NASr-21(02)

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ON THE MOTION OF ECHO I-TYPE EARTH SATELLITES

L. N. Rowell, M. C. Smith and W. L. Sibley

PREPARED FOR:
NATIONAL AERONAUTICS AND SPACE ADMINISTRATION

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PREFACE

This study was conducted under NASA Contract NASr-21(02), concerned with communication satellite technology and monitored by the Office of Space Flight Programs. In pursuing work under the contract it was necessary to determine if balloon-type communication satellites remained in orbit a sufficient time to be economically competitive with other means of communication. In this connection the present study was undertaken to determine balloon satellite lifetimes.

SUMMARY

By selecting initial orbital conditions properly, balloon-type satellites could be made to either ascend or descend in altitude very rapidly. In the example of earth satellite Echo I, during certain orientations of the satellite, perturbing forces due to solar radiation pressure and air drag oppose one another. At such times the two forces appear to be approximately equal because the perigee altitude of Echo I remains almost constant. At other times the forces due to radiation pressure and air drag operate in the same direction, and the decrease in perigee altitude is very rapid. A number of examples are given of the possibilities of using these effects to influence orbital altitude and, consequently, satellite lifetime.

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LIST OF SYMBOLS

$A_{2,0}$	oblateness constant
a	semimajor axis
e	eccentricity
GM_e	earth's gravitational constant
q	perigee altitude
r	distance between the center of earth and the satellite
r_{em}, r_{es}	distances between the center of earth and the centers of the moon and sun, respectively
ρ	air density
Ω	longitude of the line of nodes
ω	argument of perigee

I. INTRODUCTION

After the launching, in August 1960, of the balloon-type passive communication satellite Echo I, the satellite orbit was observed to change inexplicably. The present Memorandum is an attempt to explain this behavior. In this study the theoretical results are obtained from the first-order differential equations in which the rate of change of the orbital elements is equated to the perturbing accelerations.[1] These equations are integrated, with constant orbital elements, over the satellite orbital period in order to obtain the incremental changes in the orbital elements. The new orbital elements, which are obtained by adding these incremental changes to the old elements, are then used for the next integration period. This iteration procedure is carried out on an IBM 7090 computer using a program developed at Lincoln Laboratories by H. M. Jones, I. Shapiro, et. al.

The perturbing accelerations considered here are caused by the earth's bulge, the gravitational attractions of the moon and sun, atmospheric drag, and direct solar radiation. The perturbations caused by earth bulge and the gravitational attractions of the moon and sun are treated as known quantities, while the atmospheric drag and solar radiation perturbations are inferred from the observed motion of the satellite.

The program is designed for lifetime studies, i.e., it compresses long periods of satellite life into short machine computations. As a result of this time compression, mean orbital elements are obtained. The mean orbital elements represent elements averaged over 10 revolutions, which, for Echo I, represent about 0.8 day. Thus, first-order short period perturbations due to the second harmonic of the earth's gravitational potential do not appear. Also, terms in the potential resulting from the triaxial asymmetry of the earth are not included.

II. DISCUSSION OF PERTURBING ACCELERATIONS

The perturbing accelerations which have the greatest effect on the motion of a balloon-type satellite such as Echo I are:

1. Gravitational attraction of the moon.
2. Gravitational attraction of the sun.
3. Earth bulge.
4. Atmospheric drag.
5. Solar radiation pressure (direct).
6. Solar radiation pressure (reflected).
7. Magnetic drag.
8. Meteor impact.

The perturbing accelerations caused by the last three sources are found to be negligible when compared to the other accelerations.

In calculating the perturbations of close-in earth satellites due to the presence of the sun and moon, it is possible to omit terms of higher order in r/r_{em}, where r and r_{em} are the distances between the center of earth and the satellite and between the centers of the earth and moon, respectively. For a 2000 mi orbit, factors of the order of 10^{-3} are neglected.

The same approximation is made in the calculation of the sun's influence. Neglecting terms of higher order in r/r_{es}, the errors are in this case of the order of 10^{-9}. (The quantity r_{es} is the distance between the centers of the earth and sun.)

If satellites of greater than 2000 mi altitude are to be studied, further corrections in the moon's effect on the orbit must be made.

The interaction of the earth and the satellite is represented by the second through fifth harmonics of the earth's gravitational field. Values of the coefficients suggested by Kozai are used.[2]

The solar energy flux originating at the sun produces three effects:

1. **Diurnal:** Due to heating, the outer atmosphere bulges upward. The bulge lags the direction of the sun as a result of the earth's rotation. This effect is believed to increase with altitude from about 200 km to some maximum and then decrease to the boundary of the atmosphere.

2. **Solar rotation:** The flux of particles from the sun increases the density of the upper atmosphere beyond about 200 km. This effect fluctuates with the 27-day period of the sun (see Fig. 1).

3. **Transient:** A combination of both the diurnal heating and particle flux emanating from magnetic storms.

Both particle flux and the heating effect are included in the program. The Jacchia atmospheric model is used which includes these two effects.[3]

