A BRIEF STUDY OF ROCKET-POWERED MAGNETOHYDRODYNAMIC GENERATORS AND ENERGY-STORAGE DEVICES

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SUMMARY

This Research Memorandum considers the problem of supplying, on short notice, very high levels of electrical power: 1000 to 100,000 megawatts on the average, possibly delivered discontinuously to the electrical load in microsecond bursts. It would be entirely impractical to use condensers to supply such high average powers, even for a few minutes. One must therefore consider recharging the condensers between bursts from a prime source using chemical or nuclear fuel. The prime converter of the chemical or nuclear fuel must itself be as compact as possible, as well as efficient and quickly startable. For underground installations, the prime power source should also desirably be non-air-breathing.

In this report the volumetric performance of large conventional powerplants, gas-turbine units, and fuel-cells is briefly reviewed. Considerable attention is paid to magnetohydrodynamic generators, which, because of their inherent simplicity, are capable of very short start-up times and power-densities that are an order-of-magnitude greater than those obtainable from other devices. Such generators, however, require a highly energetic stream of electrically conducting gas. A ground-based chemical rocket exhaust is explored as a source for such a stream.

It is concluded that a rocket-powered MHD generator may indeed fulfill the previously stated requirements. However, the following problems remain for further investigation: (1) Finding the best means of producing sufficient ionization in the rocket exhaust for an acceptable electrical conductivity; and (2) developing electrode, insulation, and magnet materials to operate briefly in the 3000° K range.
Standard condensers are reviewed as a means of supplying individual bursts. It is pointed out that much higher energy densities are obtainable in a rotating-plasma MHD condenser, but that the device is still in the laboratory stage.
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I. INTRODUCTION

This memorandum will be concerned with the production of large pulses of electrical power. Without implying any specific requirement, it is reasonably clear that some future application may require the production of such power.

A pulse power supply should be compact (in case it must, for example, be placed underground) and capable both of storing large quantities of energy for extended periods and of delivering them at very high power levels. Inherent in the ability to store large amounts of energy is the requirement for independence from external utility supplies (i.e., the unit should be self-contained).

More specifically, attention will be devoted to means of providing high average powers (i.e., of the order of 100,000 MW) for periods up to a few minutes, with essentially instantaneous start-up, and with actual delivery at much higher rates during microsecond bursts. (As a point of reference, the total installed steady-state generating capacity of the U.S. in 1957 was about 150,000 MW). Although it may well be possible to utilize some type of nuclear unit for this purpose, this memorandum will consider only chemical-to-electric transformations.
II. ELECTRICAL AND MECHANICAL STORAGE DEVICES

In this section we will consider devices that store energy either electrically or mechanically and have a rapid-discharge capability. The first device that comes to mind is the electrical capacitor (or condenser). The energy density in an electrical field is

$$\frac{KE^2}{2}\frac{1}{\varepsilon}\frac{1}{\varepsilon}$$

where $K$ is the dielectric constant and $E$ is the electric field strength (expressed in cgs-esu units). The velocity of propagation of an electromagnetic wave through a dielectric is given by $\frac{c}{\sqrt{\varepsilon}}$, where $c$ is the velocity of light in a vacuum. This velocity, of course, determines the maximum rate at which a condenser can be discharged. Consequently the energy that can be delivered to a load in a given time interval will be proportional to $E_{\text{max}}^2\sqrt{\varepsilon}$. The condenser discharge time will therefore depend on the fashion in which the dielectric is distributed geometrically. A few approximate values for $E$ and $K$ are given in Table 1.

<table>
<thead>
<tr>
<th>Dielectric</th>
<th>$K$</th>
<th>$E$ Breakdown (volts/cm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Dry air</td>
<td>1</td>
<td>30,000</td>
</tr>
<tr>
<td>Water (pure)</td>
<td>100</td>
<td>150,000</td>
</tr>
<tr>
<td>Berium titanate</td>
<td>15 - 12,000</td>
<td>20,000 - 120,000</td>
</tr>
<tr>
<td>with additives</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

\(^{a}\text{See Page 181, Space/Aeronautics, November, 1959}\)
Barium titanate (BaTiO₃) is ferroelectric, and thus characterized by sharp transition points in the dielectric constant. Even with the upper limits for barium titanate, the capacitative energy density will amount to only about 8 joules/cc. For commercially available condensers the figure will be less than 1 joule/in³, as compared with about 4,000 joules/cc of chemical energy stored in TNT. Nevertheless, practical condenser banks can deliver their load very rapidly. For example, a capacitor bank at the Stevens Institute of Technology can transfer 80,000 joules (with 83 per cent efficiency) to a 50-Mw load in 1.5 μsec. This corresponds to an average power over the first quarter cycle of about 50,000 Mw. At 50 kv the condenser stores 500 joules/ft³, using kraft paper impregnated with pyranol as a dielectric.

Although probably of more interest in space where a suitable environment is available, a high-vacuum condenser is also worth mentioning. With this type of condenser, capacitative storage will ultimately be limited by field emission. Measurable emission usually occurs for fields of about 1 to 10 Mv/cm. The corresponding energy density will be about 5 joules/cc.

Energy can be stored at a much higher density in an inductance coil. For example, a 1,000,000-gauss magnetic field would have an energy density roughly comparable to TNT. The resulting magnetic pressure would be of the order of 560,000 psi. But even this pressure does not represent a limit; with "force free" coils a field of 10,000,000 gauss may be achieved. The main difficulty with inductance coils as storage devices is that of current switching and joule (resistance) losses.

The homopolar generator (shown schematically in Fig. 1) represents a way of obtaining effectively a very high dielectric constant. (It is
Fig. 1 — Homopolar generator
of course a rotating device.) For such a machine, \( E_r = \frac{V_\theta B_z}{C} \) where \( C \) represents the speed of light, \( E_r \) the radial induced electric field (under open circuit conditions), \( B_z \) the axial magnetic field, and \( V_\theta \) the radial velocity. Equating energy density, one obtains

\[
K\left(\frac{E_r^2}{2\pi}\right) = E_r^2/2\pi + 1/2 \rho V_\theta^2
\]

or

\[
K = 1 + 4\pi \rho C^2/P_z^2
\]

(Here \( \rho \) is defined such that \( 1/2 \rho V_\theta^2 \) times the rotor volume represents the total generator kinetic energy). This synthetic dielectric constant can run as high as \( 10^{14} \). Such a high value for \( K \) of course means that nearly all the energy is stored mechanically. The energy-storing capability of the wheel is limited by its bursting strength. A uniform-stress disk with rim can store up to about 400 joules/cc. Also, because of the limited peripheral velocity (about 100,000 cm/sec), the homopolar generator is essentially a low-voltage device. With a field of 10,000 gauss and \( V_\theta = 100,000 \) cm/sec, \( E = 10 \) volts/cm. The discharge speed of a homopolar generator, for small time intervals, would be wave-limited like that of an ordinary condenser, i.e., by the velocity of a wave passing through the rotor. Practically, however, power will be a volumetric quantity determined by the current-produced body force on the rotor. Assuming a rotor conductivity equal to that of copper, \( B = 10,000 \) gauss and \( V_\theta = 100,000 \) cm/sec, a homopolar device could possibly achieve an initial discharge rate of 60 MW/cc. Normally, discharge times are in the millisecond range.
A magnetohydrodynamic condenser has been under development in which a solid rim is replaced by a toroidal plasma filament.\(^{(5)}\) There are several advantages. First, the greatly decreased density of the rotating material, coupled with much higher attainable velocities, means that a much larger fraction of the total energy will be stored in the electric field; i.e., lower K's (from 1,000,000 to 100,000,000), shorter discharge times, and higher voltage. Second, the higher velocities allow energy densities of the same order as those obtainable in a coil. There are, however, practical difficulties involving viscosity and diffusion; in one example presented in Ref. 5, by the time maximum-charge storage was achieved, twice as much charge had leaked through the system as had been stored in recoverable form.

