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COST AND EFFECTIVENESS IMPACTS OF ORDER QUANTITY CHANGES

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PREFACE

RAND was requested by both Headquarters AFLC and AFSLP, Headquarters USAF, to investigate the problem of estimating the increased cost of setting a minimum reprocurement quantity of one year's worth of stock for all economic order quantity items. This Memorandum describes the work undertaken in response to that request. The basic content of the study and of this Memorandum was communicated informally to personnel of the Air Force Logistics Command, Requirements Division (AFLC, MCSR). This Memorandum officially documents our efforts in response to that request.

SUMMARY

Current Air Force stockage policy for Economic Order Quantity (EOQ) items at the depot level includes the following. If an item's annual issues exceed \$1,000, then less than one year's worth of stock is procured at a time. As items increase in value to \$9,000, fewer are procured at one time, but each buy must be for at least one-third of a year's supply (items worth \$10,000 also fall into this category). The policy is modified for items valued at over \$10,000.

The Air Force would like to change their policy so that one year's worth of stock could be purchased at a time on items valued at less than \$10,000 per year. The reason for contemplating the change is related to spare parts budgeting and to the difficulties attendant with making more than one procurement per item during a given year. And we are hoping that if we can compare the overall effects of a change in procurement policy with the present policy, it will be useful in determining future depot-level stockage policies for various items.

In this Memorandum, two methods are developed for estimating the increased ordering and holding costs of setting a minimum procurement quantity of one year's worth of stock on items whose annual issues are in the \$1,000 to \$10,000 category. In addition, methods are devised for estimating the decrease in the fill rate effectiveness that would result from such a change. Method 1 requires only that the items be specified by their annual issue value. Method 2 requires that items be specified both by annual demand rates and unit prices. Method 2 would be more difficult and expensive to use, although it is more accurate than the first, particularly for items with low annual demand rates.

Both methods require an acceptable estimate of the holding cost rate and reordering cost parameters, as well as estimates of the number of line items in various item specification categories. Personnel at AFLC have been collecting data on the annual demand rates and unit prices of all EOQ items so that the latter requirement should be satisfied (at least in sufficient detail for the first method) as soon as these data are sorted and summarized.

This study compares two specific stockage policies for the EOQ items that are currently ordered more than once a year. In the Memorandum, methods are presented for approximating some of the expected impacts of a systematic change in order quantities while the reorder levels and other factors are held constant. One change that can be estimated with these procedures is the variable cost of ordering and holding inventory stock. These costs, however, are functions of ordering and holding cost parameters that cannot be estimated exactly, and the parameters may change with the stockage policy change or over time.

Procedures are also presented for estimating expected changes in line item procurement actions, in inventory investment levels, and in stockouts or back orders. The estimates are based essentially upon the economic order quantity model presented in A. R. Ferguson and L. Fisher, Stockage Policies for Medium- and Low-Cost Parts, The RAND Corporation, RM-1962, April 1958. Neither policy is compared with stockage policies based upon aggregate criteria, nor has the complete economic analysis been performed that would be necessary to determine the most desirable form of stockage policy for these items.

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I. INTRODUCTION

According to the Air Force depot-level inventory stockage policy, if an Economic Order Quantity (EOQ) item's annual issues exceed \$1,000, less than one year's worth of stock is procured at a time. For various combinations of holding cost rates and reprocurement costs, this policy minimizes the total holding and reprocurement costs for the individual line items. In budgetary terms, however, it would be more convenient to procure these items only once a year. Cost and effectiveness changes or impacts should be considered in deciding whether or not to change the current EOQ policy. This Memorandum attempts to provide that input.

Compared are two specific stockage policies for the EOQ items that are currently ordered more than once a year. Methods are presented for approximating some of the expected impacts of a systematic change in order quantities while the reorder levels and other factors are held constant. One change that can be estimated with these procedures is the variable cost of ordering and holding inventory stock. These costs, however, are functions of ordering and holding cost parameters that cannot be estimated exactly, and the parameters may change with the stockage policy change or over time. Also, these costs exclude costs associated with back orders as well as costs that are not expected to vary with order quantity changes.

For these reasons, procedures are also presented for estimating expected changes in the line item procurement actions, in inventory investment levels, and in stockouts or back orders. The estimates are based upon the economic order quantity model presented in A. R. Ferguson and L. Fisher, Stockage Policies for Medium- and Low-Cost Parts, The RAND Corporation, RM-1962, April 1958. Neither policy is compared with stockage policies based upon aggregate criteria, nor has the complete economic analysis been performed that would be necessary to determine the most desirable form of stockage policy for these items (we assume, for instance, that a two-bin form of stockage would continue to be used).

Examples are presented of the cost and effectiveness impacts for various types of items specified by their annual issue values and by various combinations of holding and ordering costs. These cost and

effectiveness impact estimates are designed to be applied against data that AFLC has been collecting on the annual demand and unit cost of all items in the Air Force inventory.

Section II describes the estimating formulas and lists six primary assumptions required to make the estimates of the expected changes exact. Also discussed in Sec. II is the importance of each assumption for estimating the various expected impacts. Section III describes and compares two estimating models. RAND's JOSS computer was programmed to perform the computations in Sec. III. Section IV describes other considerations that were not incorporated in the computations given here.

^{*}JOSS is the trademark and service mark of The RAND Corporation and its computer program and services using that program.

II. CURRENT STOCKAGE POLICY AND COST IMPACTS FROM CHANGED STOCKAGE POLICY

THE CURRENT STOCKAGE POLICY

The current Air Force stockage policy for economic order and stockage policy items is specified by the EOQ formula:

(1)
$$Qv = 31.622\sqrt{A}$$
,

where

Q =the EOQ in units,

v =the unit price, and

A = the value of annual issues,

with the stipulation that the order quantity be at least one-third of the annual demand rate for any EOQ item. $\!\!\!\!\!\!\!^{\star}$

COST IMPACTS FROM CHANGED STOCKAGE POLICY

The expected annual costs of ordering and holding the depot-level stock of an EOQ item can be approximated by:

(2)
$$C(Q) = hv(Q + 1)/2 + dr/Q,$$

where

Q =the EOQin units,

h = the holding cost rate per dollar's worth
 of stock held,

v =the unit price,

d = the expected annual demand rate, and

r = the procurement setup cost;

and

Economic Order Quantity Table, AFLCR 58-42, Attachment 1, 9 April 1964, and 13 August 1965.

