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APPLICATION OF THEORY OF GAMES TO IDENTIFICATION OF FRIEND AND FOE

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APPLICATION OF THEORY OF GAMES
TO
IDENTIFICATION OF FRIEND AND FOE

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§1. Introduction.

An important problem in warfare has always been the identification of friend and foe. Regardless of the present complicatedness of I.F.F. systems, the basic problem remains the same. An observer O has detected a target. In some manner he challenges the target. The target replies with one of two signals: (F) or (E). The signal (E) may be merely the failure to signal (F).

The question we ask is what should be the observer's strategy if the enemy has discovered the system and can reply either (E) or (F). We consider, first, the case of an uncoded signal; the agreement between the observer and his friends is that a friend always signals (F). If the system has been discovered by the enemy, is it of any value? Upon what factors does this depend? How can the system be improved?

A current example of this problem is the "loyalty oath." This example illustrates a case where the system is always known to the enemy and where it is not possible to use a coded signal. Here it is certainly impossible to reach an agreement with one's friends, unknown to the enemy, to use a mixed signal: sometimes (F) and sometimes (E); the purpose of the oath is to separate friend from foe. In warfare it is possible to use a coded (or mixed) signal. It is not suprising that there is an advantage in using a coded signal. We analyze just what advantage is gained by the coded signal.

Another situation we consider is the dilemma of an observer O who must make a choice of two alternatives. He does not know the consequences of the alternatives. He signals for advice. Allowing the possibility that the advice may come from the enemy, how should he act? This is the classic problem faced by the young commoner in Frank Stockton's story, "The Lady or the Tiger?".*

To contrast the difference between this latter game and I.F.F., notice that I.F.F. corresponds to a "loyalty oath." The unidentified (uncleared) person is asked to testify concerning his own loyalty. In this game, a person is asked to testify on someone else's loyalty. If a person swears he is disloyal, you would always believe him; if he swears that someone else is loyal or disloyal, you do not know. That the games are essentially different can be seen by the results.

§2. I.F.F. Uncoded signal.

a) The game.

This game has three players: the observer O, the enemy E, and the friend F. As in bridge, the partnerships are such that it is a two-person game; O and F are partners against E. In contrast to bridge both the partner F and the opponent E can signal O. O cannot distinguish between F's signal, S(F), and E's signal, S(E). The friend always signals (F); S(F) = (F). The enemy can signal either

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* Without hesitation he followed the signal given by the princess.

(F) or (E); S(F) = (F) or (E). O receives a signal and must decide whether the signal came from F or E. This is not yet a game. We must introduce a rule for the first move. We make the first move a chance move with probability p that F signals and probability (1 − p) that E signals. The payoffs to O are:

a, if he identifies F as F

b, if he identifies F as E

c, if he identifies E as E

d, if he identifies E as F;

$a > b$, $a > d$, $c > b$, $c > d$. The corresponding payoffs to E are $\bar{a}$, $\bar{b}$, $\bar{c}$, and $\bar{d}$ with $\bar{a} < \bar{b}$, $\bar{a} < \bar{d}$, $\bar{c} < \bar{b}$, $\bar{c} < \bar{d}$.

b) <u>The solution</u>.

We make use of the notation I(E, F) = (E, E) to mean O identifies the sender of the signal (E) as E and the sender of the signal (F) as E. Thus O's strategies are:

I(F, E) = (F, E)

I(F, E) = (E, E)

I(F, E) = (F, F)

I(F, E) = (E, F) .

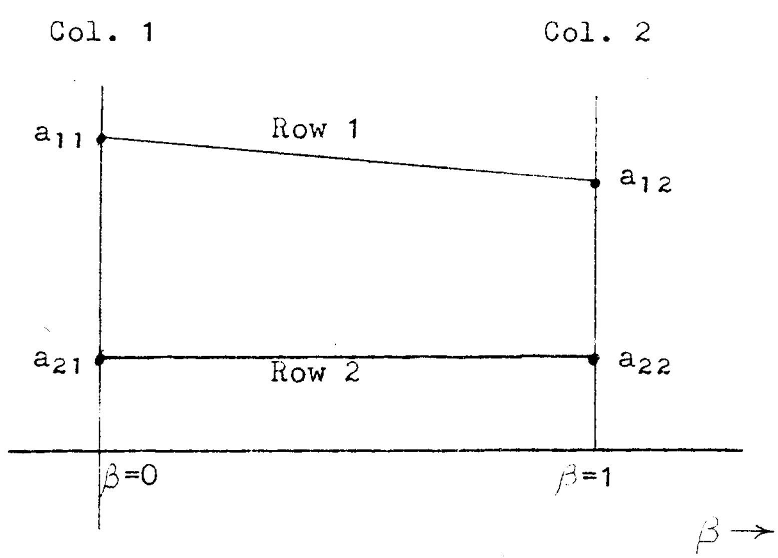
It is very easy to show that O would never use the latter two strategies. This is to be expected. When the signal (E) is received, he knows the sender is E. For the same reason E will not signal (E).

