NASr-21(02)

MEMORANDUM
RM-3296-NASA
AUGUST 1962

STABILITY OF A CLOUD OF ORBITING DIPOLES

R. H. Frick

PREPARED FOR:
NATIONAL AERONAUTICS AND SPACE ADMINISTRATION

The RAND Corporation
SANTA MONICA • CALIFORNIA

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PREFACE

The work presented in this Memorandum was done under NASA Contract NASA-21(02), monitored by the Director of Communications Studies, Office of Applications. The contract calls for technical studies of communication satellites, and RAND has given particular attention to various types of passive satellites, as exemplified by the present Memorandum.

The results should be of interest to all agencies and contractors involved in the development of passive satellite systems.

SUMMARY

This Memorandum describes a method of producing a fixed-configuration cloud of orbiting dipoles. As a passive communication system, such a cloud incorporates certain advantages of both the Echo-type balloon and the Westford belt of orbiting dipoles, in that all of the orbiting mass is confined to a relatively small volume while the reflecting area per unit mass is greatly increased by forming it into dipoles.

The method for establishing this cloud is an extension of the Westford ejection system in which dipoles are released from the circumference of a rotating cylinder. However, in the present case the axis of rotation must be maintained in the instantaneous horizontal plane at a small angle relative to the vehicle velocity vector during the ejection process, which should continue for several complete orbits. The resulting ejection is essentially in a plane normal to the vehicle velocity vector.

In the cloud formed by this method all of the ejected dipoles are contained in an ellipsoid of revolution centered on the dispensing vehicle, with the major axis along the orbit and a 4:1 ratio between the major and minor axes. The absolute value of the dimension is directly proportional to the ejection velocity. As an example, an ejection velocity of 0.5 ft/sec produces a cloud with a major axis of 1 st mi at an orbital altitude of 1500 st mi.

If the spin axis is misaligned, the vertical and lateral dimensions of the cloud are unaffected but the dimension along the orbit increases linearly with time.

As an example, the rate of change of the geocentric angle subtended by the cloud is $1.83^{\circ}/yr$ for an initial cloud dimension of 1 mi, an orbital altitude of 3000 mi and an alignment error of the spin axis equal to 1°. This rate of growth can be reduced by increasing the orbital altitude, reducing the initial size of the cloud, or reducing the alignment error.

While the analysis applies to a circular orbit, the same method could be adapted to elliptical orbits.

CONTENTS

LIST OF FIGURES

LIST OF SYMBOLS

a_y'	major axis of the dipole pattern in the $y'z'$ plane
a_z'	minor axis of the dipole pattern in the $y'z'$ plane
C_o	vehicle orbital angular momentum
ΔC	increment in dipole angular momentum after ejection
g_o	gravitational acceleration at the earth's surface
R_o	radius of the earth
r	radial distance of vehicle from the earth's center
r_o	vehicle orbital radius
r_1	radius of ejection cylinder
v_o	vehicle orbital velocity
v_1	dipole ejection velocity
$\Delta V_{xo}, \Delta V_{yo}, \Delta V_{zo}$	components of v_1 in the x, y, z system
x, y, z	dipole coordinates relative to the dispensing vehicle
x', y', z'	dipole coordinates relative to the dispensing vehicle (rotated relative to x, y, z)
x_o, y_o, z_o	initial values of x, y, z
$\bar{x}, \bar{y}, \bar{z}$	non-dimensional form of x, y, z
x_m	maximum value of x
$\dot{x}_{ss}$	steady-state rate of change of x
Δx_o	maximum cloud dimension with no alignment error
$\Delta\dot{x}$	rate of change of maximum cloud dimension

α	angle of rotation of dispensing cylinder
$\dot{\alpha}$	spin rate of dispenser
β	angle between x and x' axes
ϵ	spin axis azimuth angle relative to x-axis
ϵ_o	design value of ϵ
$\delta\epsilon$	error in ϵ
θ	vehicle orbital angle
θ_o	value of θ at ejection
$\dot{\theta}_o$	orbital angular rate of vehicle
$\Delta\theta$	geocentric angle subtended by the dipole cloud
$\delta\nu$	spin axis alignment error
ξ, η, ζ	dipole dispenser coordinate system (ξ along spin axis)
ξ_1	axial distance of dipole at time of ejection
ϕ	spin axis elevation angle
$\delta\phi$	error in ϕ

I. INTRODUCTION

In establishing passive communications satellites the problem arises of how to increase the radar cross-section per lb of material on orbit.

One solution is to place a metallized balloon on orbit, which will give a radar cross-section equal to the geometrical cross-section. The metallized balloon of Echo I has a 100-ft diameter, while Echo II will have a diameter of 140 ft. However, the resulting cross-section per lb is not as great as might be desired.

Another approach would be to place a large number of dipoles on orbit. Since a half-wave dipole has an average radar cross-section equal to about $\lambda^2/6$, this technique considerably enhances the cross-section per unit weight. The Westford experiment was an attempt to put a large number of such dipoles in orbit. It was intended to spread a band of dipoles around a full 360° of the orbit. As a result, the number of dipoles within an antenna beamwidth would be relatively low and the usable reflecting area and the radar cross-section per unit mass would be correspondingly reduced.

The present Memorandum investigates the possibility of ejecting dipoles from an orbiting body in such a way that they would form an orbiting cloud about the parent body and would not spread out around the entire orbit. If such a cloud can be formed with a dimension comparable to the antenna beamwidth, then all of the ejected mass would represent usable reflecting area, and the radar cross-section per unit mass would be correspondingly increased.

II. METHOD OF ANALYSIS

STATEMENT OF THE PROBLEM

For the purposes of this analysis it is assumed that a dipole-dispensing vehicle is placed on a circular orbit of radius r_o. The problem is to determine the ejection velocity requirements to ensure that the ejected dipoles will always remain within a fixed volume about the dispensing vehicle. It is also of interest to determine the rate of growth of the resulting dipole cloud if the ejection conditions are in error.

