shown to be a matter of very low probability.

It is the purpose of this report to estimate the magnitudes of disturbances and to determine the roll spin rate \( \omega \) required to maintain the vehicle roll axis within an angular displacement \( \theta \) from a pre-assigned orientation during the earth-moon transit period.

A specification of \( \theta = 1 \) deg will be imposed on orientation error.
Such a displacement between the vehicle approach-velocity vector and the braking-rocket thrust line will result in a touch-down velocity with a component of about 1.4 ft/sec along the approach direction and a component of about 160 ft/sec normal to it. These velocities are compatible with the range 0-500 ft/sec, within which it seems feasible to restrict the impact velocity. The other sources of impact velocity are summarized in Ref. 1.
EQUATIONS OF ROTATIONAL MOTION

The equations of motion are, from Euler's equations:

\[ \dot{\omega}_x + C \Omega \omega_y = L_x \quad (1) \]
\[ \dot{\omega}_y - C \Omega \omega_x = L_y \quad (2) \]

where \( A \) is the moment of inertia in both pitch and yaw, and \( C \) is the moment of inertia in roll. The coordinate system \((x,y,z)\) has its origin at the center of mass of the vehicle with the \( z \) axis along the vehicle roll axis. The vehicle has a roll angular rate \( \Omega \) in the \( z \) direction, but the \( z \) component of the angular velocity of the coordinate system is zero. The angular rates of vehicle and coordinate system in the \( x \) and \( y \) direction are \( \omega_x \) and \( \omega_y \). \( L_x \) and \( L_y \) are torques about the \( x \) and \( y \) axes.

The sources of torques \( L_x \) and \( L_y \) are solar radiation and gravitational gradients. These torques are treated as constants for purposes of estimating \( \Omega \).

Solution of Eqs. (1) and (2) yields

\[ \omega_x = K_1 \cos kt - K_2 \sin kt - \frac{L_y}{C \Omega} \quad (3) \]
\[ \omega_y = K_2 \cos kt + K_1 \sin kt + \frac{L_x}{C \Omega} \quad (4) \]

where

\[ k = \frac{\Omega}{\gamma} \]
\[ \gamma = \frac{A}{C} \]
The net rotations $\theta_x$ and $\theta_y$ are

$$
\theta_x = (K_1 / k) \sin kt + (K_2 / k) \cos kt - \frac{L_t}{C \Omega} + (\theta_1 - K_2 / k) \tag{5}
$$

$$
\theta_y = (K_2 / k) \sin kt - (K_1 / k) \cos kt + \frac{L_t}{C \Omega} + (\theta_2 + K_1 / k) \tag{6}
$$

with $\theta_1$ and $\theta_2$ as the initial values of $\theta_x$ and $\theta_y$.

For small values of $\theta_x$ and $\theta_y$, the net displacement $\theta$ of the roll axis is approximately the vector sum of $\theta_x$ and $\theta_y$, taken as displacements at right angles to one another. Examination of the equations for $\theta_x$ and $\theta_y$ shows that $\theta$ is the locus on a unit sphere of the end of a unit vector originating at the center of mass, and that this locus is a circle of radius

$$
r = \left[ K_1^2 + K_2^2 \right]^{1/2} / k
$$

with its center displaced from the desired point of intersection of the unit sphere and the roll axis by a distance whose $x$ and $y$ components are, for small $\theta_x$ and $\theta_y$,

$$
x_0 = \theta_1 - (K_2 / k) - \frac{L_t}{C \Omega}
$$

$$
y_0 = \theta_2 + (K_1 / k) + \frac{L_t}{C \Omega}
$$

The maximum possible value of $\theta$ is

$$
\theta_m = \theta_1 + 2K/k + \frac{L_t}{C \Omega} \tag{7}
$$
where

\[ \theta = \left( \theta_1^2 + \theta_2^2 \right)^{1/2} \]

\[ K = \left( K_1^2 + K_2^2 \right)^{1/2} \]

\[ L = \left( L_x^2 + L_y^2 \right)^{1/2} \]