The diagram of strategies with payoffs to O is:

	S(E) = (E)	S(E) = (F)
I(F, E) = (F, E)	$pa + (1-p)c = a_{11}$	$pa + (1-p)d = a_{12}$
I(F, E) = (E, E)	$pb + (1-p)c = a_{21}$	$pb + (1-p)c = a_{22}$

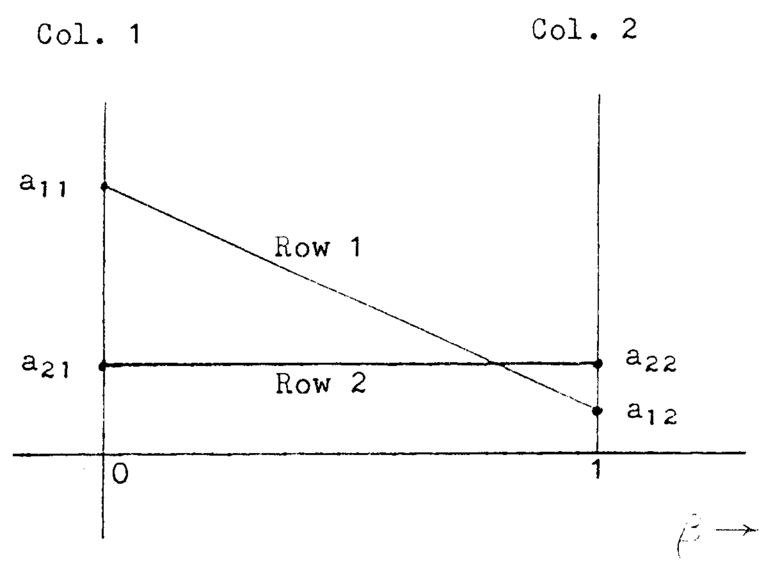
Let β be the probability that E's strategy is the second column and $(1-\beta)$ that it is the first. We then plot the payoff to O as a function of β for his two strategies row 1 and row 2. There are two cases:

Case 1. $p > p_1 = \dfrac{1}{1+\alpha}$, where $\alpha = \dfrac{a-b}{c-d}$.



Here $a_{12} > a_{22}$. It is always true that $a_{11} > a_{21}$, $a_{21} = a_{22}$. Row 1 majorizes row 2, and O's best strategy is the pure strategy I(F, E) = (F, E); he always believes the signals.

Case 2. $p < p_1$; i.e. $a_{12} < a_{22}$.



Here a_{22} is a saddle point and O's best strategy is to play row 2; he judges everyone to be the enemy.

We do not assume that this is a zero-sum game but only that E's orderings of the payoffs are the reverse of O's. From E's point of view Col. 2 majorizes Col. 1; E's best strategy is to always signal (F).

c) Discussion.

We can now conclude that when (E) has discovered how to signal (F), the uncoded I.F.F. system is useless. O must decide, on the basis of information not given by the system, either always to believe (F) or always to disbelieve (F). He must estimate whether $p > p_1$ or $p < p_1$. The estimate of p_1 depends upon $\alpha = \frac{a - b}{c - d}$; roughly speaking α is a measure of the advantage of passing a friend relative to the danger of passing an enemy. The risk in passing the enemy can become so great, $p < p_1$, that O must assume that everyone is the enemy.

The value of the game to O is

$$pa + (1-p)d, \qquad p > p_1$$

$$pb + (1-p)c, \qquad p < p_1 \; .$$

We have assumed that p is fixed by the conditions of the game; this is not strictly true; O and E may have some control of p. O might, for example, instruct his friends to stay away and make p = 0. In Fig. 1, we have plotted the value of the game to O as a function of p for the case $a < c$; it is then clear that O should make p = 0. When $a > c$, O's preference would be to have p = 1. Fig. 2 is a plot of the value of the game from E's point of view; E would like to have p close to p_1 but between p_1 and $\overline{p}_1$, except when $p_1 > \overline{p}_1$ and $\overline{a} > p_1\overline{b} + (1-p_1)\overline{c}$; here he would choose p = 1.