EJECTION VELOCITY REQUIREMENTS

Since the ejected dipoles are to remain in the vicinity of the dispensing vehicle, it is obvious that they must have orbital periods after ejection equal to that of the dispensing vehicle. Thus at intervals of one orbital period, all of the dipoles will focus at the original ejection point. The question then arises as to the desired ejection velocity pattern and the variation of the resulting dipole pattern as a function of time. To describe the behavior of the dipoles, an x, y, z-coordinate system is selected, as shown in Fig. 1, with its origin at the center of mass of the dispensing vehicle. The x-axis is in the direction of the vehicle velocity vector, the y-axis in the direction of the instantaneous vertical, and z-axis normal to the orbital plane of the vehicle. It is assumed that the dipoles are ejected by spinning them off the surface of a rotating cylinder in the manner intended in the Westford experiment. Thus the ξ, η, ξ-coordinate system in Fig. 1 is selected with the ξ-axis along the spin axis of the cylinder and the ζ-axis in the horizontal x, z plane. The

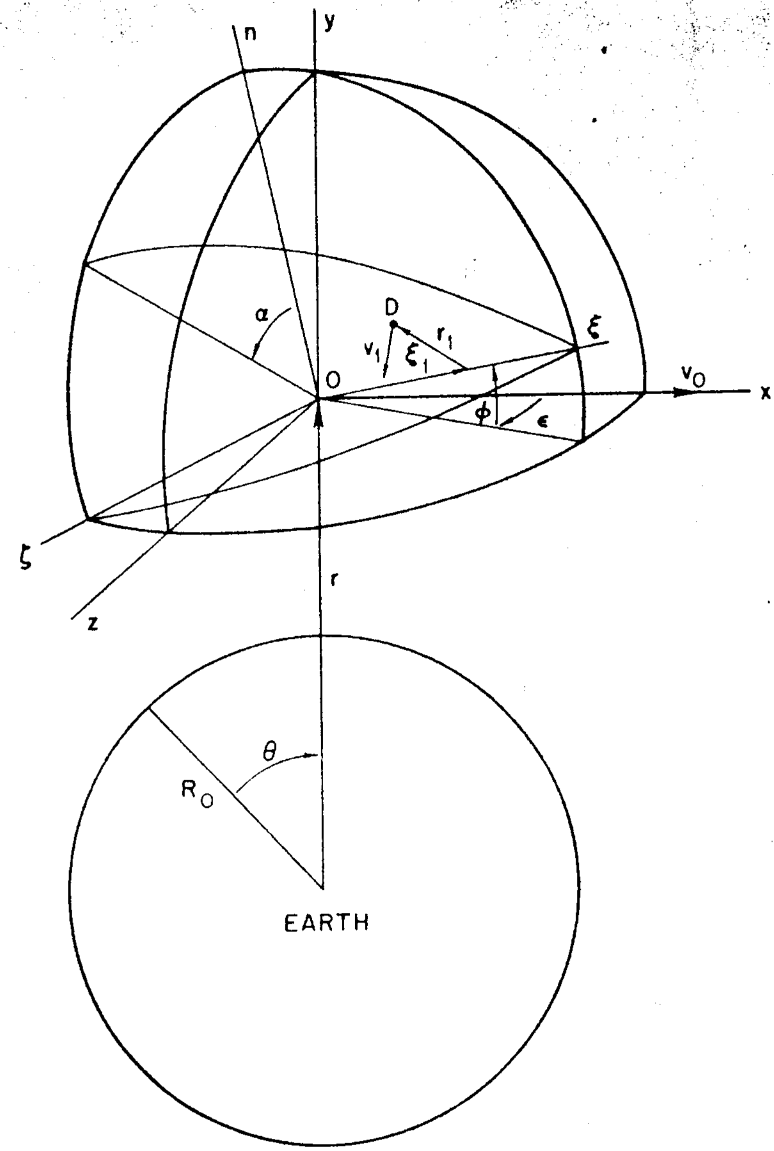

Fig. 1 — Coordinate system

position of the ξ-axis relative to the x-axis is specified by an azimuth angle ϵ and an elevation angle ϕ. The point D represents a dipole at the time of ejection as specified by an axial distance ξ_1 along the cylinder, a radial distance r_1, and a spin angle α relative to the ξ, η plane. The ejection velocity v_1 which is equal to $r_1\dot{\alpha}$ is in a direction normal to the $\xi_1 r_1$ plane. Since the coordinate systems in Fig. 1 move with the dispensing vehicle, thereby maintaining the x-axis in the direction of the vehicle velocity vector and y-axis in the vertical direction, then the coordinates x, y and z of the dipole specify its position relative to the vehicle as a function of time.

In Ref. 1, the general equations of motion of objects ejected from a ballistic vehicle were developed for such a coordinate system in the following form:

$$\frac{d^2\bar{y}}{d\theta^2} - \left[\frac{3g_oR_o^2}{C_o^2} \quad r - 4\right]\bar{y} = \frac{2\Delta C}{C_o} \tag{1}$$

$$\frac{d\bar{x}}{d\theta} + 2\bar{y} = \frac{\Delta C}{C_o} \tag{2}$$

$$\frac{d^2\bar{z}}{d\theta^2} + \bar{z} = 0 \tag{3}$$

where

$$\bar{x} = \frac{x}{r} \tag{4}$$

$$\bar{y} = \frac{y}{r} \tag{5}$$

$$\bar{z} = \frac{z}{r} \tag{6}$$

$$C_o = r_o^2 \dot{\theta}_o \tag{7}$$

$$\Delta C = 2r_o y_o \dot{\theta}_o + r_o \Delta V_{xo} \tag{8}$$

and

r_o	=	radial distance of the vehicle from the center of the earth at the time of ejection
θ_o	=	epoch angle of the vehicle in orbit at the time of ejection
r, θ	=	polar coordinates of the vehicle orbit
v_o	=	vehicle velocity at the time of ejection
$\Delta V_{xo}, \Delta V_{yo}, \Delta V_{zo}$	=	ejection velocity components
x_o, y_o, z_o	=	initial position of dipole at time of ejection
g_o	=	gravitational acceleration at the earth's surface
R_o	=	radius of the earth

For the assumed circular orbit of the dispensing vehicle, these equations of motion of objects ejected from a ballistic vehicle can be simplified considerably since

$$r \equiv r_o \tag{9}$$

$$\dot{\theta}_o^2 = \frac{g_o R_o^2}{r_o^3} \tag{10}$$

Thus Eqs. (1), (2) and (3) reduce to

$$\frac{d^2y}{d\theta^2} + y = 4y_o + \frac{2r_o \Delta V_{xo}}{v_o} \tag{11}$$

$$\frac{dx}{d\theta} + 2y = 2y_o + \frac{r_o \Delta V_{xo}}{v_o} \tag{12}$$

$$\frac{d^2z}{d\theta^2} + z = 0 \tag{13}$$

The initial conditions can be written in the following form, at $\theta = \theta_o$

$$\begin{aligned} x &= x_o & \dot{x} &= \Delta V_{xo} \\ y &= y_o & \dot{y} &= \Delta V_{yo} \\ z &= z_o & \dot{z} &= \Delta V_{zo} \end{aligned} \tag{14}$$

