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INTENSITY OF RADIATION FROM A RAYLEIGH-SCATTERING ATMOSPHERE

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PREFACE

This Memorandum reports one step further in RAND's continuing investigations in the optical properties of planetary atmospheres. Here are demonstrated the characteristics of the intensity of radiation emerging from the top or the bottom of a Rayleigh-scattering atmosphere under a variety of conditions. Although the sphericity of the planet is here ignored, the condition of Rayleigh scattering renders further approximations unnecessary. The results are useful in our interpretation of new information becoming available on planetary atmospheres.

5

ABSTRACT

Theoretically derived values of the directional intensity of radiation emerging from both the top and the bottom of a Rayleigh-scattering atmosphere are presented graphically. The model assumes a plane-parallel atmosphere illuminated by the sun, with either a completely absorbing planetary surface, or Lambert ground reflection. By using Mullikin's and Sekera's recent modification of Chandrasekhar's radiative-transfer theory, the intensities were determined for much larger values of the optical thickness of the atmosphere than were previously possible. These intensities are given here for a wide range of optical thicknesses, solar zenith angles, and directions of emergence.

ACKNOWLEDGMENTS

I would like to thank Dr. Deirmendjian and Professor Sekera for many valuable suggestions and encouragement throughout the course of this work.

CONTENTS

I. INTRODUCTION

One of the basic problems of radiative transfer in a finite sunlit planetary atmosphere is to determine the directional intensity and polarization of light emerging from its top and bottom boundaries. Unfortunately, the theoretical problem is complicated and has not yet been solved for the general case. However, the main physical characteristics of multiply scattered radiation can be illuminated and studied by the special case of Rayleigh scattering, for which an exact solution of the radiative transfer in a plane-parallel atmosphere is now available. Moreover, this solution is the only one that allows a comprehensive study of the dependence of the emergent radiations on the optical thickness in its entire domain, from small up to large values of the optical thickness. The purpose of this paper is to demonstrate the characteristics of the intensity of the emerging radiations from the upper and lower boundaries of a plane-parallel atmosphere for a wide range of external parameters (solar elevations, ground reflectivities, angle of emergence) for the entire domain of the optical thickness.

Since Chandrasekhar (1950) first developed his solution to this problem, there have been several publications showing different aspects of the solution for small and moderate optical thickness ($\tau \leq 1$). Tables of the X- and Y-functions, from which the radiation can be calculated simply, have been made by Sekera and Blanch (1952), Sekera and Ashburn (1953), and Chandrasekhar and Elbert (1954). The latter, and also Coulson et al. (1960), carried the solution one step further and published tables of the intensity and polarization parameters (Stokes parameters) for radiation emerging from the bottom and top of the atmosphere. Sekera (1957) has illustrated graphically many of the features of the polarization of the downward radiation. Coulson (1959) has shown graphs of the upward intensity and polarization. Recently Dave and Furukawa (1966) have shown a few of the main features of the intensity and polarization values graphically for larger optical thicknesses ($\tau > 1$). However, they used a more approximate method than Chandrasekhar's, based on successive evaluations of the higher orders of scattering. As Dave and Walker (1966) have pointed out, the accuracy of this approximation becomes poor for large optical thickness. This is confirmed here by comparison with our results.

Attempts to carry Chandrasekhar's exact solution to optical thickness, τ, greater than 1 at first were unsuccessful because of computational instabilities in the solutions. Then Mullikin (1962, 1965) and Sekera (1966) developed a method that removes these instabilities making solution possible for as large an optical thickness as desired. On the basis of their work, Carlstedt and Mullikin (1966) published tables of the X- and Y-functions for isotropic scattering, and Sekera and Kahle (1966) for Rayleigh scattering. In the present paper we use the latter set of values to find the directional intensity of radiation emerging from both the top and bottom of a plane-parallel Rayleigh-scattering atmosphere for larger optical thickness than was previously possible.

II. METHOD

We wish to determine the emerging radiation from a theoretical model of a scattering atmosphere illuminated by the sun. In order to make the problem mathematically tractable, Chandrasekhar (1950) found it necessary to make several simplifying assumptions. His solution is applicable to a plane-parallel atmosphere that scatters according to Rayleigh's Law, with Lambert ground reflection without true absorption or emission. But a plane-parallel atmosphere can be represented by a slab, infinite in horizontal extent. At the bottom, the reflectivity of the ground may range from 0 (total absorption) to 1 (total reflection). Lambert reflection is defined as a reflection in which the surface reflects unpolarized light uniformly in all directions, independently of the direction and polarization of the incident light.

Horizontally, the plane-parallel atmosphere is assumed to be optically homogeneous and invariant, the only variation allowed being in the vertical direction. The height dependence of the scattering can be incorporated into a single parameter, the optical thickness. This is defined as

$$\tau_\lambda = \int_0^\infty k_\lambda(h)\,\rho dh \qquad (1)$$

where k_λ is the mass scattering coefficient, ρ is the density, and h is the height. For any given total mass, the optical thickness of a Rayleigh atmosphere is strongly dependent upon the wavelength, λ (Deirmendjian, 1955).

The intensity and polarization of a radiation field can be completely described in terms of the four Stokes parameters, I, Q, U, and V (or alternatively I_ℓ, I_r, U, and V), where $I = I_\ell + I_r$ and $Q = I_\ell - I_r$. The quantity I is a measure of the total intensity, while I_ℓ and I_r are the intensities in any two mutually perpendicular directions in the plane transverse to the initial direction of propagation. In our problem we define these two directions to be that which is parallel (I_ℓ) and that which is perpendicular (I_r) to the intersection of the transverse plane with the principal plane (the plane containing the point of observation, the local vertical, and the sun). The polarization is described by Q, U, and V. The degree of polarization, or fraction of the light which is polarized, is given by

$$P = \frac{(Q^2 + U^2 + V^2)^{1/2}}{I} \tag{2}$$

The angle between the plane of polarization (the plane containing the maximum electric vector) and the direction of I_ℓ is χ, given by

$$\tan 2\chi = \frac{U}{Q} \tag{3}$$

In the plane transverse to the direction of propagation, the plane of polarization is found by measuring χ clockwise from I_ℓ. The fourth parameter, V, measures the ellipticity of the polarization. For a Rayleigh-scattering medium, illuminated by unpolarized sunlight, V is identically zero and hence the polarization of the emerging radiation is linear (Sekera, 1957).