(3)
$$C(q) = hv(q + 1)/2 + dr/q,$$

where q =the order quantity for once-a-year procurement.

The costs we are explicitly concerned with here are holding costs and reprocurement setup costs. Holding expenses include storage costs, capital costs (interest) on the money invested in stocks, and obsolescence costs. Reordering expenses are the Air Force's fixed costs of making a reprocurement action for a single line item; they consist of the paperwork required for the action plus portions of the physical handling costs that are unrelated to the size of the order. Physical handling costs related to order size are ignored here because we assume that what is procured will eventually be used up. Thus the same volume of material would eventually have to be handled regardless of the order quantity. The expected cost impact of changing the order quantity can be estimated by the differences between Eqs. 2 and 3:

(4)
$$C(q) - C(Q) = C'(q, Q) = hv(q - Q)/2 + dr(1/q - 1/Q),$$

where G'(q, Q) =the estimated cost difference attributable to changing from Q to q.

If the order quantities were continuous, that is, if they did not have to be rounded to integers, then:

(5)
$$q = d, \text{ and } Q = kd,$$

where $k = 31.622 / \sqrt{A}$,

so that

(6)
$$C'(A) = hA(1 - k)/2 + r(1 - 1/k).$$

To estimate the total (aggregate) cost change associated with the stockage policy change, multiply the cost impact per item (as computed from Eq. 6) by the total number of items in groups whose annual issue values are the same and sum up those products for all the item groups. It is possible to estimate various other aggregate results. Some that may be of interest include: the average investment in inventory over time; the average number of line-item procurement actions per year, the number of unit back orders per year, and the number of line-item stockout situations per year.

The change in the average investment, I(A), in inventory over time for one item (specified by its annual issue value) can be estimated as

(7)
$$I(A) = A(1 - k)/2.$$

The decrease in the average number of line item procurement actions per year, N(A), for one item can be estimated as

(8)
$$N(A) = 1/k - 1.$$

The average number of back orders or stockouts per year, Z(Q), for an item with a given EOQ can be expressed as

$$Z(Q) = dX/Q,$$

where X = the average number of back orders or stockouts per procurement action.

Thus if a sample estimate of total back orders or stockouts per year for the various groups of items can be obtained from data collected in a historical period when the EOQ policy was being followed, the average number of back orders or stockouts per procurement action can be estimated as

(10)
$$X = QZ(Q)/d.$$

Assuming that this number is independent of the procurement cycle's length, the number per year for the annual buy policy, Z(q), can be estimated as:

(11)
$$Z(q) = dX/q, \text{ or}$$

$$Z(q) = QZ(Q)/q.$$

Then the reduction in the effectiveness measures can be estimated by

(13)
$$Z(Q) - Z(q) = Z'(Q, q) = Z(Q) (1 - Q/q).$$

The percentage decrease in the effectiveness measures, $P(q,\,Q)$, can be estimated by

(14)
$$P(q, 0) = 100(1 - Q/q).$$

If the order quantities were continuous, then the percentage decrease in back orders and stockouts could be estimated as a function, P(A), of the value of annual issues, or

(15)
$$P(A) = 100(1-k).$$

The aggregate average investment and number of line-item procurement action estimates would be obtained by applying the estimates for individual items against the total items in the appropriate groups, as done for the cost impact estimates. The aggregate back order or stockout estimate would be obtained by multiplying the percentage errors by the total back orders or stockouts for the appropriate groups of items as specified by the annual issue values, and then summing up these products across the various item groups.

The estimating equations are designed to give approximate estimates of the expected impact values for the historical period. It is hoped that they will also provide approximate estimates for future periods. Complete accuracy in making such estimates for a future period would require the validating of a number of assumptions. It should be noted, however, that the rationale behind the EOQ levels is also based upon these same assumptions. Some assumptions have been mentioned as we developed the estimating equations, some have not. We list the primary assumptions here and then discuss the importance each has for estimating the various impacts. The assumptions are:

- (1) The true mean demand rate is known or determinable for each item.
- (2) The true mean demand rate is used in setting the order quantities.
- (3) The demand distributions are stationary over time.
- (4) The procurement lead times are independent of procurement cycle length.
- (5) The correct values of the holding cost and the reorder cost can be determined and used in the cost impact estimating equations.
- (6) The order quantities can be thought of as continuous variables.

The first three assumptions are required for all impact estimates. All are non-linear functions of the demand rate, so the true mean demand rate must be known; or, alternatively, the total items with specified mean demand rates or values of annual issues must be known to estimate the impact on the cost, investment, and total line-item procurement actions. Assumption 2 is required for two reasons: because the EOQ for the expected demand rate is not exactly equal to the EOQs' expected value resulting from all the possible stochastic demand outcomes; and also because the order frequency computed from the expected demand rate is not exactly equal to the expected order frequency resulting from a stochastic demand process. Assumption 3 is required because neither policy is designed to handle nonstationary demand distribution or demand rate situations, and the expected impacts would vary according to what changes occur over time. Also, if the demand distributions vary markedly

over time, the estimates for the historical period may have little relevance to future periods.

Since the first three assumptions are not met in the operating supply system, the impact estimates for individual items will incorporate two kinds of error: an error in estimating the expected values of the various impacts; and, due to the stochastic demand variations over time, an error in estimating the actual impacts for any specific future period. Estimates of the aggregate expected impacts for all affected EOQ items, however, should be reasonably accurate. The various sources of error in the individual item estimates can be expected to counterbalance each other to a certain extent, and the positive errors made for one item will tend to counterbalance the negative errors made for another.

Assumption 4 is required for the cost, investment and effectiveness impact estimates because systematic changes in lead times would cause changes in lead-time stock levels (investment and holding costs), and in back orders and stockouts per procurement cycle. Furthermore, items for which the assumption is known to be false would probably not be managed under either policy without some additional modification or elaboration.