For these initial conditions, the solution for Eqs. (11), (12) and (13) is given by

$$\begin{aligned} x = &- 3\left[2y_o + \frac{r_o \Delta V_{xo}}{v_o}\right](\theta - \theta_o) \\ &+ 2\left[3y_o + \frac{2r_o \Delta V_{xo}}{v_o}\right] \sin (\theta - \theta_o) \\ &- \frac{2r_o \Delta V_{yo}}{v_o}\left[1 - \cos (\theta - \theta_o)\right] + x_o \end{aligned} \tag{15}$$

$$y = 2\left[2y_o + \frac{r_o\,\Delta V_{xo}}{v_o}\right]$$

$$- \left[3y_o + \frac{2r_o\,\Delta V_{xo}}{v_o}\right]\cos\,(\theta - \theta_o)$$

$$+ \frac{r_o\,\Delta V_{yo}}{v_o}\sin\,(\theta - \theta_o) \tag{16}$$

$$z = z_o\cos\,(\theta - \theta_o)$$

$$+ \frac{r_o\Delta V_{zo}}{v_o}\sin\,(\theta - \theta_o) \tag{17}$$

The values of the initial conditions can be expressed from Fig. 1 in the form

$$x_o = \xi_1\cos\Phi\cos\epsilon - r_1\,(\sin\alpha\sin\epsilon$$

$$+ \cos\alpha\sin\Phi\cos\epsilon) \tag{18}$$

$$y_o = \xi_1\sin\Phi + r_1\cos\alpha\cos\Phi \tag{19}$$

$$z_o = \xi_1\cos\Phi\sin\epsilon + r_1\,(\sin\alpha\cos\epsilon$$

$$- \cos\alpha\sin\Phi\sin\epsilon) \tag{20}$$

$$\Delta V_{xo} = r_1\dot{\alpha}\,(-\cos\alpha\sin\epsilon + \sin\alpha\sin\Phi\cos\epsilon) \tag{21}$$

$$\Delta V_{yo} = -\,r_1\dot{\alpha}\sin\alpha\cos\Phi \tag{22}$$

$$\Delta V_{zo} = r_1\dot{\alpha}\,(\cos\alpha\cos\epsilon + \sin\alpha\sin\Phi\sin\epsilon) \tag{23}$$

An examination of Eq. (15) shows that there is a secular term in x which would cause the x-coordinate to grow without bound. However, if Eqs. (19) and (21) are substituted in the coefficient of this secular term, it is found that the steady-state rate is

$$\dot{x}_{ss} = -3\dot{\theta}_o\left[2\xi_1 \sin\phi + 2r_1 \cos\alpha \cos\phi + \frac{r_o r_1 \dot{\alpha}}{v_o}(-\cos\alpha \sin\epsilon + \sin\alpha \sin\phi \cos\epsilon)\right] \quad (24)$$

which can be made to vanish if

$$\phi = 0 \quad (25)$$

$$\sin\epsilon = \frac{2v_o}{r_o\dot{\alpha}} \quad (26)$$

As defined by Eq. (26), ϵ is a small angle since its sine is of the order of the ratio of the orbital angular rate to the spin rate, so that cos ϵ is unity. Thus the initial condition expressions in Eqs. (18) through (23) can be simplified as follows

$$x_o = \xi_1 - \frac{2r_1 v_o}{r_o\dot{\alpha}}\sin\alpha \quad (27)$$

$$y_o = r_1 \cos\alpha \quad (28)$$

$$z_o = \frac{2\xi_1 v_o}{r_o\dot{\alpha}} + r_1 \sin\alpha \quad (29)$$

$$\Delta V_{xo} = -\frac{2r_1 v_o}{r_o}\cos\alpha \quad (30)$$

$$\Delta V_{yo} = - r_1 \dot{\alpha} \sin \alpha \tag{31}$$

$$\Delta V_{zo} = r_1 \dot{\alpha} \cos \alpha \tag{32}$$

Substitution of Eqs. (27) through (32) in Eqs. (15), (16) and (17) gives

$$\begin{aligned} x = & - 2r_1 \cos \alpha \sin (\theta - \theta_o) \\ & + \frac{2r_o}{v_o} (r_1 \dot{\alpha}) \sin \alpha \left[1 - \cos (\theta - \theta_o)\right] \\ & + \xi_1 - \frac{2r_1 v_o}{r_o \dot{\alpha}} \sin \alpha \end{aligned} \tag{33}$$

$$\begin{aligned} y = & r_1 \cos \alpha \cos (\theta - \theta_o) \\ & - \frac{r_o}{v_o} (r_1 \dot{\alpha}) \sin \alpha \sin (\theta - \theta_o) \end{aligned} \tag{34}$$

$$\begin{aligned} z = & \frac{r_o}{v_o} (r_1 \dot{\alpha}) \cos \alpha \sin (\theta - \theta_o) \\ & + \left[\frac{2\xi_1 v_o}{r_o \dot{\alpha}} + r_1 \sin \alpha\right] \cos (\theta - \theta_o) \end{aligned} \tag{35}$$

If terms of the order of the dimensions of the ejecting cylinder are neglected, the solutions reduce to

$$x = \frac{2r_o}{v_o} (r_1 \dot{\alpha}) \sin \alpha \left[1 - \cos (\theta - \theta_o)\right] \tag{36}$$

$$y = - \frac{r_o}{v_o} (r_1 \dot{\alpha}) \sin \alpha \sin (\theta - \theta_o) \tag{37}$$

$$z = \frac{r_o}{v_o} (r_1 \dot{\alpha}) \cos \alpha \sin (\theta - \theta_o) \tag{38}$$

It is seen that no secular terms appear in the resulting solution; thus the maximum possible displacement of the dipole depends only on the ejection velocity $r_1\dot{\alpha}$.

If dipoles are ejected with the same velocity in all directions ($\alpha = 0^\circ$ to 360°), the resulting pattern can be described better by defining a new coordinate system x', y', z', which is rotated from the x, y, z system by an angle β about the z-axis so that

$$x' = x \cos \beta + y \sin \beta \tag{39}$$

$$y' = - x \sin \beta + y \cos \beta \tag{40}$$

$$z' = z \tag{41}$$

The angle β is determined by substituting Eqs. (36) and (37) in Eq. (39) and setting x' equal to zero so that

$$\tan \beta = \frac{2\left[1 - \cos (\theta - \theta_o)\right]}{\sin (\theta - \theta_o)} \tag{42}$$

Since β is independent of the ejection velocity $r_1\dot{\alpha}$ and the ejection angle α, all of the dipoles ejected at θ_o will remain in the y', z' plane, defined by Eq. (42). If α and β are eliminated between Eqs. (36), (37), (38), (40), (41) and (42), the resulting configuration of the dipoles in the y', z' plane is given by

$$\left[\frac{y'^2}{\sin^2(\theta - \theta_o) + 4\left[1 - \cos(\theta - \theta_o)\right]^2}\right] + \frac{z'^2}{\sin^2(\theta - \theta_o)} = \left[\frac{r_o r_1 \dot{\alpha}}{v_o}\right]^2 \tag{43}$$

which is an ellipse with semimajor axis

$$a'_y = \frac{r_o}{v_o}(r_1\dot{\alpha})\left[\sin^2(\theta - \theta_o) + 4\left[1 - \cos(\theta - \theta_o)\right]^2\right]^{1/2} \tag{44}$$

and semiminor axis

$$a'_z = \frac{r_o}{v_o}(r_1\dot{\alpha}) \sin(\theta - \theta_o) \tag{45}$$

If in addition the ejection velocity varies as the radius of the ejecting cylinder decreases, the entire area of the ellipse would be filled with dipoles.