The three remaining Stokes parameters can be expressed in terms of the X- and Y-functions and the K- and L-functions. These functions, the solutions to the integro-differential radiative-transfer equations, have been tabulated for Rayleigh scattering by Sekera and Kahle (1966). These radiation parameters depend upon the optical thickness, τ (and thus indirectly, the wavelength, λ), upon the angles of incident radiation, θ_0 (zenith angle) and φ_0 (azimuth angle), upon the angles of emerging radiation, θ, φ, and upon the ground reflectivity, A. The effect of ground reflectivity is to add a term to the intensity, I, which consequently changes the percent of polarization P, but the polarization parameters Q and U are unchanged, since a depolarizing reflection is assumed. Only the intensity, I, is studied in this paper; we intend to describe the polarization for Rayleigh scattering atmospheres of large optical thickness in a forthcoming paper.

An incident radiation of unit intensity per unit area is assumed throughout this study. Figures 1 and 2 illustrate the plane-parallel approximation. The intensity vectors of the scattered radiation emerging from the upper and lower boundaries of the atmosphere for a specific set of parameters, $\tau = 1.0$, $\theta_0 = 53.13^{\circ}$ ($\mu_0 \equiv \cos\theta_0 = 0.6$), $A = 0$, are shown in the principal plane -- the plane containing the sun, the point of observation, and the local vertical. The direct solar radiation, which decreases exponentially with optical thickness as the radiation travels through the atmosphere, is not included. Figures 1a and 2a show diagramatically how the directions of the upward and downward emergent radiation are related to the model. Figures 1b and 2b show the same data in a form more suitable for comparison and interpretation. The variation with φ, the azimuthal angle, will also be considered.

The height of the atmospheric slab in these figures is merely diagramatic. No geometric height is determined since the radiation is a function of optical thickness only. For downward radiation the actual height is practically immaterial except for the effect it has on the validity of the plane-parallel approximation. When making measurements of upward radiation, however, one must be high enough above the planet to be above substantially all of the atmosphere, yet at the same time low enough that the planetary surface below is well approximated by a plane surface.

Alternatively, for observations from great distances, the emerging radiation can be approximated by parallel radiation. Sekera and Viezee (1964) made calculations of the intensity and polarization of such radiation for $\tau \leq 1$, still using the plane-parallel approximation but moving the point of tangency to each point of the observed disk.

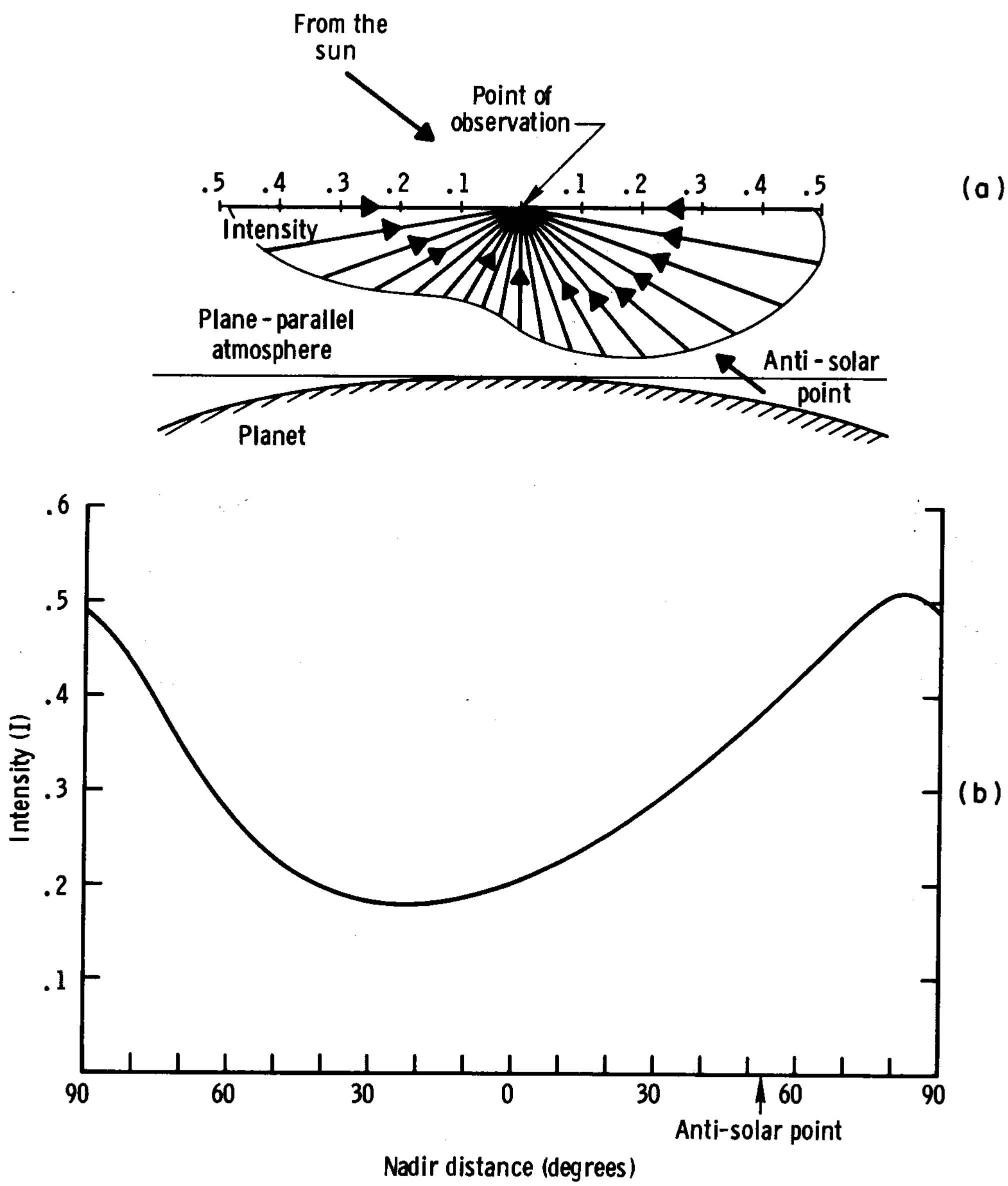

Fig. 1 -- Radiation emerging from the top of the atmosphere with $\tau = 1.0$, $\theta_0 = 53.13$, and $A = 0$; (a) vector representation, (b) graphical representation.