It can be seen from Eqs. (3) and (4) that

\[ K_1 = \omega_x(0) + \frac{L_y}{C \Omega} \]

\[ K_2 = \omega_y(0) - \frac{L_x}{C \Omega} \]

with \( \omega_x(0) \) and \( \omega_y(0) \) the initial pitch and yaw rates, respectively. Numerical evaluation of quantities below shows that the \( (L/C \Omega) \) terms are of the order of \( 10^{-6} \) of the \( \omega(0) \) terms and are, therefore, negligible for the present purpose; so

\[ K^2 = \left[ \omega_x(0) \right]^2 + \left[ \omega_y(0) \right]^2 = \omega^2 \]

very nearly, with \( \omega \) the net initial rate of the vehicle in the \( (x,y) \) plane.

The specification selected for \( \theta_m \) is about 1 deg, and it will be assumed that the initial misalignment angle \( \theta_1 \) is limited to a small part of \( \theta_m \), say \( \theta_1 \approx 0.1 \) deg. Thus we need consider only the terms of Eq. (7) that depend upon spin rate, and write

\[ \frac{2\omega Y}{\Omega} + \frac{L_t}{C \Omega} = \frac{1}{57.3} \text{ radian} \quad (8) \]

Solving for \( \Omega \) we have

\[ \Omega = (114.6) \omega_Y + (57.3) \frac{L_t}{C} \quad (9) \]
This is the basic equation to be used in determining the spin-rate requirement. In order to proceed we must establish estimates of the factors in Eq. (9).

**INITIAL PITCH/YAW RATES**

During the coast phase at the end of powered flight, the vehicle is subjected to a pitch rate of about 150 milliradians/sec to rotate it to the landing attitude.\(^{(1)}\) This is the highest sustained rate the attitude-sensing and control system will have to accommodate, and it is assumed that the net rate error will lie in the range 1-10 per cent of this 150 milliradians/sec rate. Based on this assumption, \(\omega\) is taken to be 10 milliradians/sec for purposes of estimating \(\Omega\). Such an error assignment is compatible with the performance of current flight-control rate gyros and can be compared, for example, to the rate error of about 7 milliradians/sec indicated for the attitude control system designed for the Viking RTV-N-12a.\(^{(3)}\)

Thus with \(\omega = 0.01\) radian/sec the first term of Eq. (9) is

\[
\Omega_1 = 1.15 \gamma \text{ radians/sec}
\]

\(\Omega_1\) is plotted against \(\gamma\) in Fig. 1.

**SOLAR RADIATION TORQUE**

A torque arises if sunlight is incident on the vehicle with a ray component normal to the roll axis and if the mean reflectance power of the halves of the vehicle ahead of and behind the center of mass are not exactly equal. Denoting one of the halves by the subscript 1 and the other by the subscript 2, the torques about the center of mass due to normally incident sunlight on
Fig. 1 - Required spin rate
these halves* are:

\[ L_1 = p(1 + P_1)(A/2)(\lambda/l) \]
\[ L_2 = p(1 + P_2)(A/2)(\lambda/l) \]

where

- \( p \) = radiation pressure on a perfect absorber
- \( P \) = reflecting power of the vehicle surface
- \( A = 2a\lambda \) = projected area of the vehicle side
- \( a \) = radius of vehicle
- \( \lambda \) = length of vehicle

and it has been assumed that the vehicle approximates a cylinder in shape. It is also assumed that the projected areas on either side of the center of mass are balanced within an error that is small compared with the uncertainties in reflecting power.

The net torque \( L_x \) due to the action of sunlight on the unbalanced reflecting surfaces is

\[ L_x = \left| L_1 - L_2 \right| = p(A/2)(\lambda/l) \left| P_1 - P_2 \right| \]

Letting

\[ P_1 = P + \Delta P \]
\[ P_2 = P - \Delta P \]

with \( P \) the nominal reflecting power and \( \Delta P \) the maximum expected departure from this nominal value, \( L_x \) can be written

\[ L_x = \left( p/2 \right) (P\lambda^2) F \]

*See Ref. 1, p. 509.
where \( F = \frac{\Delta P}{P} \), the maximum expected fractional uncertainty in \( P \).