Figure 2 is a three-dimensional drawing of the time history of the dipole pattern for 30^o increments for the angle $\theta - \theta_o$, while Figs. 3, 4 and 5 are the projections of these patterns in the xy, xz and yz planes, respectively.

An examination of these figures shows that if all of the dipoles were ejected at θ_o they would expand from a point into more and more elongated ellipses in the y', z' plane. Finally at $\theta - \theta_o$ equal to 180^o they would all lie in a horizontal straight line along the orbit

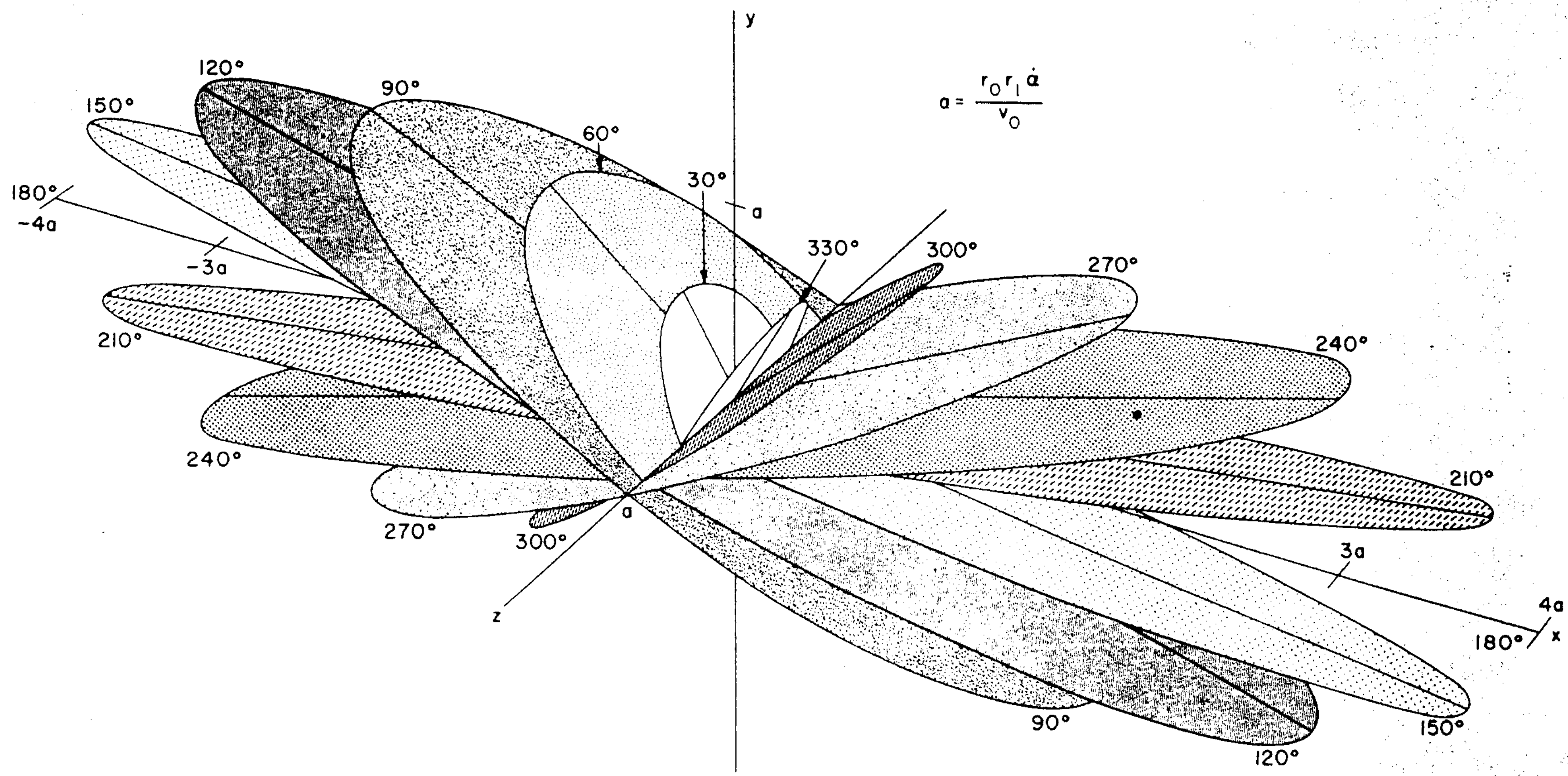

Fig. 2 — Ejected dipole pattern versus θ-θ_0

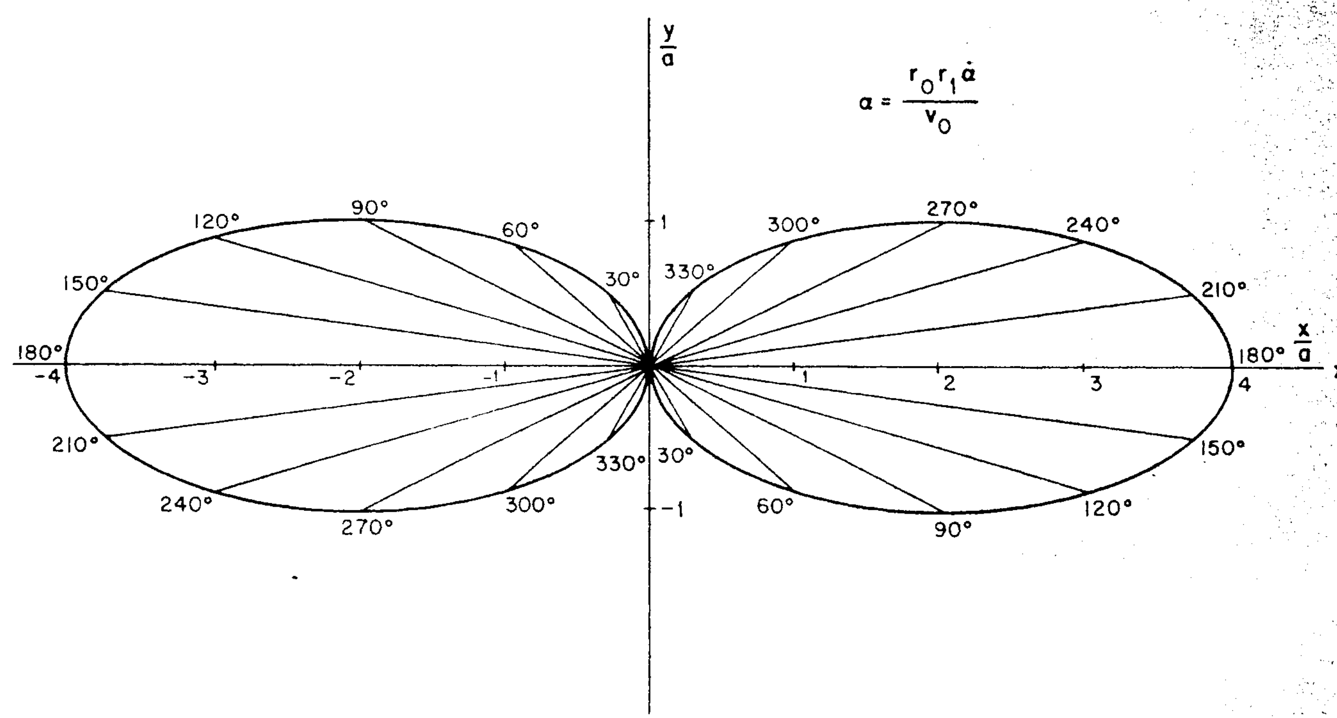

Fig. 3 — xy projection of the dipole pattern