With the possible exception of the "MHD" condenser, electrical and mechanical systems that can be easily and very rapidly discharged do not possess a high specific-energy storage capacity as compared with explosives. For this reason it is of interest to examine various direct explosive-electric conversion schemes.
III. EXPLOSIVE-ELECTRIC TRANSFORMERS

Broadly speaking, explosive-electric devices can be divided into two categories: those that use the explosive energy to distort a magnetic field, and those that involve an interaction with electrical charges. A cataloging of a number of such possibilities, together with an extensive bibliography, may be found in Ref. 6.

Explosive collapse of a magnetic field can easily be explained with reference to Fig. 2. A magnetic field is first created within the loop by, say, a switching arrangement. Then if the loop is collapsed from the right as shown, two competing phenomena will take place. The current will try to die out exponentially as in any R-L circuit, corresponding to a leakage of the enclosed magnetic flux out of the loop. At the same time flux will be swept in by the moving conductor, thus tending to build up both the flux density and current. Since for even 1 ohm and an inductance of 1 mh, the "e-folding" decay time will be 1 msec, it is clear that the loop collapse must be very rapid to achieve a buildup.\(^{(7)}\) (For a further discussion of this point, see Appendix A.) This, however, can be accomplished with an explosive collapse. One-shot devices of a type that can be represented schematically by Fig. 2 have been built in which megagauss magnetic fields are achieved. In theory, if the collapse time is very short compared to the diffusion time of the magnetic field,\(^{(8, 9)}\) a high percentage of the stored chemical energy can be transformed to the magnetic fields (better than 10 per cent has been achieved).

Explosively-driven generators thus are fairly efficient energy-converters, but they are one-shot devices and as designed to date have been primarily of use in dumping energy into very-low-resistance loops. By
Fig. 2 — Field collapse
way of comparison capacitors can be reused, can be discharged more rapidly, and can supply either high- or low-resistance loads.

A variation involves explosive demagnetization of a ferromagnetic material. Devices of this sort are described in detail in Ref. 10. One such device with a volume of 60 in.$^3$ reached a power level of about 100 kw through an 84-kilo-ohm load within 3 μsec after firing. Even higher powers could be obtained with a lower-resistance load. In such devices one can obtain about 1 joule/cc of shocked ferromagnetic material. That is, 1 cc of ferromagnetic material is required for each joule of electrical energy extracted in this fashion. But the exact theory of such devices is not well understood.

There is an analogy to the explosive collapse of magnetic fields in the forceable separation of the plates of a condenser. To be effective, separation must occur more quickly than charge leakage. This means feeding a very high resistance. Alternatively, explosive pressure could be applied to a piezoelectric material. However, materials that are purely piezoelectric (i.e., not ferro- or pyroelectric) will theoretically go dead in the hydrostatic regime. For this reason pyroelectric and ferro-electric materials are of considerable interest. Reference 11 describes a PZT-1 disk 1.6 in. in diameter and 0.028 in. thick which delivered about 1 Mw peak into a 0.1-ohm resistor. The average power was about 250 kw for under 0.3 μsec. This again corresponds roughly to 1 joule/cc of shocked ferro-electric material. If it were not for electrical breakdown, this figure might be considerably higher.

Whenever volume is an important consideration, and large amounts of energy are to be delivered in a series of pulses, Table 2 (which summarizes
the previous two sections and adds two battery examples) leads to the following conclusion. Condensers are the most likely choice among currently available high-power-density devices. However, they should be recharged between pulses from a prime chemical or nuclear source such as a steam-turbine generating plant.* (This might even involve a three-step chain: nuclear power plant, battery, and condenser.)

A large conventional generating station will have a space factor of about 20 $\text{ft}^3/\text{kW}$ of electrical output, and a volumetric fuel consumption (crude oil) of about 0.006 $\text{ft}^3/\text{kW-hr}$. A gas-turbogenerator may deliver up to approximately 7 $\text{kW}/\text{ft}^3$ with a volumetric fuel consumption (gasoline alone, not counting air) of about 0.01 $\text{ft}^3/\text{kW-hr}$. A hydrogen-oxygen fuel cell, operating at 40 per cent efficiency, can deliver about 15 $\text{kW}/\text{ft}^3$. However, a rocket-powered magnetohydrodynamic generator, because of its potentially very high power density and good efficiency, is also worthy of consideration.

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*Another way of putting it would be as follows: Condensers can store very little energy per unit volume, but deliver it very rapidly. A lot of energy per unit volume can be stored in chemical or nuclear fuel, but rapid delivery in an electrical form requires a large investment in conversion equipment; hence, a combination to supply pulses.
<table>
<thead>
<tr>
<th>Device</th>
<th>Storage Time</th>
<th>Specific Energy (electrical energy released/vol. of material)</th>
<th>Discharge Rate</th>
</tr>
</thead>
<tbody>
<tr>
<td>Electostatic condenser</td>
<td>Long</td>
<td>Current commercial condensers: (&lt; 1 \text{ joule/in.}^3)</td>
<td>Limited by the speed of light (c/\sqrt{\varepsilon_0}) in the dielectric</td>
</tr>
<tr>
<td>MHD condenser</td>
<td>Currently very short</td>
<td>Reportedly up to 4000 joules/cc of plasma(^a)</td>
<td>In the millisecond range</td>
</tr>
<tr>
<td>Nonpolar generator</td>
<td>Requires continual input to overcome frictional losses</td>
<td>400 joules/cc of wheel volume</td>
<td>Normally in the millisecond range</td>
</tr>
<tr>
<td>Solid-state explosive-electric transducers</td>
<td>Very long</td>
<td>1 joule/cc of shocked material alone; (a &quot;one-shot&quot; device)</td>
<td></td>
</tr>
<tr>
<td>Inductance coil</td>
<td>Requires continual input because of I^2R losses.</td>
<td>4000 joules/cc or more, stored in the magnetic field</td>
<td>Limited by the shock velocity in the ferroelectric or ferromagnetic material (about 5,000,000 \text{ J/m}^2/\text{sec})</td>
</tr>
<tr>
<td>High-drain-rate battery (Kapitza type)</td>
<td>Long</td>
<td>About 0.3 joule/cc</td>
<td>Theoretically also limited by speed of light through the coil; severe switching problems</td>
</tr>
<tr>
<td>Lead-acid submarine-type battery</td>
<td>Long</td>
<td>About 100 joules/cc</td>
<td>20 minutes at 80°F (1 \text{ watt/in.}^3)</td>
</tr>
</tbody>
</table>

\(^a\) 1 joule/cc = 0.0079 \text{ W-hr/cm}^2.
IV. MAGNETOHYDRODYNAMIC GENERATORS

Magnetohydrodynamic generators can be applied to any stream of an electrically conducting gas. To understand the principle of operation, we will refer to Fig. 3. An MHD generator can be considered as essentially a channel for guiding the flow of an electrically conducting gas, the top and bottom surfaces representing pole faces of a magnet and the sides being electrodes. In a crude way, the gas flow can be compared to the armature in an ordinary generator. Current flow is induced across the channel. The electrodes then collect this current. We will consider the simplest case, in which the channel area is constant and the side walls are perfect conductors.