The component of Eq. (9) due to \( L_2 \) is

\[
\Omega_2 = (57.3) \frac{L_2 t}{c} = (57.3) \frac{pFt a \lambda}{2c}
\]  

(11)

It can be seen that this equation involves the specific dimensions \( a \) and \( \lambda \), whereas Eq. (10) for \( \Omega_1 \) involves only the moment ratio \( \gamma \). It is desirable to relate \( \Omega_2 \) to \( \gamma \) also. In order to do this we assume that the vehicle mass is distributed uniformly throughout its volume. This assumption is, of course, only approximately true but is adequate, since \( \Omega_2 \) is fairly small in comparison with \( \Omega_1 \), as is seen below.

For a uniformly dense cylinder of mass \( M \) we have

\[
M = \rho \pi a^2 \lambda
\]  

(12)

with \( \rho \) the density. We also have

\[
A = M \left[ \frac{a^2}{4} + \frac{\lambda^2}{12} \right]
\]

\[
c = \frac{1}{2} M a^2
\]

\[
\gamma = \frac{C}{c} = \frac{1}{2} + \frac{2}{3} (\lambda/2a)^2
\]

\[
= \frac{1}{2} + \frac{2}{3} f^2
\]  

(13)

with \( f \) the fineness ratio \( (\lambda/2a) \).

Using Eqs. (12) and (13), Eq. (11) can be written

\[
\Omega_2 = (57.3)(pFt) \left( \frac{2 \lambda}{M} \right) \left[ \frac{3}{2} \left( \gamma - \frac{1}{2} \right) \right]^{1/2}
\]  

(14)

The factor \( (2 \lambda/M) \) is numerically related to \( \gamma \) through Eqs. (12) and (13), under the assumption of constant \( \rho \). The vehicle of particular interest here
weighs 310 lb and approximates a cylinder 80 in. long and 18 in. in diameter.\(^{(5)}\)

Thus its mass is \(M = 1.41(10^5)\) gm, and its density is \(\rho = 0.4423\) gm/cc. These parameters are considered fixed in the computation of \(\Omega_2\) for a range of \(\gamma\).

Plots of \(a\), \(\lambda\), and \(f\) as functions of \(\gamma\) are shown in Fig. 2.

Since the vehicle surface will consist of a combination of bare, bright metal and white coating,\(^{(5)}\), the reflecting power will be high. Thus for estimating purposes \(P = 1\) will be used. An unbalance \(F = 0.1\) in \(P\) is also assumed.

The nominal transit time to the moon is around 2.5 days,\(^{(6)}\) so \(t = 2.16(10^5)\) sec. The solar energy normally incident on a surface at a distance from the sun equal to the mean radius of the earth’s orbit is about 19.3 kg-cal/m²/min. This corresponds to an energy density of \(4.49(10^{-5})\) ergs/cc; so \(p = 4.49(10^{-5})\) dynes/cm². With these numerical values, Eq. (14) is

\[
\Omega_2 = 0.964 \ (10^{-3}) \lambda \ (\gamma - \frac{1}{2})^{1/2} \ \text{radian/sec} \tag{15}
\]

In this expression \(\lambda\) is in centimeters. \(\Omega_2\) is plotted against \(\gamma\) in Fig. 1.

The need to balance the radiation properties about the center of mass must be taken into account in the design of the vehicle surface.\(^{(5)}\)

**GRAVITATIONAL GRADIENT OF THE EARTH**

A torque also arises due to the gradient of the gravitational field of the earth.