Assumption 5 is immediately required only for the cost impact estimates that are functions of the two cost parameters, h and r. Having the three estimates -- change in average dollar investment over time, change in line-item procurement actions, and change in effectiveness measures -- would not help much in choosing appropriate order quantity policies without some idea about the cost parameters associated with each one (holding cost rate, reorder cost, and back order or stockout penalty). The individual estimates will be valuable, however, if there are physical, policy or budgetary limits on one or more results of the stockage policy choice. Also it may make more sense to estimate the total cost of handling so many thousands of line-item procurement actions per year than it does to attempt to measure one average or marginal reordering cost by accounting means. The same may be true in regard to holding material or to living with a given number of stockouts or back

orders per year. We shall examine the sensitivity of the cost impact estimates to changes in the cost parameters by using computational examples in Sec. III.

Assumption 6 is required for all impact estimates based only upon the value of annual issues, rather than upon the integer order quantities. We have presented formulas for cost and effectiveness impact estimates based upon integer order quantities and hence upon demand rate and unit price data in Eqs. 6 and 14. Similar formulas for estimating the impacts of investment and total line-item procurement actions could also be constructed by using the first and second terms to the right of the equal sign in Eq. 6 with the cost parameters, h and r, left off. Thus:

(16)
$$I(Q, q) = v(q - Q)/2$$
, and

(17)
$$N(q, Q) = d(1/q - 1/Q).$$

We shall use some computational examples in Sec. III to test the sensitivity of the cost and effectiveness impact estimates to this continuous order quantity assumption.

III. COMPUTED IMPACT ESTIMATES PER ITEM

METHOD 1: CONTINUOUS ORDER QUANTITIES

A first estimate of the impacts of the stockage policy change can be based upon assumed values of the item's annual issues, A. For the cost impact estimates, however, we also need to choose specific values for the two cost parameters -- holding cost rate, and reordering cost. As a starting point for such a choice we can note that the EOQ stockage factor, of 31.622 (from Eq. 1), combined with the usual economic lot-size formula, indicates a specific ratio of reorder cost to holding cost rate. The economic lot-size formula can be written as:

(18)
$$Qv = \sqrt{\frac{2r}{h}} A.$$

Comparing this with Eq. 1, we find that

(19)
$$\frac{2r}{h} = (\$31.622)^2 = \$999.50884$$
, and hence that

(20)
$$\frac{r}{h} = $499.975442$$
, or about \$500.00.

Thus for a start we can say that a holding cost rate of 10 percent and a reordering cost of \$5000 is one combination of the cost parameters that approximately matches the ratio implied by the EOQ policy. We used these cost parameter values in developing the program shown in Table 1 to perform the impact estimating computations on RAND's "JOSS" (Johnniac Open Shop System) computer. The program is also based upon Eqs. 6, 7, 8 and 14. Thus, continuous order quantities are assumed for these computations. Table 2 shows the results of the computations.

The holding cost rate and the reordering cost used for the cost impact portion of Table 2 are indicated at the top just under the title. The values of annual issues, A, are indicated in column 1. The impacts on the average investment, I(A), number of orders, N(A), and annual cost, C'(A), are indicated in columns 2, 3 and 4, and the estimated percentage impacts, P(A), on the effectiveness measures are indicated in

Table 1

```
1.00 Page.
1.03 Line.
1.04 Set h=.10.
1.05 Set r=50.00.
1.06 Type h,r in f
1.07 Type
1.10 Set A=1000.
      Type h,r in form 2.
1.11 Type form 3.
1.12 Type form 4.
1.20 Set k=31.622/sqrt(A).
1.22 Set k=1/3 if A \ge 9000.
1.25 Set I=A \cdot (1-k)/2.
1.28 Set N=(1/k) - 1.
1.30 Set C=h·A·(1-k)/2 + r·(1-1/k).
1.35 Set P = 100 \cdot (1-k).
1.40 Type A,I,N,C,P in form 5.
1.60 Set A=A+250.
1.65 Done if A>10000.
1.70 To step 1.20.
Form 1:
                   COST AND EFFECTIVENESS IMPACTS PER ITEM
Form 2:
                         For: h = 0, and r = $_{-}
Form 3:
   1
                    2
                                     3
                                                      4
Form 4:
   Α
                   I(A)
                                    N(A)
                                                     C'(A)
                                                                       P(A)
Form 5:
```

Table 2

COST AND EFFECTIVENESS IMPACTS PER ITEM

For: h = 0.10, and r = \$50.00

1	2	3	4	5
Α	I(A)	N(A)	C'(A)	P(A)
1000	•01	•00	•00	•00
1250	66,00	•12	• 70	10.56
1500	137.64	•22	2.53	18.35
1750	213.58	•32	5.21	24.41
2000	292.91	•41	8.58	29.29
2250	375.02	•50	12,50	33.33
2500	459,45	•58	16.89	36.76
2 7 50	545.86	• 66	21.67	39.70
3000	634.00	.73	26,79	42.27
3250	723.63	.80	32,22	44.53
3500	814.61	.87	37.92	46.55
3 7 50	906.7 8	•94	43.85	48.36
4000	1000.02	1.00	50.00	50.00
4250	1094.25	1.06	56.34	51.49
4500	1189.37	1.12	62.87	52.86
475 0	1285.30	1.18	69.56	54.12
5000	1381.99	1.24	76.39	55 . 28
5250	1479.38	1.29	83.37	56.36
5500	1577.42	1.35	90.48	57.36
5 7 50	1676.07	1.40	97.71	58.30
6000	1775.29	1.45	105.05	59.18
6250	1875.03	1.50	112.50	60.00
6500	19 7 5.28	1.55	120.05	60.78
6750	2075.99	1.60	127.69	61.51
7000	2177.16	1.65	135.42	62.20
7250	2278.74	1.69	143.24	62.86
7500	2380.73	1.74	151.14	63,49
7750	2483.09	1.78	159.11	64.08
8000	2585.82	1.83	167.16	64.65
8250	2688.89	1.87	175.27	65.19
8500	2792.30	1.92	183,45	65.70
8750	2896.02	1.96	191.70	66.19
9000	3000.00	2.00	200.00	66.67
9250	3083.33	2.00	208.33	66.67
9 50 0	3166.67	2.00	216.67	66.67
9 7 50	3250.00	2.00	225.00	66.67
10000	3333.33	2.00	233.33	66.67

column 5. All values (Cols. 2 to 5) are rounded to two decimal places (to the nearest cent for the two dollar and cent figures, I(A) and C'(A)). The number of orders per year and the percentage change in effectiveness impact estimates indicate decreases in this particular stockage policy change, while the investment impact estimates indicate increases. The cost impact estimates are, in a sense, a weighted combination of the average investment and the number of order impacts, with the cost parameters being the assigned weights. Thus the cost figures may be either increases or decreases, depending upon the chosen values of the cost parameters. The estimated cost impacts are all increases for chosen values of the cost parameters which, like those used for the Table 2 computations, match the parameter ratio implied by the EOQ policy.