Reference 12 discusses energy extraction from the flow of an electrically conducting gas through such a channel. The flow was assumed to be strictly one-dimensional, inviscid, without end effects, and without any energy removal from the flow except via the electrical output. Under these conditions, three parameters determine the maximum electrical power that may be drawn smoothly (without shock) from the stream. They are \( E_0 \), \( R_n \), \( \gamma \), and \( M_0 \) where

\[
\gamma = \text{ratio of specific heats of the gas at constant pressure and constant volume}
\]

\[
M_0 = \text{the initial flow Mach number (ratio of flow speed to local sonic speed) into the generator}
\]

\[
E_0 = \text{the ratio of constant transverse electrical field to } \frac{B_0 V_0}{\mu_0} \text{ (i.e., due to charge induced at the electrodes)}
\]

\[
R_n = \frac{\frac{B_0^2}{\mu_0 V_0^2}}{\mu_0 V_0^2} = \text{magnetic pressure number}
\]
Fig. 3 — MHD generator
$B_0 = \text{initial value of the transverse magnetic field}$

$\mu = \text{magnetic permeability of the gas}$

$\rho_0 = \text{initial gas density}$

$V_0 = \text{initial gas velocity}$

In the absence of any current flow, the magnetic field would be uniform. With current flow it will be distorted as illustrated in Fig. 4. We assume that the distortion causes $B$ to vary along the channel, but introduces no component in the flow direction. Then, strictly speaking, $B_0$ will depend on both the strength of the external magnet and the current distribution within the gas. If the product $BV$ were constant along the channel, then a value of $E_0 = 1/2$ would lead to maximum power density. For a channel of constant cross section $BV$ will not be constant; however, the optimum value of $E_0$ will not be far from $1/2$ (for power extraction, $0 < E_0 < 1$).

Figure 5 shows the efficiency of the simplest open-cycle generator as a function of $M_0$ and $R_n$, and it can be quite good. ($E_0 = \frac{1}{2}$)**

As $R_n$ and $M_0$ both approach $\infty$, and with $\gamma = 4/3$ and $E_0 = 1/2$, efficiency $\rightarrow \frac{5}{12}$. (The maximum value of efficiency $\rightarrow 3/7$, corresponding to $E_0 = 4/7$.) For the subsonic case, as $R_n \rightarrow \infty$, efficiency $\rightarrow 7/9$. The reasons for a limitation on the fraction of the stream's energy which can be withdrawn directly as electricity are as follows: If the entrance gas flow is subsonic, the local hydrodynamic Mach number will increase downstream. Power extraction will then be limited by the condition that the exit Mach number cannot exceed one. If the inlet flow is supersonic, the local Mach number will decrease downstream. Smooth power extraction can in this

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*This point is discussed further in Appendix B.

** Subsonically an efficiency approaching 25 per cent could be achieved by letting $E_0 \approx 1$. However this would imply a very low power-density.
Fig. 4 — Field distortion
Fig. 5 — Variation of open-cycle efficiency with $M_0$ and $R_n$. 

Limit as $R_n \to \infty$ and $M_0 \to \infty$. 

$R_n = 10$  
$R_n = 10^2$  
$R_n = 10^3$  
$R_n = 10^4$
instance be terminated by either of two events: the sonic barrier (flow in the channel cannot pass smoothly from supersonic to subsonic), or an asymptotic approach to a condition of zero current density \( j \to 0 \). (12)

The latter limitation represents the boundary between generator and motor action. If the current were to reverse, the gas flow would absorb energy; however, this cannot happen in a finite channel. Figures 6 - 9 illustrate how the flow parameters actually vary with distance along the channel.*

It is assumed in Fig. 5 that the generator channel is always sufficiently long to allow the flow to closely approach one of the two limiting conditions, \( M = 1 \) or \( j = 0 \). Also Fig. 5 is independent of the electrical conductivity of the gas, even if it varies down the channel. However, the conductivity is assumed to be independent of magnetic field effects. The electrical output per unit of channel cross-sectional area will simply equal the product of the stream specific total enthalpy, area mass flow, and efficiency. This would indicate that the inlet Mach number chosen should be as high as possible.** There is, however, a further point to be considered; namely, the physical length required to actually achieve the results of Fig. 5. This length will be inversely proportional to the electrical conductivity of the gas and the inlet velocity; it will also be approximately inversely proportional to the magnetic pressure number. (These two comments, as well as the derivation of Fig. 5, are discussed in Appendix B.) Under some conditions the electrical conductivity

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*All quantities shown in Figs. 6 - 9 have been made dimensionless in a fashion described in Appendix B; \( R_m \) is also defined there.

**This statement is based on the higher efficiencies possible with supersonic inlet conditions. There is also the point that a subsonic unit would be more sensitive to exhaust conditions.
Fig. 6 — Subsonic generator characteristics for $M_0 = 0.05$, $R_n = 100$, $E = \frac{1}{2}$, $\gamma = \frac{4}{3}$
Fig. 7—Supersonic generator characteristics for $M_o = 10$, $R_n = 0.1$, $E = \sqrt{2}$, $\gamma = 4/3$
Fig. 9—Supersonic generator characteristics for $M_0 = 3$, $R_n = 10$, $E = \frac{1}{2}$, $\gamma = \frac{4}{3}$
of a gas will depend strongly on its static temperature. Therefore, a compromise suggests itself when the aim is to maximize power density. With a supersonic stream, it may be desirable to sacrifice some increment in Mach number (and thus accept a somewhat lower limiting efficiency from Fig. 5) in order to obtain a higher stream temperature, and hence a shorter channel. An example of this will be given in the next section.

In summary, to obtain a simple open-cycle MHD unit of high power density and efficiency it is necessary to have a supersonic inlet Mach number, a high magnetic pressure number (hence as strong a magnetic field as possible), and a gas stream with high electrical conductivity and total specific enthalpy. Let us now examine the possibilities resulting from combining an MHD generator with a rocket exhaust.
V. ROCKET-POWERED UNITS

We will consider the application of an oxygen-kerosene rocket to MHD power generation. The outlet Mach number and static temperature of one current oxygen-kerosene rocket are approximately 3 and 1750°K, respectively. As will be shown later, less expansion (leading to a lower Mach number but a higher temperature) would produce a flow more suitable for passage through an MHD generator. Nevertheless, it will be instructive to consider first the application of an "off-the-shelf" rocket.

At a temperature of 1750°K the rocket exhaust will be essentially un-ionized, and therefore not electrically conducting. This can be partially overcome by "seeding" the gas with some more easily ionized material, such as potassium; i.e., introducing, for example, powdered potassium or one of its compounds into the gas stream. With a 1.0 per cent by weight 'seeding' with potassium, it should be possible to obtain a conductivity of about 0.01 mho/cm; incidentally, copper conductivity is 600,000 mho/cm. (By combining the charged-particle mobility equation with the Saha equation, one can show that electrical conductivity of a seeded gas should be proportional to $\sqrt{e^k/T}$. Here $c$ represents weight mole fraction of additive (seeding material), $T$ the gas temperature, $p$ the pressure, and $k$ a constant. With a 0.1 to 1 per cent potassium seeding by weight, this simple approach will be valid for temperatures only up to about 3000°K. At 3000°K and a conductivity of about 1 mho/cm, the conductivity will tend to increase much less rapidly and become independent of pressure. The above formulation is also limited in validity to small
values of \( c \), of the order of about 0.03 or less. The combined rocket-MHD generator would then appear as shown in Fig. 4.