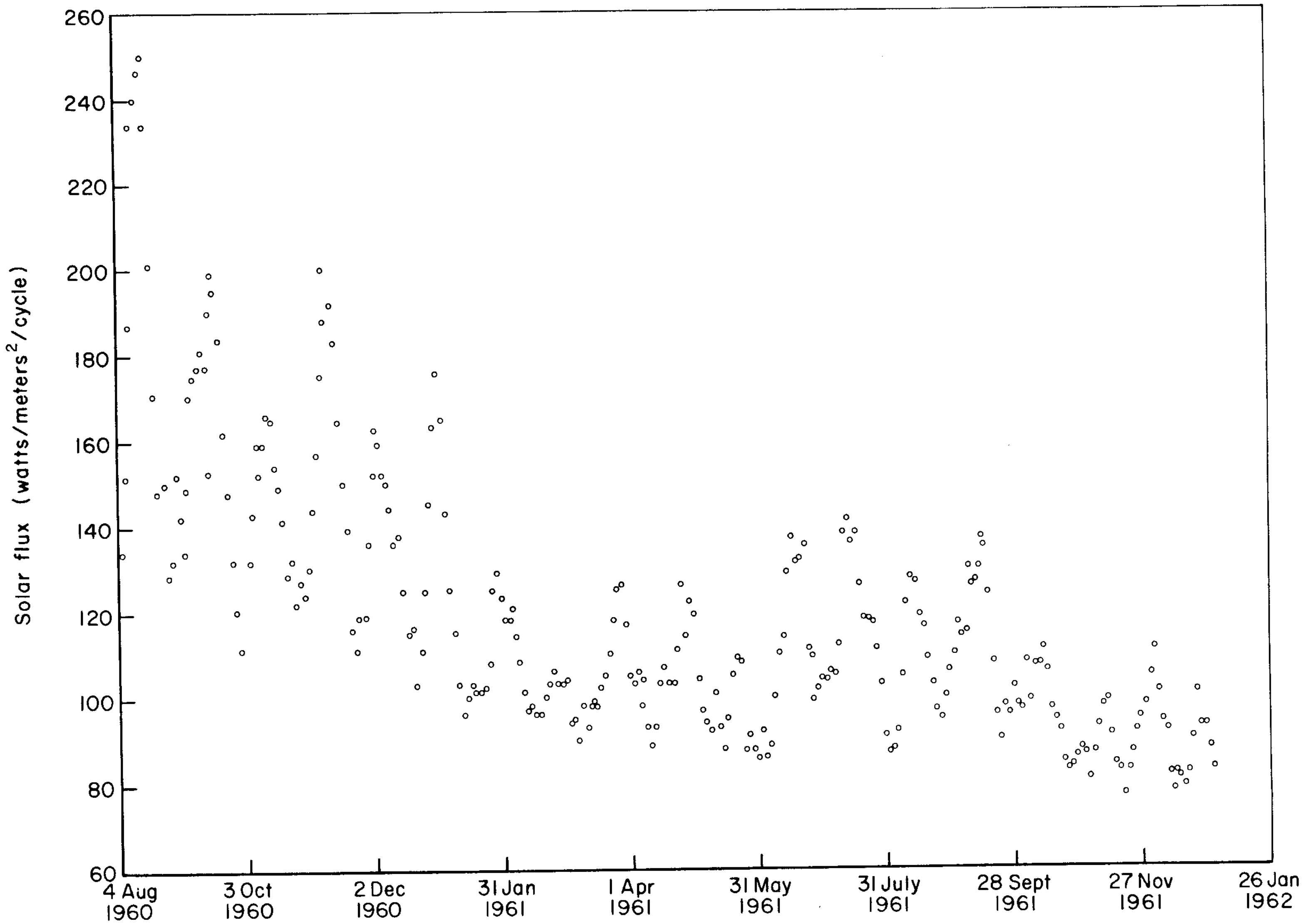

Fig. 1 — Solar flux at 10.7 cm

(data supplied by the Smithsonian Institution Astrophysical Observatory)

III. ECHO I ORBIT

The passive communication satellite, Echo I, was launched August 12, 1960, and went into an almost circular orbit with apogee about 1500 mi and perigee about 1000 mi and a period of 117 minutes.

When the satellite was launched its orbit was entirely in the sunlight. Later the orbit changed so that during a part of each orbit the satellite was in the earth's shadow. The effect of this orbital orientation was to cause the solar radiation pressure on the average to act in the same way as the air drag, i.e., to decrease the semimajor axis a, and the perigee altitude. This continued until about December 11, 1960, as illustrated in Fig. 2. During the following 10 days the drag acted alone; the satellite was entirely in daylight. Around January 1, 1961, portions of the orbit again entered the shadow of the earth but this time the radiation pressure on the average opposed the air drag, and perigee altitude and semimajor axis began to increase slightly, which indicates that the radiation pressure was actually adding energy to the orbiting system at about the same rate that the air drag was removing it.

Initially, on August 12, 1960, Echo I had a total weight of 156.995 lb including 33.34 lb of sublimating powders. One of the powders weighed 10 lb and was highly evaporative while the other weighed 23.34 lb and had a much slower rate of evaporation. Since 21 per cent of the initial total mass of the satellite was powder, it is difficult to determine accurately the time dependence of the satellite mass. On the basis of the rates of evaporation of the powder, which was assumed by Jones, et. al., nearly all of the powders would have escaped by about January 11, 1961.[4] The program