Both stockage policies result in the same order quantities for items whose annual issues amount to \$1000 so that there should be no estimated impact for these items. Table 2, however, lists a computed impact of one cent for the average investment impact for these items. This increase occurs because the ratio of \$50 to 10 percent, r/h, does not exactly match the ratio implied by the EOQ policy as indicated in Eq. 20. All estimated impacts per item increase as the value of annual issues increases from \$1,000 through \$9,000. This is due to the increasing difference between the order frequencies and hence between the order quantities for the two policies throughout this range. ferences between the policies remains constant, however, as the value of annual issues increases from \$9,000 to \$10,000, since EOO policy requires a minimum order quantity of one-third of the annual demand rate. We have not computed impact estimates for items whose annual issue values are over \$10,000. The EOQ policy does not apply, unmodified, for those items; and presumably the proposed stockage policy would not apply to them either.

The cost parameter value combination used in Table 2 is only one of an infinite number of combinations with the same ratio, r/h = 500, as implied by the EOQ policy; we have no justification for preferring this particular combination over many of the other similar combinations.

Also there is no solid justification for presuming that this ratio is necessarily correct for the Air Force wholesale supply system as it is currently operated. Methods of storage, material handling, levels computations, and data processing have changed since the EOQ policy was instituted. Capital costs have also changed, and obsolescence rates may very well have changed. Furthermore, the ratio that the EOQ policy implies was chosen to apply to all EOQ items, while here we are concerned only with the relatively small number of items whose annual issue values exceed \$1,000.

Since all items are not subject to the same cost elements (obsolescence rates, physical storage costs per dollar of inventory, or binning, picking and packing costs), different parameter values might be more appropriate for this set of items. So, for the present, we must examine the sensitivity of the cost impacts to changes in the cost parameters, and develop methods for estimating reasonable confidence limits on the aggregate cost impact of the policy change from estimates about reasonable confidence limits on the two cost parameters. The cost parameters used for this purpose are shown in Table 3.

Table 3
ORDERING COST AND HOLDING COST RATE COMBINATIONS

Cost	Combination							
Parameter	1	2	3	4	5	6		
h	.04	.07	.10	.13	.16	. 20		
r	20	35	50	65	80	100		

The ratio of ordering cost to holding cost rate, r/h, is equal to 500 for each parameter combination shown in each column of Table 3. These combinations approximately match the parameter ratio implied by the EOQ policy. Other ratios would result from combining the holding cost rates and ordering cost from different columns.

These cost parameter values probably bracket the appropriate values assuming that only the Air Force portion of the ordering setup costs are considered, $\overset{\star}{}$ and assuming that an interest rate between $2\frac{1}{2}$ and 12 percent is acceptable for this type of capital cost.

We developed the program in Table 4 to perform cost impact computations per item for any of six prespecified cost-parameter combinations. Like the first program, this one was also designed for JOSS. Table 5 contains the results of our first computations with this program. The various values of annual issues, A = \$1,000 to A = \$10,000, are listed in the first column on the left. The six cost-parameter combinations from Table 3 are shown at the top of the table; in the columns beneath these combinations are their cost impact estimates.

An examination of Table 5 indicates that increasing the holding cost rate and the reordering cost by the same factor causes the estimated cost impact to increase by that factor. For instance, doubling the holding cost rate from 10 to 20 percent and the reordering cost from \$50 to \$100 causes the impact estimates to double. So to measure the effect that any change in the cost parameter scale has on the aggregate cost impact (when both are changed by the same factor), multiply the aggregate cost impact estimate by the parameter scaling factor.

In Table 5, the cost impact estimates show the change in the scale of the two cost parameters when the ratio between them is held constant. Table 6 shows the results of using six cost-parameter combinations with different ratios (r to h), while the holding cost rate is kept constant at 10 percent. (The program in Table 4 was used again for these computations.) We have no particular reason for favoring a 10-percent holding cost rate, but we used it because the implied cost parameter ratios are easily determined integer values -- 10 times the reordering cost. For Table 6 we combined the 10-percent holding cost rate with each of the reordering cost values in Table 3.

^{*}The vendor's production setup cost may be much more than \$100 for some items, and may be virtually nonexistent for others.

Table 4

```
1.02 Set A=1000.
 1.04
       Do part 3 for i=1(1)6.
 1.06
       Type f.
 1.12
       Page.
 1.14
       Type T in form 6.
 1.21
       Туре
       Type form 1.
1.24
1.26
       Type
1.30
       Type \overline{h(1)}, h(2), h(3), h(4), h(5), h(6) in form 2.
       Type r(1), r(2), r(3), r(4), r(5), r(6) in form 3.
1.32
       Type f(1), f(2), f(3), f(4), f(5), f(6) in form 4.
1.34
1.36
       Line.
       Set k=31.622/sqrt(A).
1.40
1.42 Set k=1/3 if A \ge 9000.
1.44 Do part 4 for i=1(1)6.
1.45 Type A,C(1),C(2),C(3),C(4),C(5),C(6) in form 5.
1.46 Done if A=10000.
1.48 Set A=A+250.
1.50 To step 1.40.
3.1
       Demand h(i).
3.3
       Demand r(i).
3.5
      Set f(i)=sqrt(2\cdot r(i)/h(i)).
      Set C(i)=A \cdot h(i) \cdot (1-k)/2 + r(i) \cdot (1-1/k).
4.1
Form 1:
                                   COST IMPACTS
Form 2:
   For h = 0.
                                                             0._
                                                                         0._
Form 3:
Form 4:
   And f =
Form 5:
Form 6:
```