If we want a unit that can be stored and then "fired up" nearly instantaneously independent of any external or auxiliary power source, it is possible to consider the use of permanent magnets. Permanent magnets can produce fields up to about 10,000 gauss with a demagnetizing force of about 450 oersteds.\(^{18}\) The exhaust of the rocket being considered above has a density of about 0.01 lb/ft\(^3\); hence with a 10,000-gauss field, \( R_m = 10 \). A 30-ft long channel would then produce about 0.6 Mw of electrical power for each square foot of channel cross-sectional area. With a channel 4 ft on a side, the power output would thus be about 10 Mw at a voltage of about 1 kv. (Assuming an emissivity of 0.1, heat loss by radiation to room temperature from an area of 480 ft\(^2\) at 1750\(^\circ\)K would amount to about 2.5 Mw.) The volume of the channel itself would only be about 500 ft\(^3\), with the rocket being of the same order of magnitude. However, the volume of iron in the magnet would be about 20 times that of the channel,\(^*\) thus bringing the total volume up to around 10,000 ft\(^3\) (about 1 kw/ft\(^3\) -- comparable to some gas-turbine generators). Fuel and oxidizer consumption together would run about 1000 lb or 15 ft\(^3\)/sec. (We are assuming storage of oxygen in a liquid form.)

Incidentally, this example illustrates one prime fact about rocket-powered MHD generators: their area of application will be in the megawatt range. With the hydrodynamic and electrical characteristics of seeded rocket exhausts, and with the strongest practical magnetic fields, the channel required to approach the efficiencies by Fig. 3 will always be

\*Single-domain iron-cobalt magnets\(^{19}\) would reduce this figure by about 9/13 and may become commercially available.
fairly long. (This channel length may be thought of as an interaction
length.) At the same time the transverse dimensions must be sufficiently
great that viscous effects will not predominate, and the generated field
will be large compared to electrode drops. Hence to obtain a reasonably
efficient generator, the output must be in the megawatt range.

Returning now to our typical "off-the-shelf" rocket, the figure of
0.6 Mw/ft² does not represent the maximum possible extraction of electrical
energy from the stream. The electrical conductivity is so low that the
limiting energy extraction represented by Fig. 5 would require much more
than 30 ft. However, we can alter the rocket to make it more suitable for
our application. By decreasing the expansion and accepting a Mach number
of only two at the rocket exhaust (generator inlet), we can achieve a
static temperature of 2500°K. With the same seeding, the conductivity
would then approach 0.2 mho/cm. The power output of a magnet and channel
of the same length (but now with only about 1/3 the channel area) would
then be approximately 30 Mw (10 kw/ft³). Mass consumption would remain
the same. This now represents about 1/3 the maximum efficiency attainable
with the given inlet values of M₀ and Mₚ. (The interaction length is ap-
proximately 90 ft.) If the electrical conductivity could be tripled
while holding all else constant, about 90 Mw could be extracted in 30 ft.
Conductivities still higher would not enable one to extract more than 90 Mw,
but would allow one to do it in less than 30 ft.

By going to a more energetic reaction one can attain even higher power
densities and at the same time achieve the limiting efficiencies (i.e.,
an interaction length will be physically realizable). For example, with
the same magnetic field (10,000 gauss) and a stream representing 20 per cent
higher generator inlet power per unit area, one can hold $M_o$ and $R_p$ at the same values as above while raising $T_o$ to $3500^\circ K$. One could then obtain about 110 Mw from a channel only 8 ft long (one interaction length) and 6 ft$^2$ in area with an efficiency of approximately 7 per cent.* The power density including magnet would be about 100 Kw/ft$^3$. Alternately, one could operate at Mach 3 and a temperature of $2400^\circ K$. The interaction length would then be about 30 ft, but the efficiency would be approximately 17 per cent (from Fig. 5).

In all of the above, we have assumed a permanent magnet. An electromagnet would require separate excitation and some startup time (the inductance could be quite large); i.e., several minutes, for example, with a 100-henry 1-ohm coil. However, with an electromagnet one can obtain fields as high as 20,000 gauss. Furthermore, the volume of iron plus windings should be somewhat less than the volume of a permanent magnet—perhaps half as much. This, plus the doubling of $B_o$, would mean about 8 times the power density in the previous examples. The power consumed in the windings of the magnet itself would be in the megawatt range.

There are certainly many difficult problems to be resolved. These include:

1. **Materials problems.** The magnet must be kept well below its Curie point, while the electrode surfaces must operate hot to prevent formation of an un-ionized boundary layer and to help supply electrons at the cathode. Materials that suggest themselves for the electrodes include ZrO$_2$, TiC, and graphite in a dense form.

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*We are assuming here that the conductivity of the seeded stream at $3500^\circ K$ will be similar to that of potassium-seeded air at a similar temperature.*
2. **Boundary-layer problems.** The pressure in a supersonic generator of constant area will build up downstream. This might or might not be serious. If it is, a shaped channel (as illustrated in Fig. 10) may provide a solution.

3. **End effects.** The phrase "end effects" refers to the fact that beyond the magnetic field the gas will still be conducting, thus providing a shunt path across the load. This is illustrated in Fig. 11. One way of partially overcoming this is to extend the magnetic field beyond the conductors, as shown in Fig. 12. In Ref. 20, however, it is shown that for channel aspect ratios >3, the loss in output will not exceed about 10 percent.

In addition, there are the problems of heat-transfer, seeding, and the dynamic response of such a generator, including problems of subsidiary energy transfer to a very-rapid-delivery storage device (such as, possibly, an MHD capacitor). Concerning seeding, the problem resolves itself into finding a compound which will undergo thermal ionization quickly and appreciably in the presence of the carrier gas. Nevertheless it is encouraging to note that small-scale generators, operating on seeded gas, have been successfully run for short periods. (21)
Fig. 10 — MHD generator with non-constant cross section
Fig. 11 — End-shunting in an MHD generator
VI. CONCLUSIONS

We have shown how a simple linear MHD generator combined with a chemical rocket is suited to providing large blocks of electrical power for short periods, and on very short notice. Even with an oxygen-kerosene rocket the power-density in the generator (including rocket and magnet) compares favorably with that of the most compact conventional power plants. More energetic reactions will permit even greater power densities and, at the same time, efficiencies approaching those of fairly large conventional power plants.

Because a rocket is a self-sufficient unit in the sense that it carries its own supply of both fuel and oxidizer, the rocket-powered MHD generator is most directly comparable with a battery or H₂-O₂ fuel cell. The potential power density in a chemical rocket-powered MHD generator is about an order of magnitude greater than that obtainable (or likely to be obtained) from batteries or fuel cells under rapid-discharge conditions. (This, however, will require a reaction more energetic than oxygen and kerosene.) Efficiencies are likely to be somewhat less than those of the H₂-O₂ fuel cell. Nevertheless, for short runs, over-all volume will be dominated by the conversion equipment with the result that the rocket-MHD generator unit can be smaller. In any event the volume of MHD generator, plus rocket and energetic chemical reactants, is apt to be considerably less than the volume of any conventional battery under rapid-discharge conditions.

Future research should be directed primarily towards means of providing acceptably high electrical conductivities in the gas, and long-lived electrode surfaces.
Also, a rocket-powered open-cycle MHD generator is inherently simple, relatively insensitive to the surrounding environment (batteries and fuel cells tend to be sensitive to the ambient temperature), and can provide fairly high voltages from a single unit.

A combination of a rocket-powered MHD generator with a condenser-like rapid-delivery system appears to offer interesting possibilities for supplying peaked loads.
Appendix A

THE ROLE OF LOOP RESISTANCE IN FIELD COLLAPSE

Referring to Fig. 2, assume that the loop inductance can be expressed approximately by ky, where k is a constant. Then Ohm's law appears as

\[ k \frac{d}{dt}(yI) + RI = 0 \]

where \( t \) represents time, \( I \) the loop current, and \( R \) loop resistance (assumed constant).

This equation can be rewritten as

\[ y \frac{dT}{dt} + I \left( \frac{R}{k} + \frac{d}{dt} \right) = 0 \]

Now \( y > 0 \), and for the case in which collapse is accomplished by an initial velocity \( \frac{dy}{dt} \bigg|_{t=0} = -V_0 \) without any subsequently applied push, \((-V_0)\) represents the most negative value ever assumed by \( \frac{dy}{dt} \). This is because the loop itself will tend to expand as a result of the interaction between \( I \) and its own magnetic field. It is then clear that for \( \frac{dT}{dt} \) to be positive (a necessary condition for energy transfer to the magnetic field), \( V_0 > R/k \).