When $a < c$, both F and E may refuse to play; E will refuse to play if $\overline{c} < 0$ (i.e., to be identified is a loss to E). If $a > c$, then the probability p at which the game will be played will depend upon who has the greatest control over p, though here again E and/or F might refuse to play.

§3. I.F.F. A coded signal.

a) The game and its solution.

We make one change in the previous game. We give F the possibility of signaling (E) or (F). (O always knows when S(F) should

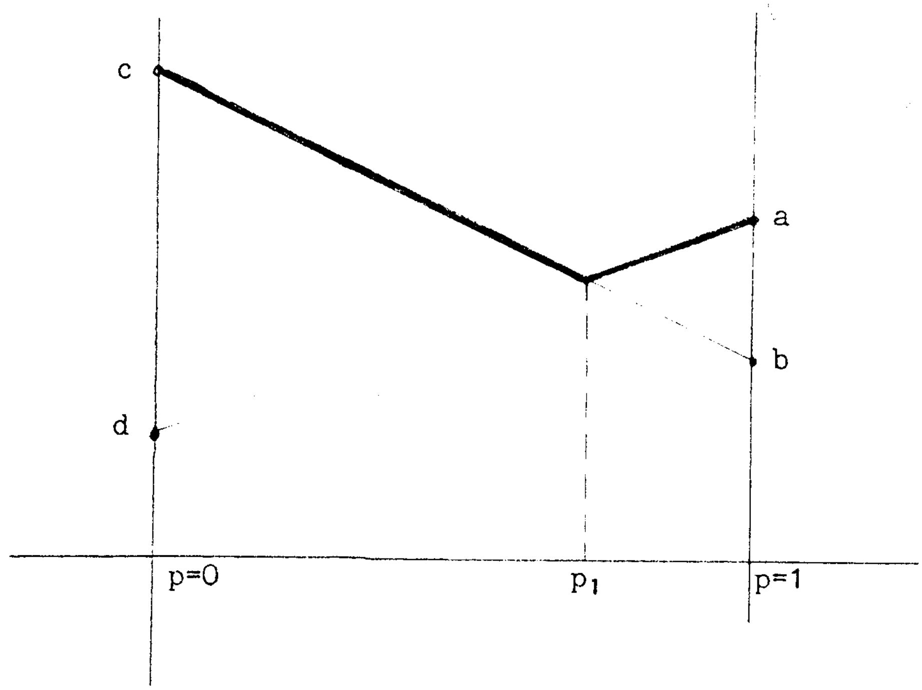


Fig. 1

Value of the game to O as a function of p. $(a < c)$

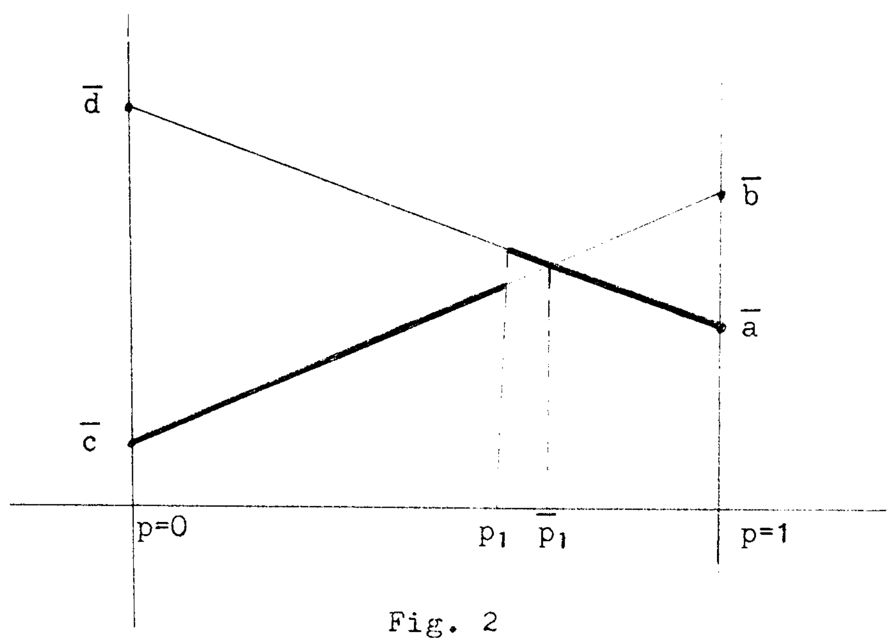


Fig. 2

Value of the game to E as a function of p.