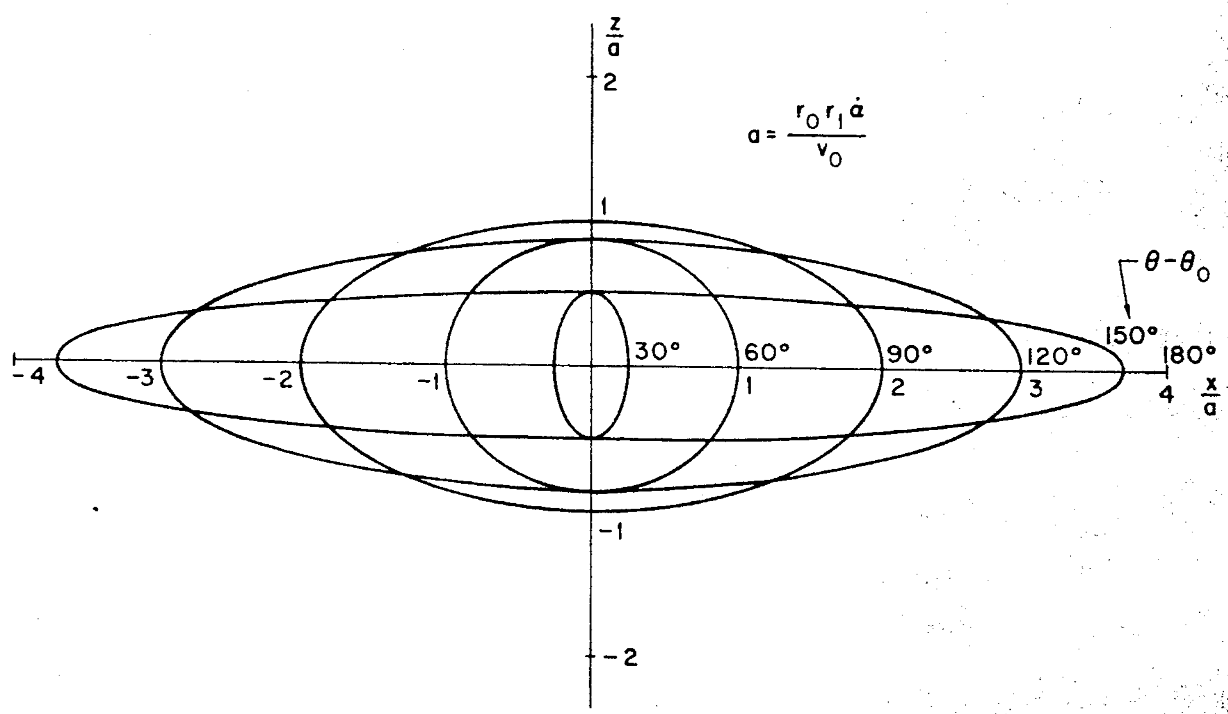

Fig. 4 — xz projection of the dipole pattern

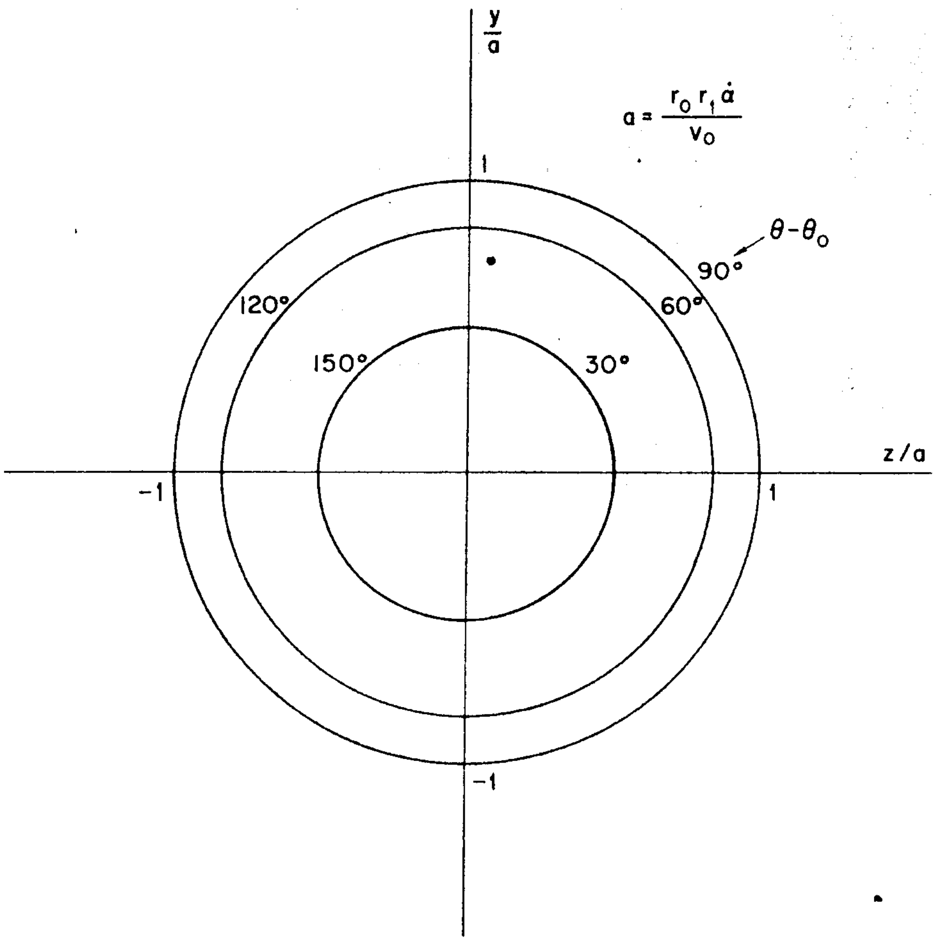

Fig. 5 — yz projection of the dipole pattern

after which the procedure would reverse and the dipoles would again focus in one point after one complete orbit. Such a fluctuating pattern is undesirable from the point of view of maintaining a constant radar cross-section. However, if the ejection process were maintained continuously for at least one complete orbit, such that θ_o varies from 0^o to 360^o, then the entire swept volume in Fig. 2 would be filled with dipoles at all times. This would produce the desired dipole cloud of fixed dimensions orbiting with the dispensing vehicle.