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*We are here assuming the validity of lumped circuit parameters. The assumptions involved in this may be found in any good text on electromagnetic theory.
Appendix B

DERIVATION OF FIG. 5

The equations of one-dimensional gas flow through an MHD generator\(^{(12)}\) may be written as

\[
\text{Momentum: } \frac{dv}{dx} + \frac{dp}{dx} - R_n R_m B j = 0
\]

\[
\text{Energy: } \frac{1}{(\gamma-1)M_o^2} \frac{dT}{dx} + \frac{1}{2} \frac{d(V^2)}{dx} - R_n R_m E_o j = 0
\]

Continuity: \( \rho V = 1 \)

State: \( \frac{p}{\gamma M_o^2} = \rho T \)

Ohm's law: \( j = E_o - BV \)

where all quantities are dimensionless and defined for this appendix as follows:

- \( V \) = flow velocity in the x direction, referred to the entrance value \( V_o \)
- \( \rho \) = density, referred to \( \rho_o \)
- \( p \) = pressure, referred to \( \rho_o V_o^2 \)
- \( B \) = magnetic inductance in a direction transverse to x, referred to \( B_o \)
- \( T \) = temperature, referred to \( T_o \)
- \( \gamma \) = ratio of specific heats of the gas
- \( x \) = distance along the channel, referred to a characteristic dimension of the channel L
\[ E_0 = \text{constant electric field, referred to } B_0 V_0 \]

\[ R_m = \mu L\sigma V_0; \quad R_n = B_0^2/\mu_0 V_0^2 \]

\[ j = \text{current density, referred to } B_0 V_0 \sigma \]

\( \sigma = \text{gas conductivity in the direction of } E_0 \)

\( \mu = \text{magnetic permeability} \)

\( M_0 = \text{entrance Mach number} \)

If \( V, p, \rho, T, j, \) and \( B \) are all regarded as unknown, this set of five equations is insufficient to determine a solution. There are now at least two courses of action. If the magnetic material bounding the channel on the top and bottom is unsaturated, then perturbations to the magnetic field produced by the current flow in the gas itself will be essentially colinear with the externally applied field. This follows from the fact that the flux lines must enter and leave the magnet faces normally. Under these circumstances one can add Maxwell's equation to the above five,

\[ \frac{dB}{dx} = -R_m j \]

This now forms a complete set. Alternatively, if the external magnetic material is saturated, then the transverse component of magnetic field \( B \) in the channel may be taken as fixed; the original five equations are then adequate. We will follow the former approach; Resler and Sears have given an approximate solution based on the second approach. In either case the factors limiting power extraction (as described in the text and again in the following paragraphs) will be the same. Regarding Fig. 5, the second approach would lead to efficiencies independent of \( R_n \); however, it
is seen that particularly in the supersonic regime this is nearly the case anyway.

Returning now to the solution of the original five equations, \( j \) can be eliminated from the first two by the use of Maxwell's equation. Then from the first four of these equations and the integral of the second, one can eliminate \( x \). With \( V \) known as a function of \( B \), \( j \) follows as a function of \( B \) from Ohm's law and then \( B \) itself as a function of \( x \) from Maxwell's equation. However, it is of interest to investigate the characterization of the flow in the \( B-V \) plane directly (this characterization is independent of the conductivity). Trajectories in this plane have the slope

\[
\frac{dB}{dV} = \frac{\left( \frac{y+1}{y-1} \right) \frac{V^2}{2} + \frac{R \epsilon E \sigma}{n} \left[ \frac{1}{(y-1)M_o^2} + \frac{1}{2} + \frac{R \epsilon E \sigma}{n} \right]}{\lambda R \epsilon E \sigma \left[ \left( \frac{y}{y-1} \right) B V \right]}
\]

Smooth power extraction must terminate when either a local Mach number of unit (represented in the \( B-V \) plane by a parabola) is reached, or as a zero-current condition (represented by a hyperbola in the \( B-V \) plane) is approached asymptotically. \(^{(12)}\) A typical \( B-V \) plane representation is shown in Fig. 13. Arrows on trajectories indicate the downstream direction in the physical flow. We see that \( \frac{dB}{dV} \rightarrow \infty \) along the hyperbola marked \( \frac{dV}{dx} = 0 \). Consequently the two intersections of this hyperbola with the Mach 1 parabola (\( dB/dV = 0 \)) represent critical points; the upper one being a saddle point, and the lower a spiral point. There is also a node at the intersection of the Mach 1 parabola with the \( B \) axis.

At a local Mach number of unity, \( T = M_o^2 V^2 \). Combining this with the
### Table 2
A COMPARISON OF CONDENSERS, INDUCTANCE COILS, EXPLOSIVE-ELECTRIC TRANSUDERS, AND BATTERIES

<table>
<thead>
<tr>
<th>Device</th>
<th>Storage Time</th>
<th>Specific Energy (electrical energy released/vol. of material)</th>
<th>Discharge Rate</th>
</tr>
</thead>
<tbody>
<tr>
<td>Electrostatic condenser</td>
<td>Long</td>
<td>Current commercial condensers: &lt; 1 joule/in.²</td>
<td>Limited by the speed of light (c/\text{r}) in the dielectric</td>
</tr>
<tr>
<td>MHD condenser</td>
<td>Currently very short</td>
<td>Reportedly up to 4000 joules/cc of plasma (^a)</td>
<td>In the millisecond range</td>
</tr>
<tr>
<td>Homopolar generator</td>
<td>Requires continual input to overcome frictional losses</td>
<td>400 joules/cc of wheel volume</td>
<td>Normally in the millisecond range</td>
</tr>
<tr>
<td>Solid-state explosive-electric transducers</td>
<td>Very long</td>
<td>1 joule/cc of shocked material alone; (a &quot;one-shot&quot; device)</td>
<td>Limited by the shock velocity in the ferroelectric or ferromagnetic material (about 5,000,000 \text{cm/sec})</td>
</tr>
<tr>
<td>Inductance coil</td>
<td>Requires continual input because of IR losses</td>
<td>4000 joules/cc or more, stored in the magnetic field</td>
<td>Theoretically also limited by speed of light through the coil; severe switching problems</td>
</tr>
<tr>
<td>High-drain-rate battery (Kapitza type)</td>
<td>Long</td>
<td>About 0.3 joule/cc</td>
<td>A few milliseconds</td>
</tr>
<tr>
<td>Lead-acid submarine-type battery</td>
<td>Long</td>
<td>About 100 joules/cc</td>
<td>20 minutes at (80^\circ\text{F}) (1 watt/in.²)</td>
</tr>
</tbody>
</table>

\(^a\) 1 joule/cc = 0.0073 \text{kw-hr/ft}³.
IV. MAGNETOHYDRODYNAMIC GENERATORS

Magnetohydrodynamic generators can be applied to any stream of an electrically conducting gas. To understand the principle of operation, we will refer to Fig. 3. An MHD generator can be considered as essentially a channel for guiding the flow of an electrically conducting gas, the top and bottom surfaces representing pole faces of a magnet and the sides being electrodes. In a crude way, the gas flow can be compared to the armature in an ordinary generator. Current flow is induced across the channel. The electrodes then collect this current. We will consider the simplest case, in which the channel area is constant and the side walls are perfect conductors.