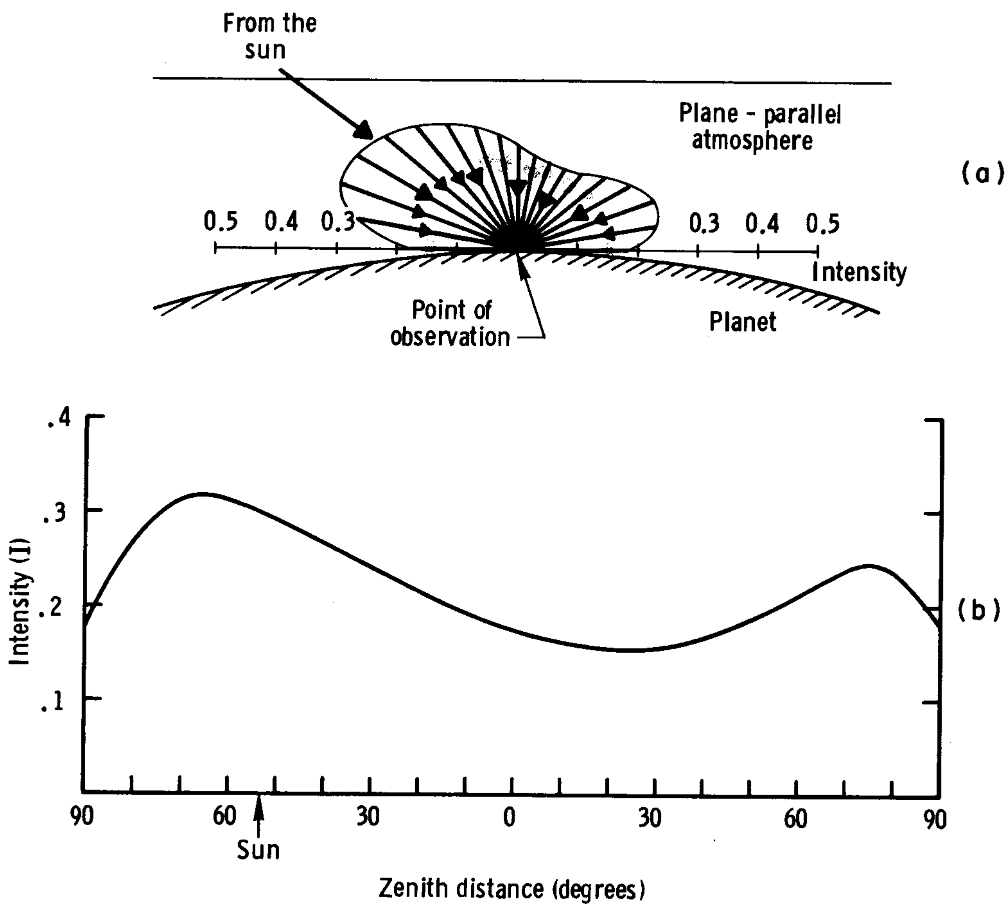

Fig. 2 -- Radiation emerging from the bottom of the atmosphere with $\tau = 1.0$; $\theta_0 = 53.13$, and A = 0; (a) vector representation, (b) graphical representation.

We hope to extend their work to larger optical thicknesses in a future study, but the method becomes less reliable as the ratio of the geometric thickness of the atmosphere to the planetary radius increases. For intermediate distances, where neither approximation is valid, as for instance at the altitude of the ATS satellites at 3 earth radii, each case must be calculated with its own particular geometry.

III. RESULTS

We wish to investigate how the intensity, I, varies with optical thickness, direction of incident light, and ground reflectivity. We will first consider the downward case, the intensity of the transmitted skylight as would be seen by an observer at the planet's surface, assuming unit incident radiation per unit area, at the top of the atmosphere.

With the sun in the zenith the intensity must, by definition, be symmetric around the local vertical; there is no azimuthal (φ) variation. Therefore only half of the principal plane need be illustrated. Figure 3 shows the variation with both optical thickness, τ, and ground reflectivity, A, for this case of the sun in the zenith. The left half of the diagram presents the intensities for A = 0 and the τ values shown; the right half shows the intensities for the same τ values and A = 0.8. In both cases we see that the maximum intensity in the zenith is at about $\tau = 2$, while at the horizon the maximum is at about $\tau = 1$. For small τ the intensity is greatest at the horizon, but by $\tau = 1$ the intensity becomes greatest at the zenith. Even for $\tau = 100$ there is still a significant amount of radiation that penetrates through the atmosphere to the planet's surface. The effect of a high ground reflectivity is to increase the intensity substantially (by roughly a factor of 2) for all values of τ, and at all zenith angles. For moderate optical thicknesses ($\tau = 0.5$ to $\tau = 4$) this increase of intensity is considerably greater near the horizon than near the zenith, significantly changing the shape of the intensity curves.

When the sun is not in the zenith, the axial symmetry about the zenith is, of course, lost, the only obvious symmetry remaining being across the principal plane. As an example of moderate solar/zenith angle, we will examine the radiation when $\theta_0 = 53.13^\circ$ ($\mu_0 = 0.6$). The intensity in the principal plane with τ as a parameter is shown in Fig. 4a for A = 0, and in Fig. 4b for A = 0.8. Both sets of curves show the same features, differing more in magnitude than in shape, with the high ground reflectivity again almost doubling the intensity. For low optical thickness, the intensity is greatest near the horizon. For