Table 5

COST IMPACTS

For h	n = 0.04	0.07	0.10	0.13	0.16	0.20
r	r = 20.00	35.00	50.00	65.00	80.00	100.00
1000	•00	•00	•00	•00	•00	.00
1250	• 28	•49	•70	•91	1.11	1.39
1500	1.01	1.77	2.53	3.28	4.04	5.05
1750	2.08	3.65	5.21	6.78	8,34	10,42
2000	3.43	6.01	8.58	11.15	13.73	17.16
2250	5.00	8.75	12.50	16.25	20.00	25,00
2500	6.75	11.82	16.89	21.95	27.02	33.77
2 7 50	8.67	15.17	21.67	28.17	34.67	43.34
3000	10.72	18.76	26.79	34.83	42.87	53,59
3 250	12.89	22.56	32.22	41.89	51.56	64.44
3500	15.17	26.54	37.92	49.29	60.67	75.83
3750	17.54	30.70	43.85	57.01	70.16	87.70
4000	20.00	35.00	50.00	65.00	80.00	100.00
4250	22.54	39.44	56.34	73.25	90.15	112.69
4500	25.15	44.01	62.87	81.73	100.59	125.74
4750	27.82	48.69	69.56	90.42	111.29	139.11
5000	30.56	53.48	76.39	99.31	122.23	152 .7 9
5250	33.35	58 .3 6	83.37	108.38	133.39	166.74
5500	36.19	63.34	90.48	117.62	144.77	180.96
5750	39.08	68.40	97.71	127.02	156.33	195.42
6000	42.02	73.54	105.05	136.57	168.08	210.10
6250	45.00	78.75	112.50	146.25	180.00	225.00
6500	48.02	84.03	120.05	156.06	192.08	240.10
6750	51.08	89.38	127.69	166.00	204.31	255.38
7000	54.17	94.80	135.42	176.05	216.68	270.85
7250	57.30	100.27	143.24	186.21	229.19	286.48
7500	60.46	105.80	151.14	196.48	241.82	302.28
77 50	63.64	111.38	159.11	206.85	254.58	318.22
8000	66.86	117.01	167.16	217.30	267.45	334.31
8250	70.11	122.69	175.27	227.85	280.43	350.54
8500	73.38	128.42	183.45	238.49	293.52	366.90
8750	76.68	134.19	191.7 0	249.20	306 .7 1	383.39
9000	80.00	140.00	200.00	260.00	320.00	400.00
9250	83.33	145.83	208.33	270.83	333.33	416.67
9500	86.67	151.67	216.67	281.67	346.67	433.33
9 7 50	90.00	157.50	225.00	292.50	360.00	450.00
10000	93.33	163.33	233.33	303.33	373.33	466.67

Table 6

COST IMPACTS

For	h = 0.10	0.10	0.10	0.10	0.10	0.10
_	r = 100.00	80.00	65.00	50.00	35.00	20.00
And	f = 44.72	rt0 • 00	36.06	31.62	26.46	20.00
1000	•00	•00	• 00	.00	•00	•00
1250	-5.2 <u>1</u>	-2.85	-1.07	•70	2.47	4.24
<u> 1</u> 500	-8.7 <u>1</u>	-4.22	85	2.53	5.90	9.27
1750	-10.93	-4.47	• 37	5.21	10.06	14.90
2000	-12.1 3	-3.85	2.36	8.58	14.79	21.01
2250	-12. 50	- 2.50	5.00	12.50	20.00	27.50
2500	-12.17	- • 55	8.17	16.89	25.60	34.32
2750	-11.25	1.92	11.79	21.67	31.54	41.42
3000	- 9.8 <u>1</u>	4.83	15.81	26 .7 9	37.78	48.76
3250	-7. 92	8.14	20.18	32.22	44.26	56.31
3500	-5.63	11.79	24.85	37.92	50.98	64.04
3 7 50	-2.98	15.7 5	29.80	43.85	5 7. 90	71.95
4000	•00	20.00	35.00	50.00	65.00	80.00
4250	3.26	24.50	40.42	56.34	72.27	88.19
4500	6.80	29.23	46.05	62.87	79.69	96.51
47 50	10.58	34.17	51.86	69,56	87.25	104.94
5000	14.59	39.31	57.85	76.39	94.94	113.48
5250	18.80	44.63	64.00	83.37	102.74	122.11
5500	23.22	50.12	70.30	90.48	110.66	130.84
5750	27.81	55 .77	76.74	97.71	118.68	139.65
6000	32,57	61.56	83.31	105.05	126.79	148.54
6250	37.50	67.50	90.00	112.50	135.00	157.50
6500	42.57	73.56	96.81	120.05	143.29	166.54
6750	47 .7 9	79.75	103.72	127.69	151.66	175.64
7000	53.13	86.05	110.74	135.42	160.11	184.80
7250	58.6 <u>1</u>	92.46	117.85	143.24	168.63	194.02
7 500	64.20	98.98	125.06	151.14	177.22	203.30
7750	69.91	105.59	1.32.35	159.11	185.87	212.63
8000	75.73	112.30	139.73	167.16	194.58	222.01
8250	81.65	119.10	147.19	175.27	203.36	231.44
8500	87.68	125.99	154.72	183.45	212.19	240.92
8 75 0	93 .7 9	132.95	162.32	191.70	221.07	250.44
9000	100.00	140.00	170.00	200.00	230.00	260.00
9 25 0	108.33	148.33	178.33	208.33	238.33	268.33
9500	116.67	156.67	186,67	216.67	246.67	276.67
9 7 50	125.00	165.00	195.00	225.00	255.00	285.00
10000	133.33	173.33	203.33	233.33	263.33	293.33

The chosen cost-parameter combinations appear at the top of each column in Table 6 as in Table 5. Immediately below these values are the stockage policy factors, f, that would replace the 31.622 factor in the EOQ policy if a similar economic lot-size policy were used with the parameter values. The minimum cost (holding and ordering costs only) order quantity would be obtained for these parameter values if the order quantity's value, Qv, were set equal to the listed factor, f, multiplied by the square root of A.