be (F) and when it should be (E).) As before, one can easily eliminate the possibility, if the signal is not the code signal for friend, that O would identify the sender as F. Plotting the table of payoffs to O we have:

	S(E) = (E)	S(E) = (F)
S(F) = (F) I(E, F) = (E, F)	pa + (1−p)c	pa + (1−p)d
S(F) = (F) I(E, F) = (E, E)	pb + (1−p)c	pb + (1−p)c
S(F) = (E) I(E, F) = (F, E)	pa + (1−p)d	pa + (1−p)c
S(F) = (E) I(E, F) = (E, E)	pb + (1−p)c	pb + (1−p)c

To E the two columns are alike and his best strategy is 1/2 Col. 1 + 1/2 Col. 2; also from symmetry O's best agreement with F is $1/2 \left\{S(F) = (F)\right\} + 1/2 \left\{S(F) = (E)\right\}$. Next O must decide whether to believe S(F) or not; if S(F) = (F), he has to decide between row 1 and row 2. The expected payoff on row 1 is:

$$pa + (1-p)\left(\frac{d+c}{2}\right);$$

on row 2 it is:

$$pa + (1-p)c\ .$$

Thus when $p > p_2 = \frac{1}{1 + 2\alpha}$, he believes S(F); when $p < p_2$, he disbelieves S(F).

b) Discussion.

To summarize, the best strategy for O is to code F's signal; half the time it is agreed that the signal for friend is (F) and half time (E). It is not surprising that when we allow the possibility of coding the signal, the best strategy says to code the signal. E is then placed in the position of not knowing which signal means friend; he must then mix his signals half and half. Coding does not, however, relieve O of the necessity of evaluating the relative danger of the situation; he must still decide whether $p > p_2$ or $p < p_2$.

More generally, for an I.F.F. with n signals, O and F should code the signals and mix the coding uniformly. E's best strategy is signal with a uniform probability distribution over the signals. When

(i) $p > p_n = \frac{1}{1 + n\alpha}$, O identifies S(F) and only S(F) as a friend's signal;

(ii) $p < p_n = \frac{1}{1 + n\alpha}$, O identifies everyone as the enemy.

Here again we have a probability level below which the I.F.F. contributes nothing to identification; coding does not eliminate the possibility that the situation can become so dangerous that you cannot afford to pass anyone.

Much has been gained by coding. In uncoded case ($n = 1$), the enemy can completely destroy the value of the I.F.F. system. With n coded signals, the system is of value as long as $p > p_n = \frac{1}{1 + n\alpha}$. Assuming everyone to be the enemy is hard on the morale of your friends and the fact that p_n decreases with n is a definite advantage of coded signals.

Let us also examine the gain in the value of the game by coding. In the n-coded case, the value of the game to O is:

$$pa + (1-p)\left[(1-\frac{1}{n})c + \frac{d}{n}\right], \qquad \text{when } p > p_n;$$

$$pb + (1-p)c, \qquad \text{when } p < p_n.$$

The gain by using an n-coded system is:

$$(1-p)\frac{c-d}{n}, \qquad \text{for } p > p_1;$$

$$p(a-b) - (1-p)\frac{c-d}{n}, \qquad \text{for } p_n < p < p_1;$$

$$0, \qquad \text{for } p < p_n.$$

Assuming that O and E have some control over p, we can ask what values of p they would prefer. O's preferences are the same as in the case n = 1. E would usually prefer p close to p_n but between p_n and $\overline{p}_n$ with the following exceptions: if

$$\overline{a} > p_n\overline{a} + (1-p_n)\left[(1-1/n)\overline{c} + \overline{d}/n\right] \quad \text{when } p_n < \overline{p}_n$$

$$\text{or} \quad \overline{a} > p_n\overline{b} + (1-p_n)\overline{c} \quad \text{when } p_n > \overline{p}_n,$$

he would prefer p = 1; he would prefer not to play. Fig. 3 is an example of this exceptional case.