If the line joining the center of mass of the earth and the center of mass of the vehicle makes an angle \(\phi\) with the vehicle roll axis, it can be shown* that a torque is produced about an axis normal to the roll axis of magnitude

\[
\frac{L}{g} = \frac{3}{2} \left[ \frac{GM}{(c - A)^2} \right] |C - A| \sin 2\phi \tag{16}
\]

*See Ref. 7, p. 370.
Fig. 2 - Vehicle dimensions
where \( M_e \) is the mass of the earth, \( G \) is the gravitational constant, \( r \) is the distance of the vehicle from the center of the earth, and \( C \) and \( A \) are the vehicle moments of inertia.

To estimate the effect of this torque we compute an approximate mean value for

\[ Y = 1/r^3 \]

by letting

\[ r \approx r_1 + \frac{r_2 - r_1}{T} t \]

where \( r_1 \) is the distance from the center of the earth at the start of the free-flight trajectory and \( r_2 \) is the distance from the center of the earth at the end of this trajectory, with \( T \) the total time interval between \( r_1 \) and \( r_2 \).

The average value of \( Y \) is

\[ \bar{Y} = \frac{1}{T} \int_0^T Y dt = \frac{1}{T} \int_0^T \frac{dt}{\left( r_1 + \frac{r_2 - r_1}{T} t \right)^3} \]

\[ \bar{Y} = \frac{r_2 + r_1}{2 \left( r_1 r_2 \right)^2} \]

The average value \( \bar{L}_g \) of \( L_g \) is then given by Eq. (15) with \( (1/r^3) \) replaced by \( \bar{Y} \). With the pessimistic assumption that \( \sin 2\phi = 1 \) for the entire time \( T \), this mean torque is

\[ \bar{L}_g = \frac{3}{4} \left[ \frac{r_2 + r_1}{r_2^2} \right] \left( \frac{1}{r_1^2} \right) G M_e |C - A| \]
The component of Eq. (9) due to \( \mathbf{I}_g \) is

\[
\Omega_3 = 57.3 \frac{\mathbf{I}_g}{c} = (57.3) \frac{3GM_e}{r_2} \frac{t}{4\left[1 - \gamma \frac{r_2 + r_1}{r_2^2 r_1^2}\right]}
\]

The initial point of the free-flight trajectory will be at an altitude of about 100 mi above mean sea level, so \( r_1 = 6.54(10^8) \) cm. At the end of free flight the vehicle is about as far from the earth’s center as is the moon, about 239,000 mi, so \( r_2 = 2.38(10^{10}) \) cm. Again, \( t = 2.16(10^5) \) sec. The product \( GM_e = 3.98(10^{20}) \) cgs. With these numerical values

\[
\Omega_3 = 0.229 \left|1 - \gamma\right| \text{ radian/sec}
\]

\( \Omega_3 \) is plotted against \( \gamma \) in Fig. 1.

**NET SPIN-RATE REQUIREMENT**

The net spin rate \( \omega \) required is the sum

\[
\omega = \omega_1 + \omega_2 + \omega_3
\]

This sum is plotted against \( \gamma \) in Fig. 1. It is apparent that the dominant factor is the spin rate \( \omega_1 \) deduced from the initial pitch/yaw rate \( \omega \).

The moment ratio \( \gamma = A/C \) is approximately 6 for the particular vehicle of interest. Therefore, the required spin rate is about 80 rpm.

It can be seen from Fig. 2 that the actual \( \gamma \) of this vehicle would correspond to a fineness ratio of 2.88 on the basis of the assumption of uniform mass distribution, whereas the actual geometrical fineness ratio is 80/18 = 4.45. While this correlation between \( \gamma \) and \( f \) is rather poor, it is adequate for the purpose of estimating \( \omega_2 \).