In Table 6, the first three columns of the estimated cost impacts per item include negative numbers. These numbers indicate that the annual buy policy would cause decreased rather than increased holding and ordering costs for the items and cost parameters. An EOQ policy based on the higher cost parameter ratios would specify larger order quantities than the current EOQ policy does. In fact, for items whose annual issues are less than the square of the listed stockage factor, the quantities for the modified EOQ policy would be greater than the items' annual demand rate and, hence, greater than the annual buy order quantities. So for the first computed column of Table 6, h = 0.10, r = 100.00, and f = 44.72 (actually $f = \sqrt{2000}$), the modified EOQ policy would cause larger buys than the annual buy policy for all items whose annual issue values are less than \$2,000.

Table 6 indicates that the estimated cost impacts per item increase as the cost-parameter ratios decrease. We choose the cost-parameter combinations for each column so that the ratios decrease from left to right. Thus the estimated cost impacts increase from left to right as they did in Table 5. The absolute values of the cost impact estimates increase with increases in the annual issue values, as occurred in Tables 2 and 5. The pattern of differences between the estimates across the columns, however, is not easily discerned from the computed values. The differences are functions of both the differences in the reordering cost parameter values and the annual issue values. The difference per item for items with a given value of annual issues can be written as

(21)
$$D(r) = (r_1 - r_2)(1 - \sqrt{A}/31.622) ,$$

where

D(r) is the difference between the estimated cost impacts when only the reordering cost parameter is changed, and

 \mathbf{r}_1 and \mathbf{r}_2 are the parameter values for the two reordering costs.

If all factors were held constant except the parameter value for the holding cost rate, the difference per item would be

(22)
$$D(h) = (h_1 - h_2)(A - 31.622 \sqrt{A})/2 ,$$

where

D(h) is the difference between the estimated cost impacts when only the parameter value for the holding cost rate is changed, and

 h_1 and h_2 are the two parameter values for the holding cost rate.

Thus, in both cases, the aggregate cost estimates for various cost parameter ratios cannot be obtained directly from one precomputed aggregate estimate and a simple function of the cost parameter values. Unlike the change of scale case, new estimates would have to be computed for each value of annual issues, and then processed against the distribution of line items the same as for the original aggregate estimate.

We computed the cost estimates per item for an additional number of cost parameter ratios to provide additional bases for such an operation. Table 7 lists the results of these computations. The JOSS program listed in Table 4 was used again. The computations were performed with the cost parameter values listed in Columns 2, 4, and 6 of Table 3. Again, Table 7 starts in the upper left with several series of negative numbers. These, like Table 6, indicate that the annual buy policy would decrease combined holding and ordering costs for items with the indicated values of annual issues, provided that

Table 7

COST IMPACTS

п.	b - 0 07					
ror	h = 0.07	0.07	9.13	0.13	0.20	0.20
	r =100.00	65.00	100.00	35.00	65.00	35.00
And	f = 53.45	43.09	39,22	23.20	25.50	18.71
1000	•00	.00	•00	•00	•00	•00
1250	-7.1 9	-3.05	-3.23	4.45	5.53	9.07
1500	-12.84	-4.98	-4.58	10.03	12.92	19.66
1750	-17.34	-6.04	-4.53	16,46	21.73	31.41
2000	-20.92	-6.42	- 3.35	23.58	31.66	44.08
2250	-23.7 5	-6.25	-1.25	31.25	42.50	57.50
2500	- 25 . 96	-5.62	1.61	39.39	54.11	71.55
2 7 50	- 27 . 62	-4.58	5.13	47.92	66.38	86.13
3000	-28.83	-3.21	9.21	56.80	79.21	101.18
3250	-29.63	-1.53	13.79	65.97	92.54	116.63
3500	-30.06	•42	18.81	75.42	106.31	132.44
3750	-30.18	2.60	24.23	85.10	120.48	148.58
4000	-30.00	5.00	30.00	95.00	135.00	165.00
4250	-29.56	7.59	36.09	105.10	149.85	181.69
4500	-28.88	10.37	42,48	115.37	164.98	198.63
4750	- 27 . 98	13.30	49.14	125.81	180.39	2 1 5.78
5000	- 26.87	16.39	56.05	136.39	196.05	233.13
5250	-25.58	19.62	63.19	147.12	211.94	250.68
5500	-24.11	22.98	70.54	157.98	228.04	268.40
5750	-22.47	26,46	78.09	168.96	244.35	286,29
6000	-20.69	30.05	85.83	180,05	260.84	304.32
6250	-18.7 5	33 .7 5	93.75	191.25	277.50	322.50
6500	-16.69	37.55	101.83	202.55	294.33	340.82
6750	-14.49	41.44	110.07	213.94	311.32	359.26
7000	-12.18	45.42	118.45	225.43	328.45	377.83
7250	-9.7 5	49,49	126.97	236.99	345.73	396.51
7500	-7. 22	53.64	135.63	248.64	363,13	415.29
7750	-4.58	57.86	144.41	260.36	380,66	434.18
8000	-1.84	62.16	153.31	272.16	398.31	453.17
8250	.99	66.52	162.32	284.02	416.08	472.25
8500	3 .91	70.95	171.44	295.95	433.95	491.42
8750	6.91	75.44	180.67	307.95	451.93	510.67
9000	10.00	80.00	190.00	320.00	470.00	530.00
9250	15.83	85.83	200.83	330.83	486.67	546.67
9500	21.67	91.67	211.67	341.67	503.33	563.33
9750	27.50	97.50	222.50	352.50	520.00	580.00
10000	33,33	103.33	233.33	363.33	536, 67	596,67

the cost parameter values at the top of those columns were appropriate for those items.

Table 7 is set up so that the estimated cost impacts are higher in the columns on the right. We believe that the parameter values and ratios used for Tables 5, 6, and 7 cover any reasonable confidence limits for the cost parameters at this time; we also believe that the aggregate cost impact estimates that would result from their use would cover any reasonable confidence limits on the true expected holding and ordering cost impact of this specific policy change. The highest and lowest estimated impacts (the first and last computed columns of Table 7) are probably somewhat outside the more reasonable confidence limits for current conditions. Cost impact estimates per item for other combinations of cost parameter values could easily be computed with the JOSS program listed in Table 4.