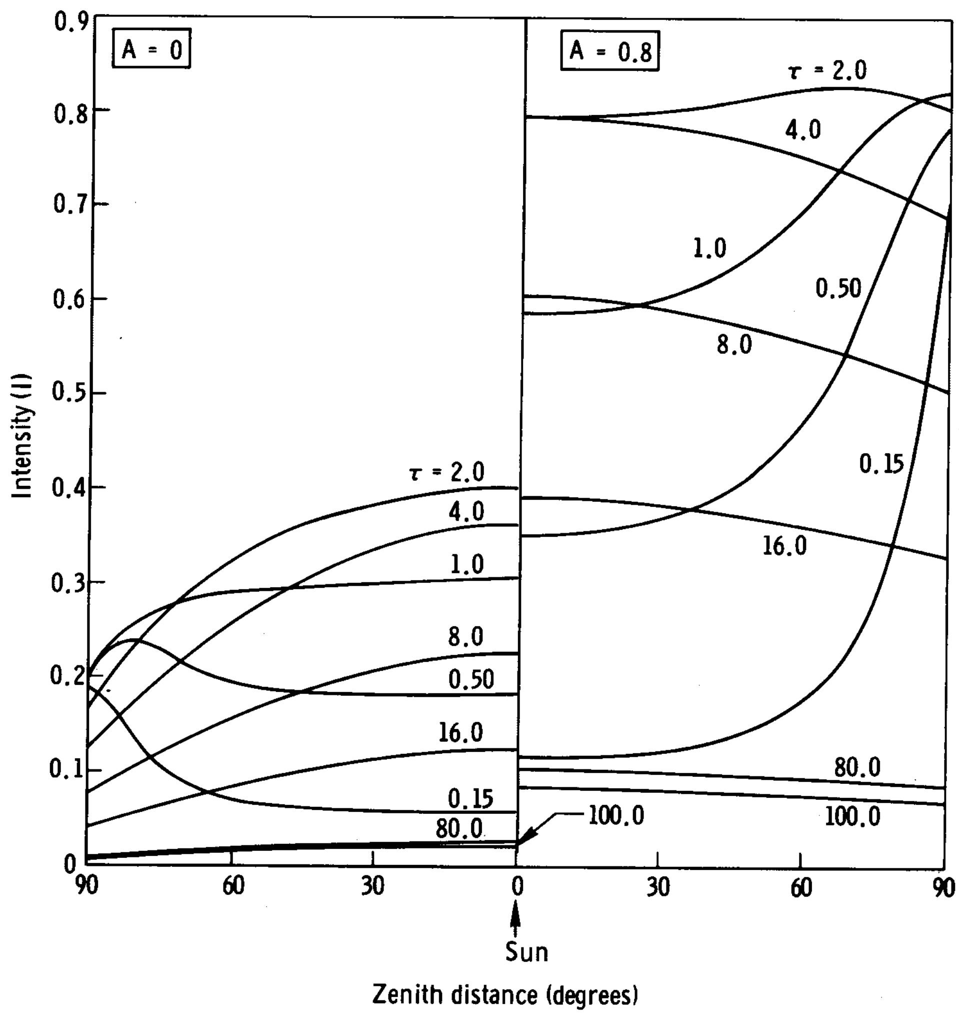

Fig. 3 -- Intensity of downward radiation with sun in the zenith.

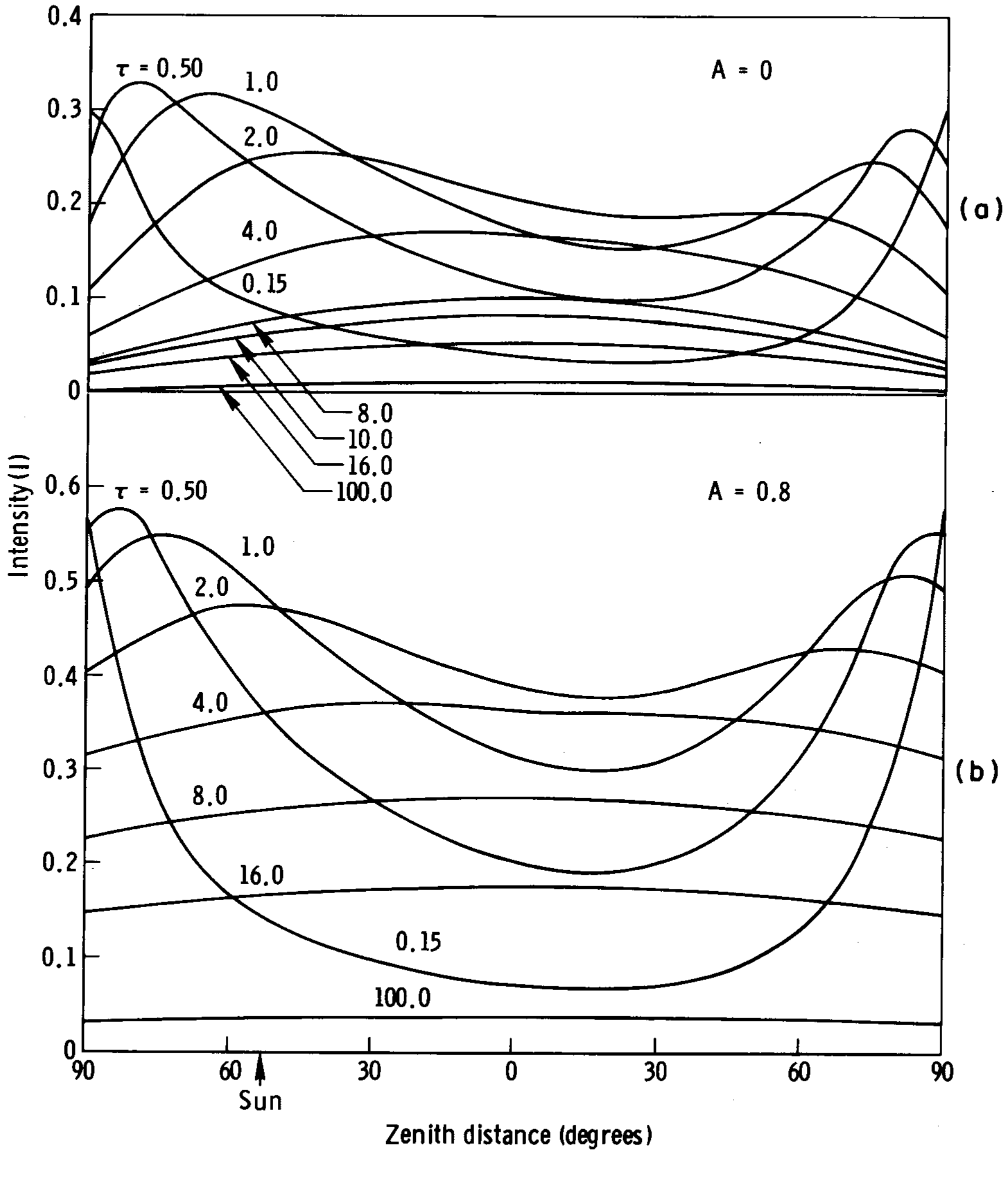

Fig. 4 -- Intensity of downward radiation with solar zenith angle θ_0 = 53.13 deg.

increasing optical thickness these two peaks of intensity move toward the zenith until, somewhere between $\tau = 2.0$ and $\tau = 4.0$, the two maxima become one, slightly to the sunward side of the zenith. By $\tau = 8$, all asymmetry from one side of the zenith to the other is essentially gone, with the peak intensity in the zenith. Comparing Figs. 3 and 4, we see that the horizon brightening at low τ is more pronounced when the sun is not in the zenith, while the average total intensity is naturally lower.

With the sun near the horizon ($\theta_0 = 84.26$, $\mu_0 = 0.1$), the decrease in average intensity continues, as one would expect (Figs. 5a and 5b -- note that the scale is 2.5 times as great as that in the other figures. Except for areas near the horizons at low optical thickness, the intensities vary quite slowly with either changing zenith distance or changing optical thickness, causing the curves to be rather flat and closely spaced.