The precession period \( \omega_0 = 2\pi/k \) is plotted against \( \gamma \) in Fig. 3.
Fig. 3 - Precession period as a function of moment ratio

Precession period, $T_p$ (sec) vs. Moment ratio, $Y$.
OTHER ATTITUDE DISTURBANCES

The moon's gravitational gradient produces a torque on the vehicle in the same manner as does the field of the earth; however, this effect would contribute a component to the required $\mathbb{I}_2$ that is about $1/61.5$ of $\mathbb{I}_3$ because of the moon's smaller mass. Since $\mathbb{I}_3$ is itself a small part of $\mathbb{I}_2$, the effect of the moon is negligible here.

It is also of interest to estimate the torque due to the gradient of the sun's field. If $L_s$ is the torque due to the sun and $L_e$ is the mean torque due to the earth, we have

$$\frac{L_s}{L_e} = \frac{M_s}{M_e} \left[ \frac{2r_1^{-2} r_2^{-2}}{r_s^{-3} (r_1 + r_2)} \right]$$

where $M_s$ is the mass of the sun and $r_s$ is the distance from the vehicle to the sun. We have, \(^{(9)}\)

$$\frac{M_s}{M_e} = 333,432$$

$$r_s = 93,000,000 \text{ miles}$$

and $r_1$ and $r_2$ are as given above. Thus

$$\frac{L_s}{L_e} = 3.06 \times 10^{-6}$$

From this figure it is clear that there is no appreciable torque due to the field of the sun.

In connection with the torques due to gravitational gradients, it should be noted that they are closely predictable, since they are well-defined
functions of vehicle parameters and the trajectory. Therefore these disturbances could be effectively eliminated as sources of attitude error by pre-computation if attitude tolerances were reduced materially below the 1 deg employed here, or if it were desirable to reduce $\Omega$ by elimination of $\Omega_3$.

During the transit from earth to moon the electrical equipment in the vehicle consumes power at a maximum sustained rate of approximately 4 watts. All of this power is ultimately radiated from the vehicle, about one watt as radio-frequency power from the transmitting antenna and the remaining 3 watts as heat from the vehicle outer surface. Thus the mean density of energy flux over the vehicle surface is about $9.23 \times 10^{-5}$ watts/cm$^2$. This is about $10^{-3}$ of the solar-flux density and would, therefore, contribute a negligible fraction of $\Omega_3$ to the total required spin rate $\Omega$.

If a meteor strikes the vehicle it will impart momentum to it. If the meteor strikes in such a fashion that the perpendicular distance from the center of mass of the vehicle to the projected line of travel of the meteor is $d$, the angular momentum imparted to the vehicle by an inelastic impact will be

$$H = mvd$$

where $m$ and $v$ are the mass and velocity of the meteor, respectively.

Solution of Eqs. (1) and (2) shows that this angular-momentum increment will add a component $\theta_\rho$ to the attitude error, where

$$\theta_\rho = \frac{H}{cd\Omega}$$

The kinetic energy of a meteor that will deliver a momentum $H$ is

$$U = \frac{1}{2}mv^2 = \frac{Hv}{2d}$$
so in order to produce an attitude error $\theta_p$ a meteor must impact with kinetic energy

$$U = \frac{C \int v \theta_p}{2d}$$

For the vehicle of Ref. 5 we have approximately $C = 3.62 \times 10^7$ gm-cm$^2$ and $\int = 8.36$ radians/sec. Since the nominal specification on attitude error is 1 deg, we can take $\theta_p = 1/57.3$ radian. The probable value of meteor velocity is about $v = 3.8 \times 10^6$ cm/sec. The maximum value $d$ can take is $\left[a^2 + (\lambda/2)^2\right]^{1/2}$, which is about 10 cm for the vehicle design of Ref. 5.