We have not examined the effects that changes in the chosen cost parameter values have on the noncost impacts of Table 2. This is justified here because we are concerned only with comparing two stockage policies -- the EOQ policy with a stockage factor of 31.622 and the annual buy policy. Eqs. 7 through 17 indicate that none of the noncost impacts are functions of the cost parameters for this specific policy change. The relative importance of the aggregate impacts (average investment, number of line-item procurement actions and effectiveness) would be affected, however by changes in the cost parameters.

It might also be interesting to compare the annual buy policy and an EOQ policy based on different cost parameter ratios. All types of impacts per item listed in Table 2 could be easily obtained by using

We should also note that the various aggregate cost impact confidence limits could be obtained by combining the aggregate impacts on the average investment and on the number of line item procurement actions with the various parameter values as weighting factors. Thus a number of confidence limits on the aggregate cost impact could be easily obtained. This would result, however, in the loss of information about which items (in terms of values of annual issues) cause the greatest aggregate cost impact shifts.

the program in Table 1, with k (program step 1.20) set equal to the appropriate stockage factor, f, divided by the square root of A. Also the impacts per item could be estimated for EOQ policies without the minimum order quantity overrides by dropping program step 1.22.

METHOD 2: INTEGER ORDER QUANTITIES

Since the order quantities must be rounded to integer values, determining the cost and effectiveness impacts on the basis of only the value of annual issues would result in some error. To get a better idea of what this error might be, we computed examples of the cost and effectiveness impacts for the rounded order quantities. Both order quantities were computed and rounded to the closest integer; then Eqs. 4 and 15 were used to compute the estimated cost and effectiveness impacts per item. Computations were run for a holding cost rate of 10 percent, a reorder cost of \$50, and for four values of annual issues -- \$2,000, \$4,000, \$6,000, and \$8,000. The demand rates were specified by:

(23)
$$d = j + 5 (2^n),$$

where n = 0, 1, 2, 3, 4, 5, 6, and 7, and

j = -0.49, 0, and +0.49.

Thus when j=0, the specified demands were integer values in multiples of five. This set of demands provides information about the errors that might occur when only the EOQ is rounded. When j=-0.49, the demands were almost half a unit less than the same integer values, so that the once-a-year order quantities were rounded up almost half a unit. When j=+0.49, the demands were almost half a unit higher than the same integer values; the once-a-year order quantities were therefore rounded down.

The cost and effectiveness impacts for rounded order quantities are shown in Tables 8 through 11. Indicated at the top of each table are the value of annual issues, A; the holding cost rate, h; and the

Table 8

COST EFFECTIVENESS IMPACTS BY d AND v

Fo	r	:
A	=	2000
h	=	.10
r	=	\$50.00

l	2	3	4	5	6	. 7
d	v	Q	q	Cost D/Item	P. B-O Drop	Cost (Q)
4.51	443.46	3	5	14.279	40.000	163.86
9.51	210.30	7	10	11.167	30,000	152.05
19.51	102.51	14	20	9.850	30.000	146.56
39.51	50.62	28	40	9.206	30.000	143.95
79.51	25.15	56	80	8.888	30.000	142,68
159.51	12.54	113	160	8.732	29.375	142.05
319.51	6.26	226	320	8.655	29.375	141.73
639.51	3.13	452	640	8.617	29.375	141.58
5.00	400.00	4	5	7.500	20,000	162.50
10.00	200.00	7	10	8.571	30.000	151.43
20.00	100.00	14	20	8.571	30.000	146.43
40.00	50.00	28	40	8.571	30.000	143.93
80.00	25.00	57	80	8.575	28.750	142.68
160.00	12.50	113	160	8.579	29.375	142.05
320.00	6.25	226	320	8.579	29.375	141.73
640.00	3.13	453	640	8.579	29.219	141.58
5.49	364.30	4	5	4.490	20.000	159.70
10.49	190.66	7	10	6.120	30.000	151.19
20.49	97.61	14	20	7.329	30.000	146.39
40.49	49.39	29	40	7.969	27.500	143.90
80.49	24.85	57	80	8.276	28.750	142.66
160.49	12.46	113	160	8.425	29.375	142.05
320.49	6.24	227	320	8.502	29.063	141.73
640.49	3.12	453	640	8.540	29.219	141.58

Table 9

COST EFFECTIVENESS IMPACTS BY d AND v

For:						
	10					
r - \$30.	00					
1	2	3	4	5	6	7
d	v	Q	q	Cost D/Item	P. B-O Drop	Cost (Q)
4.51	886.92	2	5	65.388	60.000	245.79
9.51	420.61	5	10	57.602	50,000	221.28
19.51	205.02	10	20	53.737	50,000	210.31
39.51	101.24	20	40	51.853	50.000	205.08
79.51	50.31	40	80	50,923	50.000	202.52
159.51	25.08	80	160	50.460	50.000	201.25
319.51	12.52	160	320	50,230	50,000	200.63
639.51	6.25	320	640	50.115	50.000	200.31
5.00	800.00	3	5	46.667	40.000	243.33
10.00	400.00	5	10	50,000	50.000	220.00
20.00	200.00	10	20	50.000	50.000	210.00
40.00	100.00	20	40	50.000	50,000	205.00
80.00	50.00	40	80	50.000	50,000	202.50
160.00	25.00	80	160	50.000	50.000	201.25
320.00	12.50	160	320	50.000	50.000	200.63
640.00	6.25	320	640	50.000	50.000	200.31
5.49	728.60	3	5	36.260	40.000	237.22
10.49	381.32	5	10	42.879	50.000	219.29
20.49	195.22	10	20	46.384	50.000	209.82
40.49	98.79	20	40	48.177	50.000	204.95
80.49	49.70	40	80	49.085	50.000	202.49
160.49	24.92	80	160	49.542	50.000	201.25
320.49	12.48	160	320	49.771	50.000	200.62
640-49	6.25	320	640	49 - 885	50,000	200.31