Figures 6a and 6b show the azimuthal variation of intensity for two of the cases already examined. These figures show a projection of the sky onto a plane; the diameter of the semicircle is the projection of the principal plane, concentric semicircles are of equal zenith distance, and the radial lines are of equal azimuth. Only half the circle is shown since the radiation is symmetrical about the principal plane. The contours are lines of equal intensity. Figure 6a, for $\tau = 1.0$, $\mu_0 = 0.6$ and $A = 0$ (cf. Fig. 4a), shows how the intensity varies with azimuth when the peak intensity is quite far from the horizon. Figure 6b, for $\tau = 0.15$, $\mu_0 = 0.6$ and $A = 0.8$ (cf. Fig. 4b) shows the other extreme, where the intensity is greatest on the horizon. Both figures show a slower variation of intensity with azimuth angle than with zenith angle. The peak of intensity in Fig. 6a is somewhat broader than it is high. This effect is much more pronounced in Fig. 6b. The very bright horizon continues all the way around with only a relatively slight decrease with increasing distance from the principal plane.

There is an unexpected feature in Figs. 6a and 6b that continues through all the calculations of both intensity and polarization. This is a symmetry, on the horizon only, around the $\varphi = 90^{o}$, $\varphi = 270^{o}$ line, that is, the line perpendicular to the principal plane, through the

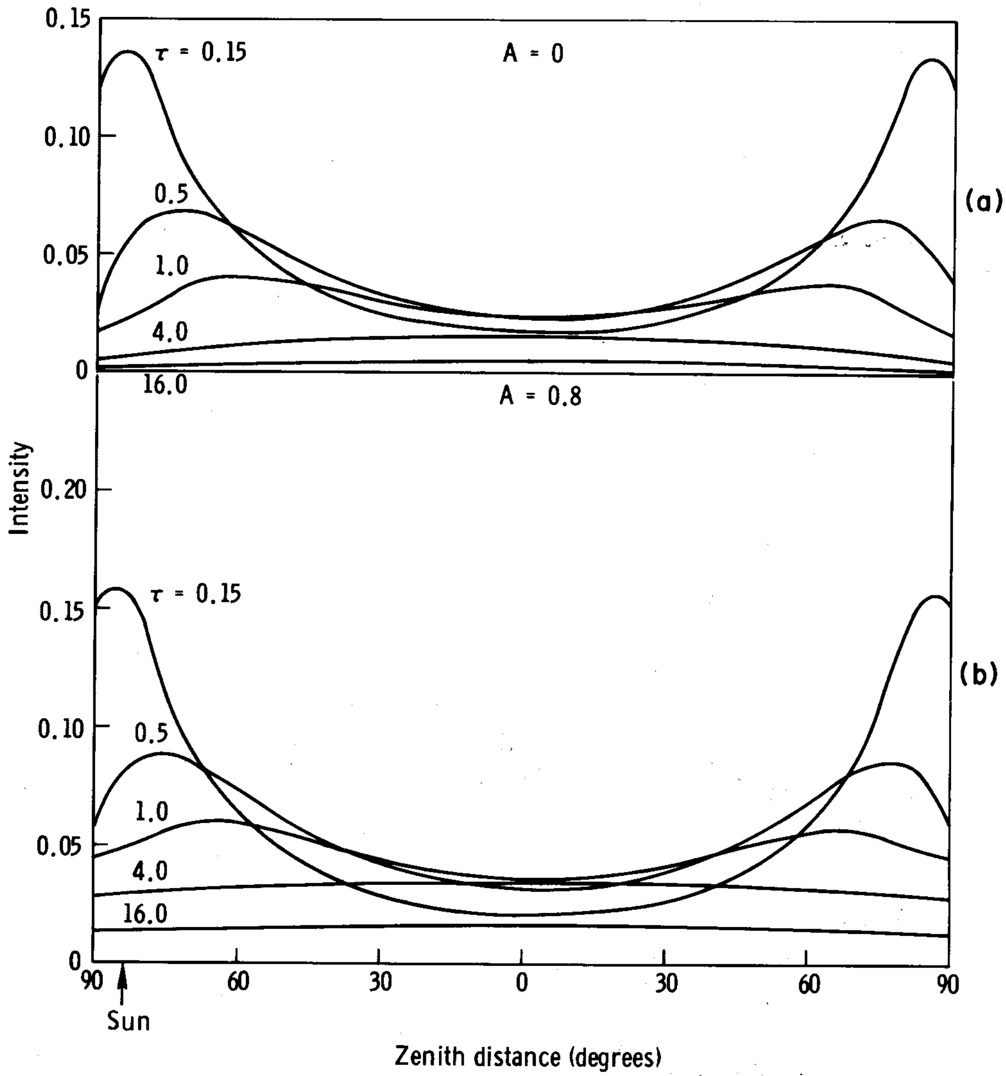

Fig. 5 -- Intensity of downward radiation with solar zenith angle θ_0 = 84.26 deg.