Reference 12 discusses energy extraction from the flow of an electrically conducting gas through such a channel. The flow was assumed to be strictly one-dimensional, inviscid, without end effects, and without any energy removal from the flow except via the electrical output. Under these conditions, three parameters determine the maximum electrical power that may be drawn smoothly (without shock) from the stream. They are $E_o$, $R_n$, $\gamma$, and $M_o$ where

$$\gamma = \text{ratio of specific heats of the gas at constant pressure and constant volume}$$

$$M_o = \text{the initial flow Mach number (ratio of flow speed to local sonic speed) into the generator}$$

$$E_o = \text{the ratio of constant transverse electrical field to } \frac{B_o}{V_0} \text{ (i.e., due to charge induced at the electrodes)}$$

$$R_n = \frac{B_o^2}{\mu_o \rho_o V_0^2} = \text{magnetic pressure number}$$
Fig. 3 — MHD generator

Magnetic field

Electrode

Gas flow

Current flow
\[ B_0 = \text{initial value of the transverse magnetic field} \]
\[ \mu = \text{magnetic permeability of the gas} \]
\[ \rho_0 = \text{initial gas density} \]
\[ V_0 = \text{initial gas velocity} \]

In the absence of any current flow, the magnetic field would be uniform. With current flow it will be distorted as illustrated in Fig. 4. We assume that the distortion causes \( B \) to vary along the channel, but introduces no component in the flow direction. Then, strictly speaking, \( B_0 \) will depend on both the strength of the external magnet and the current distribution within the gas. If the product \( BV \) were constant along the channel, then a value of \( E_0 = 1/2 \) would lead to maximum power density. For a channel of constant cross section \( BV \) will not be constant; however, the optimum value of \( E_0 \) will not be far from \( 1/2 \) (for power extraction, \( 0 < E_0 < 1 \)).

Figure 5 shows the efficiency of the simplest open-cycle generator as a function of \( M_0 \) and \( R_n \), and it can be quite good. \( (E_0 = \frac{1}{2}) \)**

As \( R_n \) and \( M_0 \) both approach \( \infty \), and with \( \gamma = 4/3 \) and \( E_0 = 1/2 \), efficiency \( \rightarrow \frac{5}{12} \). (The maximum value of efficiency \( \rightarrow 3/7 \), corresponding to \( E_0 = 4/7 \).) For the subsonic case, as \( R_n \rightarrow \infty \), efficiency \( \rightarrow 7/3 \). The reasons for a limitation on the fraction of the stream's energy which can be withdrawn directly as electricity are as follows: If the entrance gas flow is subsonic, the local hydrodynamic Mach number will increase downstream. Power extraction will then be limited by the condition that the exit Mach number cannot exceed one. If the inlet flow is supersonic, the local Mach number will decrease downstream. Smooth power extraction can in this

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** This point is discussed further in Appendix B.

** Subsonically an efficiency approaching 25 per cent could be achieved by letting \( E_0 \rightarrow 1 \). However this would imply a very low power-density.
Fig. 4 — Field distortion
Fig. 5 — Variation of open-cycle efficiency with $M_0$ and $R_n$.
instance be terminated by either of two events: the sonic barrier (flow in the channel cannot pass smoothly from supersonic to subsonic), or an asymptotic approach to a condition of zero current density \( j = 0 \). \((12)\)

The latter limitation represents the boundary between generator and motor action. If the current were to reverse, the gas flow would absorb energy; however, this cannot happen in a finite channel. Figures 6 - 9 illustrate how the flow parameters actually vary with distance along the channel.*

It is assumed in Fig. 5 that the generator channel is always sufficiently long to allow the flow to closely approach one of the two limiting conditions, \( M \rightarrow 1 \) or \( j \rightarrow 0 \). Also Fig. 5 is independent of the electrical conductivity of the gas, even if it varies down the channel. However, the conductivity is assumed to be independent of magnetic field effects. The electrical output per unit of channel cross-sectional area will simply equal the product of the stream specific total enthalpy, area mass flow, and efficiency. This would indicate that the inlet Mach number chosen should be as high as possible.**

There is, however, a further point to be considered; namely, the physical length required to actually achieve the results of Fig. 5. This length will be inversely proportional to the electrical conductivity of the gas and the inlet velocity; it will also be approximately inversely proportional to the magnetic pressure number. (These two comments, as well as the derivation of Fig. 5, are discussed in Appendix B.) Under some conditions the electrical conductivity

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* All quantities shown in Figs. 6 - 9 have been made dimensionless in a fashion described in Appendix B; \( R_m \) is also defined there.

** This statement is based on the higher efficiencies possible with supersonic inlet conditions. There is also the point that a subsonic unit would be more sensitive to exhaust conditions.
Fig. 6—Subsonic generator characteristics for $M_0 = 0.05$, $R_n = 100$, $E = \frac{1}{2}$, $\gamma = \frac{4}{3}$
Fig. 7—Supersonic generator characteristics for $M_0 = 1, R_n = 0.1, E = 0.5, \gamma = 1.4$. 

$R_{m} \times$
Fig. 9—Supersonic generator characteristics for $M_0 = 3$, $R_n = 10$, $E = \frac{1}{2}$, $\gamma = \frac{4}{3}$
of a gas will depend strongly on its static temperature. Therefore, a compromise suggests itself when the aim is to maximize power density. With a supersonic stream, it may be desirable to sacrifice some increment in Mach number (and thus accept a somewhat lower limiting efficiency from Fig. 5) in order to obtain a higher stream temperature, and hence a shorter channel. An example of this will be given in the next section.

In summary, to obtain a simple open-cycle MHD unit of high power density and efficiency it is necessary to have a supersonic inlet Mach number, a high magnetic pressure number (hence as strong a magnetic field as possible), and a gas stream with high electrical conductivity and total specific enthalpy. Let us now examine the possibilities resulting from combining an MHD generator with a rocket exhaust.
V. ROCKET-POWERED UNITS

We will consider the application of an oxygen-kerosene rocket to MHD power generation. The outlet Mach number and static temperature of one current oxygen-kerosene rocket are approximately 3 and 1750°K, respectively. As will be shown later, less expansion (leading to a lower Mach number but a higher temperature) would produce a flow more suitable for passage through an MHD generator. Nevertheless, it will be instructive to consider first the application of an "off-the-shelf" rocket.

At a temperature of 1750°K the rocket exhaust will be essentially un-ionized, and therefore not electrically conducting. This can be partially overcome by "seeding" the gas with some more easily ionized material, such as potassium; i.e., introducing, for example, powdered potassium or one of its compounds into the gas stream.\(^{(13)}\) With a 1.0 per cent by weight "seeding" with potassium, it should be possible to obtain a conductivity of about 0.01 mho/cm;\(^{(14, 15)}\) incidentally, copper conductivity is 600,000 mho/cm. (By combining the charged-particle mobility equation\(^{(16)}\) with the Saha equation,\(^{(17)}\) one can show that electrical conductivity of a seeded gas should be proportional to \(\sqrt{\frac{\tau}{P}} \cdot \tau^{5/4} \cdot \frac{e^k}{T} \cdot .\) Here \(c\) represents weight mole fraction of additive (seeding material), \(T\) the gas temperature, \(P\) the pressure, and \(k\) a constant. With a 0.1 to 1 per cent potassium seeding by weight, this simple approach will be valid for temperatures only up to about 3000°K. At 3000°K and a conductivity of about 1 mho/cm, the conductivity will tend to increase much less rapidly and become independent of pressure. The above formulation is also limited in validity to small
values of \( c \), of the order of about 0.03 or less. The combined rocket-MHD generator would then appear as shown in Fig. 4.