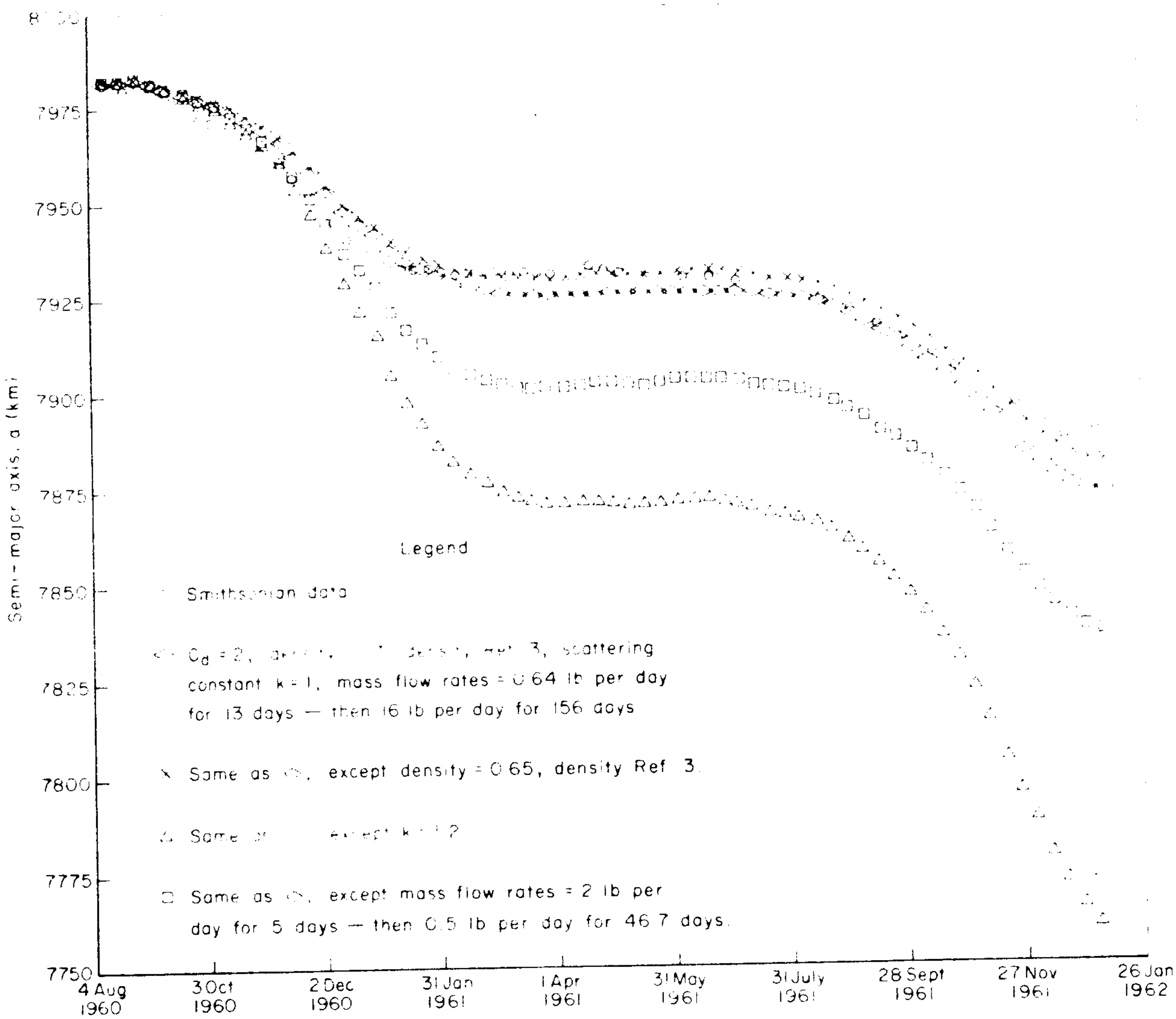

Fig. 2 — Semi-major axis

was first run using the rates of mass decrease that Jones used. These were 0.64 lb per day decrease until August 25, 1960, and then 0.16 lb per day decrease until January 11, 1961, when only a small amount of gas remained in the balloon. Figures 2 through 6 give the results of these runs.

Several computer runs were made using the orbital elements as of August 12, 1960. The atmospheric density, the mass flow rates of the gases, and the radiation pressure were varied to obtain the best agreement possible between the elements obtained from observations and those obtained theoretically.

In Figs. 2 through 6 the observed and theoretical orbital elements and other pertinent orbital data are given. A reasonably good fit to the observed orbital elements, from August 12, 1960 to August 24, 1961, was obtained by assuming that Echo I had a drag coefficient of 2, an atmospheric density of 0.7ρ, and a light-scattering constant of 1. These values differ from those used by Jones. He used a drag coefficient of 2.5, an atmospheric density of ρ, and a light scattering constant of 1. The differences in C_D and ρ results in a drag acceleration which is 0.56 of the Jones value.(4)