The dimensions of the cloud depend on the maximum value of the ejection velocity. Since the maximum dimension is in the x-direction, it is used as a measure of the cloud size. From Eq. (36) it can be shown that this maximum value x_m occurs for α equal to 90^o and for $\theta - \theta_o$ equal to 180^o, so that

$$x_m = \frac{4r_o}{v_o} (r_1\dot{\alpha}) \tag{46}$$

or in terms of the total horizontal dimension of the cloud Δx_o, then

$$\Delta x_o = 2x_m = \frac{8r_o}{v_o} (r_1\dot{\alpha}) \tag{47}$$

Thus the velocity requirement for a given value of Δx_o is given by

$$r_1\dot{\alpha} = \frac{v_o \, \Delta x_o}{8r_o} \tag{48}$$

If the orbital velocity v_o is expressed in terms of orbital radius, Eq. (48) becomes

$$r_1\dot{\alpha} = \frac{\Delta x_o}{8} \sqrt{\frac{g_o R_o^2}{r_o^3}} \tag{49}$$

From Eqs. (37) and (38) it is seen that the corresponding maximum cloud dimensions in the y- and z-directions will be $\Delta x_o/4$. Thus the cloud is essentially contained in an ellipsoid of revolution with the major axis along the x-direction and with a 4:1 ratio of axes.

CLOUD STABILITY

In the foregoing analysis, it was shown that by proper orientation of the spin axis of the dispensing cylinder a dipole cloud of fixed dimension could be produced. However, in any practical system there will be errors in the alignment of this spin axis relative to the vehicle velocity vector. Thus the question arises of the effect of such misalignment on the shape of the cloud.

If $\delta\phi$ and $\delta\epsilon$ represent the errors in this alignment, then

$$\phi = \delta\phi \tag{50}$$

$$\epsilon = \epsilon_o + \delta\epsilon \tag{51}$$

where

$$\sin \epsilon_o = \frac{2v_o}{r_o\dot{\alpha}} \tag{52}$$

$$\sin \epsilon = \frac{2v_o}{r_o\dot{\alpha}} + \delta\epsilon \tag{53}$$

$$\cos \epsilon = 1 \tag{54}$$

$$\sin \phi = \delta\phi \tag{55}$$

$$\cos \phi = 1 \tag{56}$$

Substitution of Eqs. (52) through (56) in Eq. (15) through (23) gives

$$x = -3\left[2\xi_1\,\delta\phi - \frac{r_o}{v_o}(r_1\dot{\alpha})(\delta\epsilon\cos\alpha - \delta\phi\sin\alpha)\right](\theta-\theta_o)$$

$$+ 2\left[3\xi_1\,\delta\phi - r_1\cos\alpha - \frac{2r_o}{v_o}(r_1\dot{\alpha})(\delta\epsilon\cos\alpha\right.$$

$$\left. - \delta\phi\sin\alpha)\right]\sin(\theta-\theta_o) + \frac{2r_o}{v_o}(r_1\dot{\alpha})\sin\alpha\left[1-\cos(\theta-\theta_o)\right]$$

$$+\left[\xi_1 - r_1\sin\alpha\left(\frac{2v_o}{r_o\dot{\alpha}}+\delta\epsilon\right) - r_1\,\delta\phi\cos\alpha\right] \qquad (57)$$

$$y = 2\left[2\xi_1\,\delta\phi - \frac{r_o}{v_o}(r_1\dot{\alpha})(\delta\epsilon\cos\alpha - \delta\phi\sin\alpha)\right]$$

$$-\left[3\xi_1\,\delta\phi - r_1\cos\alpha - \frac{2r_o}{v_o}(r_1\dot{\alpha})(\delta\epsilon\cos\alpha\right.$$

$$\left. - \delta\phi\sin\alpha)\right]\cos(\theta-\theta_o) - \frac{r_o}{v_o}(r_1\dot{\alpha})\sin\alpha\sin(\theta-\theta_o) \qquad (58)$$

$$z = \left[\xi_1\left(\frac{2v_o}{r_o\dot{\alpha}}+\delta\epsilon\right) + r_1\sin\alpha\right]\cos(\theta-\theta_o)$$

$$+ \frac{r_o}{v_o}(r_1\dot{\alpha})\cos\alpha\sin(\theta-\theta_o) \qquad (59)$$

If terms of the order of the dimensions of the ejecting cylinder are neglected, these equations reduce to

$$x = 3\left[2\xi_1\,\delta\phi - \frac{r_o}{v_o}(r_1\dot{\alpha})(\delta\epsilon\cos\alpha - \delta\phi\sin\alpha)\right](\theta-\theta_o)$$

$$+ \frac{2r_o}{v_o}(r_1\dot{\alpha})\sin\alpha\left[1-\cos(\theta-\theta_o)\right] \qquad (60)$$

$$y = -\frac{r_o}{v_o}(r_1\dot{\alpha}) \sin\alpha \sin(\theta - \theta_o) \tag{61}$$

$$z = \frac{r_o}{v_o}(r_1\dot{\alpha}) \cos\alpha \sin(\theta - \theta_o) \tag{62}$$

A comparison of Eqs. (60) through (62) with Eqs. (36) through (38) shows that the solutions for y and z are not affected by small errors in alignment. However, a residual secular term now appears in the solution for x, so that

$$\dot{x}_{ss} = 3\dot{\theta}_o\left[2\xi_1\ \delta\phi - \frac{r_o}{v_o}(r_1\dot{\alpha})(\delta\epsilon \cos\alpha - \delta\phi \sin\alpha)\right] \tag{63}$$

The term $2\xi_1\ \delta\phi$ is negligible in comparison with the second term in the square bracket so that

$$\dot{x}_{ss} = 3r_1\dot{\alpha}\ \delta\nu \tag{64}$$

where

$$\delta\nu = (\delta\epsilon \cos\alpha - \delta\phi \sin\alpha)_{max} \tag{65}$$

As defined, $\delta\nu$ represents the total angular error between the actual spin axis and the position specified by Eqs. (25) and (26). The rate of growth of the x-dimension of the cloud is now given by

$$\Delta\dot{x} = 2\dot{x}_{ss} = 6r_1\dot{\alpha}\ \delta\nu \tag{66}$$

By expressing $r_1\dot{\alpha}$ by means of Eq. (47), then

$$\Delta\dot{x} = \frac{4v_o \ \delta\nu \ \Delta x_o}{4r_o} \tag{67}$$

If $\Delta\theta$ is the geocentric angle subtended by the x-dimension of the cloud, Eq. (67) can be expressed in the form

$$\Delta\dot{\theta} = \frac{3}{4} \Delta x_o \ \delta\nu \sqrt{\frac{g_o R_o^2}{r_o^5}} \tag{68}$$

where Δx_o is the dimension of the cloud for $\delta\nu$ equal to zero.