If we want a unit that can be stored and then "fired up" nearly instantaneously independent of any external or auxiliary power source, it is possible to consider the use of permanent magnets. Permanent magnets can produce fields up to about 10,000 gauss with a demagnetizing force of about 450 oersteds.\(^{(18)}\) The exhaust of the rocket being considered above has a density of about 0.01 lb/ft\(^3\); hence with a 10,000-gauss field, \( R_m = 10 \). A 30-ft long channel would then produce about 0.6 Mw of electrical power for each square foot of channel cross-sectional area. With a channel 4 ft on a side, the power output would thus be about 10 Mw at a voltage of about 1 kv. (Assuming an emissivity of 0.1, heat loss by radiation to room temperature from an area of 480 ft\(^2\) at 1750°K would amount to about 2.5 Mw.) The volume of the channel itself would only be about 500 ft\(^3\), with the rocket being of the same order of magnitude. However, the volume of iron in the magnet would be about 20 times that of the channel,\(^*\) thus bringing the total volume up to around 10,000 ft\(^3\) (about 1 kw/ft\(^3\) -- comparable to some gas-turbine generators). Fuel and oxidizer consumption together would run about 1000 lb or 15 ft\(^3\)/sec. (We are assuming storage of oxygen in a liquid form.)

Incidentally, this example illustrates one prime fact about rocket-powered MHD generators; their area of application will be in the megawatt range. With the hydrodynamic and electrical characteristics of seeded rocket exhausts, and with the strongest practical magnetic fields, the channel required to approach the efficiencies by Fig. 3 will always be

\(^*\) Single-domain iron-cobalt magnets\(^{(19)}\) would reduce this figure by about 9/13 and may become commercially available.
fairly long. (This channel length may be thought of as an interaction length.) At the same time the transverse dimensions must be sufficiently great that viscous effects will not predominate, and the generated field will be large compared to electrode drops. Hence to obtain a reasonably efficient generator, the output must be in the megawatt range.

Returning now to our typical "off-the-shelf" rocket, the figure of 0.6 Mw/ft² does not represent the maximum possible extraction of electrical energy from the stream. The electrical conductivity is so low that the limiting energy extraction represented by Fig. 5 would require much more than 30 ft. However, we can alter the rocket to make it more suitable for our application. By decreasing the expansion and accepting a Mach number of only two at the rocket exhaust (generator inlet), we can achieve a static temperature of 2500°K. With the same seeding, the conductivity would then approach 0.2 mho/cm. The power output of a magnet and channel of the same length (but now with only about 1/3 the channel area) would then be approximately 30 Mw (10 kw/ft³). Mass consumption would remain the same. This now represents about 1/3 the maximum efficiency attainable with the given inlet values of $M_o$ and $R_p$. (The interaction length is approximately 90 ft.) If the electrical conductivity could be tripled while holding all else constant, about 90 Mw could be extracted in 30 ft. Conductivities still higher would not enable one to extract more than 90 Mw, but would allow one to do it in less than 30 ft.

By going to a more energetic reaction one can attain even higher power densities and at the same time achieve the limiting efficiencies (i.e., an interaction length will be physically realizable). For example, with the same magnetic field (10,000 gauss) and a stream representing 20 per cent
higher generator inlet power per unit area, one can hold $M_0$ and $R_p$ at the same values as above while raising $T_o$ to 3500$^\circ$K. One could then obtain about 110 MW from a channel only 8 ft long (one interaction length) and 6 ft$^2$ in area with an efficiency of approximately 7 per cent. The power density including magnet would be about 100 Kw/ft$^3$. Alternately, one could operate at Mach 3 and a temperature of 2400$^\circ$K. The interaction length would then be about 30 ft, but the efficiency would be approximately 17 per cent (from Fig. 5).

In all of the above, we have assumed a permanent magnet. An electromagnet would require separate excitation and some startup time (the inductance could be quite large); i.e., several minutes, for example, with a 100-henry 1-ohm coil. However, with an electromagnet one can obtain fields as high as 20,000 gauss. Furthermore, the volume of iron plus windings should be somewhat less than the volume of a permanent magnet — perhaps half as much. This, plus the doubling of $B_o'$, would mean about 8 times the power density in the previous examples. The power consumed in the windings of the magnet itself would be in the megawatt range.

There are certainly many difficult problems to be resolved. These include:

1. **Materials problems.** The magnet must be kept well below its Curie point, while the electrode surfaces must operate hot to prevent formation of an un-ionized boundary layer and to help supply electrons at the cathode. Materials that suggest themselves for the electrodes include $\text{ZrO}_2$, TiC, and graphite in a dense form.

*We are assuming here that the conductivity of the seeded stream at 3500$^\circ$K will be similar to that of potassium-seeded air at a similar temperature.*
2. Boundary-layer problems. The pressure in a supersonic generator of constant area will build up downstream. This might or might not be serious. If it is, a shaped channel (as illustrated in Fig. 10) may provide a solution.

3. End effects. The phrase "end effects" refers to the fact that beyond the magnetic field the gas will still be conducting, thus providing a shunt path across the load. This is illustrated in Fig. 11. One way of partially overcoming this is to extend the magnetic field beyond the conductors, as shown in Fig. 12. In Ref. 20, however, it is shown that for channel aspect ratios >3, the loss in output will not exceed about 10 per cent.

In addition, there are the problems of heat-transfer, seeding, and the dynamic response of such a generator, including problems of subsidiary energy transfer to a very-rapid-delivery storage device (such as, possibly, an MHD capacitor). Concerning seeding, the problem resolves itself into finding a compound which will undergo thermal ionization quickly and appreciably in the presence of the carrier gas. Nevertheless it is encouraging to note that small-scale generators, operating on seeded gas, have been successfully run for short periods.\(^{(21)}\)
Fig. 10 — MHD generator with non-constant cross section
Fig. II — End-shunting in an MHD generator
Fig. 12 — MHD generator with extended field
VI. CONCLUSIONS

We have shown how a simple linear MHD generator combined with a chemical rocket is suited to providing large blocks of electrical power for short periods, and on very short notice. Even with an oxygen-kerosene rocket the power-density in the generator (including rocket and magnet) compares favorably with that of the most compact conventional power plants. More energetic reactions will permit even greater power densities and, at the same time, efficiencies approaching those of fairly large conventional power plants.

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Future research should be directed primarily towards means of providing acceptably high electrical conductivities in the gas, and long-lived electrode surfaces.
Also, a rocket-powered open-cycle MHD generator is inherently simple, relatively insensitive to the surrounding environment (batteries and fuel cells tend to be sensitive to the ambient temperature), and can provide fairly high voltages from a single unit.

A combination of a rocket-powered MHD generator with a condenser-like rapid-delivery system appears to offer interesting possibilities for supplying peaked loads.
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Referring to Fig. 2, assume that the loop inductance can be expressed approximately by $ky$, where $k$ is a constant. Then Ohm's law appears as

$$ k \frac{d}{dt} (yI) + RI = 0 $$

where $t$ represents time, $I$ the loop current, and $R$ loop resistance (assumed constant).

This equation can be rewritten as

$$ y \frac{dI}{dt} + I \left( R + \frac{dy}{dt} \right) = 0 $$

Now $y > 0$, and for the case in which collapse is accomplished by an initial velocity $\frac{dv}{dt} _{t=0} = -V_o$ without any subsequently applied push, $(-V_o)$ represents the most negative value ever assumed by $\frac{dv}{dt}$. This is because the loop itself will tend to expand as a result of the interaction between $I$ and its own magnetic field. It is then clear that for $\frac{dI}{dt}$ to be positive (a necessary condition for energy transfer to the magnetic field), $V_o > R/k$.