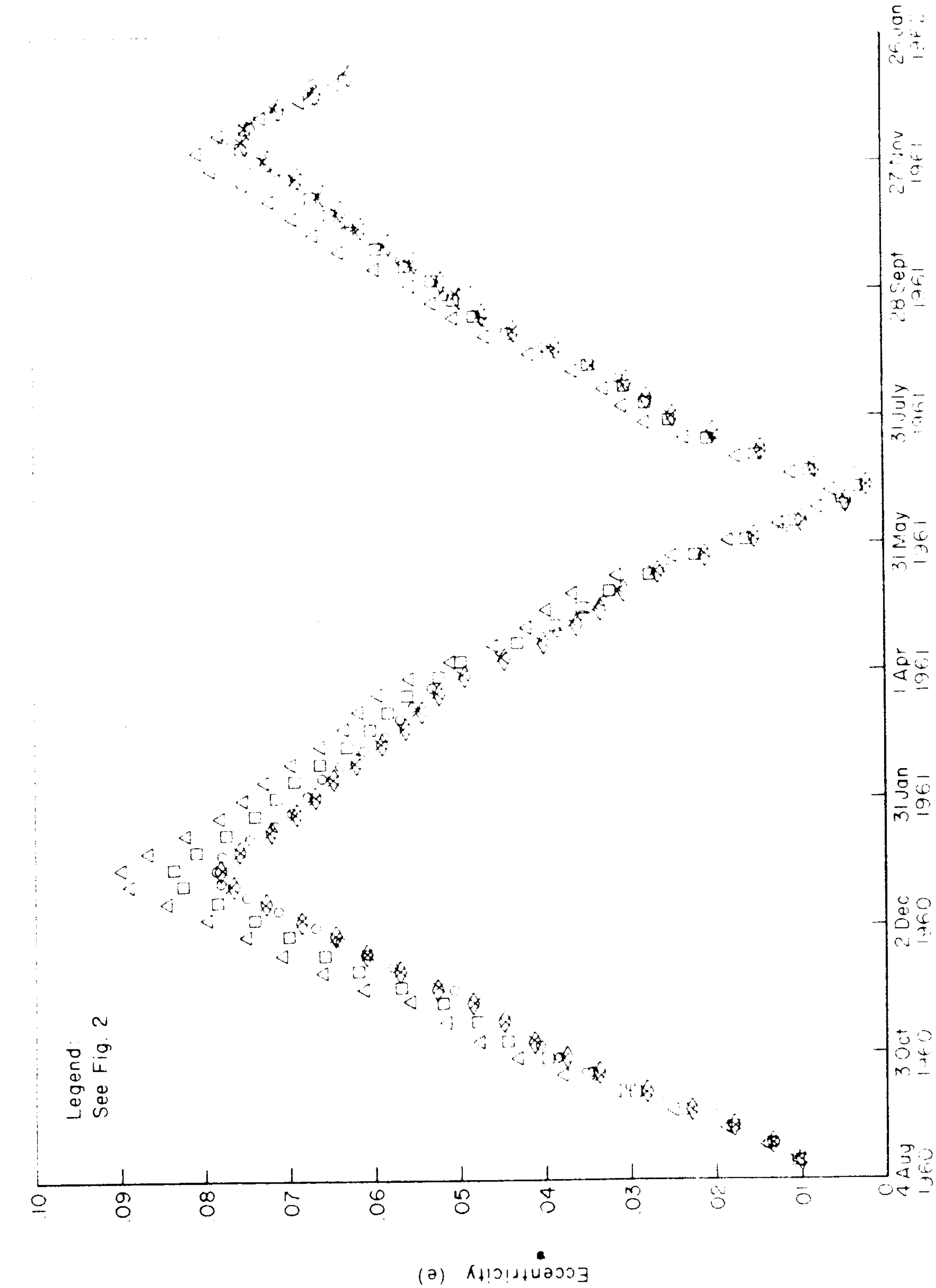

Fig. 3 — Eccentricity

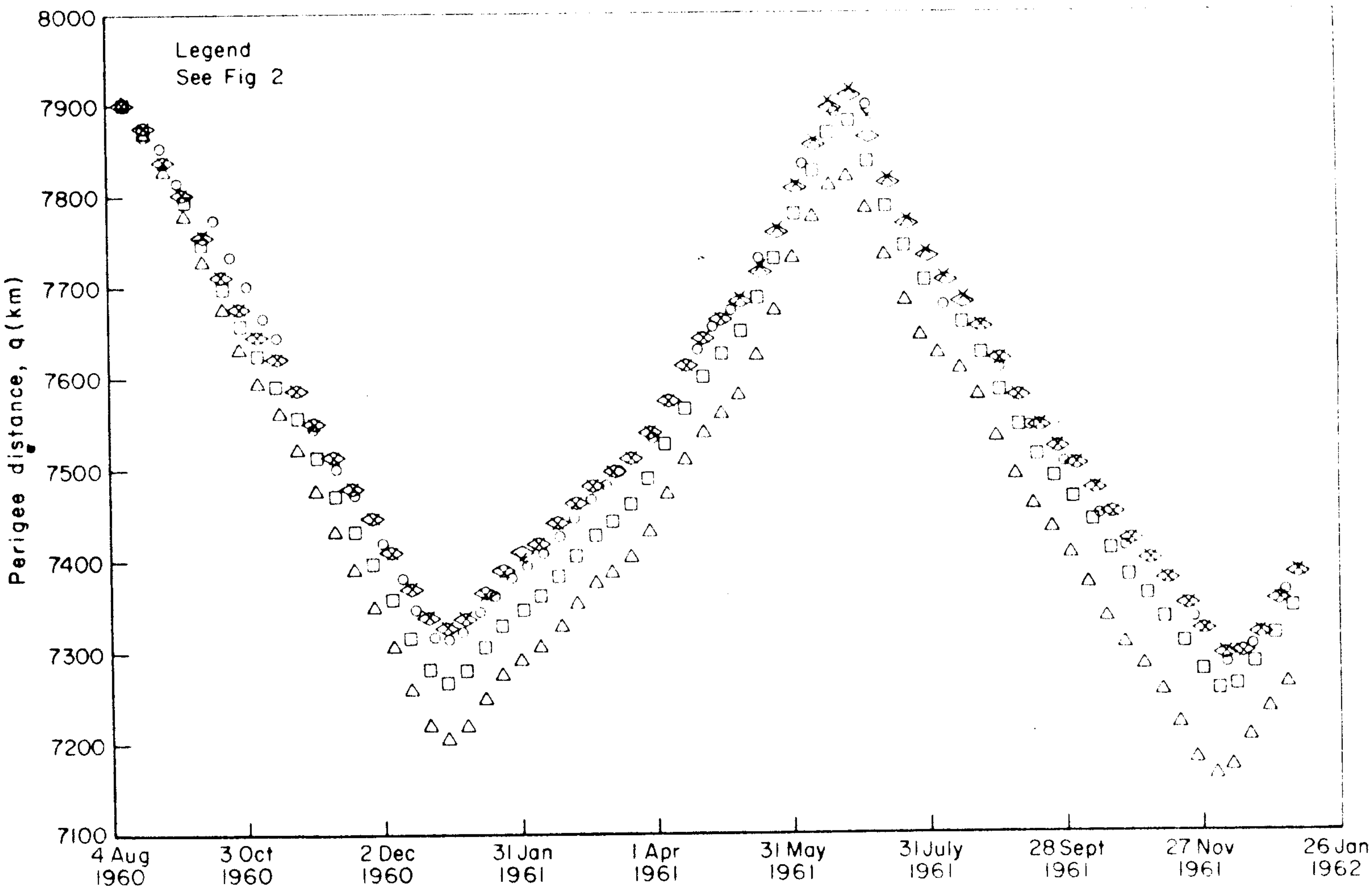

Fig. 4 — Perigee distance from earth's center

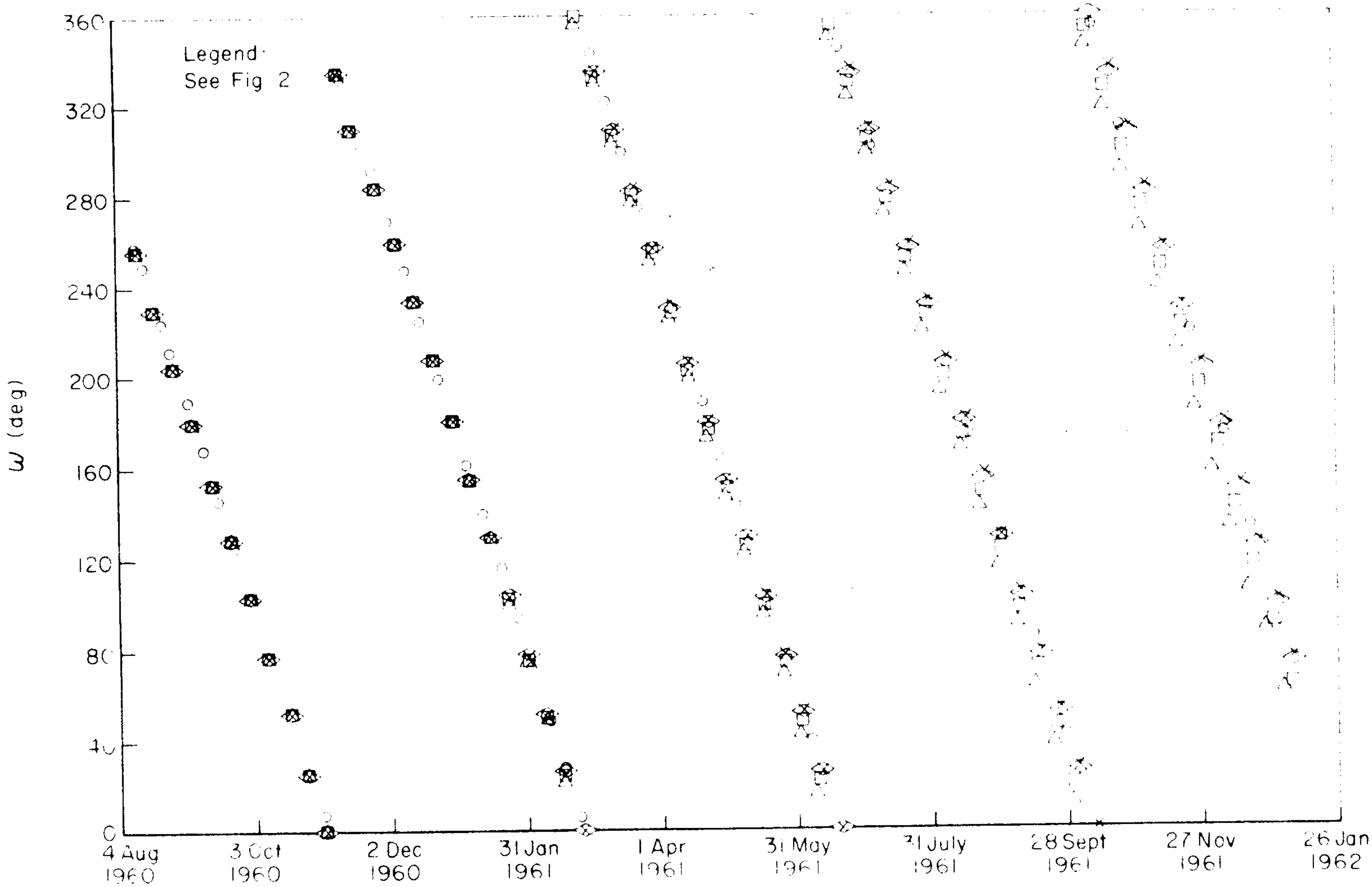

Fig. 5 — Longitude of ascending node

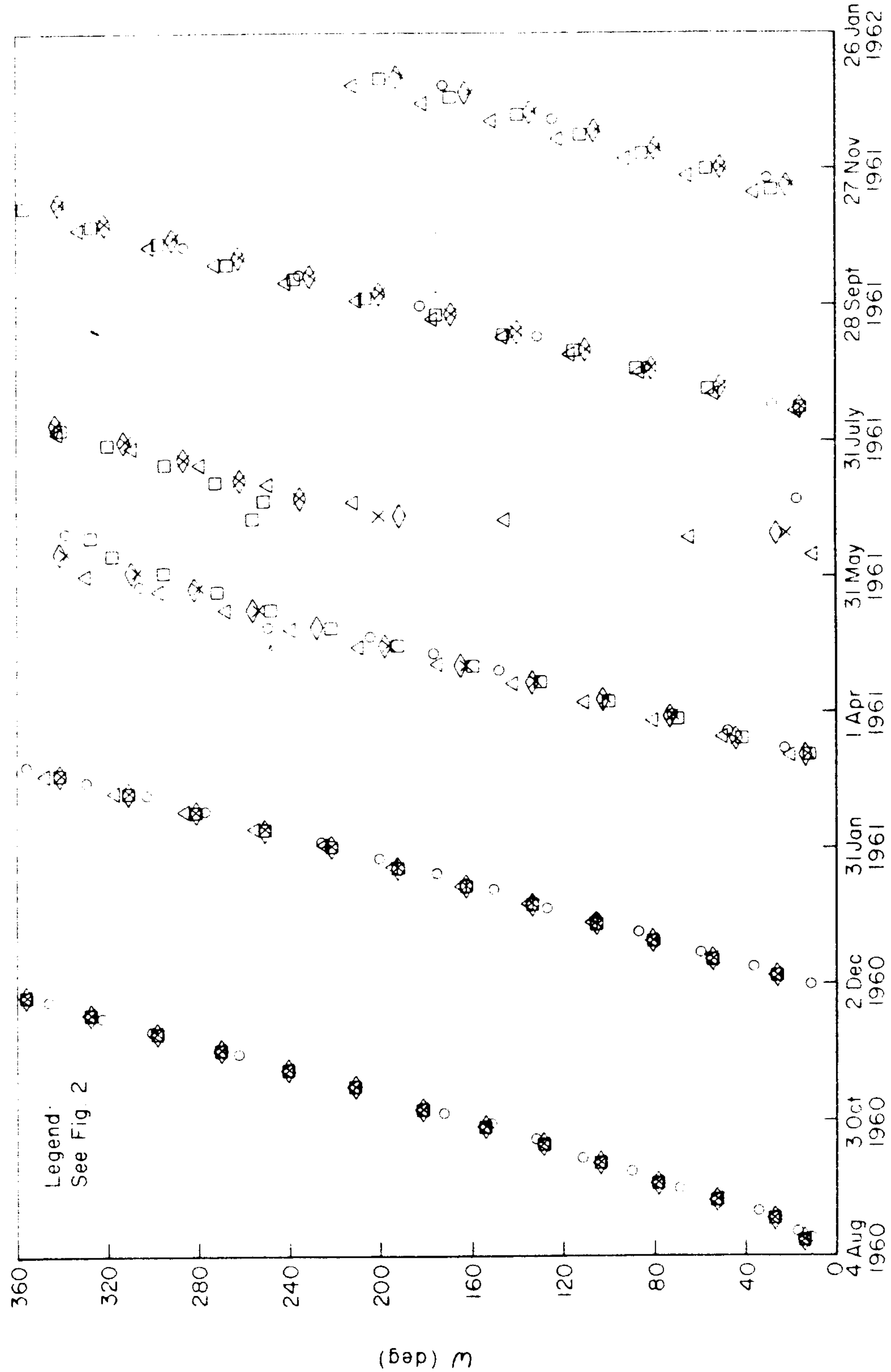

Fig. 6 — Argument of perigee

IV. LIFETIME OF ECHO I AND ECHO I-TYPE SATELLITES

The uncertainty in the present knowledge of air density permits only a low-accuracy prediction of the lifetime of balloon-type satellites. Since the perigee of the orbit of Echo I dips down into the atmosphere to about 900 km about every 300 days (Fig. 2), it is reasonable to predict that on one of these excursions the satellite will come down into the denser atmosphere and be destroyed. Depending on the atmospheric density one assumes, the "death" of Echo I will be on the fourth dip in July 1963, or the fifth dip in 1964, or the sixth dip a year later (Fig. 7). This prediction assumes that there will be no significant change in the satellite's area-to-mass ratio. Further modification in orbital predictions may be made after the correlation between sunspot activity and upper atmosphere density has been improved.

It is interesting to note that if an empty Echo I had been placed at around 2000 km and in an orbit concentric to the actual orbit of Echo I, that its orbit would increase in altitude due to the solar influence alone (Fig. 8).

The sun's influence on the orbit of an Echo I-type satellite was determined for a situation in which the satellite orbit plane was inclined at the critical angle of 243.4°, i.e., the angle for which the time rate of change of the argument of perigee is zero, from the equation

$$\dot{\omega} = \frac{3\, A_{2,0}}{a^{7/2}\, GM_e\, (1 - e^2)} \left(1 - \frac{5}{4} \sin^2 i\right)$$

and for $i = 243.4^\circ$, $\dot{\omega} = 0$. This value of i substituted in the equation for the nodal rate, i.e.,