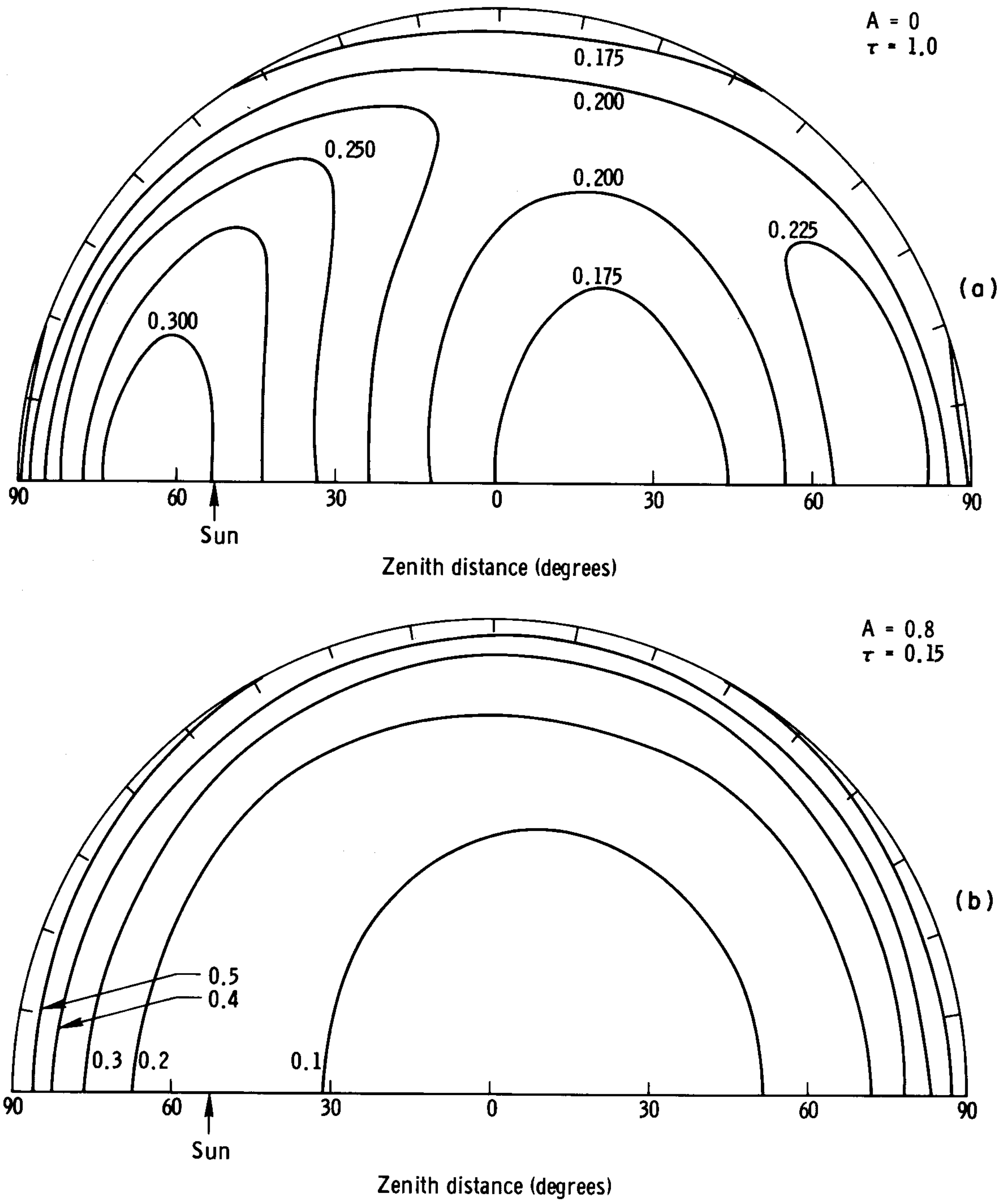

Fig. 6 -- Azimuthal variation of the intensity of downward radiation.

point of observation. In the other figures one can also see this effect, in that both end points of each curve ($\varphi = 0^o$ and $\varphi = 180^o$, $\theta = 90^o$) are at the same value. The reason for this symmetry is not clear. Since it only occurs on the horizon, where the plane-parallel approximation loses its validity for a planetary atmosphere, it will probably not be observed in a real atmosphere.

We look next at the intensity of the upward or reflected radiation as it would be seen, for example, by a satellite orbiting close to the surface. Figures 7, 8, and 9 show the reflected intensity for the same set of atmospheric parameters as shown for the transmitted intensity in Figs. 3, 4, and 5.

In Fig. 7a, for the sun in the zenith and A = 0, we see that, unlike the downward case, increasing the optical thickness monotonically increases the intensity of upward radiation at all zenith angles, although the increase is much less at the horizon than at the zenith. In Fig. 7b again with the sun in the zenith, and A = 0.8, we see that, as we might expect, the effect of a high ground reflectivity is to increase the intensity for small τ but not for large τ. At low optical thickness the shape of the intensity curves is considerably changed by large ground reflectivity, whereas when optical thickness is greater than about 16, the ground reflectivity is no longer an important factor at the top of the atmosphere; a thick atmosphere essentially eliminates the effect of the ground reflectivity upon the upward radiation. This is in contrast to the downward radiation of Fig. 3 where, since the radiation is observed at ground level, the ground reflectivity is an important factor for all τ. Not quite as expected, perhaps, is that the shape of some of these upward intensity curves in Fig. 7b is comparable to that for some of those of larger optical thickness with no ground reflectivity in Fig. 7a. For instance, the curve for $\tau = 2$ and A = 0.8 is almost identical to that for $\tau = 8$ and A = 0. A satellite that was scanning intensity at a single wavelength would be unable to distinguish, on this basis alone, between these two types of atmosphere. However, since optical thickness is so strongly dependent on wavelength, scanning on only a few wavelengths could add considerably more information -- getting into a range of optical thickness where such ambiguities