III. RESULTS AND DISCUSSION

In the preceding section the problem has been analyzed of producing an orbiting cloud of dipoles with the shape and size of the cloud remaining fixed. The ejection method postulated is the one intended for the Westford project in which dipoles embedded in a rotating cylinder are thrown off tangentially as the binding material sublimes. The analysis shows that, in order to prevent the dipoles spreading along the orbit, the spin axis of the ejecting cylinder should lie in the horizontal plane at a small angle ϵ_o relative to the vehicle velocity vector. From Eq. (49) the required spin rate of the cylinder is given by

$$\dot{\alpha}\ (\text{rpm}) = 7.823 \left(\frac{R_o}{r_o}\right)^{3/2} \frac{\Delta x_o(\text{st mi})}{r_1(\text{ft})} \tag{69}$$

and the angle ϵ_o is determined from Eqs. (26) and (48) as

$$\epsilon_o(\text{deg}) = 0.1736\ \frac{r_1(\text{ft})}{\Delta x_o(\text{st mi})} \tag{70}$$

where Δx_o is the desired maximum dimension of the cloud, and r_1 is the radius of the ejecting cylinder.

Figure 6 is a plot of $\dot{\alpha}\, r_1/\Delta x_o$ as a function of orbital radius and shows that for Δx_o equal to 1 mi and r_1 equal to 1 ft the required spin rate is less than 7.823 rpm. From Eq. (70) it is seen that the deflection angle ϵ_o is independent of orbital altitude and depends only on the ratio of the cylinder radius r_1 to the desired dimension of the cloud Δx_o. For a 1-ft radius and a 1-mi cloud dimension, ϵ_o has a value of 0.1736^o.

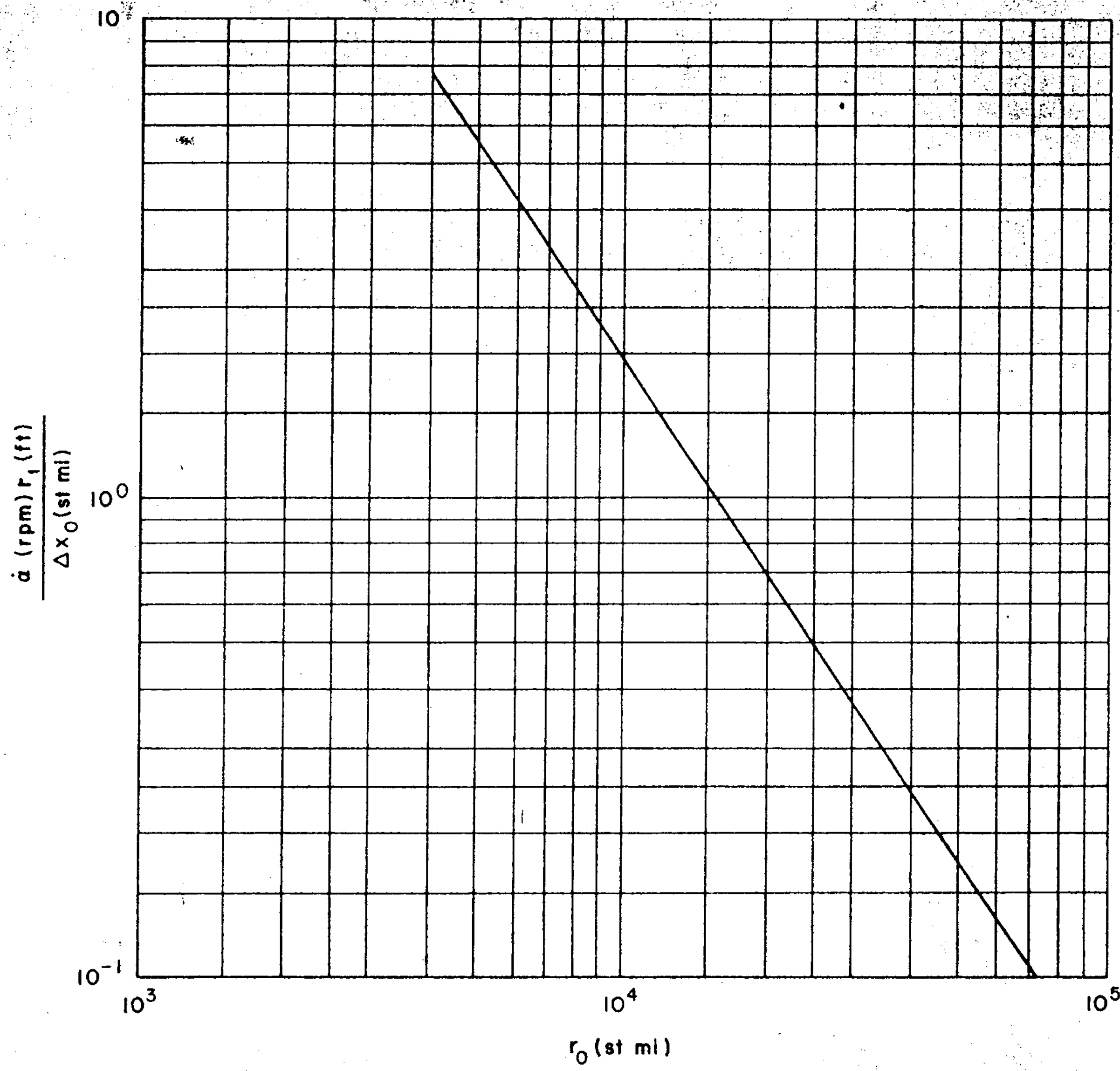

Fig. 6 — Required spin rate versus orbital radius

The reason for the deliberate misalignment of the spin axis and the vehicle velocity vector is to compensate for the fact that objects released above the vehicle orbit tend to drift to the rear, while objects released below the orbit tend to drift forward, as shown in the Appendix to Ref. 1. The deflection angle ϵ_o results in a slight reduction in the initial x-component of velocity for objects released above the vehicle orbit ($|\alpha| < 90^\circ$) and a slight increase in the x-velocity for objects released below the orbit ($90^\circ < |\alpha| < 180^\circ$). This compensating effect results in identical orbital periods for all of the ejected dipoles.

As indicated in the previous section, to produce a fixed dipole-cloud configuration, it is desirable that the ejection procedure continue throughout at least one complete orbit and preferably for several orbits. During this time the radius of the ejecting cylinder is slowly decreasing, thus the ejection velocity $r_1\dot{\alpha}$ decreases and the dipoles ejected tend to fill in the center of the cloud.

While the actual boundary of the cloud is a rather complicated closed surface, as shown in Fig. 2, all of the ejected dipoles will remain within an ellipsoid of revolution centered on the dispensing vehicle with its major axis Δx_o in the direction of the vehicle velocity vector and two equal minor axes of $\Delta x_o/4$.