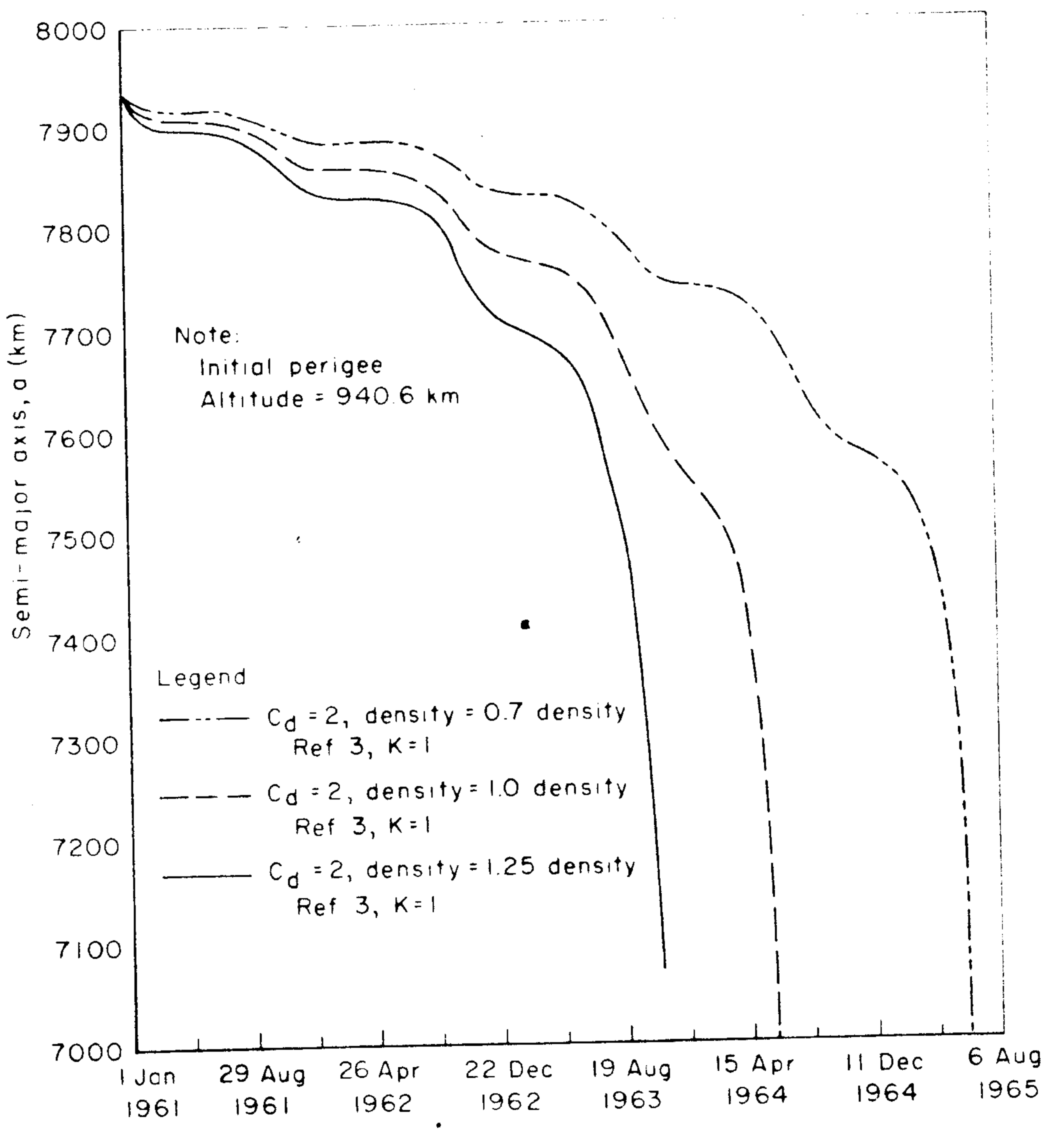

Fig. 7— Satellite lifetimes

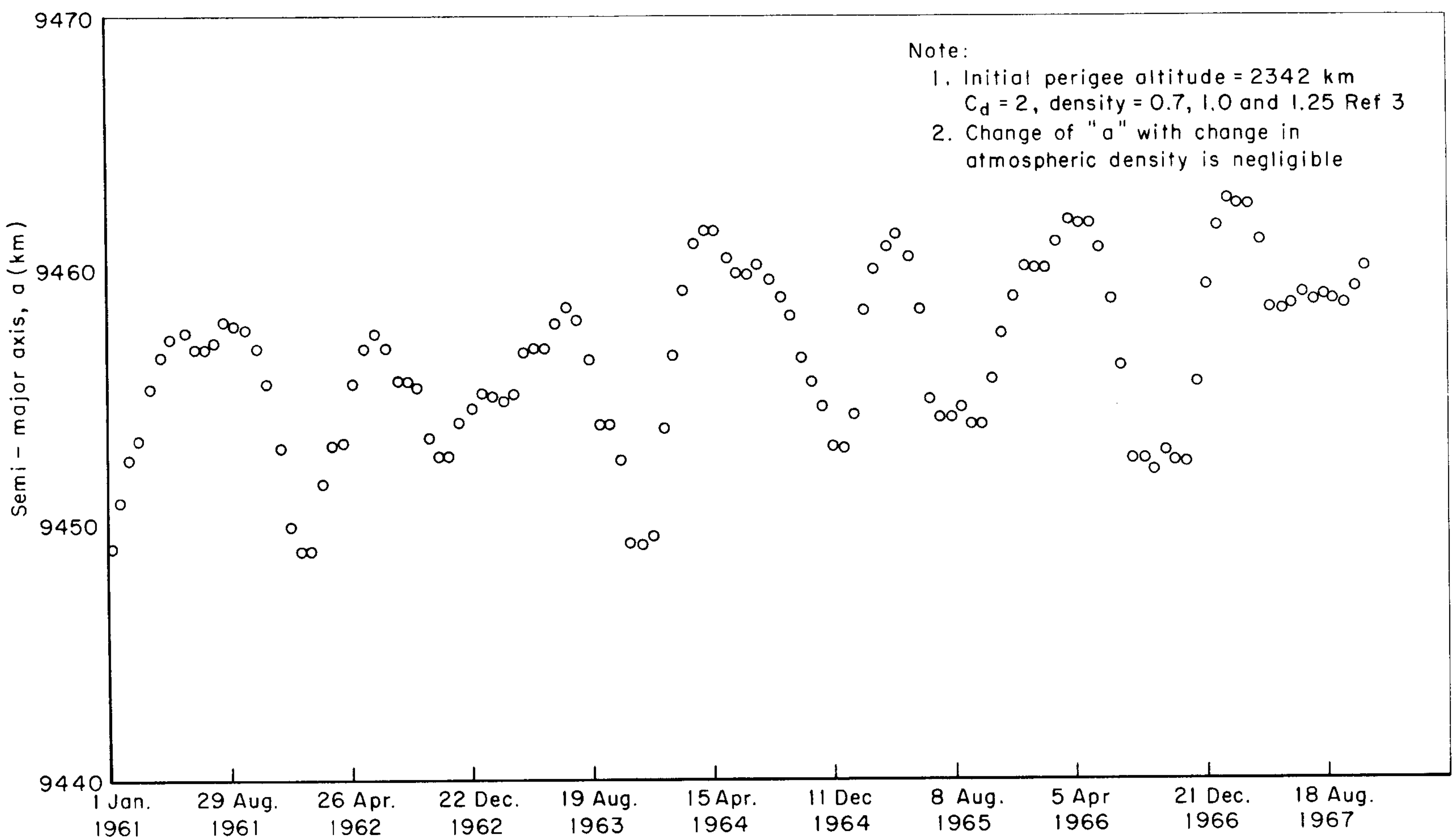

Fig. 8 — Modified Echo I orbit

$$\dot{\Omega} = \frac{3}{2} \frac{A_{2,0} \cos i}{a^{7/2} (1 - e^2) \sqrt{GM_e}}$$

gives the semimajor axis a, corresponding to any given value of $\dot{\Omega}$. For the line of nodes to have the same angular rate as the mean angular rate of the earth around the sun, set $\dot{\Omega} = -0.9856^{\circ}$ per day and compute the corresponding value of a.

Figure 9 shows the results of four runs where the only quantity varied is the initial position of perigee, i.e., the angle ω. In each case the departure date is March 21, 1961, the time of vernal equinox. On this date the direction of vernal equinox is from the east to the sun. Since the initial position of Ω is to be on the earth-sun line and between the earth and sun, then $\Omega = 0$ initially. The four curves of Fig. 9 correspond to $\omega = 0^{\circ}$, 90°, 180° and 270° initially.

Several additional computer runs were made where both air drag and the moon's attraction were included. There was no significant change in the satellite lifetime.

A simple demonstration of the effect of radiation pressure on the motion of balloon type satellites is indicated in Fig. 10. Suppose, as in the previous example, the orbital perigee remains fixed and the lines of nodes move at earth rate about the sun. Then, since the radiation pressure acts along the sun line, the satellite will lose energy during the portion of its orbit AB and gain energy along BC, Fig. 10a. If the energy gained along BC is larger than the combined loss from air drag and radiation pressure loss along AB, the satellite will remain in orbit. If, on the other hand, as indicated in Fig. 10b, the perigee is placed below the sun line, the energy loss during part of the orbit AB due to radiation will be larger than the gain along BC.