*We are here assuming the validity of lumped circuit parameters. The assumptions involved in this may be found in any good text on electromagnetic theory.
Appendix B

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The equations of one-dimensional gas flow through an MHD generator\(^{(12)}\) may be written as

**Momentum:** \[
\frac{dv}{dx} + \frac{dp}{dx} - R_n R_m B J = 0
\]

**Energy:** \[
\frac{1}{(\gamma - 1)M_o^2} \frac{dT}{dx} + \frac{1}{2} \frac{d(V^2)}{dx} - R_n R_m E_o J = 0
\]

**Continuity:** \( \rho V = 1 \)

**State:** \( \frac{\rho}{\gamma M_o} = \frac{1}{2} \rho T \)

**Ohm's law:** \( J = E_o - BV \)

where all quantities are dimensionless and defined for this appendix as follows:

- \( V = \) flow velocity in the \( x \) direction, referred to the entrance value \( V_o \)
- \( \rho = \) density, referred to \( \rho_o \)
- \( p = \) pressure, referred to \( \rho_o V_o^2 \)
- \( B = \) magnetic inductance in a direction transverse to \( x \), referred to \( B_o \)
- \( T = \) temperature, referred to \( T_o \)
- \( \gamma = \) ratio of specific heats of the gas
- \( x = \) distance along the channel, referred to a characteristic dimension of the channel \( L \)
\[ E_o = \text{constant electric field, referred to } B_o V_o \]
\[ R_m = \mu_0 V_o, \quad R_n = B_o^2/\mu_0 V_o^2 \]
\[ j = \text{current density, referred to } B_o V_o \sigma \]
\[ \sigma = \text{gas conductivity in the direction of } E_o \]
\[ \mu = \text{magnetic permeability} \]
\[ M_o = \text{entrance Mach number} \]

If \( V, p, \rho, T, j, \text{and } B \) are all regarded as unknown, this set of five equations is insufficient to determine a solution. There are now at least two courses of action. If the magnetic material bounding the channel on the top and bottom is unsaturated, then perturbations to the magnetic field produced by the current flow in the gas itself will be essentially collinear with the externally applied field. This follows from the fact that the flux lines must enter and leave the magnet faces normally. Under these circumstances one can add Maxwell's equation to the above five.

\[ \frac{dB}{dx} = -R_m j \]

This now forms a complete set. Alternatively, if the external magnetic material is saturated, then the transverse component of magnetic field \( B \) in the channel may be taken as fixed; the original five equations are then adequate. We will follow the former approach; Resler and Sears have given an approximate solution based on the second approach.\(^{16}\) In either case the factors limiting power extraction (as described in the text and again in the following paragraphs) will be the same. Regarding Fig. 5, the second approach would lead to efficiencies independent of \( R_n \); however, it
is seen that particularly in the supersonic regime this is nearly the case anyway.

Returning now to the solution of the original five equations, j can be eliminated from the first two by the use of Maxwell's equation. Then from the first four of these equations and the integral of the second, one can eliminate x. With V known as a function of B, j follows as a function of B from Ohm's law and then B itself as a function of x from Maxwell's equation. However, it is of interest to investigate the characterization of the flow in the B-V plane directly (this characterization is independent of the conductivity). Trajectories in this plane have the slope

\[
\frac{dB}{dV} = \frac{\left(\frac{V}{V_0} + \frac{L}{n_0} B\right)^2 + R E B - \left[\frac{1}{(\gamma - 1)M_o^2} + \frac{1}{2} + R E B\right]}{V R_b \left[ E_0 - \frac{\gamma - 1}{V} B V\right]}
\]

Smooth power extraction must terminate when either a local Mach number of unit (represented in the B-V plane by a parabola) is reached, or as a zero-current condition (represented by a hyperbola in the B-V plane) is approached asymptotically. A typical B-V plane representation is shown in Fig. 13. Arrows on trajectories indicate the downstream direction in the physical flow. We see that \(\frac{dB}{dV} = 0\) along the hyperbola marked \(\frac{dV}{dx} = 0\). Consequently the two intersections of this hyperbola with the Mach 1 parabola (\(dB/dV = 0\)) represent critical points; the upper one being a saddle point, and the lower a spiral point. There is also a node at the intersection of the Mach 1 parabola with the B axis.

At a local Mach number of unity, \(T = M_o^2 V^2\). Combining this with the
Fig. 13— B–V diagram for $E_o = 0.5$, $M_o = 3$, $R_n = 10$
integrals of the momentum and energy equations and the equation of state leads to the quartic

\[
\left(\frac{1}{2} + \frac{1}{(\gamma - 1)}\right) \frac{R_n^2 B^4}{4 \left(1 + \frac{1}{\gamma}\right)^2} - \frac{R_n \left[1 + \frac{R_n^2}{2} + \frac{1}{\gamma \gamma_0^2}\right]}{\left(1 + \frac{1}{\gamma}\right)^2} \left(\frac{1}{2} + \frac{1}{(\gamma - 1)}\right) B^2 + R_n E B \\
+
\left\{\frac{\gamma^2}{2(\gamma^2 - 1)} \left[1 + \frac{R_n^2}{2} + \frac{1}{\gamma \gamma_0^2}\right]^2 - \left[\frac{1}{(\gamma - 1) \gamma_0^2} + \frac{1}{2} + R_n E\right]\right\} = 0
\]

If the trajectory in the B-V plane terminates on the Mach 1 parabola (always the ultimate end, if the entrance conditions are subsonic), then this quartic will determine the exit value of B.

When the entrance conditions are supersonic the trajectory may terminate on the zero-current hyperbola. In this case the equation determining the exit value of B is given by a solution of the cubic equation

\[
B_{R_n E}^3 \left[\frac{1}{2} - \frac{(\gamma - 1)}{\gamma}\right] + B^2 \frac{(\gamma - 1)}{\gamma} \left[R_n E + \frac{1}{2} + \frac{1}{(\gamma - 1) \gamma_0^2}\right] \\
- E B \left(1 + \frac{R_n^2}{2} + \frac{1}{\gamma \gamma_0^2}\right) + E^2 \left(\frac{\gamma + 1}{2\gamma}\right) = 0
\]

By computing the roots closest to (but greater than) one, and then choosing the least between the quartic and cubic, one can determine whether the exit conditions correspond to M = 1 or j = 0. Once the exit value of B has been determined, the ratio of exit to entrance specific total enthalpy (h) will be given by
\[ \frac{h_f}{h_i} = 1 + \frac{RE_0 (1 - B^2)}{\frac{1}{(\gamma - 1)M_o^2} + \frac{1}{2}} \]

This was the manner in which Fig. 5 was prepared, and it is hence independent of conductivity. An alternate but longer procedure would be to compute the flow directly from the complete set of equations in the manner described previously. This, however, will also yield the physical length of channel required to reach maximum energy extraction; Figs. 6 - 9 resulted thusly.

The channel length required to closely approach maximum power extraction (i.e., either \( M = 1 \) or \( J = 0 \)) may be regarded as an interaction length. The interaction length will be inversely proportional to \( \sigma V_0 \). This follows from the fact that \( R_m \) can be suppressed in the five original equations plus Maxwell’s equation by replacing \( x \) everywhere by \( X = R_m x \). Integration will then determine all quantities as a function of \( X \), and the physical length corresponding to any given \( X \) will be \( \frac{X}{R_m} = \frac{X}{\mu \sigma V_0} \). The interaction length will also be approximately inversely proportional to \( R_n \). (This, incidentally, is the time significance of \( R_n \), since the efficiency is only slightly affected by it.) If Maxwell’s equation were not involved (as in Resler and Sears’ approach), this would be exactly true, since \( R_n \) could then also be suppressed in all equations. It turns out to be still a good approximation (on the basis of actual integrated cases). Heuristically, the reason is that for a given value of \( E_0 \), \( J \) and \( B \) will always assume the same initial values regardless of \( R_n \), and if they were in fact constants then only the first four equations would be needed with \( R_n \) again suppressible into \( x \).
REFERENCES