With these values, $U$ is

$$U = 9.65 \times 10^{10} \text{ ergs}$$

A meteor with this kinetic energy would burn with a visual magnitude $m_0 = 5.06$ if it entered the earth's atmosphere.\(^{(11)}\) Available observation data indicate that the number of meteors of magnitude $\leq m_0$ that enter the earth's atmosphere per day is about

$$N_e = 1.75 \times 10^8$$

Now the surface area of the earth is about $A_e = 5.6 \times 10^{15}$ ft$^2$, and the total surface area of the vehicle is about $A_v = 35$ ft$^2$. Thus the average number of meteors with kinetic energy $\geq 9.65 \times 10^{10}$ ergs impinging on the vehicle should be about

$$N_v = N_e \left(\frac{A_v}{A_e}\right) = \frac{1}{0.9114 \times 10^6} \quad \text{per day}$$

and the average number hitting the vehicle per 2.5-day earth-moon trip should be about

$$N_t = \frac{1}{3.65 \times 10^5}$$
Now only a small fraction of this number $N_v$ of meteors will produce a $\theta_p = 1$ deg, since most will act with lever arms much less than the maximum $d$. However, even on the basis of the very pessimistic assumption that all meteors striking the vehicle act with this maximum lever, $N_t$ shows that, on the average, there will be significant impact (in the sense of attitude disturbance) on one vehicle out of every 365,000 -- a small hazard, indeed.
In order that the vehicle rotation be stable in the presence of differences in yaw and pitch moments of inertia it is necessary that the roll axis be the principal axis about which the moment of inertia is either greatest or least.* If the roll axis was the principal axis of intermediate moment of inertia, the vehicle would, under even the slightest disturbance, eventually turn through 90 deg to align the angular momentum vector along one of the other two principal axes, and in this sense the rotation would be unstable. So to insure stable rotation it is necessary that the following condition not apply:

\[(1 - e) \leq \gamma \leq (1 + e)\]  \hspace{1cm} (19)

where \(e\) is the maximum expected fractional difference in the actual pitch and yaw moments of inertia.

The essence of this restriction on \(\gamma\) is simply that vehicle shapes approximating a sphere are generally to be avoided. The dimension of the vehicle along its roll axis should either be long compared with the other two principal dimensions or be definitely shorter than the other two. Usual packaging considerations would favor the first of these alternatives -- a conventional vehicle shape with the longest dimension along the roll axis.

In addition to the possibility of rotation instability, differences in pitch and yaw moments introduce a possible increase in the component of attitude error \(\theta\) due to initial angular rates. If these moments of inertia are equal, a point on the roll axis describes a circle in space with radius proportional to \(\omega\). If the moments in pitch and yaw are different, a point

*See Ref. 12, p. 66.
on the roll axis will describe an ellipse in space. If $A$, $B$, and $C$ are the actual moments in pitch, yaw, and roll, respectively, the ratio of the major and minor axes of this ellipse is the ratio of the axes of elliptical polhodes about the roll axis. This ratio is:

$$\mu = \left[ \frac{A(A - C)}{B(B - C)} \right]^{1/2}$$

Letting

$$A = B(1 + \delta)$$

$$B/C = A/C = \gamma$$

this is

$$\mu = \left[ 1 + \delta \left( \frac{2 \gamma - 1}{\gamma - 1} \right) + \delta^2 \left( \frac{\gamma}{\gamma - 1} \right) \right]^{1/2}$$

Thus an unbalance in pitch and yaw moments by a fraction $\delta$ can produce an amplification of $Q$ per cent in the initial rate-induced attitude error, where

$$Q = \left| \mu - 1 \right| 100 \text{ per cent}$$

$Q$ is plotted against $\gamma$ in Fig. 1 for $\delta = 0.1$.

Assuming a 10 per cent unbalance between the pitch and yaw moments of inertia, Fig. 4 indicates $\gamma$ should exceed about 1.5 if the attitude error due to initial rates is not to be substantially amplified. A $\gamma > 1.5$ would be compatible with the stability restriction of Eq. (19). Figure 1 shows that the required spin rate rises with increasing $\gamma$, so the upper limit on $\gamma$ will generally be determined by packaging considerations and centrifugal-load-factor limits.

*See Ref. 7, p. 102.*
Fig. 4 - Attitude error amplification, $Q$, due to a 10 per cent unbalance between pitch and yaw moments of inertia.
REFERENCES