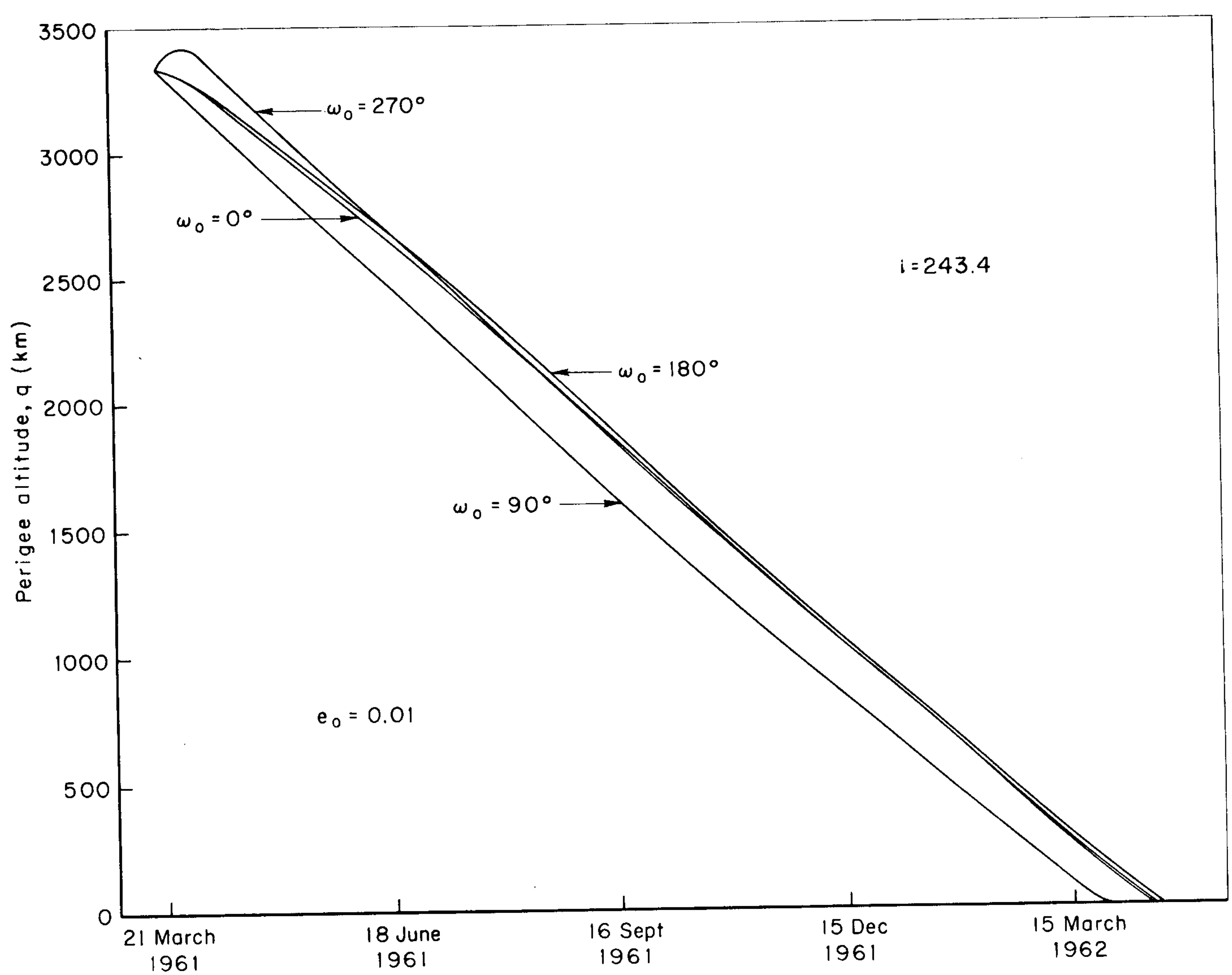

Fig. 9 — Satellite lifetimes, sun influence only

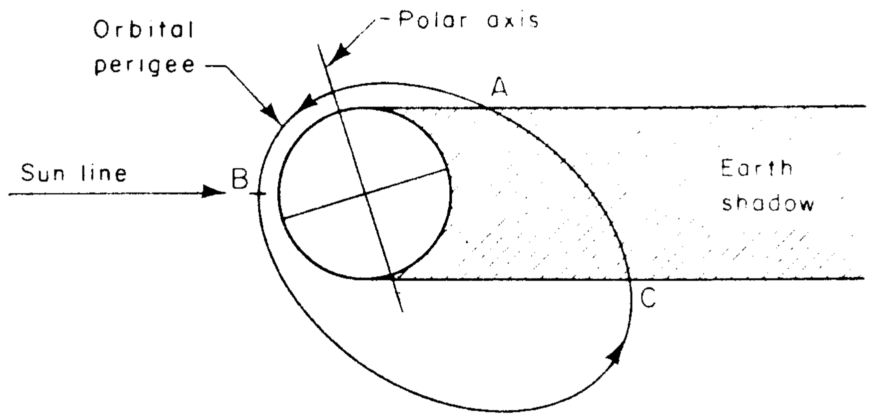

Fig. 10a — Radiation adds energy

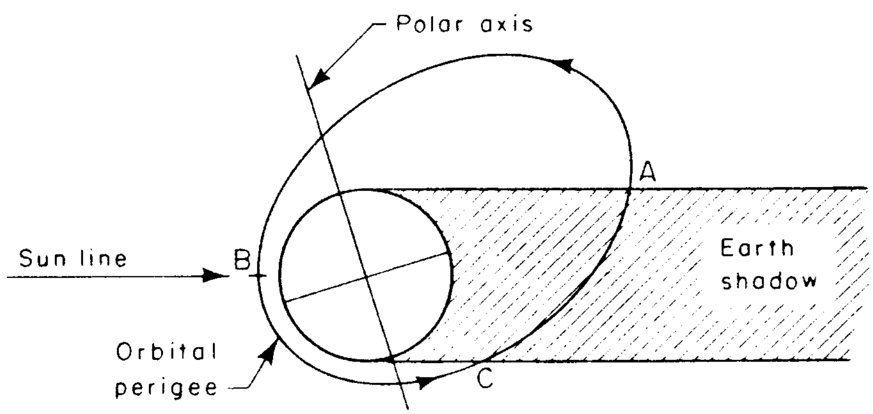

Fig. 10b — Radiation subtracts energy

With the additional air drag loss included the satellite will come down. Many other orbital orientations will give similar results but have complicated motions that are not easily described.

V. CONCLUSIONS

Aside from the apparent anomalous behavior of the orbit of Echo I during about the first four months of the satellite lifetime, the program permits a computation of the orbital elements to a high degree of agreement over a long period of time with elements obtained using observational data. Thus, if it is assumed that there are no unpredictable changes in the atmospheric drag and surface reflectivity of the satellite, then the lifetimes of Echo I and similar-type satellites can be predicted to a high degree of accuracy.

By orientating the elliptical orbits of a balloon-type satellite such that the energy gained from the solar radiation force exceeds the energy loss from drag, the satellite lifetime can be greatly increased or extended indefinitely.

If the net energy gained from the solar radiation is less than that due to drag, the satellite perigee altitude will decrease with time. If both the drag and solar radiation act in the same direction during most of the orbit, the satellite will come down rather rapidly.

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