640 49.885

50.000

200.31

640.49

6.25 320

Table 10 COST EFFECTIVENESS IMPACTS BY d AND v

For:						
	000					
h = 0.50 $r = 50	.10 .00					
2 930.						
1	2	3	4	5	6	7
d	v	Q	q	Cost D/Item	P. B-O Drop	Cost (Q)
4.51	1330.38	2	5	131,907	60,000	312.31
9.51	630.91	4	10	117.949	60.000	276.60
19.51	307.53	8	20	111.358	60.000	260.33
39.51	151.86	16	40	108.151	60.000	252.55
79.51	75.46	32	80	106.569	60.000	248.75
159.51	37.62	65	160	105.819	59.375	246.83
319.51	18.78	130 261	320 640	105.433	59.375	245.89
639.51	9.38	201	040	105.243	59.219	245.42
5.00	1200.00	2	5	105.000	60.000	305.00
10.00	600.00	4	10	105.000	60.000	275.00
20.00	300.00	8	20	105.000	60.000	260.00
40.00	150.00	16	40	105.000	60.000	252.50
80.00	75.00	33	80	105.038	58,750	248.71
160.00	37.50	65	160	105.048	59.375	246.83
320.00	18.75	131	320	105.050	59.063	245.89
640.00	9.38	261	640	105.051	59.219	245.42
5.49	1092.90	2	5	81.584	60.000	301.18
10.49	571.97	4	10	92.917	60.000	274.12
20.49	292.83	8	20	98.858	60.000	259.83
40.49	148.18	17	40	101.937	57.500	252.45
80.49	74.54	33	80	103.529	58.750	248.68
160.49	37.39	66	160	104.282	58,750	246.82
320.49	18.72	131	320	104.669	59.063	245.89
640.49	9 . 37	261	640	104.859	59.219	245.42

Table 11 COST EFFECTIVENESS IMPACTS BY d AND ν

For:						
	000					
	.10					
r = \$50	•00					
1	2	. 3	4	5	6	7
đ	V	Q	P	Cost D/Item	P. B-O Drop	Cost (Q)
4.51	1773.84	2	5	198.425	60.000	378.83
9.51	841.22	3	10	183.477	70.000	326.74
19.51	410.05	7	20	175.948	65.000	303.38
39.51	202.48	14	40	171.505	65.000	292.97
79.51	100.62	28	80	169.314	65.000	287.88
159.51	50.15	56	160	168.226	65.000	285.36
319.51	25.04	113	320	167.694	64.688	284.09
639.51	12.51	226	640	167.425	64.688	283.47
5.00	1600.00	2	5	165.000	60.000	365.00
10.00	800.00	4	10	165.000	60.000	325.00
20.00	400.00	7	20	167.143	65.000	302.86
40.00	200.00	14	40	167.143	65.000	292.86
80.00	100.00	28	80	167.143	65.000	287.86
160.00	50.00	57	160	167.149	64.375	285.35
320.00	25.00	113	320	167.157	64.688	284.09
640.00	12.50	226	640	167.157	64.688	283.47
5.49	1457.19	2	5	136.229	60.000	355.83
10.49	762.63	4	10	150.114	60.000	321.78
20.49	390.43	7	20	158.650	65.000	302.53
40.49	197.58	14	40	162.859	65.000	292.79
80,49	99.39	28	80	164.991	65.000	287.85
160.49	49.85	57	160	166.086	64.375	285.34
320.49	24.96	113	320	166.621	64.688	284.09
((0 /0	10 /0	006	410			

283,47

226

640

166.889

64.688

12.49

640,49

reordering cost, r. The first four columns show the annual demand rates, the unit prices, and the two order quantities, respectively. Column 5 indicates the expected cost increase per line item attributable to the proposed policy change. Column 6 gives the expected percentage drops in back orders and stockouts per item, and Col. 7 shows the cost of holding and ordering per item when the EOQ policy is followed.

For all items represented in Tables 8 through 11, the use of Method 1 would result in a single cost impact estimate and a single effectiveness percentage impact estimate. For instance, the comparable Method 1 cost estimate for items in Table 8 is listed in Table 5 in the line for A = 2000, column 4, where the estimate for all these items is \$8.58. The difference between the estimates in Table 5 and Table 11 indicates the type of errors that would be made using Method 1, since it is based upon an incorrect assumption that procurement quantities are continuous. The errors per item tend to be greater when both order quantities have to be rounded than when only the EOQs have to be rounded.

The errors indicated for the rounded once-a-year order quantities however, are more extreme than would be expected in normal practice. And the negative errors would have a strong tendency to balance out the positive errors. All in all, we would expect the overall error caused by using the simpler unrounded EOQ computations (Eqs. 4 and 15 as exemplified by Tables 3 through 7) to be within the expected error range caused by: inaccurate cost parameter estimates, and the stochastic nature of the demand rate, the number of back orders, and the number of stockouts per procurement action.

The cost and effectiveness impact estimates are larger for items with higher annual issue values because the once-a-year order quantity is a more serious violation of the economic order principle for those items. Method I would have to be used to make these estimates if only data regarding annual issue values of EOQ items are available. Method 2 could be used if data regarding the annual demand rates and unit prices of the EOQ items are available. Method 2 would result in somewhat more precise estimates, but it would require more computation without any certainty that the cost impact estimate was more accurate in an absolute sense.

IV. OTHER CONSIDERATIONS

We have not explicitly discussed a number of other considerations for and against the proposed change in order quantity policy. If we knew how to estimate them correctly, many other concepts could be embodied in choosing the cost parameters, h and r, or in choosing confidence limits for those parameters. For instance, considerations related to the political, budgetary, or workload difficulties of ordering more than once a year could be accounted for by increasing the ordering cost when more than one order per year is expected. This would tend to increase the order quantities up to one year's worth of stock, which would make the once-a-year policy economical for more EOQ items.

The advantages of having liquid cash later in the year to buy items that unexpectedly turn out to be in short supply could be accounted for by increasing the holding cost rate. This would reduce the minimum cost order quantities and make the once-a-year order quantities less desirable. In the concepts presented here, however, we cannot estimate the possible future costs of dropping an EOQ policy that apparently reduces measurable costs simply because another policy is more convenient. Future costs would depend on the efficiency of the stockage policies that could be devised and implemented in the coming years as well as on the effect that back-slipping on economical principles now might have on the acceptance of such proposals in the future.

	·		