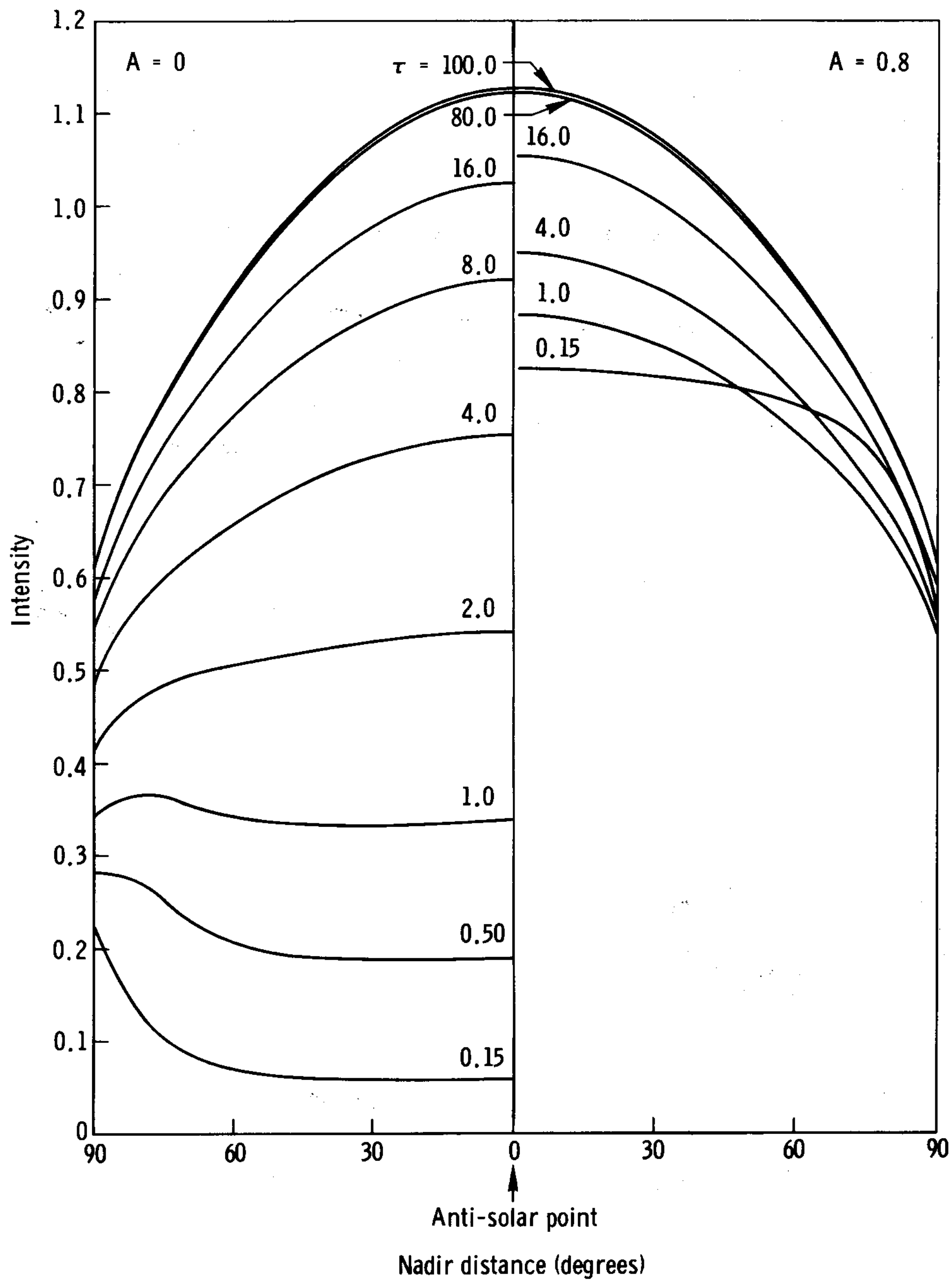

Fig. 7 -- Intensity of upward radiation with sun in the zenith.

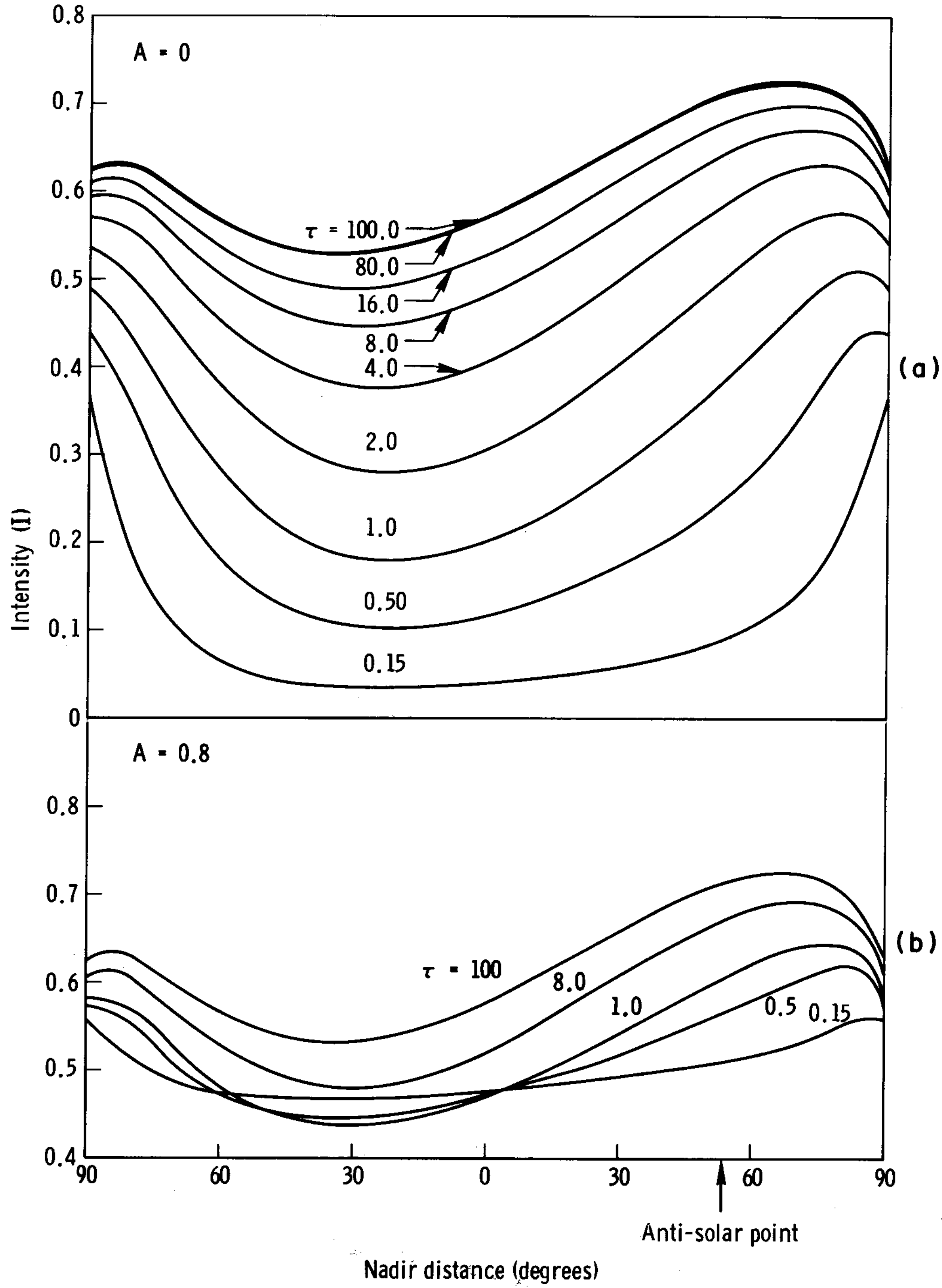

Fig. 8 -- Intensity of upward radiation with solar zenith angle θ_0 = 53.13 deg.