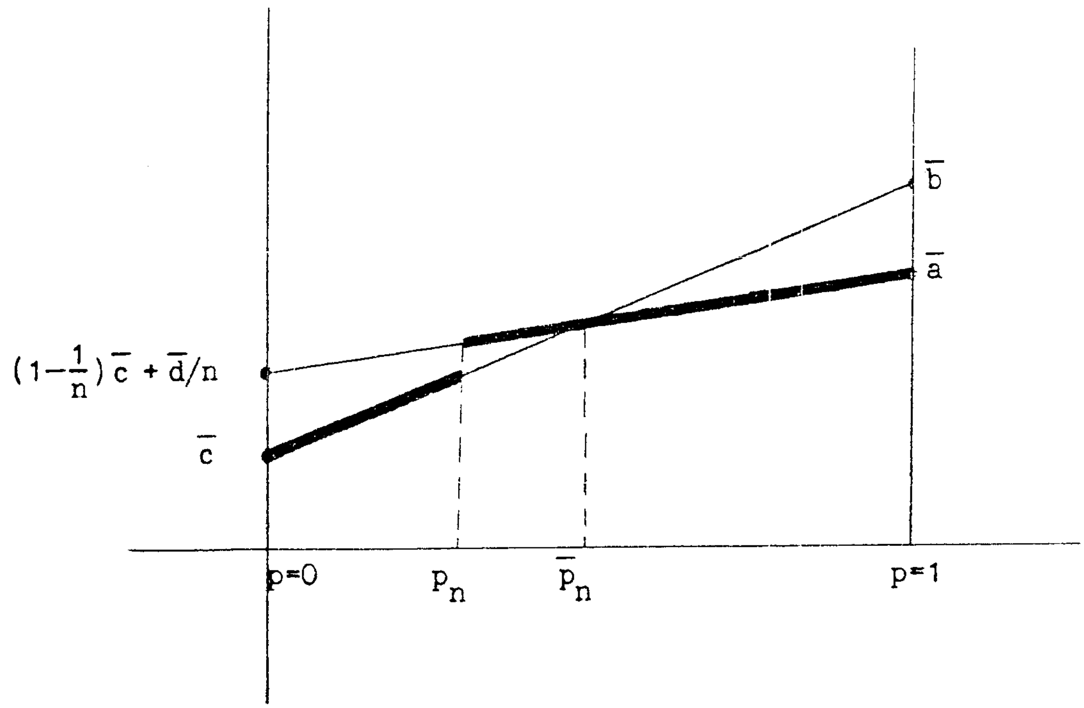


Fig. 3

An example of a case where the maximum value of the game to E occurs at $p = 1$.

§4. Request for information. Uncoded Signal. (Lady or the Tiger?)

a) The game.

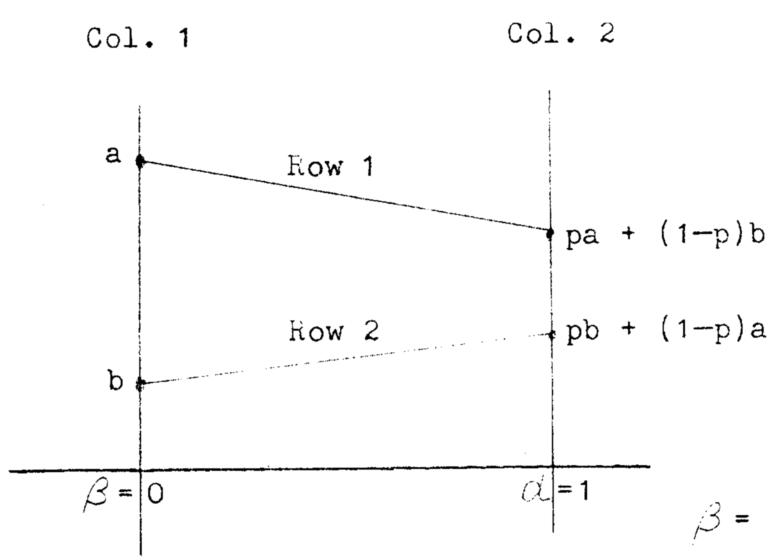
The game is that O must select one of two alternatives: one of which pays off a, the other b; $a > b$. He receives a signal which points to one of the two alternatives. The signal may be from either friend or foe. He knows that his friend would point to a. The enemy might do either. The rule for signaling is that the probability is p for a friend's signal and (1–p) for an enemy's signal.