Obviously such a dispensing method would require a fairly precise attitude sensing and control system in order to maintain the desired spin-axis orientation relative to the vehicle velocity vector and the instantaneous vertical, as determined by Eqs. (50) and (51). If the spin axis deviates from its desired orientation by an angle $\Delta\nu$, Eq. (68)

shows that the cloud will spread along the orbit such that the rate of change of the subtended geocentric angle $\Delta\theta$ is given by

$$\Delta\dot{\theta}(\text{deg/yr}) = 7.414 \left(\frac{R_o}{r_o}\right)^{5/2} \Delta x_o(\text{st mi})\ \delta\nu(\text{deg}) \qquad (71)$$

Figure 7 is a plot of $\Delta\dot{\theta}$ as a function of orbital radius r_o. It is seen that for r_o equal to 7000 mi, Δx_o equal to 1 mi, and $\delta\nu$ equal to 1^o, the cloud grows at the rate of 1.83^o per yr. The permissible magnitude of $\Delta\dot{\theta}$ is dependent on the beamwidths of the sending and receiving antennas, as well as the required lifetime of the system. While it is beyond the scope of this Memorandum to specify an acceptable rate of growth $\Delta\dot{\theta}$, it can be seen from Eq. (71) that $\Delta\dot{\theta}$ can be reduced by: 1) increasing the orbital radius r_o; 2) decreasing the initial cloud dimension Δx_o; and 3) reducing the spin-axis alignment error $\delta\nu$.

While the present analysis has been concerned only with dipole ejection from a circular orbit, it should be possible to adapt this type of dispensing method to any arbitrary elliptical orbit.

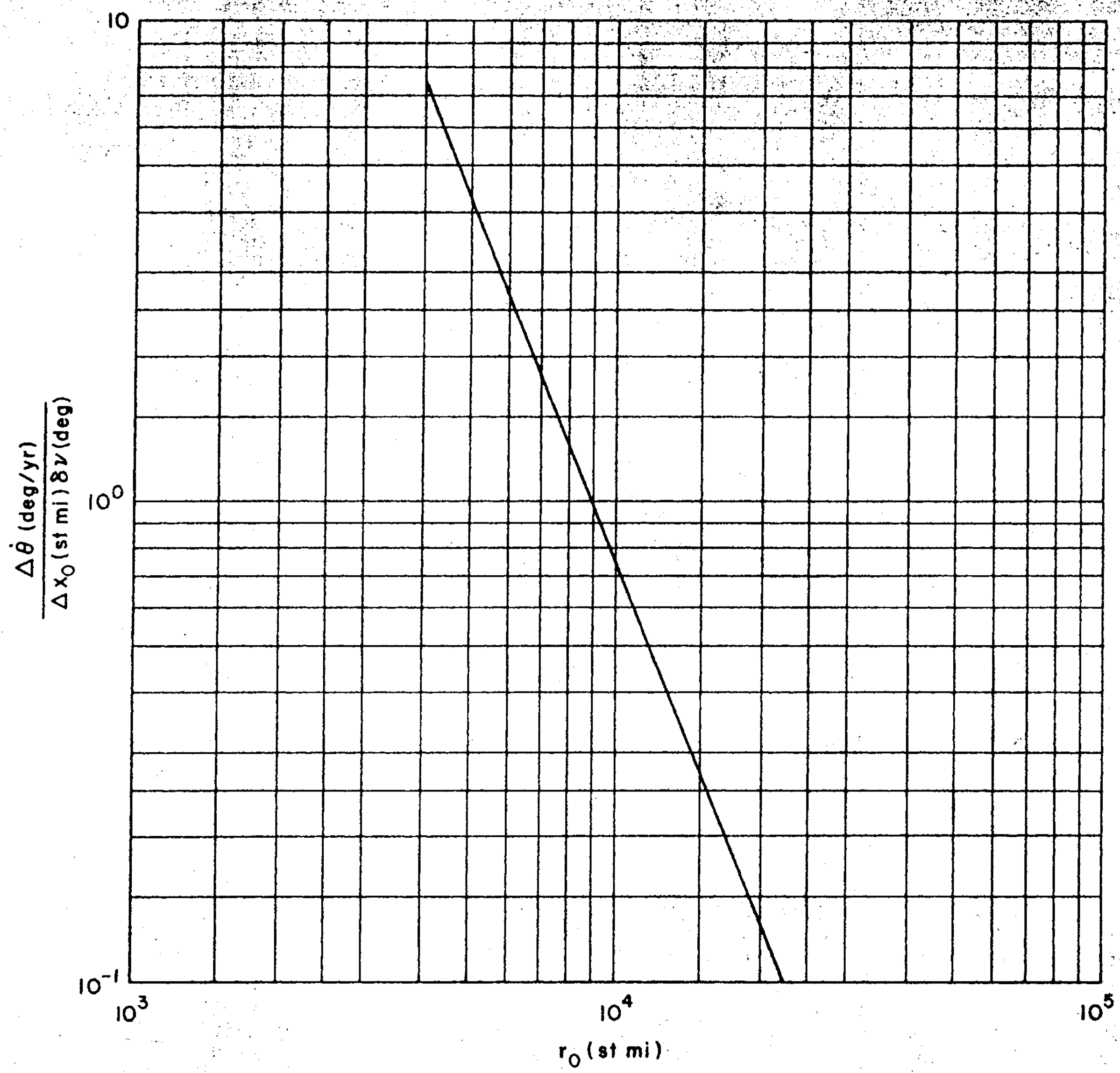

Fig. 7 — Rate of growth of dipole cloud as a function of orbital radius

IV. CONCLUSIONS

As a result of the analysis presented in this Memorandum, the following conclusions can be stated:

o Dipoles can be ejected from an orbiting vehicle in such a way that they form an orbiting cloud of fixed size and shape centered on the ejecting vehicle.

o Such a cloud would considerably enhance the radar cross-section per unit mass on orbit as compared with either orbiting balloons or the Westford belt of needles in orbit.

o The desired ejection velocity pattern can be achieved by spinning the dipoles from a rotating cylinder, similar to the Westford dispenser, but with the spin axis maintained in the instantaneous horizontal plane and making a small angle with the vehicle velocity vector.

o This method of dispensing compensates for the fact that some dipoles are ejected from positions slightly above or below the basic vehicle orbit.

o In order to produce the desired fixed-configuration cloud, the dispensing procedure should be done continuously for at least one complete orbit and preferably for several orbits.

o The resulting cloud is essentially contained in an ellipsoid of revolution centered on the vehicle with a major axis along the vehicle velocity vector and two minor axes which are one-fourth the major axis. The absolute value of these dimensions is directly proportional to the ejection velocity.

o If the attitude sensing and control system results in misalignment of the spin axis of the dispenser, the vertical and lateral dimensions of the cloud are unaffected but the dimension along the orbit increases linearly with time.

o The acceptable rate of growth is dependent on the beamwidths of the sending and receiving antennas, as well as the desired lifetime of the system. This rate of growth can be reduced by either increasing the orbital radius, reducing the initial dimension of the cloud, or by reducing the spin axis alignment error.

o While the analysis presented applies to a circular orbit, it should be possible to adapt this dispensing method to produce a stable cloud on any arbitrary elliptical orbit.

REFERENCE

1. Frick, R. H., Motion of Objects Ejected from an ICBM or a Satellite Vehicle (U), The RAND Corporation, RM-1701-PR, 1 May 1956 (Confidential).