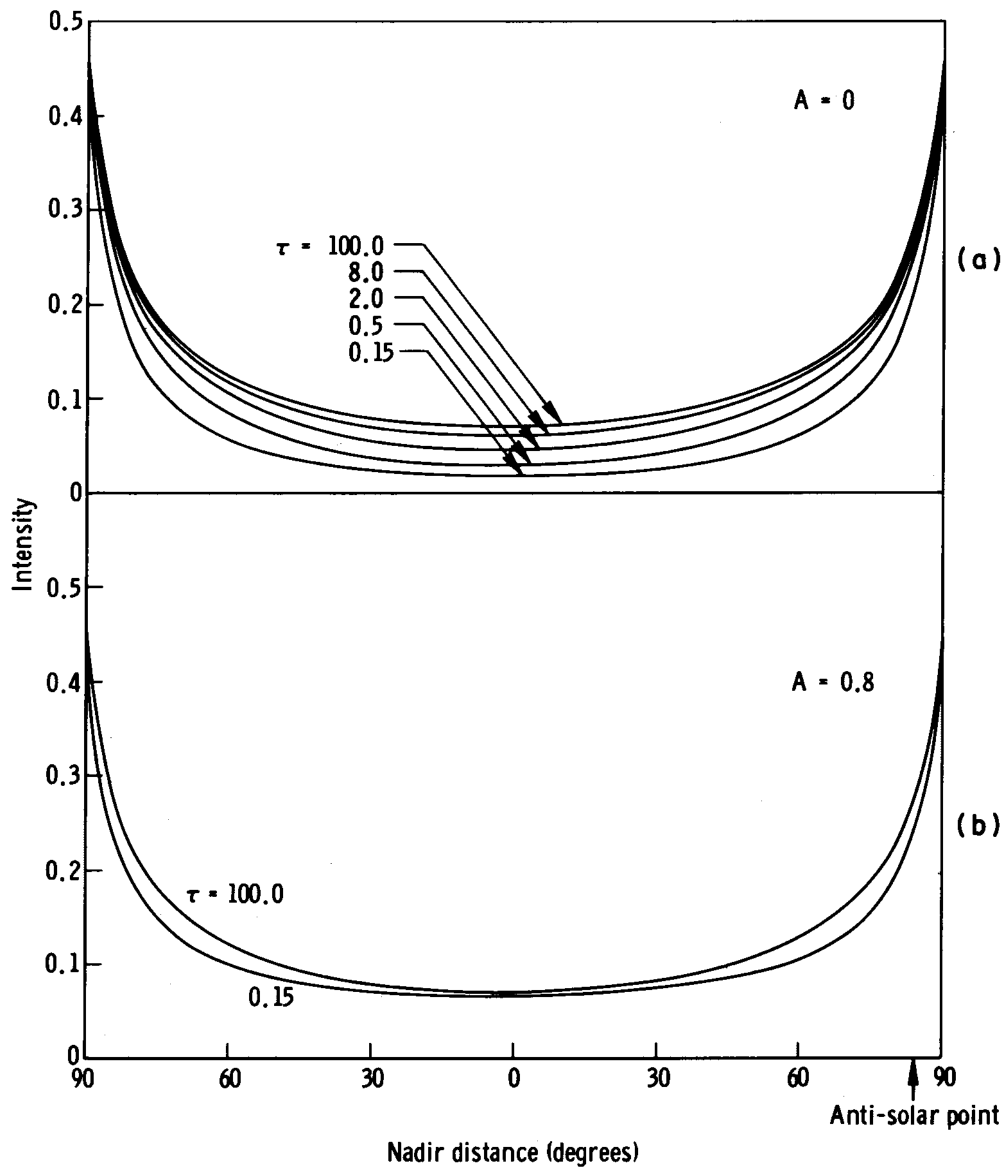

Fig. 9 -- Intensity of upward radiation with solar zenith angle θ_0 = 84.26 deg.

no longer exist. Observation of the polarization might also help distinguish between the two types.

Figures 8a and 8b show similar results for the solar/zenith angle of 53.13° (μ_0 = 0.6) with A = 0 and A = 0.8, respectively. Again the intensity increases with optical thickness. At low optical thickness, the high ground reflectivity distorts the intensity curves while at moderate optical thickness, its effect is similar to an increase of optical thickness. At large optical thickness the effect of ground reflectivity becomes negligible. Unlike the downward transmitted radiation shown in Fig. 4, the asymmetry across the zenith is present here for all optical thicknesses.

When the sun is low on the horizon (θ_0 = 84.26, μ_0 = 0.1) we see, looking at Figs. 9a and 9b, that the intensity is high at the horizon and low at the zenith, with very little asymmetry across the zenith. But the interesting feature is how the intensity is remarkably independent of both optical thickness and ground reflectivity. Presumably the calculated radiation must principally comprise light scattered in the nearly forward and nearly backward directions in the topmost part of the atmosphere. Comparing Fig. 9 with Fig. 5, the downward radiation for the same case of large solar/zenith angle, we see that, except right at the horizon, the transmitted intensity at very low optical thickness, τ = 0.15, is rather like the reflected intensities for all optical thicknesses. This lends support to the idea of only the topmost layers contributing very much to the reflected radiation, at this nearly grazing incidence.

IV. DISCUSSION

The intensity figures given here represent a fairly complete graphic description of the intensity of radiation emerging from a Rayleigh-scattering atmosphere. These intensities were calculated from linear combinations of the scattering functions tabulated by Sekera and Kahle (1966). Since those functions were shown to be accurate to at least five figures, the intensities should be very nearly that accurate also.

Other published values of the intensity for $\tau > 1$ for Rayleigh scattering are those given by Dave and Furukawa (1966), which were calculated by the evaluation of successive orders of scattering, and which hence become less accurate with increasing τ. Comparing their Fig. 4 with our Fig. 4a (showing the downward intensity for $\mu_0 = 0.6$, with no ground reflectivity), we see very good agreement for $\tau < 1$. Although the figures are rather difficult to read accurately, it appears that their curves for $\tau = 1$ and $\tau = 2$ may cross each other a little closer to the zenith than do ours. At $\tau = 10$ there is a large discrepancy. Their intensity curve goes above the 0.1 level for more than 30° on either side of the zenith, while the greatest value we find for the intensity at $\tau = 10$ is 0.084. Although Dave and Furukawa have two other figures similar to ours which show the upward intensity it is hard to make further direct comparisons because of difficulty in accurately reading the numerical values from figures.

A Rayleigh-scattering atmosphere is only an approximation, of course, to any real planetary atmosphere. The applicability of this approximation and its limitations have been discussed in detail elsewhere (see for example Chandrasekhar, 1950; Deirmendjian, 1957, 1959; Rozenberg, 1966; Sekera, 1957). It is hoped that even when a real atmosphere differs considerably from a Rayleigh-scattering atmosphere, useful information can be gained by comparing it with, and noting the differences from, the theoretical model.

Earlier, in a closely related study (Kahle, 1968) we examined the global radiation from a Rayleigh atmosphere. Global radiation is the total scattered radiation emerging from a point on the boundary in all outward directions, either upward or downward -- the hemispherical integral of the directional radiation shown here. With the additional study of the polarization which we intend to complete soon, we will have a complete graphical description of the radiation emerging from a Rayleigh-scattering atmosphere, complementing the numerical description of Sekera and Kahle (1966).

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