This is the situation which confronted the young commoner in Frank Stockton's story, "The Lady or the Tiger?". He must choose between two doors: behind one is a tiger, behind the other a beautiful young lady. The princess points to one of the doors. The introduction of the probability p is to account for the good and bad side of the princess.

We use S(F) = (a), etc., to mean the friend points to the alternative (a) that pays off a, etc. Here the friend and O have agreed that S(F) = (a). The matrix is then:

	S(E) = (a)	S(E) = (b)
O believes	$(1-p)a + pa = a$	$pa + (1-p)b$
O disbelieves	b	$pb + (1-p)a$

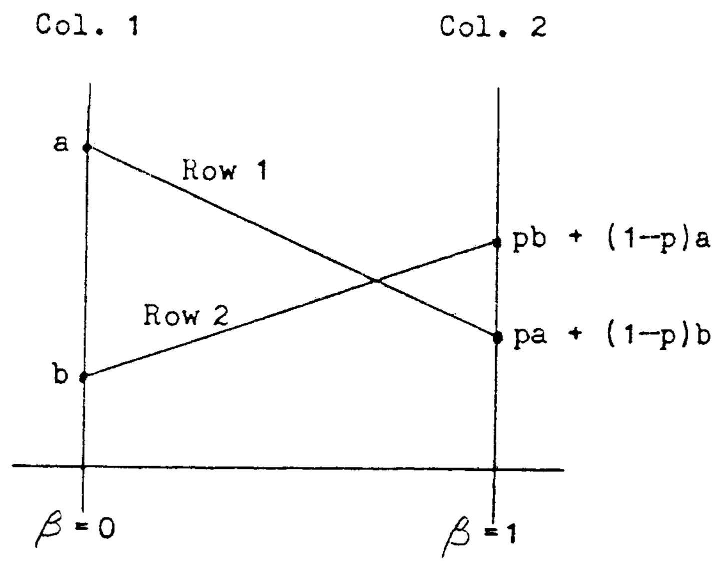
<u>Case</u> 1. $p > 1/2$.



β = prob. that E uses Col. 1.

Hence for p > 1/2, O plays row 1: he believes the signal points to (a).

Case 2. $p < 1/2$.



For $p < 1/2$, we see by symmetry that O's best strategy is the mixed one: 1/2 row 1 + 1/2 row 2; half the time he believes and half the time he disbelieves.

b) Discussion.

Again the best strategy depends upon p in a discontinuous fashion. If the enemy gives directions more than half the time, he can make your friends' signals worthless. He does this by so mixing his true and false signals, that half of the total signals are false and half true. If $p > 1/2$, the best the enemy can do is to make the ratio of false signals as close to half as he can by always signaling (b). You gain by the greater percent of true signals by believing all signals. If the "good—bad princess" is more than "half bad," the commoner should act as though she were all bad. Evil has triumphed. Since he followed her directions without hesitation, he acted as he should if he judged her to be more than "half good." We assume he preferred the lady to the tiger.

§5. Request for information.

a) The game is the same as before except that O and F agree that under a code S(F) = (a) and S(F) = (b). The matrix is now:

	S(E) = (a)	S(E) = (b)
S(F) = (a) O believes	a	pa + (1—p)b
S(F) = (a) O disbelieves	b	pb + (1—p)a
S(F) = (b) O disbelieves	pa + (1—p)b	a
S(F) = (b) O believes	pb + (1—p)a	b

Row 2 is majorized by row 3 and row 4 is majorized by row 1; he will not use these strategies and will act as though the signal is always from a friend. By the symmetry of row 1 and row 3, O's best strategy is 1/2 row 1 + 1/2 row 3.

b) Discussion.

Here coding has achieved something that it did not accomplish in the previous cases; with coding O's best strategy no longer depends upon p. Essentially, coding has placed E in the position of not knowing the payoff of the two alternatives; his best strategy is to signal half the time to one and half to the other. Now O is always able to benefit by F's signal even though p be small.

To O the value of the game is now

$$\frac{p+1}{2}\,a + \frac{(1-p)}{2}\,b\ ;$$

the gain by coding is

$$\frac{1-p}{2}\,(a-b) \qquad \text{for} \qquad p > 1/2$$

and

$$\frac{p(a-b)}{2} \qquad \text{for} \qquad p < 1/2\ .$$